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Abstract

We use an overlapping generations model with physical and human capital, and two reproductive periods to explore how fertility decisions may differ in response to economic incentives in early and late adulthood. In particular, we analyze the interplay between fertility choices—related to career opportunities—and wages, and investigate the role played by work experience and investment in both types of capital. We show that young adults postpone parenthood above a certain wage threshold and that late fertility increases with work experience. The long run trend is either to converge to a low productivity equilibrium, involving high early fertility, investment in physical

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capital and relatively low income, or to a high productivity equilibrium, where households postpone parenthood to invest in their human capital and work experience, with higher late fertility and higher levels of income. A convergence to the latter state would explain the postponement of parenthood and the mitigation or slight reversal of fertility decrease in some European countries in recent decades.

JEL classification: E21, J11, J13

Keywords: fertility, postponement, work experience, overlapping generations

1 Introduction

Ongoing changes in fertility, family formation and relationship patterns since the 1960s in developed economies have come to be known as the second demographic transition, SDT (cf. Sobotka (2008)). The SDT is typically described in the economic literature as a sharp decline in total fertility rates — below the replacement rate of 2.1 children per women — and delayed parenthood, *i.e.* an increase in the mean age of mothers. These trends can be explained by improved access for women to tertiary education and the labor market, combined with the widespread availability of efficient contraception and the erosion of marriage. But alongside these seemingly related fertility patterns, trends in Age-Specific Fertility Rates (ASFRs) provide additional interesting insights.[1] In particular, in European countries, two separate trends emerge as depicted in Figure 1: i) A gradual decline in fertility in young women (20 to 29 years of age); ii) An initial decline in fertility followed by an increase over the past three decades for older women (30 to 39 years of age). These recent trends suggest a shift in demographic dynamics in European economies.

[1] Our main source is the United Nations 2022 World Population Prospect database, which covers the period 1950-2020 and provides age-specific fertility rates for European countries among others. ASFRs are measured as the number of births to women in a particular single age, divided by the number of women in that age. https://population.un.org/wpp/Download/Standard/Fertility/. For the sake of clarity, we choose to report four provided ASFR trends for four wide European areas, namely Northern (including Denmark, Estonia, Finland, Iceland, Ireland, Latvia, Lithuania, Norway, Sweden, United Kingdom), Western (including Austria, Belgium, France, Germany, Luxembourg, Netherlands, Switzerland), Southern (including Albania, Croatia, Greece, Italy, Portugal, Serbia, Slovenia, Spain) and Eastern Europe (including Bulgaria, Czech Republic, Hungary, Poland, Romania, Slovakia, Ukraine).

Historically, until the 1980s, age-specific fertility rates exhibited a consistent decline, consequently resulting in a decrease in total fertility rates. However, from this period onward, there has been a reversal in late fertility trends, potentially halting the declining trajectory of total fertility rates and opening the possibility for a period of stagnation or even a slight increase in total fertility rates (see Figure 2). Therefore, as well as reflecting a postponement of parenthood, we argue that these ASFR patterns shed light on the mitigation of the decrease or slight rebound of fertility, which has been poorly investigated to date.[2] This is of particular interest to countries concerned about their low fertility rates (cf. Doepke and Tertilt (2016)).

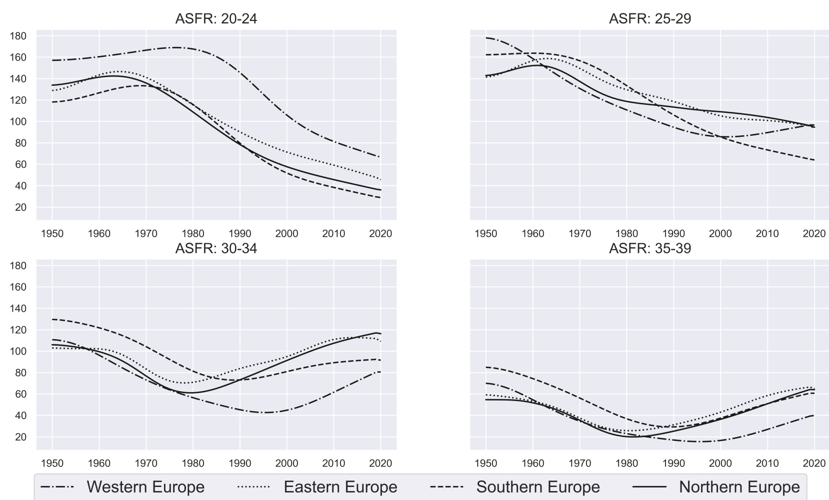


Figure 1: Smoothed estimates of Age-Specific Fertility Rates by groups.[3]

The aim of this paper is to provide a theoretical underpinning for these empirical findings. Our main objective is to identify the economic mechanisms underlying recently observed income-fertility relationship in high-income economies (Doepke et al. (2022)). To that end, we analyze the postponement-fertility nexus in a model where individuals choose when and how many children to have, and may invest in physical or human capital. Re-

[2]Some recent contributions, including Yakita (2018), Ohinata and Varvarigos (2020) or Dioikitopoulos and Varvarigos (2023), explain a fertility rebound.

[3]To obtain the smoothed estimates, we fit a local linear trend model and use the estimated parameters into a Kalman smoother.

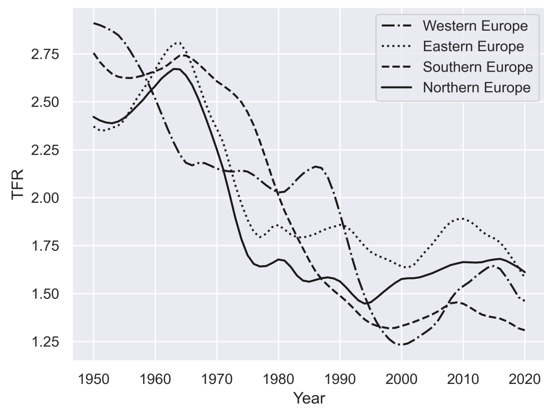


Figure 2: Smoothed estimates of the Total Fertility Rates by groups.

sults suggest possible positive relationships between late fertility and both wages and human capital, and reproduce a slight fertility rebound or a dampened fertility decrease observed in rich industrialized countries. They also highlight the crucial role played by education and career investment, with respect to capital investment, to understand recent fertility trends.

In an overlapping generations (OLG) setup with two reproductive periods, we explore how fertility decisions differ in response to economic incentives in early and late adulthood. We focus in particular on the interplay between childbearing, career choices (education/work experience), type of investment (physical/human capital), and wages. Young adults can spend their wages on consumption, time-consuming child-rearing activities, or investment in either physical or human capital. When wages are low, young adults invest in physical capital, not in human capital. As wages increase, early fertility increases leading to a loss in their working experience meanwhile education expenditures are null, which further hinders late fertility. When wages become sufficiently high, young adults choose to spend on education expenditure rather than early savings while they reduce early fertility, which allows for an extended work experience (learning-by-doing). These two channels contribute to accumulating human capital, which enhances future income but also late fertility. Therefore, having children at a young age compromises career opportunities, which might translate into a loss of future earnings (wage penalty), loss of skills during job interruptions, and/or loss of experience due to reduced working hours (Adda et al. (2017)). This theoretical trade-off is consistent with empirical results which show that young mothers suffer a greater wage penalty than older mothers, particularly those on the lowest wages. For instance, Miller (2011) has shown that delaying motherhood is associated with an increase in labor markets earnings of around 9% per year of delay, while Budig and England (2001) calculate a wage penalty of 7% per child. Later in life, old adults can once again use their income to either consume, save, or raise children. Interestingly, we find that early fertility and investment in human capital are substitutes, while investment in human capital and late fertility are complements. Indeed, although career development may appear incompatible at first glance with high late fertility rates, as pointed out by Sobotka et al. (2011), Hazan and Zoabi (2014), Bar et al. (2018), d'Albis et al. (2017) and Nitsche and Brückner (2021), higher-earning, more educated women can nowadays combine late childbearing with continued investment in their professional careers, because they can afford childcare. For instance, Nitsche and Brückner (2021) found that highly educated women in the US born in the 1960s and 70s were more likely to combine family and professional responsibilities with child bearing than their counterparts in previous cohorts, leading them to catch up with the fertility levels of their less-educated counterparts. Similarly, d'Albis et al. (2017) show that young women in Europe are more likely to subsequently start a family if childbearing is postponed for education rather than because of limited access to the labor market. Finally, Hazan and Zoabi (2014) and Bar et al. (2018) provide evidence that marketization of child care increases fertility of women with higher wage. It is also a source of flattening the relationship between income and fertility. This complementarity of late fertility and human capital is at the core of our model.

In our general equilibrium model, wages are endogenous and outcomes depend on total factor productivity. Within this framework, two types of stationary equilibria emerge. When productivity is low, corresponding wages are reduced, prompting households to prioritize physical capital accumulation and early fertility. This focus on early savings limits the accumulation of work experience. In contrast, when productivity is sufficiently high, the higher levels of income encourage households to postpone fertility, investing instead in their careers through education and work experience, before having children in late adulthood. In this configuration, the increase in late fertility may compensate the decrease in early fertility, leading to higher total fertility. This pattern is in line with observations of relatively higher late ASFR and the phenomenon of slight rebound or flattening of fertility rates discussed at the beginning of this Introduction.

Our paper contributes to the literature on the joint dynamics of economic and demographic variables over time. Introducing endogenous fertility choices into growth models leads to the empirically well-established decrease in fertility with advanced development (Barro and Becker (1989), Galor and Weil (1996, 2000), Bhattacharya and Chakraborty (2012)). In these theoretical models, the accumulation of physical or human capital leads to a reduction in fertility through the so-called quantity-quality trade-off, since greater development implies larger returns on education and higher opportunity costs for childbearing. However, while this literature focuses on the number (*quantum*) of births and its interaction with development, it neglects an equally important feature of demographic dynamics which is the timing of parenthood (the *tempo* of births).

The timing of births and postponement of parenthood have been investigated more recently (d'Albis et al. (2010), Pestieau and Ponthière (2014, 2015), Sommer (2016), de la Croix and Pommeret (2021)), although these studies consider the number of births as given. The question of timing is important because the reproductive period is limited and fertility decreases with age; the timing of births is therefore a major driver of demographic dynamics. Existing studies mainly highlight the negative effects of postponement on fertility rates. Our contribution is to show instead that postponement can lead to higher fertility rates. Following Iyigun (2000) and d'Albis et al. (2018), households in our model can choose both the timing and the number of children they have, but while d'Albis et al. (2018) and Iyigun (2000) find that total fertility rates continue to decrease, our model reproduces the observed postponement of childbearing and the possibility of a fertility rebound, driven by higher late fertility. Recently, Dioikitopoulos and Varvarigos (2023) offers a theoretical contribution in which a change in preferences is required to replicate both the fertility rebound and the postponement phenomenon. The mechanism they highlight is therefore different than ours, and they do not adopt a dynamic general equilibrium model with physical capital accumulation.

Finally, our paper is also related to a strand of the literature that focuses on the emergence of the fertility rebound without being interested in the timing of adulthood. Ohinata and Varvarigos (2020), for instance, propose a growth model in which the fertility rebound emerges as the final stage of a three-phase process of demographic change and economic development.

This final stage stems from an accumulation of human capital, which leads to a fertility rebound through a strong income effect. Our model also differs from Yakita's (2018), in which external childcare production allows for a fertility rebound as women's wages increase. Interestingly, Futagami and Konishi (2019) exhibit a positive relationship between income and fertility due to a positive income effect in particular for sufficiently high levels of wages. This paper relies on crucial assumptions on the nature of child-rearing costs (in terms of goods) and a positive effect of wage on longevity. With respect to this literature, we offer a theoretical framework in which we introduce two types of assets which play a crucial role to understand the interactions between income and fertility choices. In addition, we contribute to this literature by showing that the rebound or flattening can be explained by increased late fertility.

The remainder of the paper is organized as follows. In Section 2, we discuss some stylized facts. Section 3 presents the model and the choices available to agents. Section 4 presents the fertility and investment behaviors. In Section 5, we define the intertemporal equilibrium and analyze the equilibrium, the existence and uniqueness of a steady state in the two distinct regimes. Section 6 provides a complete picture of the long-term dynamics of the economy and explains the motivations for postponing parenthood. Section 7 provides a quantitative analysis of the theoretical model to replicate the development pattern over time and performs a comparative static exercise before concluding in Section 8. Technical details are provided in Appendices.

2 Stylized facts

Before presenting the theoretical mechanisms underlying the joint fertility trends mentioned in the introduction (the postponement of parenthood and the slight rebound or flattening of fertility), we show how recent patterns in fertility rates are related to a selection of economic variables, focusing on European economies.

In the introduction, we showed in Figure 1 how ASFRs in European countries for women over 30 years of age follow a U-shaped curve over the period 1950–2020, decreasing from 1950 to the mid 1980s and then rising back up. Total fertility rates also seem to be increasing, after reaching a minimum in the mid 2000s. In contrast, the ASFRs of younger women over the same period have continued to decrease. Young European women now have fewer children than their counterparts did in the 1970s and 80s, while older women have more, leading overall to a moderate fertility rebound. The main objective of this paper is to identify the economic mechanisms that have been driving these recent trends.

These demographic changes indeed occurred in the context of a continuous increase in wealth per capita, challenging the well-established relationship between fertility and income in the demographic transition. In this regard, interesting insights can be drawn from Figure 3, where we have plotted age-specific fertility rates against potentially related economic variables in 2017 for a cross section of European countries.[4]

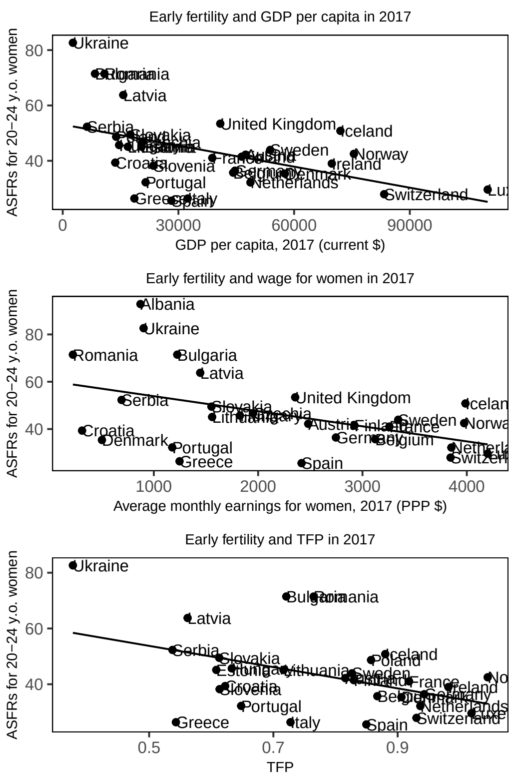


Figure 3: Early and late fertility in European countries as a function of various economic variables in 2017.

