The Deadweight Loss of Short-Time Work *

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updated version

Abstract

Since the Great Recession, OECD countries have increased the generosity of short-time work (publicly subsidised reductions in working hours) in order to reduce employment fluctuation along business cycles. This paper combines macroeconomic evidence and modelling to identify and quantify the effect of monetary incentives to use short-time work on employer-employee matching. Using local projections and a novel narrative dataset for France, I show that past reductions in the cost paid by firms for using short-time work have increased the programme's deficit and reduced the number of hours worked by registered workers, while having a modest effect on unemployment. I construct a labour market to show the trade-off of STW programmes between creating incentives for firms to participate in the programme and incentives for firms to reduce the number of hours worked. I then calibrate the model using French data. Results show that a 50% increase in the cost of using short-time work would increase the number of hours worked by enrolled workers leading to a 25% raise in the output they produce and almost cancel the public deficit generated by the programme while having a small effect on employment.

JEL Classification: E24; H21; J22; J24; J65;

Keywords: Short-Time Work; Deadweight loss; Optimal Policy;

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1 Introduction

Short-Time Work (STW) have become a pillar of European economic policy, particularly in times of crisis. Following the success of German STW during the Great Recession (Rinne and Zimmermann, 2012), the programme is now a common tool in the hands of European policymakers and is used to protect the labour market when GDP falls. Policymakers frequently change the rules and incentives of STW with a common trend of increasing public spending over the years to increase the attractiveness of the programme. This paper aims to highlight the role and consequences of monetary incentives to take up the programme, which has so far been ignored in the literature.

Short-Time Work is a job retention scheme that allows firms to temporarily reduce their labour costs when they face business difficulties. Firms in the scheme reduce working hours and pay a wage based on the new reduced hours basis. Workers then receive a lower wage but are compensated for the loss. The compensation is co-financed by the firm and the government. The compensation and the share paid by the firm are the only monetary incentives to use the programme.

In this paper, I document, demonstrate and estimate the deadweight loss caused by financial incentives to take up STW. I show that increasing the generosity of the programme affects both the incentives to enter STW for job at risks - the extensive incentives - and the incentives to reduce the number of hours worked for worker-firm pairs already in the programme - the intensive incentives - which ultimately affects output and public expenditure. STW *per se* create a deadweight loss, as they cannot simultaneously set the optimal intensive and extensive incentives, unless the programme is coupled with wage subsidies. Even without wage subsidies, I identify a cutoff below which reducing the financial incentives of the current STW programme would improve the cost-effectiveness of the programme by increasing the output produced by short-time workers, reducing public spending, while having little effect on unemployment. I investigate the French STW - which is one of the less generous programmes - and find that a 50% increase in the cost of using short-time work would increase the number of hours worked by enrolled workers leading to a 25% increase in the output produced by short-time workers, and reduce the public deficit generated by the programme by 90%, while increasing the firing rate by only 0.03 p.p.

The paper documents the deadweight loss of STW through three interrelated steps. First, the estimation of a local projection to measure the impact of changes in the STW programme on macroeconomic aggregates. Second, the construction a dynamic labour market with hours model to theoretically demonstrate the impact of changes in the STW programme. Third, the calibration of the model to measure the theoretical predictions.

I construct a new narrative dataset where I record all changes in monetary incentive to use STW with French law reports between 2008 and 2024. The French programme is one of the oldest, has been modified several times in the past two decades and is one of the less generous. It therefore allows me to explore the effect of different variations in the generosity of the programme and the critics of its generosity are likely to apply to other countries. I merge the narrative database with quaterly data on employment, hours worked, public expenditure and number of workers on STW. Using this combined dataset, I run a local projection and find that past decreased in the cost paid by firms increased the share of short-time workers in the programme by 0.04 p.p., reduced the number of hours worked per worker enrolled by 30 hours per quarters, and increased public expenditure by 2% enrolled with no decrease in unemployment.

