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The Priced Survey Methodology: Theory

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Abstract

In this paper, I introduce a novel methodology to conduct surveys. The priced survey methodology (PSM). Like standard surveys, priced surveys are easy to implement, and measure social preferences on numerical scales. The PSM's design draws inspiration from consumption choice experiments, as respondents fill out the same survey several times under different choice sets. I extend Afriat's theorem and show that the Generalized Axiom of Revealed Preferences is necessary and sufficient for the existence of a concave, continuous, and single-peaked utility function rationalizing answers to the PSM. I apply the PSM to a sample of online participants and show that most respondents are rational when answering the PSM. I estimate respondents' single-peaked utility functions and draw several implications on their social preferences.

JEL C9, D91, C44

Keywords: Decision Theory, Revealed Preference, Social Preferences, Behavioral Economics, Survey.

1 Introduction

One of the most fundamental contributions of economics to the history of thought is its ability to explain choices by a set of preferences. These preferences encompass consumption goods, attitudes towards risk, time, and social aspects. Specifically, social preferences include aspects such as altruism, identity, environmental concerns, political inclinations,

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and perceptions of fairness and justice. Although a substantial theoretical literature in economics has focused on the recoverability of preferences over consumption goods (Andreoni and Miller (2002)), risk (Choi et al. (2007, 2014); Halevy, Persitz and Zrill (2018)), and time (Dziewulski (2018)), far less is known about social preferences. As a result, the existing measures of social preferences either implicitly assume the existence of a utility representation that internalizes social aspects of decision-making, or rely on surveys (Falk et al. (2018)). While these measures are valuable in different contexts, they leave several important points unanswered. To start, little is known about the rationality axioms sustaining the existence of a utility representation for social preferences. Hence, there is no scientific rationale explaining the use and selection of specific utility functions to describe social preferences. As for surveys, they give a snapshot of societal preferences, so the decision mechanisms behind survey answers are unknown. For example, it is not possible to distinguish authentic responses from the ones influenced by the way questions are framed. Moreover, surveys fall short of offering a reliable ground for comparing data, as recently shown by Bond and Lang (2019).

In this paper, I introduce a novel methodology to measure social preferences, the Priced Survey Methodology (PSM). While it maintains the simplicity of traditional surveys, the PSM's design draws inspiration from economic concepts of revealed preferences and consumer demand. The fundamental novelty of the PSM is its capacity to allow social scientists to recover the preference ordering embedded in survey responses.

The design of the PSM is close to experiments built to recover preferences from choices on linear budget sets (Andreoni and Miller (2002), Choi et al. (2007), Choi et al. (2014), Fisman et al. (2015) Halevy, Persitz and Zrill (2018)) as respondents fill the same survey multiple times under different choice sets. Choi et al. (2007), Choi et al. (2014) and Halevy, Persitz and Zrill (2018) study risk preferences using portfolio choices of Arrow securities. In every round of the PSM, each given participant is not greeted with a blank slate but rather a predetermined default answer. She can however adjust this default to better match her preferences. Participants have a finite pool of credits in each round, and adjusting their answer from the default depletes this pool. The credit cost for changes isn't constant but varies between rounds. With this design, it is as if subjects were "buying" goods when they move from the default.

Example.Deontology vs Utilitarianism. On a scale from 0 to +10, where 0 indicates that you strongly disagree and +10 that you strongly agree, to what extent do you agree with these statements:

1. Human rights should never be compromised, even if it leads to larger societal harm.

2. The needs of the many should sometimes take precedence over the rights of the few.

In the PSM, respondents answer this survey several times, under different choice sets. In each round, the default belongs to one of the four corners of the choice set, and moving from the corners is costly.

The first key contribution of this paper is theoretical. I extend Afriat's theorem, and show that the Generalized Axiom of Revealed Preferences (GARP) is necessary and sufficient for the existence of a concave, continuous, and **single-peaked** utility function rationalizing survey answers. There are several important implications. First, rather than interpreting the *cardinal answers* subjects provide to a traditional survey, the PSM enables to estimate - and interpret *utility parameters*, which are related to the *ordinal relations* between all possible answers to the survey. This is more than a simple interpretation difference. It is a key improvement, as taking numerical values at face value can be misleading Bond and Lang (2019). In contrast, utility parameters offer a ground for comparing social preferences. Moreover, by estimating the peak of the utility function behind survey answers, experimenters have access to a new measure of a respondent's ideal answer to a survey. This measure is estimated using ordinal relations between survey answers, thereby not subject to issues inherent to cardinal interpretations of scales. Finally, with the PSM, it is possible to measure valuable aspects of social preferences that cannot be captured with traditional surveys such as how people navigate moral dilemmas.

The second key contribution of this paper is to apply the PSM to a sample of 100 online respondents. All participants had to answer a PSM consisting of nine rounds and two questions measuring altruistic and self-interested preferences. I measure the decision-making quality by evaluating the consistency of individual choices with the Generalized Axiom of Revealed Preference (GARP), using the CCEI Index (Critical Cost Efficiency Index). I find that respondents reach an average CCEI score of 92%. This is even higher than CCEI scores measured in the consumption choice environment. Second, I used the individual-level data to estimate the following single-peaked utility function:

$$u^i(q) = -\sum_{s=1}^{S} \frac{1}{2} a_s^i \left(q_s - b_s^i\right)^2.$$

Vector $\mathbf{b^i}_{s \in \mathcal{S}}$ measures respondent i's ideal answer. It offers a robust alternative to the "ideal point" measured directly through the (cardinal) answer that respondents give to traditional surveys. The key difference with traditional surveys is that the ideal point $\mathbf{b^i}$ is measured using all rounds of the PSM. Moreover, as $\mathbf{b^i}$'s estimation is based on the revealed preference information of the PSM, it is robust to the issues inherent to cardinal interpretations of survey answers (Bond and Lang (2019)). Vector $\mathbf{a^i}$ measures the *relative* importance of different social preferences for respondent i, as measured through survey questions. Concretely, a respondent might answer that she is very altruistic but gives a low weight to altruism, as compared to selfishness.

Although the low number of participants make it difficult to interpret the results, it seems that respondents ideal point, as estimated through the PSM, does not correspond to the answers respondents provide to the traditional survey. Moreover, several aspects of social preferences that cannot be captured by traditional surveys seem to arise. Older respondents seem to be particularly more concerned about their selfish and altruistic values than younger respondents.

2 Related Literature

The PSM is close to experiments built to recover preferences from choices on linear budget sets (Andreoni and Miller (2002), Choi et al. (2007), Choi et al. (2014), Fisman et al. (2015) Halevy, Persitz and Zrill (2018)) as respondents fill the same survey multiple times under different choice sets. Choi et al. (2007), Choi et al. (2014) and Halevy, Persitz and Zrill (2018) study risk preferences using portfolio choices of Arrow securities. Andreoni and Miller (2002) and Fisman et al. (2015) are closer to the PSM in spirit as they seek to recover altruistic preferences using modified versions of the dictator game. In all previous experiments, the designs are meant to capture monotonic preferences. Since respondents have ideal points when answering surveys (Coombs (1964); Jebb, Ng and Tay (2021); Thurstone (1928)), preferences behind survey answers cannot be monotonic, so the previous designs cannot be applied to recover preferences behind survey answers. A key contribution of this paper is to build on the previous designs so that the PSM can enable social scientists to recover single-peaked preferences over survey answers. The PSM can therefore capture a broader set of social preferences than existing studies.