The left and right panels focus respectively on early and late fertility,

[4]Results are reported for all 32 European countries considered in Figures 1 and 2.

captured by the ASFRs of 20–24-year-old and 30–34-year-old women. These two measures of fertility are plotted from top to bottom against GDP per capita,[5] average monthly earnings for women,[6] and a measure of total factor productivity.[7] The comparison with wages is relevant given the known effect of wages on fertility choices (and of fertility choices on wage profiles), and the importance of wages in our theoretical model. The correlation with total factor productivity is shown because it is used in our model to identify long-term equilibria.

The relationships in the left panel are clearly negative and those on the right, clearly positive. This suggests that the negative fertility–income relationship of the post-Malthusian era still prevails in young women: in Europe in 2017, the richer the economy, the lower the early fertility rate. However, the right panel shows that this relationship does not hold anymore for older women: the more they earn for instance, the higher the late fertility rate is. This means that the relationship between fertility rates and economic factors has changed in recent decades and that differences in wages, income and/or productivity may at least partially explain the discrepancies in demographic dynamics between countries observed at the macroeconomic level. Recent economic and demographic studies have highlighted the variance in fertility trends in Europe since the mid 20th century. In particular, total fertility rates have been increasing since the mid 2000s in richer European countries, the so-called high fertility belt (see for instance Frejka and Sobotka (2008), Myrskylä et al. (2009), Luci-Greulich and Thévenon (2014)). This suggests in turn that late fertility is a key driver of this (modest) rebound in total fertility rates. Our theoretical model reproduces these phenomena at a micro level and also reveals different long-run equilibria that provide an overall view of European economies. The first stationary equilibrium is characterized by low income, high early fertility and low late fertility, while the second equilibrium features higher per capita income, higher late fertility and larger career investments.

These trends are also illustrated in the following panel analysis, using

[5] World Bank. GDP per capita current $, The World Bank Group, https://data.worldbank.org/indicator/NY.GDP.PCAP.CD.

[6] International Labour Organization, "ILO modeled estimates database ILOSTAT, https://ilostat.ilo.org/data/, expressed in units of PPP.

[7] Penn World Tables. International comparisons of production, income and prices 10.0, https://www.rug.nl/ggdc/productivity/pwt/, University of Groningen, expressed in current PPPs (USA = 1).

annual World Bank data on GDP per capita, annual fertility data from the United Nations 2022 World Population Prospect database and a measure of human capital provided by Barro and Lee (2013). Four age-specific categories of ASFRs were regressed against the average years of schooling for women for 32 European countries from the 1950s onward and we add a control variable which is the GDP per capita. More precisely, the model that is estimated is given by:

$$ASFR_{i,c,t} = \alpha_c + \beta_0\, year\_sch_{i,c,t} + \beta_1\, (year\_sch_{i,c,t})^2 + \beta_2\, GDP\_per\_cap. + \varepsilon_{c,t}$$

with $i = 20-24, 25-29, 30-34, 35-39$ the five-year interval of women' age, c the country, t the year, α_c which accounts for country fixed-effects and $\varepsilon_{c,t}$ is the error term. We run three types of models, for which the predicted values are plotted in Figure 4 (Details about the data and the methodology, as well as regression results are provided in Appendix A.): (1) A Panel OLS with both country and time-fixed effects; (2) A Panel OLS model without time-fixed effect; (3) A Random Effect model.

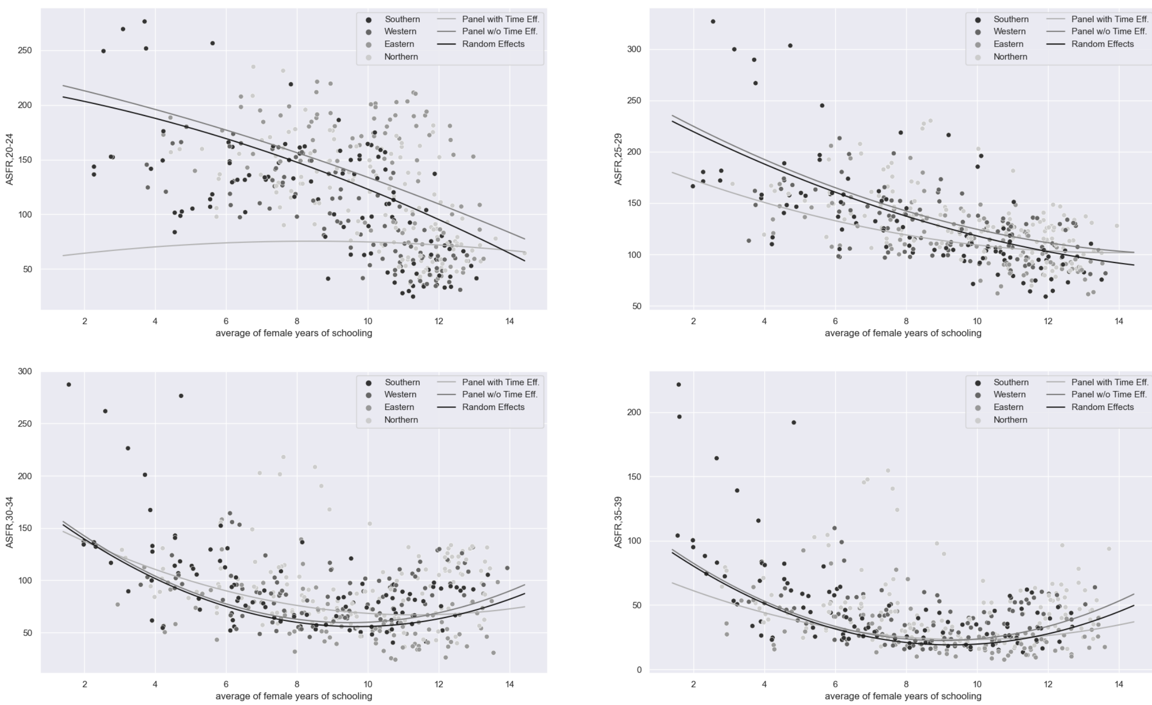


Figure 4: Predicted values.

From these Panel estimations, we can observe that early fertility (top of the grid) decreases with human capital thereby confirming the substitutability between the two at a macro level at least. On the contrary, late fertility (bottom of the grid) displays a clear humped-shape pattern with the average years of schooling for women. We then deduce that the complementary between the investment in human capital occurs only for a sufficiently high level of years of schooling (around 10 years in average). This precisely captures the investment in (tertiary) education that we aim at modeling in our theoretical framework since we consider choices of education of young adults.

3 The model

We use a dynamic general equilibrium model with both physical capital and human capital and endogenous fertility. More precisely, we consider a three-period OLG model where time is discrete and indexed by $t = 0, 1, 2...$ and in which generations born at date t are of size N_t. There are two types of agents: firms and households.

3.1 Production

A continuum of firms of unit size exists and produces a final good, Y_t, using both physical capital, K_t, and labor, L_t. For the sake of simplicity, we assume a Cobb-Douglas technology, *i.e.* $Y_t = AK_t^\alpha L_t^{1-\alpha}$, with $A > 0$ the total factor productivity and $\alpha \in (0, 1)$ the share of physical capital in the production process. $k_t \equiv K_t/L_t$ is the capital-labor ratio, w_t the wage and r_t the interest rate. Profit maximization gives:

$$w_t = (1-\alpha)Ak_t^\alpha \equiv w(k_t) \quad (1)$$

$$r_t = \alpha A k_t^{\alpha-1} \equiv r(k_t) \quad (2)$$

In the following, we assume complete depreciation of capital, meaning that the interest factor on capital is given by $R_t = r_t$.

3.2 Household behavior

Households live for three periods: early adulthood, late adulthood and the old age. Importantly in this model, those households have a two-shot reproductive period: they may have children during both early and/or late adulthood. Therefore, during early adulthood, households – young adults – earn a labor income, w_t, and make decisions regarding the number of children to have, n_{1t}, their consumption, c_{1t}, and investments in two distinct assets: human capital and physical capital. To do so, they may allocate resources toward education expenditures denoted by h_{t+1} or initiate early savings denoted by k_{t+1}^Y. During late adulthood, households – old adults – can once again choose to have children, n_{2t+1}, save by accumulating physical capital, k_{t+2}^O, and consume, c_{2t+1}. In the third period of life, households are retired and consume their late adulthood savings, c_{3t+2}. Basically, in this framework, all the mechanisms we highlight capture the choices typically made by women within a household regarding education, careers and fertility.

Households derive utility from consumption and from parenting in the first two periods of life. Preferences are represented by a utility function, which is additively separable between consumption and child-rearing:

$$\ln c_{1t} + \delta_1 \ln (\mu_1 + n_{1t}) + \beta \left[\ln c_{2t+1} + \delta_2 \ln (\mu_2 + n_{2t+1}) + \beta \ln c_{3t+2} \right] \tag{3}$$

where $\delta_i \geqslant 0$ measures the preference toward having children with $i = 1, 2$ and $\beta \in (0, 1)$ is the discount factor. As in Baudin et al. (2015, 2020), the parameters $\mu_i \geqslant 0$ allow for corner solutions in fertility choices. During early adulthood, the households' budget constraint writes as:

$$c_{1t} + k^Y_{t+1} + h_{t+1} = w_t(1 - \phi_1 n_{1t}) \tag{4}$$

with $\phi_1 > 0$ the time cost per child. We assume that child-rearing is a time-consuming activity while investing in human or physical capital incurs costs in terms of goods. We differ in this regard from d'Albis et al. (2018), who overlook the direct cost of education in household budgets and instead incorporate a disutility to human capital investment. Also, in contrast to Iyigun's (2000) model where households refrain from working in the first period and must choose between dedicating time to education or child-rearing, our model introduces more general and complex household trade-offs. In addition, children are not considered perfect substitutes, which generalizes the setup of Iyigun (2000). Finally, with regards to the existing literature, we introduce an additional asset, which is physical capital into the decision-making process.

Drawing insights from the literature on the career costs of motherhood, particularly the wage penalties associated with early motherhood, we thus introduce *human capital* denoted by $\tilde{h}_{t+1}$. On the one hand, this variable captures accumulated job experience during early working life, as a form a learning-by-doing phenomenon. Naturally, this experience is gained when early fertility is curtailed, freeing up time resources for work. On the other hand, investing h_{t+1} in higher education for instance provides an other channel for households to augment their human capital. Formally, the human capital reached when old adult is considered to be a weighted sum of first-period working time and investment in education:

$$\tilde{h}_{t+1} = \kappa h_{t+1} + (1 - \kappa)(1 - \phi_1 n_{1t}) \equiv \tilde{h}(h_{t+1}, n_{1,t}) \tag{5}$$

with $\kappa \in (0, 1)$ the relative weight of education as a component of human capital.[8] Throughout the paper, we may alternatively use the term "career investment" in place of "human capital investment." This substitution is appropriate as it reflects choices aimed directly at shaping one's career trajectory, such as acquiring work experience or pursuing higher education.

In our framework, firms request labor time to produce the good. During young adulthood, households who have not yet accumulated work experience have a unit labor productivity, meanwhile their labor time is reduced by the time spent to rearing children activities: They earn $(1-\phi_1 n_{1t})w_t$. During late adulthood, the household benefits from its human capital accumulated during the previous period, leading to increased productivity, equal to $1+\epsilon h_{t+1}$. In addition, the household may have n_{2t+1} children. Overall, the time available and offered to firms is equal to $1-\phi_2 n_{2t+1}+\epsilon h_{t+1}$ for a given wage rate w_{t+1}. This should be understood to convey a broader idea that the opportunity cost of having children can be, at least partially, disconnected from workers abilities in the mid-working life. Therefore, investing in the career when young allows for a productivity gain which does not imply an increasing rearing cost of children. On the one hand, following the literature, it seems that such a framework may echo the papers by Kemnitz and Thum (2015), Yakita (2018) among others who point out that late fertility may be enhanced as soon as the (more educated) female wage rate becomes higher than the price of an available external child care, which would cost in our model $w_{t+1}\phi_2 n_{2t+1}$. On the other hand, early fertility choices may well affect the wage profile but it is not the case for late fertility, since there is no further career choices to be made in late adulthood.

The budget constraints during late adulthood and retirement are given by:

$$
\begin{aligned}
c_{2t+1} + k^O_{t+2} &= w_{t+1}(1+\epsilon\tilde{h}_{t+1} - \phi_2 n_{2t+1}) + R_{t+1}k^Y_{t+1} && (6)\\
c_{3t+2} &= R_{t+2}k^O_{t+2} && (7)
\end{aligned}
$$

with $\phi_2 > 0$ the time cost per child during late adulthood.[9] Our set-up is therefore in line with the empirical literature (*e.g.* Budig and England

[8] As a first approximation, and for tractability reasons, we assume working time and education to be perfect substitutes.

[9] Note that in our setup, the time cost of raising children is only incurred for newborns, infants and toddlers, so that children born in the previous period no longer carry a time cost for their parents. An alternative approach would be to consider that children born in a previous period live with their parents and share consumption spending, as in Pestieau and Ponthière (2014).

(2001), Caucutt et al. (2002), Miller (2011), Olivetti (2006), Herr (2016)), which highlights the career costs of motherhood and a higher wage penalty associated with early fertility. Essentially, diminishing early parenthood reduces first period utility but i) it favors work experience and ii) frees the resources available for investing in education or physical capital.

Households maximize their utility (3) given their budget constraints (4)-(7) but also the positivity constraints $h_{t+1} \geqslant 0$, $k^Y_{t+1} \geqslant 0$, $n_{1t} \geqslant 0$ and $n_{2t+1} \geqslant 0$. The choices of young households are governed chiefly by two types of trade-offs: between consumption and assets on the one hand, and between consumption and parenthood on the other hand.

$$\frac{1}{c_{1t}} \geq \beta\frac{R_{t+1}}{c_{2t+1}} \tag{8}$$

$$\frac{1}{c_{1t}} \geq \beta\frac{w_{t+1}\epsilon\kappa}{c_{2t+1}} \tag{9}$$

$$\frac{w_t\phi_1}{c_{1t}} \geq \frac{\delta_1}{\mu_1 + n_{1t}} - \beta\frac{w_{t+1}\epsilon(1-\kappa)\phi_1}{c_{2t+1}} \tag{10}$$

where equations (8), (9) and (10) hold as equality if $k^Y_{t+1} > 0$, $h_{t+1} > 0$ and $n_{1t} > 0$. For old adults, choices reflect their intertemporal consumption smoothing behavior and the trade-off between consumption and late parenthood:

$$\frac{1}{c_{2t+1}} = R_{t+2}\beta\frac{1}{c_{3t+2}} \tag{11}$$

$$\frac{w_{t+1}\phi_2}{c_{2t+1}} \geq \frac{\delta_2}{\mu_2 + n_{2t+1}} \tag{12}$$

with equality if $n_{2t+1} > 0$.

Now, let's delve into a more detailed examination of these choices, beginning with those made by the old adults.