I model a dynamic labour market *à la* Diamond-Mortensen-Pissarides with wage rigidities and a random productivity shock affecting worker-firm pairs. To reduce layoffs, a policymaker can modify the *laisser faire* equilibrium through three instruments: the labour tax, the short-time costs paid by the firm, and the short-time compensation received by the worker. I solve the program of the policy maker who maximises the total surplus generated by labour market matches under a public budget constraint. I find that the optimal short-time cost resolves the trade-off between increasing the number of hours worked by short-time workers - the intensive incentive - and lowering the firing threshold - the extensive incentive. The deadweight loss of STW comes from the common ST cost paid by all firms, which provides a similarly intensive incentive to use the programme for different productivity gains. Reducing the short-time cost below its optimal level leads to an increase in public expenditure and a deadweight loss due to the reduced number of hours worked by workers already enrolled in the programme, which is not compensated by reduced dismissals. The public deficit then affect the surplus and the equilibrium in future periods due to an increase in taxation.

Finally, I calibrate the model using French data in 2024 and during the Covid-19 recession. In this section, I derive the numerical value of the optimal short-run cost and compare it with its current value. I do this for recession and expansion periods. I find that in both cases the short-run cost is set too low. In expansion periods, the cost should be set at 60% of the wage, which is currently 32%. Following a drop in labour productivity of 15% as during the the Covid-19, I find that the cost should be set at 30% of wages, during the last recession the short-time cost was reduced to 0%. In both periods, the low cost of the programme had a negative impact on the aggregate surplus generated and on the public budget. The estimates are in line with the local projections and the prediction of the model, i.e. the French ST cost is already below its optimal level, so that its reduction has a small impact on unemployment but a large impact on hours worked and the public deficit.

Related literature. The paper is the first to examine the impact of ST costs on STW consumption and the labour market. So far, the literature has focused on ST compensation (Tilly and Niedermayer, 2016; Giupponi and Landais, 2023). This is probably due to the fact that ST costs are not paid in the programme studied, e.g. in Germany. My setup is closely related to Teichgräber et al. (2022), which models the allocation of hours and the information asymmetry problem faced by the policymaker to derive an optimal policy. In contrast to my approach, this paper does not discuss the role of the STW policy, but rather models a social planner that does not allow to estimate, measure and compare the existing programme with the optimal policy.

I propose a new measure of the cost-effectiveness of the STW programme by comparing it to the estimated second-best policy. One strand of the literature compares the observed cost of job retention with the cost of unemployment benefits that would have led to job destruction (Borowczyk-Martins and Lalé, 2016; Giupponi and Landais, 2023). In the paper, I show that as long as the ST compensation is less than or equal to the unemployment benefit, STW is *per se* less costly for the government. I propose to compare the observed policy with the second-best policy to measure the inefficiency of high public spending on STW.

In this respect, the paper is related to Cahuc et al. (2021), Balleer et al. (2016) and Albertini et al. (2022), that criticise the adverse selection effect of the STW. Cahuc et al. (2021) show that a large proportion of French firms entered the programme when they were not facing a decline in activity sufficient to threaten their jobs during the Great Recession. Similarly, Albertini et al. (2022) measures overconsumption in hours during Covid-19.Balleer et al. (2016) shows that the relaxation of STW eligibility criteria increased the number of ST workers but had no effect on employment in Germany. In this paper, I estimate that an increase in STW costs would reduce adverse selection and overuse by reducing the incentives to use the programme for firms facing small reductions in activity. For firms at the top of the productivity distribution, this increase in ST cost is equivalent to a reduction in the leniency of the programme as modeled by Balleer et al. (2016).

The paper is organised as follows. Section 2 introduces motivating evidence. Section 3 presents the model. Section 4 presents the theoretical results and the optimal STW policy. Section 5 shows the simulation results.

2 Short-Time Work Facts

In this section, I document STW programmes and policies in Europe and then focus on the French case to estimate the effect of change in the programmes on macroeconomic aggregates. I compile a new narrative dataset and run a local projection to present three main stylized facts: 1- Policymakers lower the cost of using short-time work for firms during recessions; 2- There is an increase in public spending on short-time work; 3- Lowering the cost of short-time work reduces the number of hours worked per worker, increases the number of workers on short-time work, increases public spending of the programme and has no significant effect on employment.