The economic literature on the recoverability of preferences from repeated choices started with Afriat's theorem Afriat (1967), who established that cyclical consistency was necessary and sufficient to recover preferences that rationalize consumption choice. Various proofs of this theorem exist in the literature (e.g., Varian (1982), Polisson and Renou

(2016), Chambers and Echenique (2016)). The theorem has been extended to risk preferences (Polisson, Quah and Renou (2020)), or to more general choice environments (Forges and Minelli (2009), Nishimura, Ok and Quah (2017)). One key aspect of Afriat's theorem is that the utility function rationalizing choices is monotonic. This property comes from the assumption that $\geq$ constitutes an exogenous pre-order of the consumption sets. That is, absent constraints on choice, any individual's consumption will tend to infinity. While this assumption can be justified in the analysis of consumption choices, it cannot reasonably hold for social preferences embedded in survey responses. My key contribution to the literature here is be to extend Afriat's theorem to study single-peaked preference domains.

3 Design and Theory

Notations. Let's consider a PSM with S questions, and $\mathcal{K} = \{1, \ldots, K\}$ denotes a set of rounds. I denote $X(s)$ the set of possible answers to question s, and $X = \Pi_{s\in\mathcal{S}}X(s)$ the set of possible answers to the survey $\mathcal{S}$, with $X \subset \mathbb{R}^S$. Let $X^o = \{q^k\}_{k\in\mathcal{K}} \subset X$ denotes the set of observations, and $\mathcal{A}$ the set of subsets of X. I assume that the answer of respondent i to question s belongs to the integer scale $X(s) = \{0, \ldots, N(s)\}$, although all the results extend to close, countable, and compact sets. The default answer in the kth round is $o^k \in C(X)$, where $C(X)$ is the set of corners of X. For example, if $X = \{0, \ldots, 10\} \times \{0, \ldots, 10\}$, there are four corners, $C(X) = \{(0, 0), (10, 0), (10, 10), (0, 10)\}$.

Design. The PSM has three key design features. First, each respondent answers the same survey K times under different choice sets. Second, in any round k, respondents are presented with a default answer that belongs to one of the corners. Respondents have a budget in tokens R. Deviating from the default answer is costly. It is as if subjects were "buying" goods with their budget when they move from the default. Third, in any round k, the default answer is randomly selected and belongs to one of the corners.

Figure 1 below represents a PSM with three rounds and two questions, both on the set $\{0, \ldots, 10\}$. A respondent's answer to the PSM in round k can be represented as a vector q^k of two integers between 0 and 10. In round 1, represented by Panel (a), the default answer is the point $(10, 0)$. If the respondent submits this as her final answer, she would express a strong agreement with question 1, and a strong disagreement with question 2. The choice set in this round includes all possible answers in the square. Round 1 is then similar to a standard survey of Likert scale questions starting at the default $(10, 0)$. In round 2, represented by Panel (b), the default answer is the origin $(0, 0)$. If the respondent

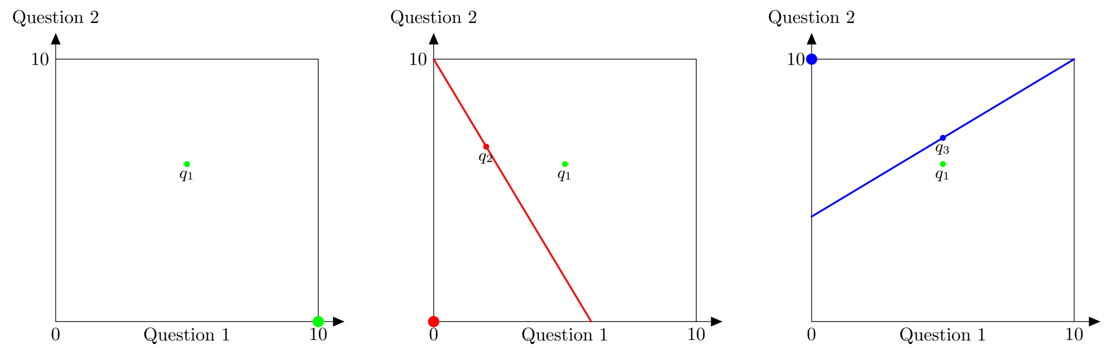

Figure 1: (a) Round 1 (b) Round 2 (c) Round 3

submits this as her final answer, she will express strong disagreement with both statements 1 and 2. She might want to move from this default to a point closer to her ideal point but only has limited options. She chooses to answer q^2 in round 2, where she expresses slightly more agreement with question 1 than with question 2. In round 3, represented by panel (c) of Figure 1, the default answer is the corner $(0, 10)$. If the respondent submits the default answer, she would express strong disagreement with question 1, and strong agreement with question 2. She might want to move to an answer closer to her ideal point, but again has limited options. She chooses q^3 in round 3.

Choice sets. The choice sets are designed as follows. In round 1, respondents are asked to answer the survey when all answers are included in the choice set, as represented in Figure 1 panel (a). In the following rounds, the choice sets are such that the answer to round 1, q^1, is *never* attainable. The basic idea is that if a subject is rational when answering the PSM, she would seek to give an answer in any round that is as close as possible to her answer to the first round q^1. In round 2 for example, she would increase her answers to both question 1 and question 2 in order to express a more neutral answer, similar to what she did in round 1. Having this key feature of the design in mind, I need to introduce some minimum formalization before discussing the link between PSM and the standard consumption choice environment.

I denote o the lowest corner of $C(X)$. In the previous example, the lowest corner is $o = (0, 0)$. To alleviate the notation, when I denote q without a subscript, I mean the coordinate of q in the coordinate system whose origin is the lowest corner of $C(X)$. Moving from the corners is costly. $p_{s,k}$ denotes the price in tokens of marginally changing the answer from the default of observation k, question s. In the coordinate system with origin o^k, the choice set of observation k can be defined as follows:

$$B^k = \{q_{o^k} \in X \text{ such that } q_{o^k}.p^k \leq R\}, \tag{1}$$

with q_{o^k} the coordinates of q in the coordinate system with origin o^k, and $q_{o^k}.p^k$ the scalar product between q_{o^k} and p^k. A dataset at the individual level will be denoted $D = \{q^k, B^k\}_{k \in \mathcal{K}}$ in the rest of this document.

The similarity with the standard consumption choice environment is straightforward from equation (1). In the PSM, it is as if subjects were "buying" goods when they move from the default. Since the ideal answer q^1 is not part of the budget set in round k, subjects should "saturate" their budget set in any given observation $k > 1$, behaving like (rational) consumers in the different coordinate systems. This is a key intuition behind the recoverability of preferences in the PSM, as it implies that the standard toolkit of consumer choice analysis can be applied in the different coordinate systems.

Axiomatization of survey answers.

I seek to understand when a respondent's behavior is compatible with rational choice. Formally, a preference relation $\succcurlyeq$ *weakly rationalizes* the dataset D if for all observation k and $y \in X$, $p^k.q^k_{o^k} \geq p^k.y_{o^k}$ implies $q^k \succcurlyeq y$. If no restriction is placed on $\succcurlyeq$, then any dataset is weakly rationalizable (we can let $\succcurlyeq$ indicate indifference among all the elements of X).