3.3 Choices of old adults

From equations (7) and (11) we have $c_{2t+1} = k^O_{t+2}/\beta$ and, using the budget constraint (6), we obtain the optimal levels of savings and consumption for old adults:

$$c_{2t+1} = \frac{1}{1+\beta}[w_{t+1}(1+\epsilon\tilde{h}_{t+1} - \phi_2 n_{2t+1}) + R_{t+1}k^Y_{t+1}] \quad (13)$$

$$k^O_{t+2} = \frac{\beta}{1+\beta}[w_{t+1}(1+\epsilon\tilde{h}_{t+1} - \phi_2 n_{2t+1}) + R_{t+1}k^Y_{t+1}] \quad (14)$$

As expected therefore, late savings and consumption both increase with income. As for late fertility, equations (7), (11) and (12) can be combined to show that the number of children is given by $\delta_2 k^O_{t+2} \leq w_{t+1}\beta\phi_2(\mu_2 + n_{2t+1})$. Substituting equation (14) into this last inequality gives:

$$\delta_2(1+\epsilon\tilde{h}_{t+1}) - \phi_2\mu_2(1+\beta) + \delta_2\frac{R_{t+1}k^Y_{t+1}}{w_{t+1}} \leqslant n_{2t+1}\phi_2(1+\beta+\delta_2) \quad (15)$$

with equality if $n_{2t+1} > 0$, in which case n_{2t+1} is defined by:

$$\begin{aligned} n_{2t+1} &= \frac{\delta_2(1+\epsilon\tilde{h}_{t+1}) - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} + \frac{\delta_2}{\phi_2(1+\beta+\delta_2)}\frac{R_{t+1}k^Y_{t+1}}{w_{t+1}} \\ &\equiv n_2(\tilde{h}_{t+1}, k^Y_{t+1}) \end{aligned} \quad (16)$$

Late fertility is an increasing function in $\tilde{h}_{t+1}$ and k^Y_{t+1}. Indeed, any increase in the late adulthood income, that goes through either a higher level of human capital or augmented early savings, enhances the affordability of child-rearing activities for old adults (positive income effect). Furthermore, it is noteworthy that late fertility experiences a negative relationship with the future wage rate w_{t+1}, but solely if households had invested in physical capital during the preceding period. When a unit of saving is invested when young, it yields R_{t+1} when individuals reach old age, which can be expressed in terms of future labor income as R_{t+1}/w_{t+1}. However, if one household rather opts for investing in higher education, the return on the investment remains constant. Specifically, the expected return of one additional unit of education is equal to $\epsilon\kappa w_{t+1}$. Therefore, the return to education relatively to labor income equates to $\epsilon\kappa$. In the following, we exclude the possibility for old adults to remain childless for all $\tilde{h}_{t+1}, k^Y_{t+1} \geqslant 0$:

Assumption 1 $\delta_2 > \phi_2\mu_2(1+\beta)$.

As mentioned above, this expression of late fertility suggests a complementarity between late fertility and investment in any form of asset. This is because households have an incentive to invest in human or physical capital as means of covering the costs associated with raising more children in late adulthood. However, anticipating on the subsequent sections, this relationship appears to be more powerful in the case of human capital and late fertility, as it is not depreciated by the negative effect embedded through w_{t+1}/R_{t+1}. This possibly stronger and positive relationship between career investment and fertility may appear at first sight to contradict the longstanding negative link between fertility and female labor force participation, which is a typical feature of the demographic transition. However, our theoretical result is consistent with recent, mostly empirical studies, which emphasize that the compatibility of career and family is a crucial determinant of current fertility trends in high-income countries (Doepke et al., 2022). In particular, these studies show that the latest cohorts of US and European women tend to postpone fertility to invest in education or their careers, but without necessarily reducing their fertility intentions (see Sobotka et al., 2011; d'Albis et al., 2017; Goldin, 2021; Nitsche and Brückner, 2021). In the literature, these results can be explained in part by better access to childcare services and/or the increased bargaining power of women in households, particularly for more highly educated/skilled women. In our model, career investment will be associated with higher late fertility and higher labor income in late adulthood, accurately capturing this phenomenon.

3.4 Choices of young households and emergence of different regimes

By inspection of equations (8) and (9), a young household invests in early saving rather than in education if its return is the largest one, *i.e.* $R_{t+1} > w_{t+1}\epsilon\kappa$. This is equivalent to the relative return of education with regards to physical capital being less than one, $\epsilon\kappa x_{t+1} < 1$, with $x_{t+1} \equiv \frac{w_{t+1}}{R_{t+1}}$.

3.4.1 Early fertility choices

On the one hand, let us first examine the case where $x_{t+1} < 1/(\epsilon\kappa)$. Hence, $k^Y_{t+1} > 0$, $h_{t+1} = 0$ and inequality (10) is equivalent to:

$$n_{1t} \geq \frac{\delta_1 - \phi_1\mu_1 - \delta_1 k^Y_{t+1}/w_t - \phi_1\mu_1\epsilon(1-\kappa)x_{t+1}/w_t}{\phi_1(1+\delta_1+\epsilon(1-\kappa)x_{t+1}/w_t)} \equiv n^K_1(w_t, x_{t+1}, k^Y_{t+1}) \quad (17)$$

From equation (17), we observe that an interior solution for early fertility, even though households do not engage in early saving, requires that $x_{t+1}/w_t < \frac{\delta_1-\phi_1\mu_1}{\epsilon(1-\kappa)\phi_1\mu_1} \equiv a_0$. Since $\kappa \in (0,1)$, this entails:

Assumption 2 $\delta_1 > \phi_1\mu_1$

Therefore, early fertility is a decreasing function of k_{t+1}^Y and it is positive when households invest in physical capital, but only if early savings remain below a specified upper bound. Formally, equation (17) holds as an equality if:

$$k_{t+1}^Y < \left(1 - \frac{\phi_1\mu_1}{\delta_1}\right) w_t - \frac{\phi_1\mu_1}{\delta_1} x_{t+1}\epsilon(1-\kappa) \equiv \overline{k}^Y(w_t, x_{t+1}) \tag{18}$$

In this configuration, equation (17) defines $n_{1t} = n_1^K(w_t, x_{t+1}, k_{t+1}^Y)$, which is increasing with respect to its first argument and decreasing with the two other ones.

On the other hand, when $x_{t+1} > 1/(\epsilon\kappa)$, a young household invests in education rather than early saving. Hence, $k_{t+1}^Y = 0$, $h_{t+1} > 0$ and inequality (10) is equivalent to:

$$n_{1t} \geq \frac{1}{\phi_1} \frac{w_t(\delta_1 - \phi_1\mu_1) - \delta_1 h_{t+1} - \phi_1\mu_1\frac{1-\kappa}{\kappa}}{w_t(1+\delta_1) + \frac{1-\kappa}{\kappa}} \equiv n_1^H(w_t, h_{t+1}) \tag{19}$$

We may notice that early fertility is also decreasing function of the investment in education. Therefore, to guarantee that early fertility may be positive when households invest in their career, an upper bound on h_{t+1} should be consistently defined. Formally, equation (19) holds as an equality if:

$$h_{t+1} \leq w_t - \frac{\phi_1\mu_1}{\delta_1}\left(w_t + \frac{1-\kappa}{\kappa}\right) \equiv \overline{h}(w_t) \tag{20}$$

In this configuration, equation (19) defines $n_{1t} = n_1^H(w_t, h_{t+1})$, which is increasing with respect to its first argument and decreasing with the second one.

An interesting insight from these early fertility choices lies in the importance played by the parameter κ. In both configurations, the lower the parameter κ, the higher the human capital reached when old adult through work experience – and for a given level of early fertility. As a direct consequence a lower κ results in an increase in the future labor income thereby reducing incentives for early saving and early fertility, which in turn, simultaneously drives a higher consumption. Let us now explore the choices of early savings or education by young adults.

3.4.2 Early savings and education

Using equations (4), (5), (13), (16) and (17) with $h_{t+1} = 0$, inequality (8) is equivalent to:

$$\begin{aligned}
&k^Y_{t+1}\left[x_{t+1}\epsilon(1-\kappa) + w_t\right]\left[1+\delta_1+\beta(1+\beta+\delta_2)\right] \\
&\geq \beta(1+\beta+\delta_2)(1+\phi_1\mu_1)w_t^2 \\
&-x_{t+1}w_t\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)(1-\beta(1+\beta+\delta_2)) + (1+\delta_1)(1+\phi_2\mu_2)\right] \\
&-x_{t+1}^2\epsilon(1-\kappa)\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)+(1+\phi_2\mu_2)\right] \equiv \Lambda(w_t, x_{t+1}) \qquad (21)
\end{aligned}$$

Obviously, when it holds as an equality, this equation (21) defines k^Y_{t+1} as a function of x_{t+1} and w_t, *i.e.* $k^Y_{t+1} \equiv k^Y(w_t, x_{t+1})$.

Similarly, using equations (4), (5), (13), (16) and (19) with $k^Y_{t+1} = 0$, inequality (9) is equivalent to:

$$\begin{aligned}
&h_{t+1}\left(\frac{1-\kappa}{\kappa} + w_t\right)\left[1+\delta_1+\beta(1+\beta+\delta_2)\right] \\
&\geq \beta(1+\beta+\delta_2)(1+\phi_1\mu_1)w_t^2 \\
&-\frac{w_t}{\epsilon\kappa}\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)(1-\beta(1+\beta+\delta_2)) + (1+\delta_1)(1+\phi_2\mu_2)\right] \\
&-\frac{1-\kappa}{\epsilon\kappa^2}\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)+(1+\phi_2\mu_2)\right] \equiv \Lambda(w_t, 1/(\epsilon\kappa)) \qquad (22)
\end{aligned}$$

When equation (22) holds as an equality, it defines h_{t+1} as a function of w_t, *i.e.* $h_{t+1} \equiv h(w_t)$. Finally, equation (21) is equivalent to:

$$\begin{aligned}
&\frac{k^Y_{t+1}}{w_t}\left[1+\delta_1+\beta(1+\beta+\delta_2)\right] \geqslant \beta(1+\beta+\delta_2)(1+\phi_1\mu_1) \qquad (23) \\
&-\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)+1+\phi_2\mu_2\right]\frac{x_{t+1}}{w_t} - \frac{\delta_1(1+\phi_2\mu_2)x_{t+1}/w_t}{1+\epsilon(1-\kappa)x_{t+1}/w_t} \equiv \Xi\left(\frac{x_{t+1}}{w_t}\right)
\end{aligned}$$

with $\Xi\left(\frac{x_{t+1}}{w_t}\right)' < 0$, $\Xi(0) > 0$ and $\Xi(+\infty) = -\infty$. This means that there exists $\chi > 0$ such that $\Xi\left(\frac{x_{t+1}}{w_t}\right) > 0$ for all $\frac{x_{t+1}}{w_t} < \chi$. In other words, when this equation (23) holds as an equality, $k^Y_{t+1} > 0$ if $\frac{x_{t+1}}{w_t}$ is not too high. It also implies that k^Y_{t+1}/w_t is increasing in w_t, *i.e.* k^Y_{t+1} is increasing in w_t and is a superior good.

Additionally, we note that equation (22) is similar to (21) considering $x_{t+1} = 1/(\epsilon\kappa)$ and substituting k^Y_{t+1} by h_{t+1}. This implies that when x_{t+1}

reaches $1/(\epsilon\kappa)$, there is a continuity between k^Y_{t+1} and h_{t+1}, *i.e.* $k^Y(w_t, 1/(\epsilon\kappa)) = h(w_t)$. Hence, we deduce from the previous analysis that h_{t+1} displays the same properties as k^Y_{t+1}.

Once we have defined the optimal choices of either early savings or education, we can show that those choices are compatible with positive fertility ones:

Lemma 1 *Let*

$$\underline{\delta}_1 \equiv \phi_1\mu_1[1 + \beta(1 + \beta + \delta_2)] \tag{24}$$

Under Assumptions 1-2 and $\delta_1 > \underline{\delta}_1$*, early fertility and late fertility are always positive for all* $\frac{x_{t+1}}{w_t} < \chi$*,* $k^Y_{t+1} > 0$ *and* $h_{t+1} > 0$*.*

Proof. See Appendix B. ■

From Lemma 1, we deduce that households never choose to remain childless because the utility associated with parenthood is sufficiently high. In addition, as early fertility is always positive we may highlight the substitutability between early fertility and the investment in any form of capital. This stand in contrast to the complementary relationship between late fertility and both physical and human capital emphasized previously. Nonetheless, this statement is consistent with the hypothesis formulated in Introduction by providing support for the rationale behind recent patterns observed in developed economies. Fertility behaviors do not respond the same way to economic variables, like education or savings, depending on the life-period considered.

To provide a complete characterization of the economy we may go back to equations (1) and (2) to get that $x_{t+1} = w_{t+1}/R_{t+1} = A_x w_{t+1}^{1/\alpha}$, with $A_x \equiv \alpha^{-1}(1-\alpha)^{\frac{\alpha-1}{\alpha}} A^{-\frac{1}{\alpha}}$. Then, the arbitrage condition between saving or education, $x_{t+1} \leqslant 1/(\epsilon\kappa)$, is equivalent to $w_{t+1} \leqslant \underline{w}$, where:

$$\underline{w} \equiv \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}A}{(\epsilon\kappa)^\alpha} \tag{25}$$

Using all the previous results of this section, we can now establish that the economy is characterized by two different regimes and summarize our main findings in the Proposition below:

Proposition 1 *Under Assumptions 1-2 and* $\delta_1 > \underline{\delta}_1$*, there exists* $\underline{A} > 0$ *such that for* $A > \underline{A}$*, the economy is characterized by:*

1. *A low-income regime where* $k^Y_{t+1} = k^Y(w_t, x_{t+1}) > 0$*,* $h_{t+1} = 0$*,* $n_{1t} = n_1^K(w_t, x_{t+1}, k^Y_{t+1}) > 0$ *and* $n_{2t+1} = n_2(\tilde{h}_{t+1}, k^Y_{t+1}) > 0$ *for* $x_{t+1} < 1/(\epsilon\kappa)$ *or* $w_{t+1} < \underline{w}$*;*

2. *A high-income regime where* $k^Y_{t+1} = 0$*,* $h_{t+1} = h(w_t) > 0$*,* $n_{1t} = n_1^H(w_t, h_{t+1}) > 0$ *and* $n_{2t+1} = n_2(\tilde{h}_{t+1}, 0) > 0$ *for* $x_{t+1} > 1/(\epsilon\kappa)$ *or* $w_{t+1} > \underline{w}$*.*

Proof. See Appendix C. ■

We should first note that h_{t+1} is positive for higher values of x_{t+1}/w_t compared to k^Y_{t+1}, thus explaining why such a configuration is referred to as the *high-income* regime – the other one being the *low-income* regime. Indeed, k^Y_{t+1} is increasing with the first-period of life wage w_t but decreasing with old-age discounted income x_{t+1}, and becomes equal to h_{t+1} when $x_{t+1} = 1/(\epsilon\kappa)$. When $x_{t+1} > 1/(\epsilon\kappa)$, physical capital investment is replaced by career investment, which increases with w_t. The economy is thus characterized by two regimes contingent upon the level of the old age discounted wage. Nonetheless, in both regimes, households choose to enter parenthood when young adult although early fertility declines with the investment in any type of asset, be it early saving or education. On the contrary, late fertility is made more affordable thanks to previous investments.