2.1 Legal Background

STW is a labour market policy that allows employers to temporarily reduce the working hours of their employees during periods of economic downturn or temporary business disruptions. The aim is to avoid layoffs by temporarily reducing working hours rather than cutting jobs. STW programs generally include the following key elements. First, eligibility: employers must meet certain criteria to be eligible for short-time work programs, such as experiencing a significant decline in business activity through no fault of their own. In order to access the STW program, employers must prove to a public authority that they meet the eligibility criteria. Second, reduced working hours: if access to the STW program is granted, employers can reduce the regular working hours of their employees and pay them only on the basis of this reduced number of working hours. Third, wage compensation, workers receive partial wage compensation for the hours lost due to reduced working hours (compensation co-financed by the government and employers). Fourth, duration: STW is limited to a specific period, such as several weeks or months. Fifth, employer obligations: employers participating in short-time work programs may have certain obligations, such as retaining employees for a certain period of time.

Short-time work policies work through four channels. First, they change eligibility criteria. In 2008, for example, Spanish public policymakers abolished administrative approval for reductions in the working week for economic reasons of between 10% and 70%. Balleer et al. (2016) find that the change in eligibility criteria in Germany during the Great Recession has mostly had a windfull effect on unemployment. Second, they affect the maximum use and duration. In Germany, during the Covid-19 crisis, the maximum duration was extended from 12 to 21 months. Third, ST compensation (ST compensation) is being increased. In Portugal, during the Covid-19 crisis, the level of ST compensation was raised from 60 to 92% of the gross hourly wage for some workers. Finally, policies usually reduce the participation of firms to ST compensation (ST compensation). This reduce the short-time cost (ST cost) paid by firms in order to increase the benefits of STW use for firms. In 2009 in France, the cost of STW was reduced to 0 for the majority of workers, firms were only forced to pay the ST cost for high wages. Those changes in ST compensation and ST cost are the only monetary incentives to use the programme.

These four channels of STW policy aim to make the STW program more attractive, all examples given in the previous paragraph are anecdote evidence but curious readers can consult ETUC (2020) reports for an exhaustive description of policies during the Covid-19. It worked: in most European countries, the use of STW reached unprecedented levels during the Great Recession and the Covid-19 (figure B.2). In Germany, for example, short-time workers saw their working hours halved on average during the Great Recession. The huge increase in the use of ST compensation, as well as the difference between the cost of ST compensation and the level of ST compensation, comes at a significant public cost. Between 2008 and 2009 in France, public expenditure on ST compensation rose from less than 20 million euros to almost 300 million euros. However, as in most European countries, access to STW programs became more restricted after the great recession. The same pattern was observed during the Covid-19 with a much stronger magnitude.

STW policies then restrict access and incentives to use the programme after the recession. In figure B.1 we see that after Covid-19 the ST costs for companies increased in most countries. Nevertheless, there is a general trend for the programme to become more generous and more used over time. Over 10 years, between 2007 and 2017, the number of ST workers in Europe increased by 61% and the amount spent by policymakers on each ST worker increased by 46% (table A.1)[1].

2.2 Database Construction

In this section, I estimate the effect of change in monetary incentives to use STW on the French labour market. Focusing on one country allows me to clear the heterogeneity issues due to differences between programs, institutionnal background and economic trends in cross country analysis. The French programme is one of the oldest and has been modified several times so it provides sufficient variations to estimate the effect of change in ST cost and compensation on the labour market. In addition, France has followed the common European trends in STW program and policies so the observation made on France is likely to be valid externally.

This paper presents a new quaterly narrative database on major shocks on the french STW programme over 2008-2024. The dataset is built in two steps. First, I record all legislative actions mentioned in the official website of the French government for the publication of legislation, regulations, and legal

[1]Computations were made for four countries: Germany, France, Italy and Spain.

information[2]. The legislation website record all the change in the programme including change in ST cost, ST compensation, and procedure to apply to the programme. In a second step, I create two binary variables registering for an increase (+1) or a decrease (-1) in ST cost paid by firms and in ST compensation received by workers.