Definition 1 *A preference relation $\succcurlyeq$ is c-monotonic with respect to the order pair $(\geq, >)$ if for any round $k \in \mathcal{K}$ and any pair $(x, y) \in B^k$, $x_{o^k} \geq y_{o^k}$ iff $x \succcurlyeq y$ and $x_{o^k} > y_{o^k}$ iff $x \succ y$, with $\succ$ the strict part of $\succcurlyeq$.*

c-monotonicity generalizes the standard concept of monotonicity to account for monotonicity with respect to all coordinate systems. Consider panel (b) of Figure 2. When the respondent answers q^3 in round 3, she could have chosen q^2 by spending strictly less than she did in round 2. We therefore cannot conclude that the respondent regards the two choices as exactly equivalent. These two observations provide a refutation of the hypothesis that the respondent is rational and her preferences c-monotonic. As usual in the revealed preference literature, a preference relation can be characterized through a utility function. A utility function $u : X \to \mathbb{R}$ weakly rationalizes the data if for all k and $y \in X$, $p^k.q^k_{o^k} \geq p^k.y_{o^k}$ implies that $u(q^k) \geq u(y)$.

One key aspect of the PSM is that $\geq$ does not constitute an exogenous pre-order of the set of possible survey answers. Indeed, respondents have ideal points when answering surveys Coombs (1964); Jebb, Ng and Tay (2021); Thurstone (1928), so the axiomatization of choice used in the consumer choice environment cannot be applied. My working assumption is that given that the ideal point q^1 never belongs to the choice sets after round 1, in each coordinate system that originates with a corner answer, respondents should saturate their budget constraint and behave rationally.[1]

Definition 2 *The subjective pre-order of set $X(s)$ denoted $\trianglerighteq_s$ is such that in round k,*

$$\trianglerighteq_s = \begin{cases} \geq & \text{if } \forall q \in B(p^k, R), q_s \leq q_s^1 \\ \leq & \text{if } \forall q \in B(p^k, R), q_s \geq q_s^1 \end{cases}$$

Any round k falls in one of the two cases highlighted in Definition 2, by design of the choice sets. Definition 2 says that the natural pre-order of $X(s)$ increases with $q_s \in X(s)$ when increasing q_s is costly, and decreases with q_s when decreasing q_s is costly. Concretely, take an integer scale $\{0, \ldots, 10\}$ for question s. In any round, the default for question s is either 0 or 10. If the default is 0 in round k, increasing the answer is costly. The respondent would perceive that 0 is lower than 1, which is lower than 2, ..., lower than n, lower the highest answer she can possibly give to question s, and which is lower than q_s^1 by construction. Reciprocally, if the default is 10 in round k, decreasing the answer from 10 is costly. The respondent would perceive that 10 is ranked *below* 9, ranked *below* 8, and so forth, until the lowest possible answer she can give to question s, which is ranked lower than q_s^1. The following corollary is direct from Definition 2.

Corollary 1 *In the coordinate system with origin $o_k \in C(X)$, in round k, the subjective pre-order of set $X(s)$ is $\trianglerighteq_s = \geq$.*

This corollary says that in the coordinate system that takes as origin the default answer of round k, Definition 2 in fact means that $\geq$ is the natural pre-order of the set of alternatives $B(p^k, R)$. As a result of this Corollary - which is direct from Definition 2 - it is possible to apply the rationality axioms used in the standard consumption choice environment and follow the exact same formalization to recover preferences. Here are the rationality axioms in the different coordinate systems:

[1] This is what would happen if the choice correspondence was single-peaked (Bossert and Peters (2009)).

Definition 3 *For subject $i \in \mathcal{I}$, an observed bundle $q^k \in X$ is*

1. *directly revealed preferred to a bundle q, denoted $q^k R^0 q$, if $p_k q^k_{o^k} \geq p_{i,k} q_{o^k}$ or $q_{o^k} = q^k_{o^k}$.*

2. *directly revealed strictly preferred to a bundle q, denoted $q^k P^0 q$, if $p_k q^k_{o^k} > p_{i,k} q_{o^k}$ or $q_{o^k} = q^k_{o^k}$.*

3. *revealed preferred to a bundle q, denoted $q^k R q$, if there exists a sequence of observed bundles $(q^j, \ldots, q^m)$ such that $q^k R^0 q^j$, $\ldots q^m R^0 q$.*

4. *revealed preferred to a bundle q, denoted $q^k P q$, if there exists a sequence of observed bundles $(q^j, \ldots, q^m)$ such that $q^k R^0 q^j$, $\ldots q^m R^0 q$. and at least one of them is strict.*

Note that if the default was o in any round, Definition 6 would reduce to the standard rationality axioms assumed in the consumer choice environment. The following definition generalizes the standard cyclical consistency condition established by Varian (1982), so that it holds in all coordinate systems:

Definition 4 *A dataset $D = \{q^k, B^k\}_{k \in \mathcal{K}}$ satisfies the general axiom of revealed preference (GARP) if for every pair of observed bundles, $q^k R q$ implies not $q P^0 q^k$.*

Again, in the coordinate system where $o^k = o$, Definition 4 reduces to the standard definition of GARP from Varian (1982). Figure 2 illustrates the (direct) revealed preference relations inherent to the PSM. Panel (a) represents the two last rounds of Figure 1. Here, when the respondent chooses q^3, q^2 is also in her budget set, as $p^3.q^2_{o(2)} < p^3 q^3_{o(3)}$. q^3 is directly revealed preferred to q^2. In panel (b), in round 2, the respondent chooses q^2 although q^3 is available, thereby revealing that she prefers q^2 to q^3. In round 3, she chooses q^3 although q^2 is available, thereby revealing that she prefers q^3 to q^2. Since $q^3 \neq q^2$, these two answers violate GARP (and WARP as well).

One important feature of this design of the PSM is that irrationality can be assessed using standard indices of the revealed preference literature such as the Critical Cost Efficiency Index (Afriat (1972)), Varian's index (Varian (1990)), or the Money Pump Index (Echenique, Lee and Shum (2011)). As I will show next, utility functions rationalizing the data might be singled-peaked. I define a single-peaked function below:

Definition 5 *A function $f : X \to \mathbb{R}$ is said single-peaked if*

- *There exists a point $y^* \in X$ such that $f(y) \leq f(y^*)$ for any $y \in X$.*

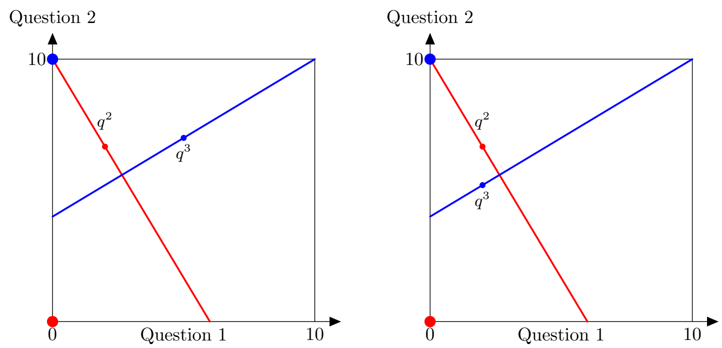

Figure 2: Revealed Preferences in the PSM

- *For any $x, y \in X$ such that $x_c \leq y_c \leq y_c^*$ for $c \in C(X)$, $f(x_c) \leq f(y_c)$.*

The second condition means that if it is possible to rank x, y, y^* as $x_c \leq y_c \leq y_c^*$ in a given coordinate system c, then $f(x) \leq f(y)$ as x is further away than y in the coordinate system c. The following theorem extends Afriat's theorem Afriat (1967):

Theorem 1 *The following conditions are equivalent:*

1. *D has a c-mononotonic weak rationalization.*

2. *The data satisfy GARP.*

3. *There are strictly positive real numbers U^k and λ^k, for each k such that*

$$U^k \leq U^l + \lambda^l p^l (q^k_{o(l)} - q^l_{o(l)}) \tag{2}$$

   for each pair of observations $(q^k, B^k), (q^l, B^l)$ in D.