Beyond these direct effects of the young age investments on fertility, it might be relevant to explore in more details how the latter overall evolves with changes in the income. Indeed, as demonstrated previously, both the young and old adult labor income determine asset holding, which in turn influences fertility. In the following section we therefore focus on the income-fertility relationship, at young and old age.

4 Fertility and investment behaviors

Using results from Proposition 1, we aim at disentangling the direct income effect on fertility choices from the indirect ones – through asset holding – at each reproductive age and over the life cycle, in each regime. By doing so, we are also able examine how asset holding evolve with the life-cycle labor income.

4.1 Low-income regime

First, both the young age and the discounted old age wages directly impact early fertility, but in the opposite direction. On the one hand, a rise in the w_t boosts early fertility through a positive income effect while, on the other hand, as already mentioned, an increase in the future discounted wage x_{t+1} involves a higher opportunity cost of early fertility through the loss of work experience.

Second, upon households engaging in early saving, these two labor incomes indirectly influence early fertility, through their impact on asset holding. The young age labor income exerts a negative impact on early fertility, which is primarily attributed to a substitution effect between fertility and early saving. This substitution effect operates through the positive impact of the current wage on asset holding. As a result, the relationship between early fertility and current wage may experience a shift, depending on the dominance of either the income effect or the substitution effect. In addition, the future discounted wage influences early fertility choices through its direct impact on the level of early saving. This time, this effect goes in the opposite direction. Indeed, a rise in the future wage reduces incentives to save thus freeing resources to afford child care when young. Finally, both young age and discounted old age wages influence early fertility directly and indirectly, but it is difficult to know ultimately in which direction.

To clarify which effect dominates, let us introduce a new variable $z_{t+1} \equiv w_{t+1}^{\frac{1}{\alpha}}/w_t$, which is proportional to the ratio of the future discounted wage over the current wage. This relative wage measure proves useful to explore the relationship between asset holding or fertility over the life-cycle and income. It allows us to derive clear-cut results as the labor income is consistently defined during the young and the old adult ages. In particular, using equation (23), we can first express early saving over income as a function of z_{t+1}:

$$\frac{k_{t+1}^Y}{w_t} = \frac{1}{1+\delta_1+\beta(1+\beta+\delta_2)}[\beta(1+\beta+\delta_2)(1+\phi_1\mu_1) \tag{26}$$

$$-[\epsilon(1-\kappa)(1+\phi_1\mu_1)+1+\phi_2\mu_2]A_x z_{t+1} - \frac{A_x z_{t+1}\delta_1(1+\phi_2\mu_2)}{1+\epsilon(1-\kappa)A_x z_{t+1}}] \equiv \widetilde{k}(z_{t+1})$$

Then, we deduce the expression of early fertility substituting this expression above into equation (17):

$$n_{1t} = \frac{1}{\phi_1}\frac{\delta_1 - \phi_1\mu_1 - \delta_1\widetilde{k}(z_{t+1}) - \phi_1\mu_1\epsilon(1-\kappa)A_x z_{t+1}}{1+\delta_1+\epsilon(1-\kappa)A_x z_{t+1}} \equiv n_1^L(z_{t+1}) \qquad (27)$$

Using equation (5), we also have $\tilde{h}_{t+1} = (1-\kappa)(1-\phi_1 n_1^L(z_{t+1}))$ and we substitute this expression into equation (16) to obtain the new expression for late fertility:

$$\begin{aligned} n_{2t+1} &= \frac{\delta_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z_{t+1})) + A_x^{-1}\widetilde{k}(z_{t+1})/z_{t+1}] - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \\ &\equiv n_2^L(z_{t+1}) \qquad (28) \end{aligned}$$

Finally, using equations (26), (27) and (28), we can assess the role played by this relative wage measure on the changes in fertility and investment behaviors in the low-income regime. We show the following lemma:

Lemma 2 *Under Assumptions 1-2 and $\delta_1 > \underline{\delta}_1$, $\widetilde{k}(z_{t+1})$ and $n_2^L(z_{t+1})$ are decreasing with z_{t+1}, while $n_1^L(z_{t+1})$ is increasing with z_{t+1}.*

Proof. See Appendix D. ■

This lemma underscores the opposite answer of either early or late fertility to a rise in the relative wage. Notably, early fertility experiences an increase while late fertility shows a decline, just like early saving. Indeed, the increase in the relative wage is detrimental to the accumulation of physical capital thereby diverting resources towards early fertility, at least partially, and further implying a loss of work experience. This effect exacerbates a subsequent reduction in the old age labor income, in addition of the reduced capital income. Overall, households' income during late adulthood is lowered, contributing to a decline in late fertility. Notice that all these interconnected channels align in the same direction, implying an unquestionable reduction in late fertility. Conversely, the relationship between early fertility and the relative wage suggests only a slight positive global effect.

Since $z_{t+1} = w_{t+1}^{\frac{1}{\alpha}}/w_t$, a sequence of increasing wages through time can be characterized by a sequence of increasing relative wage. Lemma 2 shows that in the low-income regime, it will be characterized by a slight increase of early fertility, but a decrease of late fertility, meaning that in the low-income regime there is no postponement.

4.2 High-income regime

In the high-income regime, investment in asset holding occurs through education or work experience. In addition, as explained above, the return on the investment is proportional to the future wage, which means that the discounted future labor income is constant. Hence, all the mechanisms that rely on the effect of the future wage vanish implying that the young people's wage plays a crucial role. Using equation (19), we have $n_{1,t} = n_1^H(w_t)$, where:

$$n_1^H(w_t) \equiv \frac{1}{\phi_1} \frac{w_t(\delta_1 - \phi_1\mu_1 - \delta_1 h(w_t)/w_t) - \phi_1\mu_1 \frac{1-\kappa}{\kappa}}{w_t(1+\delta_1) + \frac{1-\kappa}{\kappa}} \tag{29}$$

Using equation (5), we can redefine human capital reached at the old age $\tilde{h}(h(w_t), n_1^H(w_t)) = \tilde{h}(w_t)$ as a function of the young people's wage w_t only. Finally, using equation (16), we deduce that $n_{2,t+1} = n_2(\tilde{h}(w_t), 0) \equiv n_2^H(w_t)$.

Lemma 3 *Under Assumptions 1-2 and $\delta_1 > \underline{\delta}_1$, $\tilde{h}(w_t)$ and $n_2^H(w_t)$ are increasing with w_t, while $n_1^H(w_t)$ is decreasing with w_t.*

Proof. See Appendix E. ■

In the high-income regime, early fertility declines while late fertility increases as young households get richer. In particular, despite a positive income effect, it turns out that the substitution effect dominates, so that an increase in the current wage translates into a larger investment in education that is detrimental to early fertility. As a consequence, work experience is boosted through two channels that reinforce each other: i) A lower early fertility that frees more time; ii) education expenditure. We can now easily assess the effect of the young people's labor wage on late fertility. Because work experience grows with w_t, so does late fertility, which becomes an increasing function of the young people's wage.

In this regime, early and late fertility respond in an opposite direction to an increase in the current wage. Interestingly and regarding our discussion at the end of the previous subsection, early and late fertility evolve in a reversed way to an increasing sequence of wages with respect to the low-income regime. Following an increase in wages, households prefer to postpone the entry to parenthood to the late adulthood.

4.3 Life-cycle fertility

Finally, we may now assess how life-cycle fertility, denoted by m_t, is related to wages, as early and late fertility respond in an opposite direction to the rise in wages in both regimes. Let us first define life-cycle fertility for a young household born at date t as the sum of early and late fertility: $m_t = n_{1,t} + n_{2,t+1}$. Then, we may distinguish the life-cycle fertility in the low-income regime, denoted by $m_t = m^L(z_{t+1})$ from the life-cycle fertility in the high-income regime, $m_t = m^H(w_t)$. We show that:

Lemma 4 *Under Assumptions 1-2 ,* $\delta_1 > \underline{\delta}_1$*, and* $1 < \frac{\delta_2 \epsilon (1-\kappa)\phi_1}{\phi_2(1+\beta+\delta_2)}$*,* $m^H(w_t)$ *increases in* w_t*, while* $m^L(z_{t+1})$ *decreases in* z_{t+1}*.*

Proof. See Appendix F. ■

These results reflect the patterns in fertility behaviors observed empirically from the 60's in developed countries, in particular in Europe as illustrated in the Introduction: Total fertility is first declining as economies get richer and then starts increasing slightly. This last period being called the fertility rebound. In our model, switching from one regime to the other one entails divergent fertility behaviors that may help to justify this phenomenon. In the low-income regime, keeping in mind that z_{t+1} may increase following an increase of wages through time, total fertility decreases as wages increase and this could correspond to the end of the demographic transition. As soon as a sufficiently high level of wage is reached, young adult households start investing in education rather than physical capital. This induces both a reduced early fertility, but increasing late and total fertility. Then, it appears that the fertility rebound relies on the accumulation of human capital.

In the following, Section 5 provides the definition of the intertemporal equilibrium and its analysis in each regime. The existence and the uniqueness of a steady state in each regime is carefully proved. Readers less inclined towards technical details may opt to proceed directly to Section 6, where we summarize and discuss the main findings of the paper.

5 Equilibrium analysis of the two different regimes

We start by defining an intertemporal equilibrium. We then investigate the existence and uniqueness of the steady state in each regime.

5.1 Intertemporal equilibrium

The population size of the next generation equals the sum of early fertility weighted by the number of young adults plus late fertility weighted by the number of old adults. The population dynamics is therefore described as follows:

$$N_{t+1} = N_t n_{1t} + N_{t-1} n_{2t} \tag{30}$$

If we denote by $n_t = N_{t+1}/N_t$, population growth (of successive generations) can be expressed as follows[10]:

$$n_t = n_{1t} + \frac{1}{n_{t-1}} n_{2t} \tag{31}$$

Recall also that K_t is the aggregate stock of physical capital used in the production process in period t. It is equal to the sum of the capital held by young and old adult households at the same date. The market clearing condition satisfies the following equality:

$$K_{t+1} = N_t k_{t+1}^Y + N_{t-1} k_{t+1}^O \tag{32}$$

As for the labor market, the market clearing condition can be written:

$$L_t = N_t(1 - \phi_1 n_{1t}) + N_{t-1}(1 + \epsilon \tilde{h}_t - \phi_2 n_{2t}) \tag{33}$$

while the wage $w_t = w(k_t)$ is given by equation (1). Combining equations (32) and (33) gives:

$$[n_t(1 - \phi_1 n_{1t+1}) + 1 + \epsilon \tilde{h}_{t+1} - \phi_2 n_{2t+1}] k_{t+1} = \frac{1}{n_{t-1}} k_{t+1}^O + k_{t+1}^Y \tag{34}$$

[10]Note that the population growth rate differs from the total fertility rate, defined as the sum of early and late fertility at a given date ($\text{TFR}_t = n_{1t} + n_{2t}$).

In addition, substituting equation (16) into equation (14) yields:

$$k_{t+1}^O = \frac{\beta}{1+\beta+\delta_2}[w_t(1+\epsilon\tilde{h}_t+\phi_2\mu_2)+R_t k_t^Y] \tag{35}$$

Substituting equations (35) into equation (34) and using $k_t = [w_t/((1-\alpha)A)]^{1/\alpha}$ leads to:

$$\begin{aligned}[n_t(1 \quad - \quad \phi_1 n_{1t+1}) + 1 + \epsilon\tilde{h}_{t+1} - \phi_2 n_{2t+1}]\left[\frac{w_{t+1}}{(1-\alpha)A}\right]^{1/\alpha} \\ = \frac{\beta}{n_{t-1}(1+\beta+\delta_2)}[w_t(1+\epsilon\tilde{h}_t+\phi_2\mu_2)+R_t k_t^Y] + k_{t+1}^Y \end{aligned} \tag{36}$$

Under Assumptions 1-2, $\delta_1 > \underline{\delta}_1$ and $A > \underline{A}$, equations (31) and (36) define an intertemporal equilibrium, with:

1. $k_{t+1}^Y/w_t = \widetilde{k}(z_{t+1})$, $h_{t+1} = 0$, $n_{1t} = n_1^L(z_{t+1})$ and $n_{2t+1} = n_2^L(z_{t+1})$ for $w_t < \underline{w}$;
2. $k_{t+1}^Y = 0$, $h_{t+1} = h(w_t)$, $n_{1t} = n_1^H(w_t)$ and $n_{2t+1} = n_2^H(w_t)$ for $w_t > \underline{w}$.

5.2 Low-income regime and investment in physical capital ($w_t < \underline{w}$)

We start by defining an intertemporal equilibrium in this low-income regime. Using (26)-(28) and (31), we have:

$$\begin{aligned} n_t &= n_1^L(z_{t+1}) \\ &+ \frac{\delta_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z_t)) + A_x^{-1}\widetilde{k}(z_t)/z_t] - \phi_2\mu_2(1+\beta)}{n_{t-1}\phi_2(1+\beta+\delta_2)} \end{aligned} \tag{37}$$

Using equation (36), we get:

$$\begin{aligned} &[n_t(1-\phi_1 n_1^L(z_{t+2})) + \frac{1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z_{t+1}))+\phi_2\mu_2}{1+\beta+\delta_2}(1+\beta) \\ &-\frac{\delta_2}{1+\beta+\delta_2}\frac{\widetilde{k}(z_{t+1})}{A_x z_{t+1}}]\frac{z_{t+1}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} = \frac{\beta}{n_{t-1}(1+\beta+\delta_2)}[1 \\ &+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z_t)) + \phi_2\mu_2 + \frac{\widetilde{k}(z_t)}{A_x z_t}] + \widetilde{k}(z_{t+1}) \end{aligned} \tag{38}$$

Given equations (26) and (27), the system (37)-(38) drives the dynamics in the low-income regime. More specifically, this is a three-dimensional system, (n_{t-1}, z_t, z_{t+1}), with two predetermined variables n_{t-1} and z_t. Then, we can deduce the dynamics of wage using $w_{t+1} = z_{t+1}^{\alpha} w_t^{\alpha}$.

A steady state is a solution $n_{t-1} = n_t = n$ and $z_t = z_{t+1} = z_{t+2}$ solving equations (37) and (38). Note that $z = w^{\frac{1-\alpha}{\alpha}}$, which implies that, at a steady state, z is an increasing function of the wage. Using Lemma 2, we deduce that n_2^L and $\widetilde{k}$ decrease with the relative wage, whereas n_1^L increases with it.

Equation (37) gives:

$$n = \frac{1}{2}\left[n_1^L(z) + \sqrt{\Delta^L(z)}\right] \equiv I_{11}(z) \tag{39}$$

where $\Delta^L(z)$ is given in Appendix G. We use equation (37) to substitute $1/n$ into (38). We obtain:

$$n = \frac{\Omega_{11}(z)}{\Omega_{21}(z)} \equiv I_{21}(z) \tag{40}$$

where $\Omega_{11}(z)$ and $\Omega_{21}(z)$ are also given in Appendix G.

A steady state in this low-income regime is a solution $z \in (0, \underline{z})$ to the equation $n = I_{11}(z) = I_{21}(z)$, with $\underline{z} \equiv \underline{w}^{\frac{1-\alpha}{\alpha}}$. The following proposition summarizes our results:

Proposition 2 *Under Assumptions 1-2, δ_1 and δ_2 sufficiently high, ϕ_2 low enough, there exists a steady state (w^L, n^L), or equivalently (z^L, n^L), with $w^L \in (0, \underline{w})$ $(z^L \in (0, \underline{z}))$ if A is sufficiently close to $\underline{A}$. Moreover, there exists $\underline{\epsilon} > 0$ such that this steady state is unique for $\epsilon > \underline{\epsilon}$.*

Proof. See Appendix G. ■

This proposition shows that if the productivity is low, there exists a unique steady state with a relatively low wage. This result is illustrated in Figure 5. As it is shown in Appendix J, this steady state is a saddle under a reasonable parametrization of the model. Therefore, if the wages are low enough, young households invest in physical capital rather than education and they substitute early fertility to physical capital in the long-run.

5.3 High-income regime and investment in human capital ($w_t > \underline{w}$)

Using equations (5), (16) and (29), in the high-income regime, equation (31) becomes:

$$\begin{aligned} n_t &= n_1^H(w_t) \\ &+ \frac{\delta_2 - \phi_2\mu_2(1+\beta) + \delta_2\epsilon[\kappa h_t + (1-\kappa)(1-\phi_1 n_1^H(w_{t-1}))]}{n_{t-1}\phi_2(1+\beta+\delta_2)} \end{aligned} \quad (41)$$

and equation (36):

$$\begin{aligned} &[n_t(1-\phi_1 n_1^H(w_{t+1}))(1+\beta+\delta_2) + (1+\beta)(1+\phi_2\mu_2 + \epsilon\kappa h_{t+1} \\ &+\epsilon(1-\kappa)(1-\phi_1 n_1^H(w_t))] \left[\frac{w_{t+1}}{(1-\alpha)A}\right]^{1/\alpha} \\ &= \frac{\beta w_t}{n_{t-1}} \left[1 + \phi_2\mu_2 + \epsilon\kappa h_t + \epsilon(1-\kappa)(1-\phi_1 n_1^H(w_{t-1}))\right] \end{aligned} \quad (42)$$

where $h_{t+1} = h(w_t)$ is defined by equation (22). Equations (41), (42) and $h_{t+1} = h(w_t)$ define a three-dimensional dynamic system, (w_t, h_t, n_{t-1}), where h_t and n_{t-1} are predetermined at time t. This system governs the behavior of the economy when wages are sufficiently high, $w_t \geqslant \underline{w}$.

A steady state is a solution $n_t = n$, $w_t = w$ and $h_t = h$ solving equations (41), (42) and $h = h(w)$. Let $\Delta^H(w)$ be given in Appendix H. Using equation (41), we deduce:

$$n = \frac{1}{2}\left(n_1^H(w) + \sqrt{\Delta^H(w)}\right) \equiv I_{12}(w) \quad (43)$$

then, we substitute $1/n$ using (41). As a consequence, equation (42) yields:

$$n = \frac{\Omega_{12}(w)}{\Omega_{22}(w)} \equiv I_{22}(w) \quad (44)$$

where $\Omega_{12}(w)$ and $\Omega_{22}(w)$ are given in Appendix H.

A steady state in this regime is a solution to $n = I_{12}(w) = I_{22}(w)$. The following proposition summarizes our main result:

Proposition 3 *Under Assumptions 1-2, $\delta_1 > \underline{\delta}_1$, δ_2 sufficiently high, there exists $\overline{w}$ greater than $\underline{w}$ such that:*

1. *If $A > A_1$, there exists a steady state (w^H, n^H), with $w^H \in (\underline{w}, \overline{w})$. If ϕ_2 and α are low, this steady state is unique.*

2. *If A is significantly lower than A_1, there is no steady state for $w > \underline{w}$.*

Proof. See Appendix H. ■

The existence and uniqueness of a steady state with high income requires a sufficiently high productivity A. This result is depicted in Figure 5. In Appendix J, we show that under a reasonable parametrization of the model, this steady state is stable. Therefore, if the wage is sufficiently high, the economy converges to a steady state where households invest in education and choose to postpone having children.

6 What makes households choose or choose not to postpone parenthood?

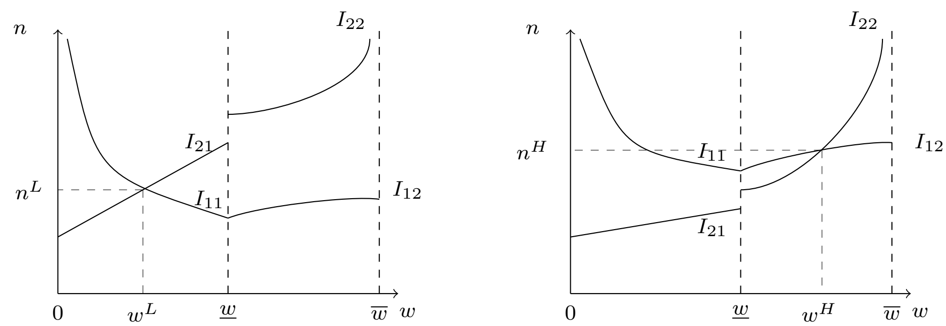
Our objective is to identify the underlying factors that drive one economy to converge toward either a long-run equilibrium in which young households choose to have children and invest in physical capital, or a steady state in which young adults invest in their career and postpone parenthood. The previous sections suggest that the productivity parameter A is pivotal in this analysis. It follows from Propositions 2 and 3 that:

Proposition 4 *Under Assumptions 1-2, if δ_1 and δ_2 are sufficiently high, ϕ_2 and α are low enough, and $\epsilon > \underline{\epsilon}$:*

1. *If A is sufficiently close to $\underline{A}$, there exists a unique steady state (w^L, n^L), with $w^L \in (0, \underline{w})$;*

2. *If $A > A_1$, there exists a unique steady state (w^H, n^H) with $w^H \in (\underline{w}, \overline{w})$.*

The two different scenarios outlined in Proposition 4 are depicted in Figure 5. If the productivity is low, the economy cannot converge to a long-run equilibrium with high wages and where young adults postpone having children to increase their human capital. The unique steady state involves lower wages and positive investment in physical capital only over the life cycle.



(a) A sufficiently close to $\underline{A}$ and $\epsilon > \underline{\epsilon}$. (b) $A > A_1$.

Figure 5: Existence and uniqueness of a steady state

The lower level of wage at the steady state may be justified by: i) the relatively lower supply of labor time since young individuals have higher early fertility; ii) a global lower level of capital since early fertility induces a wage penalty that may negatively impact the old adult age's income, reducing late savings. In contrast, when productivity becomes high enough, the regime in which young households neglect career investment is no longer stable. Therefore, the economy might converge toward a long-run equilibrium with higher wages, postponement of childbearing, and investment in education when young, favoring an increase in future labor income, and investment in physical capital when older.

Another way to interpret this proposition is to consider a strong positive productivity shock, such that A passes over A_1. The steady state then shifts from the low to the high income regime and households' behaviors modify accordingly. The economy may finally end up in an equilibrium where young adults prefer to have children later in the course of life, following the investment in their careers.

To further examine what happens following an increase in total factor productivity (A), the question we may address now is how stationary fertility and wage evolve following an increase of the productivity in each regime. We start by studying the low income regime, and more precisely the steady state (w^L, n^L):

Corollary 1 *Under Assumptions 1-2, δ_1 and δ_2 sufficiently high, ϕ_2 low enough, A sufficiently close to $\underline{A}$, and $\epsilon > \underline{\epsilon}$, w^L increases with A, while the effect of A on the population growth factor is ambiguous.*

Proof. See Appendix I. ■

An increase in productivity clearly entails a rise in the equilibrium wage. However, the effect on the stationary population growth is more contrasted. The complexity arises from the opposite relationships between fertility behaviors at each reproductive age and the stationary relative wage, along with the direct effect of productivity on fertility. To elaborate, it's essential to consider the key variable influencing fertility behaviors, in the low-income regime: the ratio between the old adult's discounted wage and the young people's wage. For a given level of the wage, this ratio is of course negatively affected by the productivity and, therefore, decreases following a positive shock on productivity. This positive shock on productivity may, at least partially, be compensated by a higher relative wage. However, the final effects on early and late fertility are not a priori clear-cut. As a result, the overall effect on the population growth factor is ambiguous.

Consider now that a steady state exists in the high-income regime. Following an increase of the productivity A, $n = I_{22}(w)$ moves down whereas $n = I_{12}(w)$ does not vary. Since both $I_{12}(w)$ and $I_{22}(w)$ are increasing and since $I_{22}(w)$ is steeper at the steady state, a higher A implies an increase of both fertility and wage at the steady state (w_H, n_H):

Corollary 2 *Under Assumptions 1-2, $\delta_1 > \underline{\delta}_1$, δ_2 sufficiently high, ϕ_2 and α low, and $A > A_1$, both w^H and n^H increase with A.*

This corollary shows that, in the high-income regime with investment in education, a higher productivity supports higher long-run wage and population growth. Any increase in productivity involves a rise in the wage which pushes up investment in education, as well as work experience. As a consequence, human capital is enhanced, leading to an increase in late fertility alongside a decrease in early fertility. Indeed, human capital and late fertility are complements whereas human capital and early fertility are substitutes. Finally, everything goes as if the stronger effect on late fertility outweighs the negative effect on early fertility, resulting in an increase in total fertility. This occurs despite households having fewer children in early adulthood, thereby fostering demographic growth. This results is in accordance with the empirical evidence of Section 2 and the possibility of fertility stagnating or rebounding, justified by human capital accumulation and the positive relationship between late fertility and income. This is also consistent with the results highlighted by Frejka and Sobotka (2008), Myrskylä et al. (2009), Sobotka (2017), Yakita (2018), Ohinata and Varvarigos (2020).

In addition, Proposition 4 provides an overview of the diversity of fertility trends in European countries over the past few decades (Frejka and Sobotka (2008), Myrskylä et al. (2009)). While Northern and Western European nations constitute a so-called "high fertility belt", with relatively low early fertility and higher late fertility rates supported by relatively high income levels, Eastern and Southern European countries continue to have relatively high early fertility and low late fertility, with lower total fertility rates. These trends match the features of an equilibrium close to $\underline{w}$ for Southern and Eastern European economies and are in line with the high-income equilibrium for Northern and Western European countries.

7 Quantitative analysis

In the following, we provide a quantitative analysis of our theoretical framework in order to illustrate the effects of a positive productivity shock on the steady states and the long-run pattern of development of the economy. This analysis also allows us to highlight the differences between early and late fertility behaviors in each regime and therefore the crucial role played by human capital in explaining the recent trends in fertility behavior.

7.1 Calibration

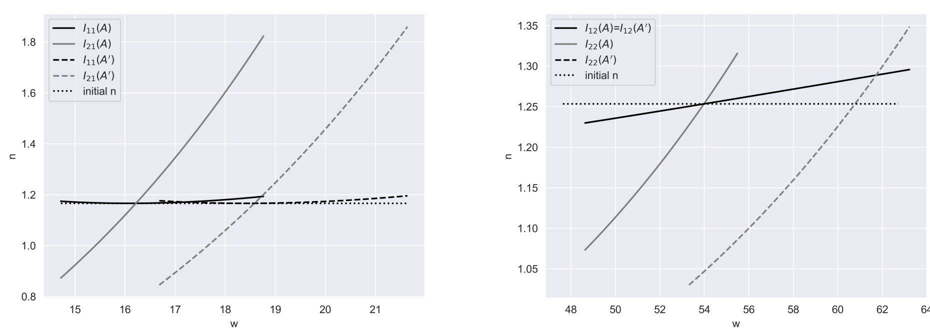
To do so, we assume that a period represents 10 years. In addition, we consider an annual value of $\beta_{annual} = 0.96$ which gives a 4 per cent returns on asset and we derive that $\beta = 0.66$. We also set $\alpha = 0.3$ to get a capital share of income of 30 %. The remaining parameters δ_1, δ_2, ϕ_1, ϕ_2, ϵ, κ and A are calibrated to match the high-income regime steady state with three empirical targets: the demographic growth factor ($n = 1.1$), the total fertility rate ($TFR = n_1^h + n_2^h = 1.5$), the time spent with children ($\phi_1 n_1^h = 0.21$ and $\phi_2 n_2^h = 0.11$) and the wage premium ($\epsilon \tilde{h} = 0.90$). We also constrain A such that the high-income steady state wage rate is at least 10 % above $\underline{w}$. Table 1 summarizes the calibration of the model.

β	δ_1	δ_2	ϕ_1	ϕ_2	μ_1	μ_2	ϵ	κ	α	A^H	A^L
0.66	0.5	0.105	0.41	0.137	0	0	0.07	0.834	0.3	37.36	15

Table 1: Model calibration

7.2 Comparative statics

From Figure 6, we can observe that, consistently with Corollaries 1 and 2, a 10 percent permanent positive shock on total factor productivity entails an increase in the stationary wage in both regimes, albeit the low-income wage remains lower than its high-income counterpart. Regarding the rate of population growth, we find a clear positive effect in the high-income regime, as expected, while the low-income regime shows insignificant changes. Specifically, as reported in Table 2, in the high-income regime, the increase in the population growth rate stems from the rise in late fertility, which counterbalances the decrease in early fertility.



(a) Low-income equilibrium. (b) High-income equilibrium.

Figure 6: The effects of a productivity shock on stationary wages and population growth factors.

In addition, these simulations allow for some insightful comparisons between the two stationary population growth rates. Notably, the population growth factor is higher in the high-income regime compared to the low-income regime. Therefore this simulation illustrates the fertility rebound discussed in the introduction, primarily attributed to the more pronounced rise in late fertility within the high-income regime.

	Low-income regime		High-income regime	
	pre-shock	post-shock	pre-shock	post-shock
n_1	0.74	0.74	0.59	0.58
n_2	0.50	0.50	0.83	0.92
n	1.17	1.17	1.25	1.29

Table 2: Estimations

7.3 Long-run dynamics

Finally, we are able to represent the transition path between the low-income regime and the high-income one, building upon our theoretical framework. To do so, we use the definition of the intertemporal equilibrium in each regime and we compute the non-linear solution starting from the low-income steady state as the initial situation where $A = A^L$ and we let A grow monotonically to its ending stationary value A^H (see Appendix J for more details on the solution method). As depicted in Figure 7, the simulated economy reaches the high-income steady state after 11 periods and switches to the high-income regime after 40 years (assuming that a period lasts for 10 years). The wage w_t is always above $\underline{w}$ afterwards.[11] At the time of the switch, the population growth rate collapses before gradually increasing to its stationary value. Despite this sharp initial decrease, we can observe that the last observation of the population growth rate is higher than its initial value. This result is in line with the theoretical results established in the previous sections. Finally, given the equilibrium path of w_t, we can compute the trajectories of early and late fertility over time. Consistent with our findings, early fertility decreases along the transition while the reverse is true for late fertility. Given our calibration, while early fertility starts being higher initially, it ends up being lower than late fertility in the last periods.