4. *D has a single-peaked, continuous, concave utility function that rationalizes the data.*

If all observations start from the origin o, Theorem 1 reduces to the standard version of Afriat's theorem. What is remarkable here is that accounting for different origins, Afriat's theorem can be generalized and D admits a rationalizing utility function that is single-peaked.

Before detailing the proof, it is useful to observe that given a solution to the system of inequalities 3 in Theorem 1, one can write down a rationalizing utility function as follows:

$$u(x) = \min\{U^k + \lambda^k p^k.(x_{o^k} - q^k_{o^k}) : k = 1, \ldots, K\}. \tag{3}$$

This utility function is illustrated in Figure 3. On the left, in Figure 3, is a dataset satisfying GARP (WARP in that case). On the right are indifference curves corresponding to the utility function defined, as above, from solutions to the Afriat inequalities for that dataset. These indifference curves clearly represent single-peaked preferences. As the indifference curve becomes closer to the peak, the corresponding utility level becomes higher.

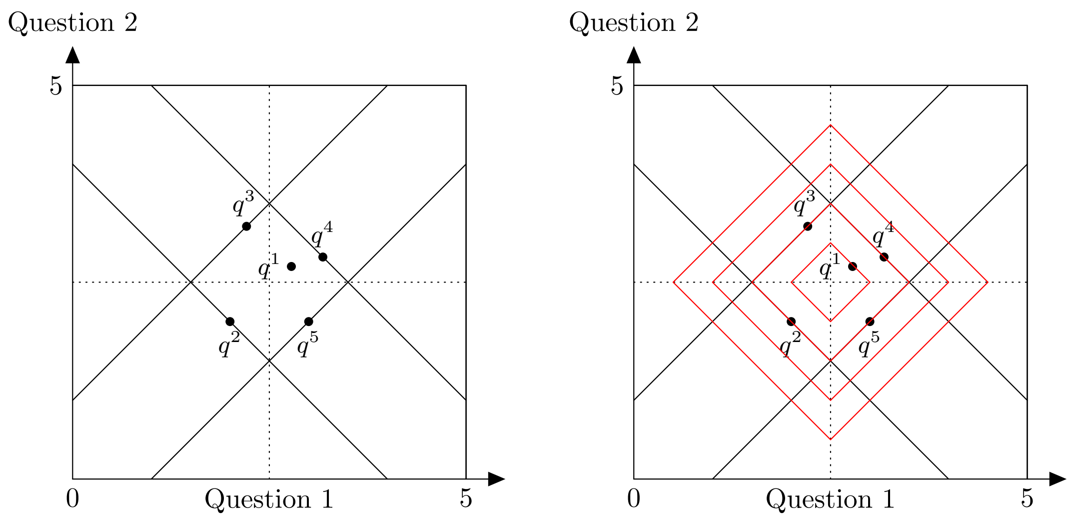

Figure 3: Dataset satisfying GARP (left panel) Indifference curves for single-peaked utility (3) (right panel).

The utility that I have depicted is not smooth. Introducing smoothness is not crucial, as for the standard consumption choice environment, but might be important in the applications, as smooth utility functions might be estimated, using predicted answers. I give several elements of proof of Theorem 1 at the end of the proposal. In particular, I show that 2 implies 1 and that D admits a single-peaked, continuous, concave utility function that rationalizes the data. The rest of the proof is provided in the Appendix, and remains close to the proof of Afriat's Theorem.

Proof. Below, I demonstrate that $3 \Rightarrow 4$. The rest of the proof of the theorem can be found in the Appendix.

Define a utility function as (3). It can be demonstrated that u is a weak rationalization. First, $u(q^k) = U^k$ for all round k as (5) implies that $U^k = U^k + \lambda^k p^k(q^k_{o(k)} - q^k_{o(k)}) \leq U^l + \lambda^l p^l(q^k_{o(l)} - q^l_{o(l)})$. Second, for any round k, let y be such that $p^k.q^k_{o^k} \geq p^k.y_{o^k}$. We have that $u(q^k) \geq u(y)$, because

$$u(q^k) = U^k \geq U^k + \lambda^k p^k.(y_{o^k} - q^k_{o^k}) \geq u(y).$$

The preference represented by u is a weak rationalization of the data.

Proof that the utility function (3) is single-peaked. The function given in (3) is the minimum of continuous, concave functions, and hence is itself continuous, and concave. To prove that it is single-peaked, first, I prove that $u(q^1) \geq u(q^l)$ for any round $l \in \mathcal{K}$. Indeed, since $u(q^1) = \min\{U^k + \lambda^k p^k.(q^1_{o^k} - q^k_{o^k}) : k = 1, \ldots, K\}$, there exists a round k such that

$$u(q^1) = U^k + \lambda^k p^k.(q^1_{o^k} - q^k_{o^k}).$$

As $p^k q^1_{o^k} > p^k q^l_{o^k}$ for any round l,

$$u(q^1) \geq U^k + \lambda^k p^k.(q^l_{o^k} - q^k_{o^k}) \geq U^l = u(q^l),$$

where the second inequality follows from (5). I am now going to prove the second element of Definition 5. Take $x, y \in X$ such that there exists $c \in C(X)$ and either $x_c \leq y_c \leq q^1_c$, or $y_c \leq x_c \leq q^1_c$. Assume without loss of generality that $y_c \leq x_c \leq q^1_c$. I will demonstrate that $u(x) \geq u(y)$. $u(x) = U^k + \lambda^k p^k.(x_{o^k} - q^k_{o^k})$ for some k. Two cases need to be distinguished. Case 1: $o^k = c$. Hence,

$$\begin{aligned} u(y) = U^z + \lambda^z p^z.(y_{o^z} - q^z_{o^z}) &\leq U^k + \lambda^k p^k.(y_{o^k} - q^k_{o^k}) \\ &\leq u(x) \end{aligned}$$

as $p^k y_{o^k} = p^k y_c \leq p^k x_c = p^k x_{o^k}$. In the second case, $o^k \neq c$. Hence,

$$\begin{aligned} u(x) &= U^k + \lambda^k p^k.(x_{o^k} - q^k_{o^k}) \\ u(x) &\geq U^z + \lambda^z p^z (x_c - q^z_{o^z}) \end{aligned}$$

as $p^k x_{o^k} \geq p^k x_c$ when $c \neq o^k$.[2] The last inequality implies

$$u(x) \geq U^z + \lambda^z p^z.(y_c - q^z_{o^z}) = u(y), \tag{4}$$

[2] This follows from the observation that x is either lower or higher than the peak. Since it is lower than the peak in the coordinate system originating in c, $p^k x_c$ is lower or equal to the scalar product of p^k with x_q for any $q \in C(X)$.

as $p^z x_c \geq p^z y_c$. This concludes the proof that the utility function (3) is singled-peaked.