8 Conclusion

This paper investigates the relationship between fertility decisions and economic variables including earnings and productivity in a framework with

[11] Note that our numerical solution displays an average absolute relative residuals of 0.004 and 0.07 for n_t and w_t respectively.

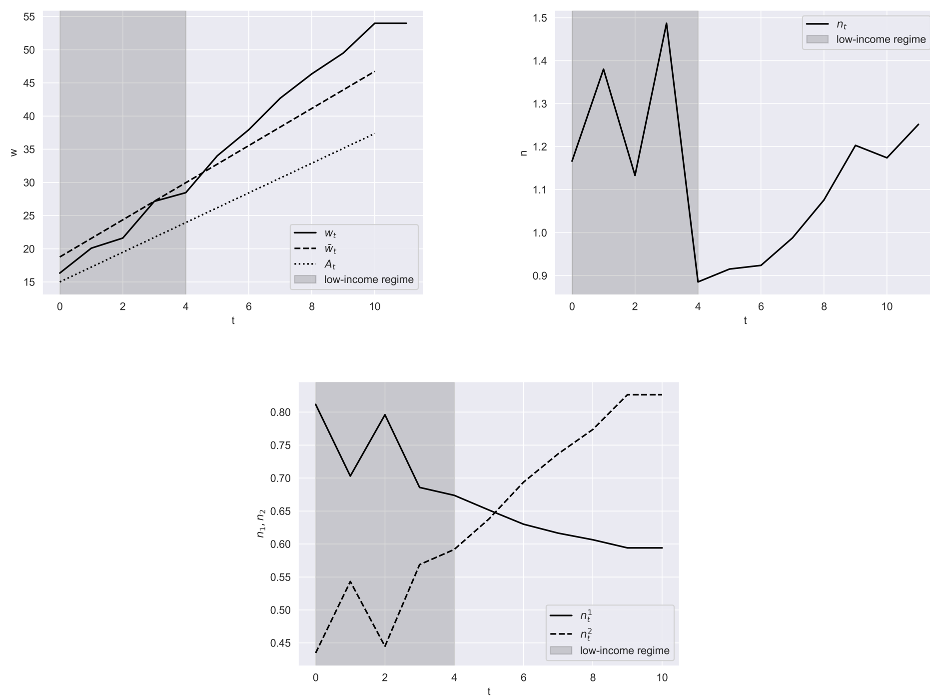


Figure 7: Equilibrium path from the low-income to the high-income regime.

physical and human capital. The main aim is to explain that ASFR of young women is decreasing while ASFR of older women is increasing, which may explain the fertility rebound in some high-income countries. In our model, households choose both how many children to have (the quantum of births) and when to have them (the tempo of births). Moreover, a young adult can invest in physical capital or education.

When the wages are low, young people invest in physical capital only. Following an increase of wages, early fertility increases, while late fertility decreases. After a given level of wage, young adults switch their investment in physical capital to an investment in education. It emerges that early fertility and investment in human capital are substitutes, but that late fertility and human capital are complements. If productivity is low, incomes are lower, households rather have children in early adulthood and save through physical capital. If productivity is high in contrast, incomes are higher and households choose to postpone childbearing to invest in human capital. Hence, our analysis provides an explanation for the fertility flattening or rebound currently observed at least in some European countries, which is driven by an increase in late fertility and investment in the career when young.

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Appendices

A Empirical Analysis

To run the panel regressions we first built a data set containing the ASFRs (four age intervals, $i = 20-24, 25-29, 30-34, 35-39$), the GDP per capita (measured in PPP, 2015 US $), and the average years of schooling for women for 32 European countries indexed by c and listed in footnote 1 over 14 periods of 5-year interval indexed by t, covering the whole range of 1950-2015 years. Indeed, from the data set on educational attainment for women provided by Barro and Lee (2013), we have available the average years of schooling for women aged of 20-24, 25-29, 30-34 and 35-39 years old, every five years from 1950. To build consistent observations for the dependent variable, we comput a 5-year rolling mean for both ASFRs and GDP variables, each corresponding to the considered year in the Barro and Lee (2013) data set so that, for each regression we have run, we have 249 observations. We perform three regression models, for which results are reported in Tables 3 and 4 below:

	20-24 (1)	**20-24 (2)**	**20-24 (3)**	**25-29 (1)**	**25-29 (2)**	**25-29 (3)**
Dep. Var.	ASFR	ASFR	ASFR	ASFR	ASFR	ASFR
Estimator	RandomEff.	PanelOLS	PanelOLS	RandomEff.	PanelOLS	PanelOLS
Obs.	249	249	249	249	249	249
Cov. Est.	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted
R-squared	0.6655	0.7244	0.0700	0.5440	0.5910	0.1098
R-Squared (Within)	0.7138	0.7244	-0.2494	0.5869	0.5910	0.3513
R-Squared (Between)	-0.2659	-0.8910	-0.5123	0.1049	-0.0586	0.2185
R-Squared (Overall)	0.4810	0.3639	-0.3129	0.3272	0.2774	0.2707
F-statistic	162.48	187.53	5.1146	97.422	103.09	8.3843
P-value (F-stat)	0.0000	0.0000	0.0020	0.0000	0.0000	0.0000
==========	==========	==========	==========	==========	==========	==========
const	215.73	228.20	56.412	254.86	261.68	198.30
	(9.9396)	(10.904)	(2.9124)	(16.715)	(17.615)	(8.4566)
yr_sch	-5.4540	-7.1542	4.5259	-18.734	-19.702	-13.922
	(-1.1453)	(-1.5364)	(1.3106)	(-5.6913)	(-5.9841)	(-3.5600)
yr_sch_sq	-0.3821	-0.2281	-0.2696	0.5055	0.5989	0.5022
	(-1.4824)	(-0.8847)	(-1.4891)	(2.8136)	(3.2695)	(2.4889)
GDP_cap	-0.0010	-0.0015	0.0006	-8.474e-05	-0.0003	0.0002
	(-8.4997)	(-9.0336)	(3.5723)	(-0.6874)	(-1.9749)	(1.1612)
==========	==========	==========	==========	==========	==========	==========
Effects		Entity	Entity Time		Entity	Entity Time

T-stats reported in parentheses

Table 3: Model Comparison

	30-34 (1)	**30-34 (2)**	**30-34 (3)**	**35-39 (1)**	**35-39 (2)**	**35-39 (3)**
Dep. Var.	ASFR	ASFR	ASFR	ASFR	ASFR	ASFR
Estimator	RandomEff.	PanelOLS	PanelOLS	RandomEff.	PanelOLS	PanelOLS
Obs.	249	249	249	249	249	249
Cov. Est.	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted	Unadjusted
R-squared	0.3323	0.3614	0.2201	0.3946	0.4362	0.3008
R-Squared (Within)	0.3597	0.3614	0.2135	0.4321	0.4362	0.3528
R-Squared (Between)	0.2890	0.2342	0.3067	0.1572	0.0116	0.2283
R-Squared (Overall)	0.2661	0.2643	0.2149	0.2633	0.2389	0.2420
F-statistic	40.648	40.362	19.191	53.240	55.197	29.256
P-value (F-stat)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
=======	=======	=======	=======	=======	=======	=======
const	188.64	192.66	170.75	118.42	121.55	83.201
	(14.376)	(14.934)	(8.8581)	(15.130)	(15.977)	(7.7878)
yr_sch	-27.400	-28.085	-18.310	-21.420	-22.036	-12.322
	(-9.5074)	(-9.6032)	(-5.8235)	(-12.063)	(-12.290)	(-6.9262)
yr_sch_sq	1.4122	1.4808	0.8073	1.1543	1.2244	0.6318
	(8.8436)	(8.9144)	(4.9385)	(11.401)	(11.623)	(6.5210)
GDP_cap	0.0007	0.0006	0.0004	0.0003	0.0002	0.0004
	(5.8235)	(3.6962)	(2.4382)	(4.4220)	(2.1638)	(3.6135)
=======	=======	=======	=======	=======	=======	=======
Effects		Entity	Entity Time		Entity	Entity Time

T-stats reported in parentheses

Table 4: Model Comparison

B Proof of Lemma 1

Substituting k^Y_{t+1} given by equation (21) into inequality (18) is equivalent to check that: $k^Y(w_t, x_{t+1}) < \overline{k}^Y(w_t, x_{t+1}) \Leftrightarrow$

$$\begin{aligned}
&\Lambda(w_t, x_{t+1}) < w_t^2 \left[1 + \delta_1 + \beta(1 + \beta + \delta_2)\right] \left(1 - \frac{\phi_1 \mu_1}{\delta_1}\right) \\
&+ w_t x_{t+1} \left[1 + \delta_1 + \beta(1 + \beta + \delta_2)\right] \epsilon(1 - \kappa) \left(1 - 2\frac{\phi_1 \mu_1}{\delta_1}\right) \\
&- x_{t+1}^2 \left[1 + \delta_1 + \beta(1 + \beta + \delta_2)\right] \epsilon^2 (1 - \kappa)^2 \frac{\phi_1 \mu_1}{\delta_1}
\end{aligned} \tag{B.1}$$

This inequality in fact compares two quadratic functions. If we examine each coefficient associated with w_t^2, $w_t x_{t+1}$ and x_{t+1}^2 on both sides of the inequality (B.1), we can deduce that the ones of the function $\Lambda(w_t, x_{t+1})$ are all the lowest if $\delta_1 > \phi_1 \mu_1 [1 + \beta(1 + \beta + \delta_2)] \equiv \underline{\delta}_1$. Hence, for $\delta_1 > \underline{\delta}_1$, $k^Y(w_t, x_{t+1}) < \overline{k}^Y(w_t, x_{t+1})$. Using equation (23), we also know that $k^Y_{t+1} > 0$ for all $x_{t+1}/w_t < \chi$ and $k^Y_{t+1} = 0$ otherwise. Nevertheless, we may check that $x_{t+1}/w_t < a_0$ to ensure that early fertility is positive. Using equation (23) and the definition of a_0, we can show that $\Xi(a_0) < 0$ is equivalent to:

$$\delta_1 > \phi_1 \mu_1 \frac{\epsilon(1 - \kappa)(1 + \beta(1 + \beta + \delta_2)) + 1 + \phi_2 \mu_2}{\epsilon(1 - \kappa) + 1 + \phi_2 \mu_2}$$

which is satisfied because $\delta_1 > \underline{\delta}_1$. This directly implies that $\chi < a_0$. Therefore, if $\delta_1 > \underline{\delta}_1$ and $\frac{x_{t+1}}{w_t} < \chi$, $k^Y_{t+1} > 0$ and $n_{1t} > 0$ for all $k^Y_{t+1} > 0$.

Note also from equations (18) and (20) that $\overline{h}(w_t) = \overline{k}^Y(w_t, 1/(\epsilon\kappa))$. Hence, $h_{t+1} = h(w_t) < \overline{h}(w_t)$ is equivalent to $k^Y(w_t, 1/(\epsilon\kappa)) < \overline{k}^Y(w_t, 1/(\epsilon\kappa))$, *i.e.* to inequality (B.1) with $x_{t+1} = 1/(\epsilon\kappa)$. This is satisfied for $\delta_1 > \underline{\delta}_1$, which means that $n_{1t} > 0$ for all $h_{t+1} > 0$ as well. Finally, under Assumption 1, $n_{2,t+1} > 0$ for all $k^Y_{t+1}, h_{t+1} > 0$.

C Proof of Proposition 1

Let us consider the low-income regime where $k^Y_{t+1} = k^Y(w_t, 1/(\epsilon\kappa)) > 0$ and $h_{t+1} = 0$ for $x_{t+1}/w_t < 1/(\epsilon\kappa w_t)$. When $x_{t+1}/w_t = 1/(\epsilon\kappa w_t)$, $k^Y_{t+1} = h_{t+1}$, *i.e.* $k^Y(w_t, 1/(\epsilon\kappa)) = h(w_t)$, and otherwise, $h_{t+1} > 0$ and increases with w_t. Since $x_{t+1} = w_{t+1}/R_{t+1} = A_x w_{t+1}^{1/\alpha}$, $x_{t+1} = 1/(\epsilon\kappa)$ is equivalent to $w_{t+1} = \underline{w}$, where $\underline{w}$ is given by equation (25).

Hence $k^Y(w_t, 1/(\epsilon\kappa)) = h(w_t) \geqslant h(\underline{w})$, because $h(w_t)$ is an increasing function. We should then ensure that it is strictly positive. Using equation (22) and substituting w_t by $\underline{w}$, this is satisfied if $\Lambda(\underline{w}, \frac{1}{\epsilon\kappa}) > 0$:

$$\begin{aligned}
&\beta(1+\beta+\delta_2)(1+\phi_1\mu_1)\frac{\alpha^{2\alpha}(1-\alpha)^{2(1-\alpha)}A^2}{(\epsilon\kappa)^{2\alpha}} \\
&-\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}A}{(\epsilon\kappa)^{1+\alpha}}\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)(1-\beta(1+\beta+\delta_2))+(1+\delta_1)(1+\phi_2\mu_2)\right] \\
&-\frac{1-\kappa}{\epsilon\kappa^2}\left[\epsilon(1-\kappa)(1+\phi_1\mu_1)+1+\phi_2\mu_2\right] > 0
\end{aligned}$$

This inequality can be defined as a polynomial of degree 2 in A such that there exists $\underline{A} > 0$ and $h(\underline{w}) > 0$ for $A > \underline{A}$. It also implies that all relevant values of x_{t+1}/w_t that are to be considered are such that $x_{t+1}/w_t < \chi < a_0$, because $\Xi\left(\frac{x_{t+1}}{w_t}\right)' < 0$.