■

4 Application

4.1 Data Description

I conducted a Priced Survey Methodology with 100 online participants recruited through Amazon Mechanical Turk. All participants had to answer a PSM consisting of the two following survey questions measuring altruism and self-interest:

On a scale from 0 to +10, where 0 indicates that you strongly disagree and +10 that you strongly agree, to what extent do you agree with these statements:

1. Individuals should primarily look after their own well-being before concerning themselves with the well-being of others.

2. Helping others, even when there's no direct benefit to oneself, is a fundamental value that people should live by.

Participants filled out this survey for 9 consecutive rounds. They were paid \$1 for completing the nine rounds and a short sociodemographic survey of seven questions. Table 1 provides summary statistics of individual characteristics.

Budget sets. In the first round, subjects can choose any answer to both questions on the $\{0, \ldots, 10\}$ scale. In the 8 following rounds, subjects are randomly presented with 8 different choice sets. The choice set of respondent i in round $k \geq 1$, B_i^k, depends on i's initial answer:

$$B_i^k = \{q_{o^k} \in X \text{ such that } q_{o^k}.p^k \leq q^0_{i,o^k}.p^k - 2\},$$

with q_i^0 denoting i's answer to round 0. That way, in any round $k \geq 1$, q_i^0 is not attainable but close, as it would have been with 2 more tokens. The price vectors are chosen so that budget sets intersect many times. As seen next, these aspects of the design imply that it is possible for respondents to violate rationality. This is important, as if subjects behave rationality, it is not an artifact of the design, but a feature of their decisions. Finally, the rounds are designed such that two out of eight rounds start from each of the four corners.[3]

[3] For each pair of rounds starting from a given corner, there are two symmetric price vectors: $p^k = (1, 2)$, and $p^l = (2, 1)$ for $k, l \in \{1, \ldots, 8\}$.

Variables	Number of Participants
Female	23
Age	
18-34	61
35-49	34
50-64	5
Education	
Low	17
Medium	14
High	69
Household income	
under 20000	8
20000-34999	9
35000-49999	28
50000-74999	45
75000+	10
Occupation	
Paid work	81
House work	11
Retired	2
Students	6
Marital status	
married	88
single	12
Number of children	
none	7
1	46
2	46
2+	1
Observations	100

The low, medium, and high education levels correspond to primary, secondary, and university education, respectively. House annual income are in $.

Table 1: Sociodemographic Variables.

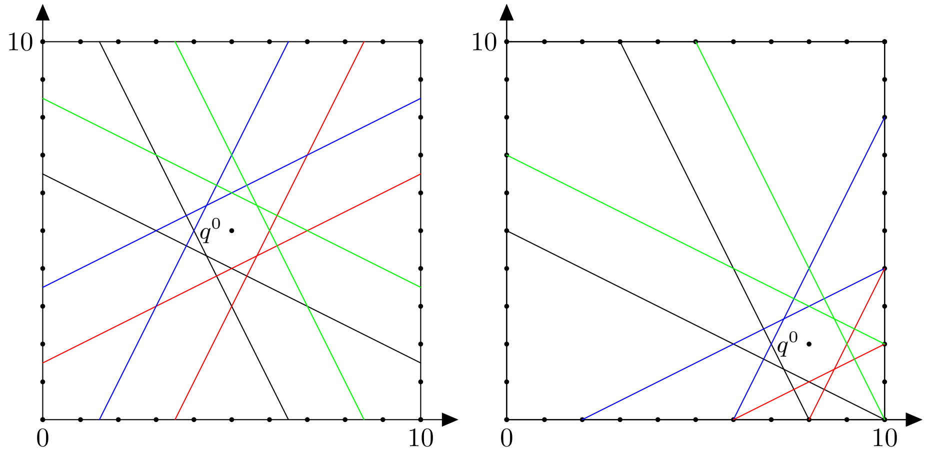

Figure 4: Examples of budget sets

Figure 4 gives two designs. In the left panel, the respondent initially provides a neutral answer to round 0. Her budget sets in the following eight rounds are computed such that $(5,5)$ is never attainable but close, and two out of eight rounds start from each of the four corners. The black, blue, green, and red budget lines correspond to the rounds where the default answer is $(0,0)$, $(0,10)$, $(10,10)$, and $(10,0)$ respectively. In the right panel, the respondent initially provides a more asymmetrical answer, agreeing with question 1 and disagreeing with question 2.

4.2 Checking rationality

I begin by looking at respondents' rationality when they fill out the PSM. For that, I introduce a slightly extended version of the Critical Cost Efficiency Index (CCEI) introduced by Afriat (1972).

Definition 6 *For subject $i \in \mathcal{I}$, and $e \in [0,1]$, an observed bundle $q^k \in X$ is*

1. *e-directly revealed preferred to a bundle q, denoted $q^k R_e^0 q$, if $ep_k q_{o^k}^k \geq p_{i,k} q_{o^k}$ or $q_{o^k} = q_{o^k}^k$.*

2. *e-directly revealed strictly preferred to a bundle q, denoted $q^k P_e^0 q$, if $ep_k q_{o^k}^k > p_{i,k} q_{o^k}$ or $q_{o^k} = q_{o^k}^k$.*

3. *e-revealed preferred to a bundle q, denoted $q^k R_e q$, if there exists a sequence of observed bundles $(q^j, \dots, q^m)$ such that $q^k R_e^0 q^j$, $\dots q^m R_e^0 q$.*

4. *e-revealed preferred to a bundle q, denoted $q^k P_e q$, if there exists a sequence of observed bundles $(q^j, \ldots, q^m)$ such that $q^k R_e^0 q^j$, $\ldots q^m R_e^0 q$. and at least one of them is strict.*

and

Definition 7 *Let $e \in [0,1]$. A dataset $D = \{q^k, B^k\}_{k\in\mathcal{K}}$ satisfies the general axiom of revealed preference ($GARP_e$) if for every pair of observed bundles, $q^k R_e q$ implies not $q P_e^0 q^k$.*

Afriat's inconsistency index is

$$e^* = \max\{e \in [0,1] : \{q^k, B^k\}_{k\in\mathcal{K}} \text{ satisfies } GARP_{\mathbf{e}}\}. \tag{5}$$

Afriat's inconsistency index is the most prevalent in the literature, and measures the extent of utility-maximizing behavior in the data. The main idea behind this index is that if expenditures at each observation are sufficiently "deflated", then violations of GARP will disappear. The closer is the index to 1, the smaller it is necessary to shrink any budget to avoid GARP violation.

The violations of revealed preferences are summarized in Table 2. The average value of the CCEI index is 92%. As a comparison, Choi et al. (2014) finds an average CCEI of 88% in a standard consumption choice environment. Varian (1990) suggests a significance threshold of 95% for the CCEI index. Hence, even in uncontrolled and online experimental settings, subjects appear to behave rationally in the PSM. The average number of GARP violations in the sample is 2.3, with about 60% of the respondents with 0 GARP violations.