D Proof of Lemma 2

Using equation (26), we easily deduce that:

$$\begin{aligned}
\widetilde{k}'(z_{t+1}) \quad = \quad & \frac{-1}{1+\delta_1+\beta(1+\beta+\delta_2)}[(\epsilon(1-\kappa)(1+\phi_1\mu_1)+1+\phi_2\mu_2)A_x \\
&+\frac{A_x\delta_1(1+\phi_2\mu_2)}{[1+\epsilon(1-\kappa)A_xz_{t+1}]^2}] < 0
\end{aligned} \tag{D.2}$$

Then, using (27), we get that the sign of $n_1^{L\prime}(z_{t+1})$ is given by:

$$n_1^{L\prime}(z_{t+1}) = \frac{\Theta(z_{t+1})}{\phi_1[1+\delta_1+\epsilon(1-\kappa)A_xz_{t+1}]^2}, \tag{D.3}$$

with $\Theta(z_{t+1})$ defined as follows:

$$\begin{aligned}
\Theta(z_{t+1}) \quad = \quad & -\delta_1\widetilde{k}'(z_{t+1})[1+\delta_1+\epsilon(1-\kappa)A_xz_{t+1}] \\
& -\epsilon(1-\kappa)A_x\delta_1[1+\phi_1\mu_1-\widetilde{k}(z_{t+1})]
\end{aligned} \tag{D.4}$$

Using both equations (26) and (D.2), this expression above is equivalent, after some computations, to:

$$\begin{aligned}
\Theta(z_{t+1}) \quad = \quad & \frac{\delta_1(1+\phi_2\mu_2)A_x}{[1+\delta_1+\beta(1+\beta+\delta_2)][1+\epsilon(1-\kappa)A_xz_{t+1}]^2}[(1+\delta_1)^2 \\
& +2\epsilon(1-\kappa)(1+\delta_1)A_xz_{t+1}+\epsilon^2(1-\kappa)^2A_x^2z_{t+1}^2] > 0
\end{aligned} \tag{D.5}$$

Since $n_1^{L\prime}(z_{t+1}) > 0$ and $\widetilde{k}'(z_{t+1}) < 0$, we easily deduce that $n_2^{L\prime}(z_{t+1}) < 0$.

E Proof of Lemma 3

Using (23), $h(w_t)/w_t$ is given by equation (26) substituting $\widetilde{k}(z_{t+1})$ by $h(w_t)/w_t$ and $A_x z_{t+1}$ by $1/(\epsilon\kappa w_t)$. We immediately deduce from (D.2) in Appendix D, that $h(w_t)/w_t$ is increasing in w_t.

Comparing (29) to (27), we observe that $n_1^H(w_t)$ has the same expression than $n_1^I(z_{t+1})$ substituting $\widetilde{k}(z_{t+1})$ by $h(w_t)/w_t$ and $A_x z_{t+1}$ by $1/(\epsilon\kappa w_t)$. We deduce from Appendix D that $(n_1^H)'(w_t) < 0$.

Then, it comes easily that $\tilde{h}(h(w_t), n_1^H(w_t)) = \tilde{h}(w_t)$ is increasing in w_t and $n_{2,t+1} = n_2(\tilde{h}(w_t), 0) = n_2^H(w_t)$ is an increasing function of w_t.

F Proof of Lemma 4

$$\frac{\partial m^L(z_{t+1})}{\partial z_{t+1}} = \frac{\partial n_1^L(z_{t+1})}{\partial z_{t+1}}\left(1 - \frac{\delta_2\epsilon(1-\kappa)\phi_1}{\phi_2(1+\beta+\delta_2)}\right) + \frac{\delta_2 A_x^{-1}[\frac{\partial \tilde{k}(z_{t+1})}{\partial z_{t+1}} z_{t+1} - \tilde{k}(z_{t+1})]}{\phi_2(1+\beta+\delta_2)z_{t+1}^2}$$

A sufficient condition to ensure that $\frac{\partial m^L(z_{t+1})}{\partial z_{t+1}}$ is negative is $1 < \frac{\delta_2\epsilon(1-\kappa)\phi_1}{\phi_2(1+\beta+\delta_2)}$. If this condition is satisfied, we also have that $\frac{\partial m^H(w_t)}{\partial w_t} > 0$:

$$\frac{\partial m^H(w_t)}{\partial w_t} = \frac{\partial n_1^H(w_t)}{\partial w_t}\left(1 - \frac{\delta_2\epsilon(1-\kappa)\phi_1}{\phi_2(1+\beta+\delta_2)}\right) + \frac{\delta_2\kappa\epsilon}{\phi_2(1+\beta+\delta 2)}\frac{\partial h(w_t)}{\partial w_t}$$

G Proof of Proposition 2

As preliminary results, we have:

$$\Delta^L(z) \equiv n_1^L(z)^2 + 4\frac{\delta_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z)) + A_x^{-1}\widetilde{k}(z)/z] - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \tag{G.6}$$

and

$$\Omega_{11}(z) \equiv [\frac{1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z))+\phi_2\mu_2}{1+\beta+\delta_2}\frac{(1+\beta)z}{[(1-\alpha)A]^{\frac{1}{\alpha}}}$$
$$-\widetilde{k}(z)\frac{(1+\beta)(1-\alpha)+\delta_2}{(1-\alpha)(1+\beta+\delta_2)}]$$
$$[\delta_2(1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z))+A_x^{-1}\widetilde{k}(z)/z)-\phi_2\mu_2(1+\beta)]$$
$$+n_1^L(z)\beta\phi_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z))+\phi_2\mu_2+A_x^{-1}\widetilde{k}(z)/z] \quad \text{(G.7)}$$
$$\Omega_{21}(z) \equiv \beta\phi_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z))+\phi_2\mu_2+A_x^{-1}\widetilde{k}(z)/z]$$
$$-(1-\phi_1 n_1^L(z))\frac{z}{[(1-\alpha)A]^{\frac{1}{\alpha}}}$$
$$[\delta_2(1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z))+A_x^{-1}\widetilde{k}(z)/z)-\phi_2\mu_2(1+\beta)] \quad \text{(G.8)}$$

A steady state in the low income regime is a solution $z \in (0, \underline{z})$ such that $I_{11}(z) = I_{21}(z)$. When z tends to 0, we have:

$$\widetilde{k}(0) = \frac{\beta(1+\beta+\delta_2)(1+\phi_1\mu_1)}{1+\delta_1+\beta(1+\beta+\delta_2)}$$
$$n_1^I(0) = \frac{\delta_1-\phi_1\mu_1-\delta_1\widetilde{k}(0)}{\phi_1(1+\delta_1)}$$

which means that both have a finite value. Using (40)-(G.8), we deduce that:

$$I_{21}(0) = n_1^L(0) - \widetilde{k}(0)\frac{\delta_2}{\beta\phi_2}\frac{(1+\beta)(1-\alpha)+\delta_2}{(1-\alpha)(1+\beta+\delta_2)}$$

which has a finite value, whereas using (39), we get $I_{11}(0) = +\infty$. Since $I_{11}(0) > I_{21}(0)$, there exists a steady state in the interval $(0, \underline{z})$ if $I_{21}(\underline{z}) > I_{11}(\underline{z})$.

From the proof of Proposition 1, when A tends to $\underline{A}$, $k^Y(\underline{z})$ tends to 0, and therefore, $\widetilde{k}(\underline{z})$ tends to 0 too. Using (G.7) and (G.8), we have:

$$\Omega_{11}(\underline{z}) \equiv \frac{1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(\underline{z}))+\phi_2\mu_2}{1+\beta+\delta_2}\frac{(1+\beta)\underline{z}}{[(1-\alpha)A]^{\frac{1}{\alpha}}}$$
$$[\delta_2(1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(\underline{z}))-\phi_2\mu_2(1+\beta)]$$
$$+n_1^L(\underline{z})\beta\phi_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(\underline{z}))+\phi_2\mu_2]$$
$$\Omega_{21}(\underline{z}) < \beta\phi_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(\underline{z}))+\phi_2\mu_2]$$

When A tends to $\underline{A}$, this implies that:

$$
\begin{aligned}
I_{21}(\underline{z}) &> n_1^L(\underline{z}) + \frac{1+\beta}{\beta}\frac{\underline{z}}{[(1-\alpha)A]^{\frac{1}{\alpha}}}n_2^L(\underline{z}) \\
&= n_1^L(\underline{z}) + \frac{1+\beta}{\beta}\frac{\alpha^{1-\alpha}}{A(\epsilon\kappa)^{1-\alpha}(1-\alpha)^{2-\alpha}}n_2^L(\underline{z})
\end{aligned}
$$

This is higher than $I_{11}(\underline{z}) = \frac{1}{2}\left[n_1^L(\underline{z}) + \sqrt{n_1^L(\underline{z})^2 + 4n_2^L(\underline{z})}\right]$ if and only if:

$$
\left(\frac{1+\beta}{\beta}\frac{\alpha^{1-\alpha}}{A(\epsilon\kappa)^{1-\alpha}(1-\alpha)^{2-\alpha}}\right)^2 n_2^L(\underline{z}) + \left(\frac{1+\beta}{\beta}\frac{\alpha^{1-\alpha}}{A(\epsilon\kappa)^{1-\alpha}(1-\alpha)^{2-\alpha}}\right) n_1^L(\underline{z}) > 1
$$

which is satisfied if δ_2 is sufficiently high and ϕ_2 sufficiently low. This shows the existence of the steady state.

We focus now on the uniqueness. Consider the function $n = I_{11}(z)$. Using (G.6) and (39), we have that $I_{11}'(z) < 0$ is equivalent to:

$$
\begin{aligned}
&2\frac{\delta_2}{\phi_2(1+\beta+\delta_2)}\left[A_x^{-1}\frac{\widetilde{k}'(z)z - \widetilde{k}(z)}{z^2} - \phi_1(n_1^L)'(z)\epsilon(1-\kappa)\right] \\
&< -\phi_2(n_1^L)'(z)\left[n_1^L(z) + \sqrt{\Delta^L(z)}\right]
\end{aligned}
$$

Since $\widetilde{k}'(z) < 0$ and $(n_1^L)'(z) > 0$, this inequality is satisfied under the assumption that ϕ_2 is low enough.

Using (40)-(G.8), $I_{21}(z)$ can be rewritten as $I_{21}(z) = \widetilde{\Omega}_{11}(z)/\widetilde{\Omega}_{21}(z)$, with:

$$
\begin{aligned}
&\widetilde{\Omega}_{11}(z) \equiv [\frac{1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z)) + \phi_2\mu_2}{1+\beta+\delta_2}\frac{(1+\beta)z}{[(1-\alpha)A]^{\frac{1}{\alpha}}} \\
&-\widetilde{k}(z)\frac{(1+\beta)(1-\alpha)+\delta_2}{(1-\alpha)(1+\beta+\delta_2)}] \\
&+n_1^L(z)\beta\phi_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z)) + \phi_2\mu_2 + A_x^{-1}\widetilde{k}(z)/z] \\
&[\delta_2(1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z)) + A_x^{-1}\widetilde{k}(z)/z) - \phi_2\mu_2(1+\beta)]^{-1} \quad \text{(G.9)} \\
&\widetilde{\Omega}_{21}(z) \equiv \beta\phi_2[1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z)) + \phi_2\mu_2 + A_x^{-1}\widetilde{k}(z)/z] \\
&[\delta_2(1+\epsilon(1-\kappa)(1-\phi_1 n_1^L(z)) + A_x^{-1}\widetilde{k}(z)/z) - \phi_2\mu_2(1+\beta)]^{-1} \\
&-(1-\phi_1 n_1^L(z))\frac{z}{[(1-\alpha)A]^{\frac{1}{\alpha}}} \quad \text{(G.10)}
\end{aligned}
$$

With ϕ_2 low enough, the derivative of $\widetilde{\Omega}_{11}(z)$ is given by the derivative of the first term in (G.9), and the derivative of $\widetilde{\Omega}_{21}(z)$ is given by the derivative of the last term in (G.10). Therefore, since $\widetilde{k}'(z) < 0$, $I_{21}(z)$ is increasing if the derivative of $(1-\phi_1 n_1^L(z))z$ with respect to z is positive.

Using (27), we deduce that $[(1-\phi_1 n_1^L(z))z]/dz > 0$ is equivalent to:

$$(1+\phi_1\mu_1)[1+\epsilon(1-\kappa)A_x z](1+\delta_1) + (1+\phi_1\mu_1)\epsilon(1-\kappa)A_x z[1+\delta_1 \\ +\epsilon(1-\kappa)A_x] + \delta_1(1+\delta_1)\widetilde{k}(z) + \delta_1\widetilde{k}'(z)z[1+\delta_1+\epsilon(1-\kappa)A_x z] > 0 \tag{G.11}$$

Using (26), we have:

$$\widetilde{k}'(z)z = -\frac{1}{1+\delta_1+\beta(1+\beta+\delta_2)}[(\epsilon(1-\kappa)(1+\phi_1\mu_1)+1+\phi_2\mu_2)A_x z \\ +\frac{A_x z\delta_1(1+\phi_2\mu_2)}{[1+\epsilon(1-\kappa)A_x z]^2}] \tag{G.12}$$

Substituting this expression in inequality (G.11), we obtain:

$$(1+\phi_1\mu_1)\delta_1\epsilon(1-\kappa)A_x z + \delta_1(1+\delta_1)\widetilde{k}(z) \\ +\delta_1\frac{1+\delta_1+\epsilon(1-\kappa)A_x z}{1+\delta_1+\beta(1+\beta+\delta_2)}[(1+\phi_1\mu_1)(1+\epsilon(1-\kappa)A_x z) \\ \frac{1+\delta_1+\beta(1+\beta+\delta_2)}{\delta_1} - [\epsilon(1-\kappa)(1+\phi_1\mu_1)+1+\phi_2\mu_2]A_x z \\ -\frac{A_x z\delta_1(1+\phi_2\mu_2)}{[1+\epsilon(1-\kappa)A_x z]^2}] > 0 \tag{G.13}$$

Since $\widetilde{k}(z) > 0$, this inequality is satisfied if:

$$(1+\phi_1\mu_1)(1+\epsilon(1-\kappa)A_x z)\frac{1+\delta_1+\beta(1+\beta+\delta_2)}{\delta_1} > \\ [\epsilon(1-\kappa)(1+\phi_1\mu_1)+(1+\phi_2\mu_2)(1+\delta_1)]A_x z \tag{G.14}$$

This inequality is satisfied for $A_x z = 0$. When $A_x z = A_x \underline{z} = 1/[\alpha^\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}(\epsilon\kappa)^\alpha A]$, inequality (G.14) writes:

$$\alpha^\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}(\epsilon\kappa)^\alpha A\frac{1+\delta_1+\beta(1+\beta+\delta_2)}{\delta_1} + \epsilon(1-\kappa)\frac{1+\beta(1+\beta+\delta_2)}{\delta_1} \\ > (1+\delta_1)\frac{1+\phi_2\mu_2}{1+\phi_1\mu_1}$$

This last inequality is satisfied if ϵ is high enough. Then, inequality (G.14) is satisfied for all $z \in (0, \underline{z})$. We deduce that $(1 - \phi_1 n_1^L(z))z$ is increasing in z and there exists a value $\underline{\epsilon} > 0$ such that $I_{21}(z)$ is increasing for all $\epsilon > \underline{\epsilon}$ and ϕ_2 low enough. Since $I_{11}(z)$ is decreasing, the steady state is unique.

H Proof of Proposition 3

As preliminary results, we have:

$$\Delta^H(w) \equiv n_1^H(w)^2 + 4\frac{\delta_2[1 + \epsilon\kappa h(w) + \epsilon(1-\kappa)(1 - \phi_1 n_1^H(w))] - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \tag{H.15}$$

and

$$\Omega_{12}(w) \equiv \Gamma_a(w) + \Gamma_b(w)\frac{w^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} \tag{H.16}$$

$$\Omega_{22}(w) \equiv \Gamma_c(w) - \Gamma_d(w)\frac{w^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} \tag{H.17}$$

with

$$\Gamma_a(w) \equiv [1 + \epsilon\kappa h(w) + \epsilon(1-\kappa)(1 - \phi_1 n_1^H(w)) + \phi_2\mu_2]n_1^H(w)\beta\phi_2 \tag{H.18}$$

$$\begin{aligned}\Gamma_b(w) \equiv\ & \frac{1 + \epsilon\kappa h(w) + \epsilon(1-\kappa)(1 - \phi_1 n_1^H(w)) + \phi_2\mu_2}{1+\beta+\delta_2}(1+\beta) \\ & [\delta_2(1 + \epsilon\kappa h(w) + \epsilon(1-\kappa)(1 - \phi_1 n_1^H(w))) - \phi_2\mu_2(1+\beta)]\end{aligned} \tag{H.19}$$

$$\Gamma_c(w) \equiv \beta\phi_2[1 + \epsilon\kappa h(w) + \epsilon(1-\kappa)(1 - \phi_1 n_1^H(w)) + \phi_2\mu_2] \tag{H.20}$$

$$\begin{aligned}\Gamma_d(w) \equiv\ & (1 - \phi_1 n_1^H(w))[\delta_2(1 + \epsilon\kappa h(w) + \epsilon(1-\kappa)(1 - \phi_1 n_1^H(w))) \\ & -\phi_2\mu_2(1+\beta)]\end{aligned} \tag{H.21}$$

From Lemma 3, we know that $n_1^H(w)$ is a decreasing function and therefore, $1 - \phi_1 n_1^H(w)$ is increasing in w.

Using (23), we deduce that $h(w)/w$ is defined by:

$$\begin{aligned}\frac{h(w)}{w}[1 + \delta_1 + \beta(1+\beta+\delta_2)] =\ & \beta(1+\beta+\delta_2)(1+\phi_1\mu_1) \\ & -\frac{\epsilon(1-\kappa)(1+\phi_1\mu_1) + 1 + \phi_2\mu_2}{\epsilon\kappa w} - \frac{\delta_1(1+\phi_2\mu_2)}{\epsilon(\kappa w + 1 - \kappa)}\end{aligned} \tag{H.22}$$

which implies that $h(w)/w$ is increasing in w. We also observe that $h(w)/w$ is bounded above by $\beta(1+\beta+\delta_2)(1+\phi_1\mu_1)/[1+\delta_1+\beta(1+\beta+\delta_2)]$. This implies that $1-\phi_1 n_1^H(w)$ is also bounded above by a finite value.

Let us focus on $\Omega_{22}(w)$. Since $h(w)$, $1-\phi_1 n_1^H(w)$ and $w^{\frac{1-\alpha}{\alpha}}$ are increasing, $\Omega_{22}'(w) < 0$ for δ_2 high enough, and $\Omega_{22}(+\infty) < 0$. In addition, using (25), $\Omega_{22}(\underline{w}) > 0$ is equivalent to:

$$A > \left[\frac{\Gamma_d(\underline{w})}{\Gamma_c(\underline{w})}\right] \frac{\alpha^{1-\alpha}}{(1-\alpha)^{2-\alpha}(\epsilon\kappa)^{1-\alpha}} \tag{H.23}$$

Since $\Gamma_d(w)/\Gamma_c(w) < (1-\phi_1 n_1^H(w))/(\beta\phi_2)$ and $1-\phi_1 n_1^H(w)$ is bounded above, there exists a $A_0 > 0$ such that inequality (H.23) is satisfied for $A > A_0$.

This means that there exists $\overline{w} > \underline{w}$ such that $\Omega_{22}(\overline{w}) = 0$ and $\Omega_{22}(w) > 0$ for all $w \in (\underline{w}, \overline{w})$.

Using (H.16), we observe that $\Omega_{12}(w)$ is strictly positive for all $w \in (\underline{w}, \overline{w})$. This implies that for δ_2 high enough, $I_{22}(w) > 0$ is defined for all $w \in (\underline{w}, \overline{w})$, with $I_{22}(\underline{w}) > 0$ finite and $I_{22}(\overline{w}) = +\infty$.

Since $I_{12}(w)$ is strictly positive and finite for all $w \in (\underline{w}, \overline{w})$, we especially have $I_{12}(\overline{w}) < I_{22}(\overline{w})$. Therefore, there exists a steady state in this regime if $I_{12}(\underline{w}) > I_{22}(\underline{w})$, i.e. $\Omega_{12}(\underline{w}) < I_{12}(\underline{w})\Omega_{22}(\underline{w})$. This is equivalent to:

$$A > Z(\underline{w}) \frac{\alpha^{1-\alpha}}{(1-\alpha)^{2-\alpha}(\epsilon\kappa)^{1-\alpha}} \tag{H.24}$$

with

$$Z(\underline{w}) \equiv \frac{\Gamma_b(\underline{w}) + I_{12}(\underline{w})\Gamma_d(\underline{w})}{I_{12}(\underline{w})\Gamma_c(\underline{w}) - \Gamma_a(\underline{w})} \tag{H.25}$$

where $I_{12}(\underline{w})\Gamma_c(\underline{w}) - \Gamma_a(\underline{w}) > 0$ because $I_{12}(\underline{w}) > n_1^H(\underline{w})$. Using (H.18), (H.19) and (H.20), we have $\Gamma_a(\underline{w}) = n_1^H(\underline{w})\Gamma_c(\underline{w})$ and:

$$\Gamma_b(\underline{w}) = \frac{1+\beta}{1+\beta+\delta_2} \frac{\Gamma_c(\underline{w})}{\beta\phi_2} [\delta_2(1+\epsilon\kappa h(\underline{w}) + \epsilon(1-\kappa)(1-\phi_1 n_1^H(\underline{w}))) - \phi_2\mu_2(1+\beta)]$$

We deduce that:

$$Z(\underline{w}) = \frac{1}{I_{12}(\underline{w}) - n_1^H(\underline{w})} [\frac{1+\beta}{1+\beta+\delta_2} \frac{1}{\beta\phi_2} [\delta_2(1+\epsilon\kappa h(\underline{w})$$
$$+\epsilon(1-\kappa)(1-\phi_1 n_1^H(\underline{w}))) - \phi_2\mu_2(1+\beta)] + I_{12}(\underline{w}) \frac{\Gamma_d(\underline{w})}{\Gamma_c(\underline{w})}]$$

Since $h(w)/w < \beta(1+\beta+\delta_2)(1+\phi_1\mu_1)/[1+\delta_1+\beta(1+\beta+\delta_2)]$ and $\Gamma_d(w)/\Gamma_c(w) < (1-\phi_1 n_1^H(w))/(\beta\phi_2)$, we get:

$$Z(\underline{w}) < \frac{1}{I_{12}(\underline{w}) - n_1^H(\underline{w})} \frac{1}{\beta\phi_2}[\frac{1+\beta}{1+\beta+\delta_2}[\delta_2(1+\epsilon\kappa\frac{\beta(1+\beta+\delta_2)(1+\phi_1\mu_1)}{1+\delta_1+\beta(1+\beta+\delta_2)}\underline{w}$$
$$+\epsilon(1-\kappa)(1-\phi_1 n_1^H(\underline{w}))) - \phi_2\mu_2(1+\beta)] + I_{12}(\underline{w})(1-\phi_1 n_1^H(w))] \qquad \text{(H.26)}$$

Using (H.15) and (43), we can further show that $I'_{12}(w) > 0$ is equivalent to:

$$\frac{2\delta_2}{1+\beta+\delta_2}[\epsilon\kappa h'(w) - \epsilon(1-\kappa)(n_1^H)'(w)] > -\phi_2 (n_1^H)'(w)\left[n_1^H(w) + \sqrt{\Delta^H(w)}\right]$$

which is satisfied for ϕ_2 low enough.

In this case, the denominator $I_{12}(\underline{w}) - n_1^H(\underline{w})$ is increasing in $\underline{w}$. Since $1-\phi_1 n_1^H(w)$ is bounded above by a finite value, the right-hand side of inequality (H.26) is smaller than a function $\widetilde{Z}(\underline{w})$ characterized by $\widetilde{Z}'(\underline{w})\underline{w}/\widetilde{Z}(\underline{w}) < 1$.

Therefore, inequality (H.24) is satisfied if:

$$A > \widetilde{Z}(\underline{w}) \frac{\alpha^{1-\alpha}}{(1-\alpha)^{2-\alpha}(\epsilon\kappa)^{1-\alpha}} \qquad \text{(H.27)}$$

where $\underline{w}$ given by (25) linearly increases with A. Since $\widetilde{Z}'(\underline{w})\underline{w}/\widetilde{Z}(\underline{w}) < 1$, there exists a unique $A_1 (> A_0)$ such that inequality (H.27) is satisfied if $A > A_1$. This proves the existence of a steady state.

By inspection of (H.16), we note that when ϕ_2 and α are low, $\Omega_{12}(w)$ is increasing. Since $\Omega_{22}(w)$ is decreasing, this implies that $I_{22}(w)$ is strongly increasing and convex, while we have shown that $I_{12}(w)$ is weakly increasing. This ensures the uniqueness of the stationary solution.

I Proof of Corollary 1

We recall that $A_x = \alpha^{-1}(1-\alpha)^{\frac{\alpha-1}{\alpha}} A^{-\frac{1}{\alpha}}$, *i.e.* decreases with A. Using (26)-(28), we observe that A_x enters in the expression of $\widetilde{k}(z)$, $n_1^L(z)$, and $n_2^L(z)$ as a multiplicative component of z. Using Lemma 2, this implies that $n_1^L(z)$ increases with A_x, and $\widetilde{k}(z)$ and $n_2^L(z)$ decreases with A_x. Since A appears in these expressions through A_x only, it means that $n_1^L(z)$ decreases with A, and $\widetilde{k}(z)$ and $n_2^L(z)$ increases with A.

For the same reason, since we have shown that $I_{11}(z)$ is decreasing (see the proof of Proposition 2), $I_{11}(z)$ is decreasing with A_x, and therefore increasing with A.

We note that $\frac{z}{[(1-\alpha)A]^{\frac{1}{\alpha}}} = A_x z\alpha(1-\alpha)$. Substituting this expression in $\Omega_{11}(z)$ and $\Omega_{21}(z)$ given by equations (G.7) and (G.8), we deduce that A_x enters as a multiplicative component of z in $I_{21}(z)$. Since we have shown in the proof of Proposition 2 that $I_{21}(z)$ is increasing , $I_{21}(z)$ is increasing with A_x, and therefore decreasing with A.

We deduce from these observations that z^L and then w^L are increasing in A. The effect on n^L is ambiguous and will depend on the slope of $I_{11}(z)$ and $I_{21}(z)$.

J Additional Details about the Quantitative Analysis

From equations (37)-(38) and (41)-(42), we (implicitly) define the perfect-foresight intertemporal equilibrium:

$$\begin{array}{ll} g^L(z_{t+1}, z_t, z_{t-1}, n_{t-1}, n_{t-2}, A^j) = 0 & \text{if } w_t < \underline{w} \\ g^H(w_t, w_{t-1}, w_{t-2}, n_{t-1}, n_{t-2}, A^j) = 0 & \text{else} \end{array} \tag{J.28}$$

In order to obtain a one-period lag system, we add $\tilde{w}_t = w_{t-1}$ and $\tilde{z}_t = z_{t-1}$ to consider $\tilde{g}^j(x_t^j, x_{t-1}^j, A^j) = 0$ for $j = L, H$, where $x_t^H = [n_{t-1}, w_t, \tilde{w}_t]$ and $x_t^L = [n_{t-1}, z_t, \tilde{z}_t]$.

Using our calibration in Section 7, we compute the steady state by solving $\tilde{g}^j(x^j, x^j, A^j) = 0$ with x^j the steady state solutions.

To study the local stability in the neighborhood of the steady state in both regime, the vectorized linear model writes:

$$C^j dx_t^j = B^j dx_{t-1}^j \tag{J.29}$$

with $C \equiv \frac{\partial \tilde{g}^j(x^j, x^j, A^j)}{\partial x_t}$ and $B \equiv -\frac{\partial \tilde{g}^j(x^j, x^j, A^j)}{\partial x_{t-1}}$.

From (J.29), we derive the eigenvalues. Based on our calibration in Section 7, we obtain Figures 8. As stated after Propositions 2 and 3, the steady state is locally a saddle in the low-income regime and it is a sink in the high-income regime steady state.

We can perform a comparative statics exercise by varying A^j (e.g. a 10 per cent increase). Figure 6 replicates the results stated in Corollaries 1 and 2.

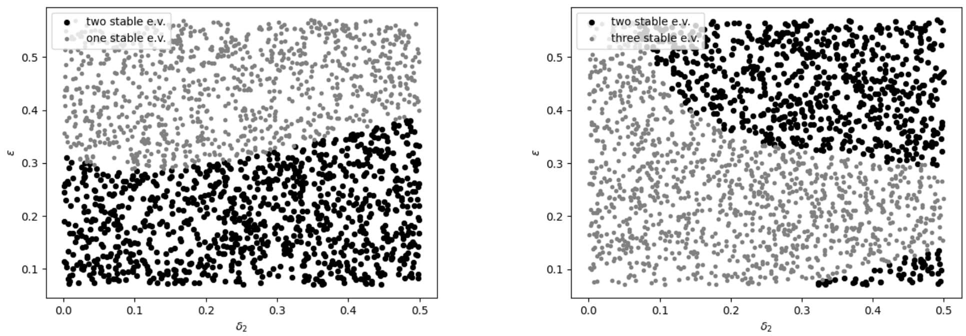

Figure 8: Number of stable eigenvalues. Left: low-income regime. Right: high-income regime

To solve for the transition dynamics between the low-income and the high-income regime, we use the extended-path method.[12] In system (J.28), we allow A to be a time-dependent exogenous process $A = A_t$, $t = 0, 1, .., T$. We also substitute $z_t = \frac{w_t^{\frac{1}{\alpha}}}{w_{t-1}}$ in the low-income regime. Taking these changes into account, an equilibrium path satisfies:

$$\begin{array}{l} g^L(w_{t+1}, w_t, w_{t-1}, w_{t-2}, n_{t-1}, n_{t-2}, A_t, A_{t-1}) = 0 \quad \text{if } w_t < \underline{w} \\ g^H(w_t, w_{t-1}, w_{t-2}, n_{t-1}, n_{t-2}, A_t) = 0 \quad \text{else} \end{array} \tag{J.30}$$

for $t = 0, 1, .., T$. The extended-path method defines this problem as a $2 \times (T + 1)$ system of non-linear equations with given initial and terminal conditions n_{-1} and w_T, and exogenous path of A_t solved by vectors $\mathbf{w} = (w_0, w_1, ..., w_{T-1})$ and $\mathbf{n} = (n_0, n_1, ..., n_{T-1})$ satisfying (J.30).

[12] See Judd (1998) for further details on the extended-path method.