The relatively high rationality of the respondents raises the question of how easy it is to violate rationality in this PSM design. Bronars (1987) designed a test that answers this question. The test measures the probability that a respondent with a random behavior would violate GARP in the consumption choice environment. Bronars' test is widely applied in the literature on consumer choice and revealed preferences (Cox (1997), Mattei (2000), Andreoni and Miller (2002)). For example, Cox (1997) reports a Bronars power of 0.49 in a study of three consumption goods and seven budget rounds. With 11 budgets and two goods, Andreoni and Miller (2002) report a Bronars power of 78%. Since each individual faces different choice sets in the PSM, it is possible to perform Bronars's test for each respondent. Column 3 of Table 2 reports the summary statistics of Bronars power test in the PSM. On average, the Bronars power is 54%, meaning that if answers to the PSM were made randomly, out of 1000 simulated choices, 54% would violate rationality. Hence, the Bronars' power of this PSM design is relatively low. One reason why it is hard

	CCEI	GARP	Bronars	Time
Mean	0.92	2.31	0.54	285
Std	0.14	3.92	0.12	143
p5	0.6	0.00	0.31	91
p25	0.86	0.00	0.46	159
p50	1.00	0.00	0.56	294
p75	1.00	3.00	0.64	406
p95	1.00	11.40	0.66	492

Column 1 gives the summary statistics of Afriat's Critical Cost Efficiency Index (CCEI). Column 2 gives the summary statistics for the number of GARP violations. Column 3 gives the summary statistics for the Bronars' index, and Column 4 gives the summary statistics for the time to complete the experiment in seconds.

Table 2: Summary Statistics: Rationality

to achieve high indices in the PSM is that budget sets originate from different corners. Hence, they inherently intersect less than when they all originate from the same corner, as in the consumption choice environment. Future research might look at different designs that achieve higher Bronars scores. Increasing the Bronars's scores might however require more complicated designs where inattentive respondents might violate rationality more often. Indeed, as reported in column 4, one interesting aspect of this design is that the average response time is less than five minutes, so respondents might not be too inattentive.

4.3 Estimating Preferences

Given that subjects' answers are close to rational, it is worth recovering preferences behind survey answers. For subjects that do not consistently give corner answers, I will estimate the following single-peaked functional form:

$$u^i(q) = -\sum_{s=1}^{S} \frac{1}{2} a_s^i \left(q_s - b_s^i\right)^2 . \tag{6}$$

Vector $\mathbf{b^i}_{s\in\mathcal{S}}$ measures respondent i's ideal answer. It offers a robust alternative to the "ideal point" measured directly through the (cardinal) answer that respondents give to traditional surveys. The key difference with traditional surveys is that the ideal point $\mathbf{b^i}$ is measured using all rounds of the PSM. Moreover, as $\mathbf{b^i}$'s estimation is based on the revealed preference information of the PSM, it is robust to the issues inherent to cardinal interpretations of survey answers (Bond and Lang (2019)). Vector $\mathbf{a^i}$ measures the *relative* importance of different social preferences for respondent i, as measured through survey questions. Concretely, a respondent might answer that she is very altruistic but gives a low weight to altruism, as compared to selfishness. The responses of such respondents might typically be hard to interpret in standard surveys. [complete]

With this specification, indifference curves are smooth and have an elliptic shape. Although more general specifications can be found, this one has the advantage of giving simple functional forms for the optimal answer to question s in round k.

$$q_{1,o^k} = \alpha b_1 + (1-\alpha)\frac{R - p_2 b_2}{p_1}, \tag{7}$$

with $\alpha = \frac{a_1/p_1^2}{a_1/p_1^2 + a_2/p_2^2}$, and $q_{2,o^k} = (R - p_1 q_{1,o^k})/p_2$. It is as if a respondent was weighting providing an answer to question 1 close to b_1 versus providing an answer to question 2 close to b_2. If α is high, the respondent prefers to give an answer close to b_1 in observation k and diverges from b_2 when she answers question 2. Note that since $R > b_1 p_1 + b_2 p_2$ by design, $q_{s,o^k} < b_s$ for any $s \in \{1, 2\}$. Here, a respondent will never entirely "sacrifice" one question. She would rather try to answer to both questions as close as she can to her ideal answer (b_1, b_2). This property is akin to the taste for variety property of the CES specification commonly used in consumer theory. In fact, equation (6) can be seen as a special case of a single-peaked CES specification.

Table 3 reports the summary statistics of the utility parameters, estimated using the non-linear least square method. It seems that respondents 50 and older seem to give higher weight to answering both the altruistic question and the self-interest question, as both a_1 and a_2 are significantly higher. As there are only five of these respondents, however, this correlation might be spurious. Moreover, it seems that higher education is associated with less agreement with both the selfish and the altruistic statements. Interestingly, interpreting the results of columns 5 and 6, a social scientist could have argued that more education is associated with less selfish attitudes. Interpreting columns 3 and 4, higher education seems to be associated with both less selfish attitudes, and less altruistic attitudes.

Finally, since b_s^i is the utility parameter that corresponds to a respondent i's ideal answer to question s, it is interesting to compare its value with respondent i's answers to the survey

	a_1	a_2	b_1	b_2	q_1^0	q_2^0
Constant	2.279	-0.666	4.220	7.703***	6.287**	9.521***
	(2.376)	(4.784)	(3.204)	(2.700)	(2.608)	(2.763)
Age						
35-49	0.667	0.274	-0.904	-0.565	-0.068	0.029
	(0.519)	(1.044)	(0.699)	(0.589)	(0.569)	(0.603)
50-64	5.683***	13.283***	2.444	-1.562	-1.991	-2.531
	(1.355)	(2.728)	(1.827)	(1.540)	(1.488)	(1.576)
Male	-0.107	-1.134	-0.135	-0.963	0.070	0.216
	(0.572)	(1.152)	(0.772)	(0.650)	(0.628)	(0.665)
Education						
Medium	0.478	-0.102	−2.375**	−2.515**	-1.163	−2.084**
	(0.858)	(1.727)	(1.157)	(0.975)	(0.942)	(0.997)
High	-0.253	0.813	-1.521	−1.382*	-0.792	−1.409*
	(0.688)	(1.384)	(0.927)	(0.781)	(0.755)	(0.800)
Single	-0.540	-2.274	-1.634	-0.584	0.474	-0.234
	(0.956)	(1.924)	(1.289)	(1.086)	(1.049)	(1.111)
Income						
35000-49999	1.436	3.068*	0.411	-0.123	-1.326	-0.654
	(0.875)	(1.761)	(1.179)	(0.994)	(0.960)	(1.017)
50000-74999	0.685	1.224	1.387	1.138	-0.498	-0.358
	(0.825)	(1.661)	(1.112)	(0.937)	(0.906)	(0.959)
>75000	1.138	3.168	-1.223	-0.110	0.095	0.128
	(1.119)	(2.252)	(1.508)	(1.271)	(1.228)	(1.301)
<20000	0.519	2.446	1.786	0.384	-0.293	-0.975
	(1.208)	(2.432)	(1.629)	(1.373)	(1.326)	(1.405)
Observations	93	93	93	93	93	93
R^2	0.248	0.306	0.197	0.207	0.125	0.138

7 participants were excluded, as their decisions in the PSM were significantly less rational than the decisions of the other respondents. $\mathbf{a^i}$ and $\mathbf{b^i}$ are the parameters of the utility model (6) estimated using PSM data and a NLLS method. q_s^0 is the answer of subject i to statement s in round 0, where i faces no constraint on her choice set. * ($p < 0.05$), ** ($p < 0.01$), ***($p < 0.001$).

Table 3: Utility parameters explained by Sociodemographic Variables

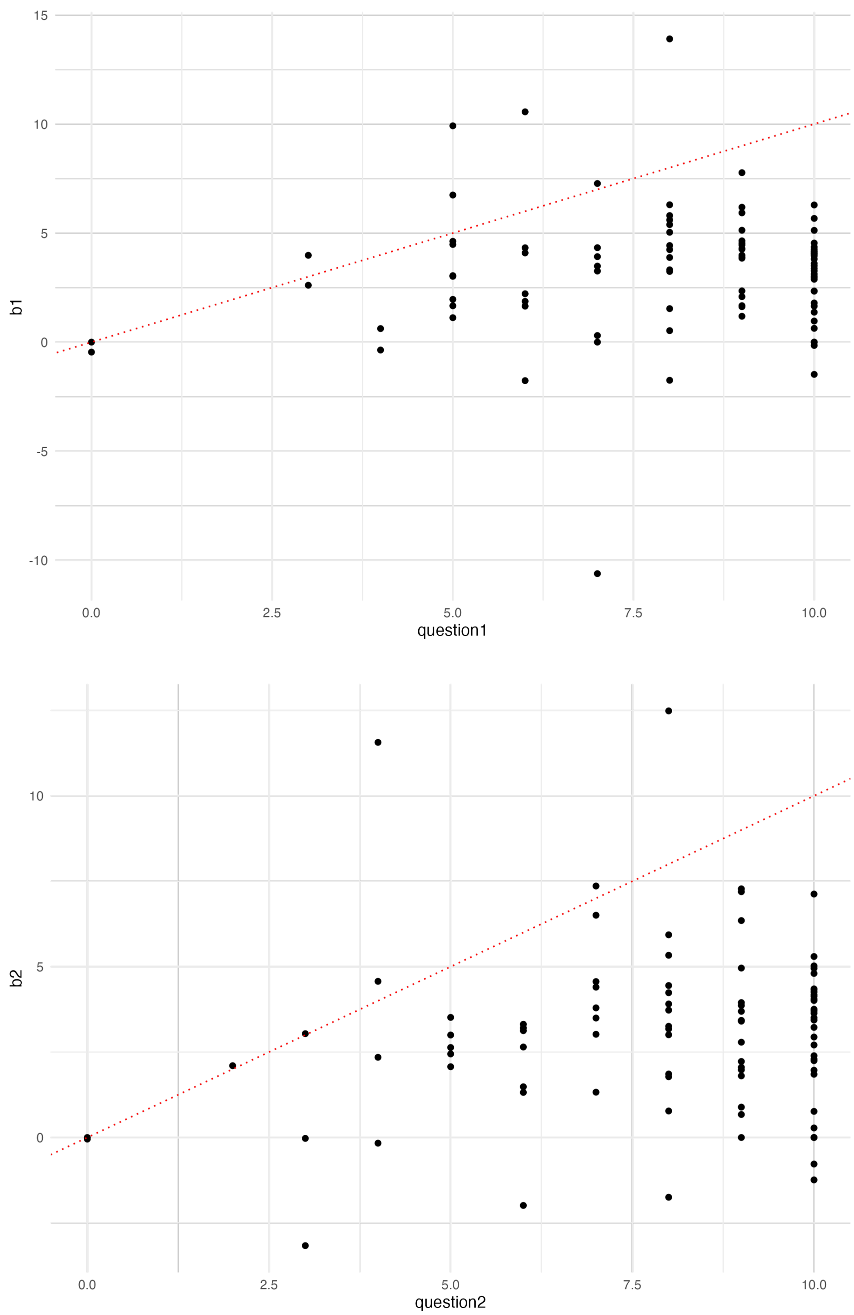

Figure 5: Correlation between (a) b_1 and q_1^0 (b) b_2 and q_2^0.

when her choice set is not constrained. Figure 5 represents the correlations between $b_{s,i}$ and $q^0_{s,i}$, respondent i's answer to the survey when her choice set is not constrained. As can be seen from Figure 5, there is no correlation between $b_{s,i}$ and $q^0_{s,i}$. This intriguing result might be due to the uncontrolled experimental conditions. However, taken at face value, this result would suggest that the answers provided to traditional surveys might in fact be poor predictors of the ideal points that subjects have on scales, as estimated through the PSM.

5 Discussion

In this paper, I introduced a novel methodology to measure social preferences - the priced survey methodology. It consists of giving subjects various opportunities to fill out the same survey under different choice sets.

The first key contribution of this paper is theoretical. I extend Afriat's theorem, and show that the Generalized Axiom of Revealed Preferences (GARP) is necessary and sufficient for the existence of a concave, continuous, and single-peaked utility function rationalizing survey answers. There are several important implications. First, rather than interpreting the *cardinal answers* subjects provide to a traditional survey, the PSM enables to estimate - and interpret *utility parameters*, which are related to the *ordinal relations* between all possible answers to the survey. This is more than a simple interpretation difference. It is a key improvement, as taking numerical values at face value can be misleading Bond and Lang (2019). In contrast, utility parameters offer a ground for comparing social preferences. Moreover, by estimating the peak of the utility function behind survey answers, experimenters have access to a new measure of a respondent's ideal answer to a survey. This measure is estimated using ordinal relations between survey answers, thereby not subject to issues inherent to cardinal interpretations of scales. Finally, with the PSM, it is possible to measure valuable aspects of social preferences that cannot be captured with traditional surveys such as how people navigate moral dilemmas.

The second key contribution of this paper is to apply the PSM to a sample of 100 online respondents. All participants had to answer a PSM consisting of nine rounds and two questions measuring altruistic and self-interested preferences. I measure the decision-making quality by evaluating the consistency of individual choices with the Generalized Axiom of Revealed Preference (GARP), using the CCEI Index (Critical Cost Efficiency Index). I find that respondents reach an average CCEI score of 92%. Moreover, I used the individual-level data to estimate a smooth, concave, and single-peaked utility function. Although the low number of participants make it difficult to interpret the results, it seems that respondents ideal point, as estimated through the PSM, does not correspond to the answers respondents provide to the traditional survey.

Other potentially useful applications are not discussed in this paper. First, the dynamics of revealed preferences over social preferences could be assessed, following works in demand analysis by Crawford (2010), or Demuynck and Verriest (2013). The intergenerational transmission of social preferences could also be assessed, by implementing priced surveys in the appropriate samples. There are several important studies on revealed preferences for multi-person demand behavior (e.g., Chiappori (1988), Brown and Matzkin (1996), Cherchye, de Rock and Vermeulen (2007, 2010, 2011)). Accounting for multi-person demand for social preferences could be a starting point to an empirical analysis of social interactions and social preferences in a general equilibrium framework.

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Appendix

A Proof of Theorem 1

The proof that $4 \Rightarrow 1$ is direct, and $3 \Rightarrow 4$ has been proven in the main text. It remains to be proven that $2 \Rightarrow 3$, and $1 \Leftrightarrow 2$.

Proof that $2 \Rightarrow 3$**.** The following is a constructive proof that follows the standard proof of Afriat's Theorem, as detailed by (Chambers and Echenique, 2016, p.45).

Consider the revealed preference pair $(\succcurlyeq^R, \succ^R)$ restricted to X^0. GARP implies that there is a preference relation $\succcurlyeq$ on X^0 such that $x \succcurlyeq y$ when $x \succcurlyeq^R y$, and $x \succ y$ when $x \succ^R y$. Partition X^0 according to the equivalence classes of $\succcurlyeq$. That is, let $I_1, \ldots, Y_J$ be a partition of X^0 such that $x \sim y$ for $x, y \in I_j$ and $x \succ y$ if $x \in I_j$, $y \in I_h$ and $j > h$.

Define $(U^k, \lambda^k)_{k \in \mathcal{K}}$ recursively. Let $U^k = \lambda^k = 1$ if $x^k \in I_J$.

Suppose that we have defined (U^k, λ^k) for all $x^k \in \bigcup_{h=j+1}^J I_h$. We can choose V_j such that, for all $x^l \in I_j$ and $x^k \in \bigcup_{h=j+1}^J I_h$,

$$V_j < U^k$$

and

$$V_j < U^k + \lambda^k p^k.(x^l_{o^k} - x^k_{o^k}). \tag{A.1}$$

Set $U^l = V_j$ for all l with $x^l \in I_j$.

Given this choice of U^l, if $x^k \in \bigcup_{h=j+1}^J I_h$, then $U^l < U^k$. Moreover, since $p^l.(x^k_{o^l} - x^l_{o^l}) > 0$, it is possible to characterize λ^l as:

$$\lambda^l = \max_k \frac{U^k - U^l}{p^l.(x^k_{o^l} - x^l_{o^l})} \geq 0 \tag{A.2}$$

where the max is taken over the values of k such that $x^k \in \bigcup_{h=j+1}^J I_h$.

The chosen $(U^k, \lambda^k)_{k \in \mathcal{K}}$ satisfy the system of inequalities (5). Indeed, let k and l be such that $x^k \in I_j$, and $x^l \in I_h$ with $j > h$. Then (A.1) ensures that

$$U^l \leq U^k + \lambda^k p^k.(x^l_{o^k} - x^k_{o^k}).$$

and equation (A.2) ensures that

$$U^k \leq U^l + \lambda^k p^l.(x^k_{o^l} - x^l_{o^l}).$$

If k and l are such that $x^k, x^l \in I_j$, then $U^k = U^l$, so $U^k \leq U^l + \lambda^k p^l.(x^k_{o^l} - x^l_{o^l})$ because $p^l.(x^k_{o^l} - x^l_{o^l}) \geq 0$, and $U^l \leq U^k + \lambda^k p^k.(x^l_{o^k} - x^k_{o^k})$ because $p^k.(x^l_{o^k} - x^k_{o^k}) \geq 0$

Proof that $1 \Leftrightarrow 2$**.** The proof below closely follows the steps of the proof of Theorem 3.1 in (Chambers and Echenique, 2016, p. 37).

Theorem 2 *In any observation k, $(\geq, >)$ is an acyclic order pair, and for any $(x, y) \in B^k$, it satisfies $x_{o^k} > y_{o^k} \geq z \Rightarrow x_{o^k} > z_{o^k}$. There exists a preference relation which is c-monotonic with respect to the order pair $(\geq, >)$ and which weakly rationalizes the data iff $(\succcurlyeq^R, \succ^R)$ satisfies GARP.*

Assume that $\succcurlyeq$ is c-monotonic and weakly rationalizes the data D. Assume moreover that $(\succcurlyeq^R, \succ^R)$ does not satisfy GARP. Hence, there exists a sequence of observations in D $x^1, \ldots, x^L$ such that

$$x^1 \succcurlyeq^R \cdots \succcurlyeq^R x^L \text{ and } x^L \succ^R x^1$$

As $x^L \succ^R x^1$ and B^L is comprehensive in the coordinate system with origin o^L, there exists $z \in B^L$ such that $x^L \succcurlyeq^R z$ and $z_{o^L} > x^1_{o^L}$.

Since $\succcurlyeq$ weakly rationalizes D, $x^1 \succcurlyeq \cdots \succcurlyeq x^L \succcurlyeq z$. By c-monotonicity, since $z_{o^L} > x^1_{o^L}$, $z \succ x^1$, a contradiction that $\succcurlyeq$ is a preference relation.

Conversely, assume that $(\succcurlyeq^R, \succ^R)$ is an acyclic order pair. Let's show that $(\succcurlyeq^R \cup \geq, >)$ is also an acyclic order pair. To do that, we make first three key observations. For any observation k, and pair $(x, y) \in B^k$

1. $x \succcurlyeq^R y$ and $y_{o^k} \geq z_{o^k} \Rightarrow x \succcurlyeq^R z$.

2. $x \succcurlyeq^R y$ and $y_{o^k} > z_{o^k} \Rightarrow x \succ^R z$.

3. $x \succ^R y$ and $y_{o^k} \geq z_{o^k} \Rightarrow x \succ^R y$.

Assume that $(\succcurlyeq^R \cup \geq, >)$ is not acyclic and let $Q = \succcurlyeq^R \cup \geq$. There exists a sequence of observations in D such that $x^1 Q \ldots Q x^L$ and $x^L_{o^L} > x^1_{o^1}$. Without loss of generality, the cycle can be rewritten as

$$x^1_{o^1} \geq x^2_{o^1}, x^2_{o^2} \geq x^3_{o^2}, x^3_{o^3} \geq x^4_{o^3}, \ldots x^{p-1}_{o^{p-1}} \geq x^p_{o^{p-1}},\ x^p \succcurlyeq^R \cdots \succcurlyeq^R x^L, \text{ and } x^L_{o^L} > x^1_{o^L}.$$

If $p \neq L$, $x^{L-1} \succcurlyeq^R x^L$ and $x^L o^L > x^1_{o^L}$ implies, from observation 2, that $x^{L-1} \succ^R x^1$. Repeating the same reasoning using the inequality $x^1_{o^1} \geq x^2_{o^1}$, we obtain $x^{L-1} \succ^R x^2$.

Repeating again in an iterative way, we obtain $x^{L-1} \succ^R x^p$. Hence, $x^p \succcurlyeq^R \cdots \succcurlyeq^R x^{L-1} \succ^R x^p$, contradicting that $(\succcurlyeq^R, \succ^R)$ is acyclic. This implies that $(\succcurlyeq^R \cup \geq, >)$ is an acyclic order pair.

As $(\succcurlyeq^R \cup \geq, >)$ is an acyclic order pair, there is a preference relation $\succcurlyeq$ such that $\succcurlyeq^R \subseteq \succcurlyeq$, $\geq \subseteq \succcurlyeq$, and $> \subseteq \succ$ (Theorem 1.5 in (Chambers and Echenique, 2016, p. 7)). As a consequence, $\succcurlyeq$ is c-monotonic with respect to $(\geq, >)$ and weakly rationalizes D.