I count 15 reforms between 2008 and 2024 (Appendix A.1). In most cases, reforms of ST compensation and ST costs happened in the same quarter. The direction of the reforms is not straightforward due to the complexity of the French programme. In B.3, I plot the ST cost paid by the firm according to the wage of the worker in STW in 2009. The cost paid by firms is non-linear and depends on the size of the firm, the type of STW programme in which it is enrolled and the wage of the worker. In addition, before 2021 the cost of STW was a fixed cost and then changed to a proportion of the worker's remuneration. In figure 2, I plot the cost paid by the firm and the compensation received by the worker for an hour of STW consumed when the firm has less than 250 employees and the worker has the average wage (in 2023, 17 euros per hour). Between 2009 and 2022, all but one of the reforms have increased the generosity of the programme; after the Covid-19 recession, the share of compensation paid by the firm has increased. Reforms do not necessarily affect all worker-firm pairs. In my narrative database, the binary variable takes a value of +1 (-1) if the generosity of the scheme has increased (decreased) for at least one worker-firm match.

I merge the narrative database with French quarterly data on STW consumption. This dataset is produced by the Statistical Department of the French Labor Ministry (*DARES*) and is on public access. It includes the number of workers on short-time work, the number of hours consumed, the number of firms in STW, the number of STW demand, and the amount of public expenditure between 2008 and 2024. Finally, I include quarterly GDP growth from OECD.

French STW consumption follows the trend observed for European countries, it is strongly counter-cyclical and has increased over time (figure 1). French data allows me to observe the intensive use of the programme measured as the number of hours decreased per workers. The extensive and intensive use of the programme increased during recessions and over-time. STW consumption and public expenditure also increased over time with a higher consumption per worker and higher number of workers after the great recession than before, same thing for the Covid-19 crisis. The amount of public expenditure per STW hour consumed is also counter-cyclical and increased overtime. This can be easily explained in figure (**??**) where I plot the evolution of ST compensation and ST cost through years. The gap between the two captures the public expenditure. It increased during recessions and overtime. Overall, during recessions and over time, STW use becomes more widespread across the labour force and the number of hours worked when

[2] https://www.legifrance.gouv.fr

participating in the programme declines.

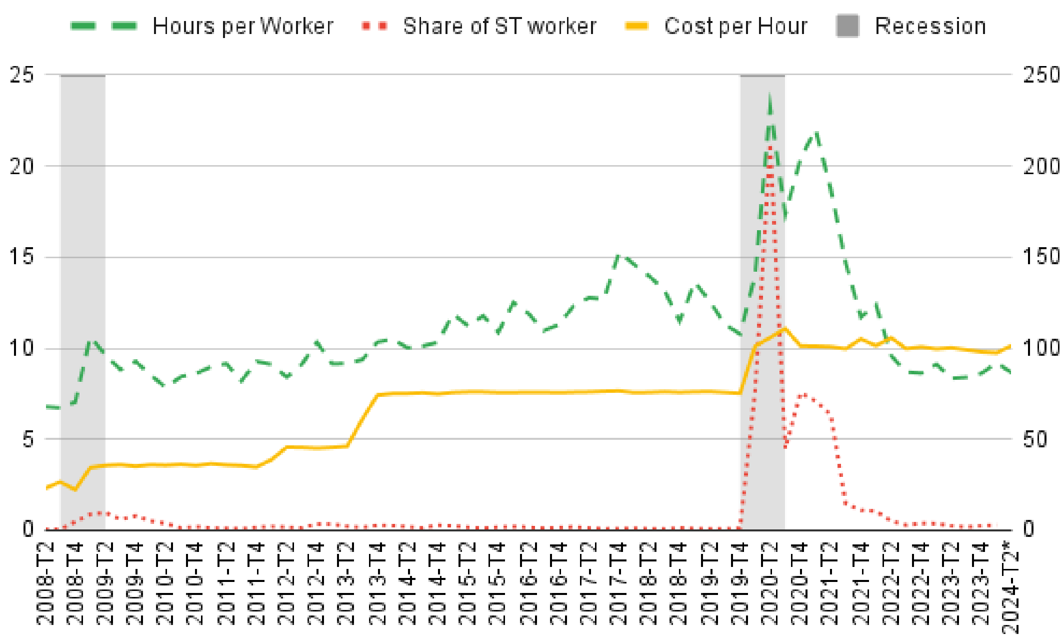

Figure 1: STW Consumption, Public Expenditure and GDP growth in France, 2008-2023

Sources: DARES, OECD. The graph plots the public STW expenditure per hour (green line, left axis), the number of hours of STW consumed per worker (red line, right axis), and the share of short-time worker in the labour force (blue line, right axis). Axis have a logarithmic scale. Grey areas correspond to recessions.

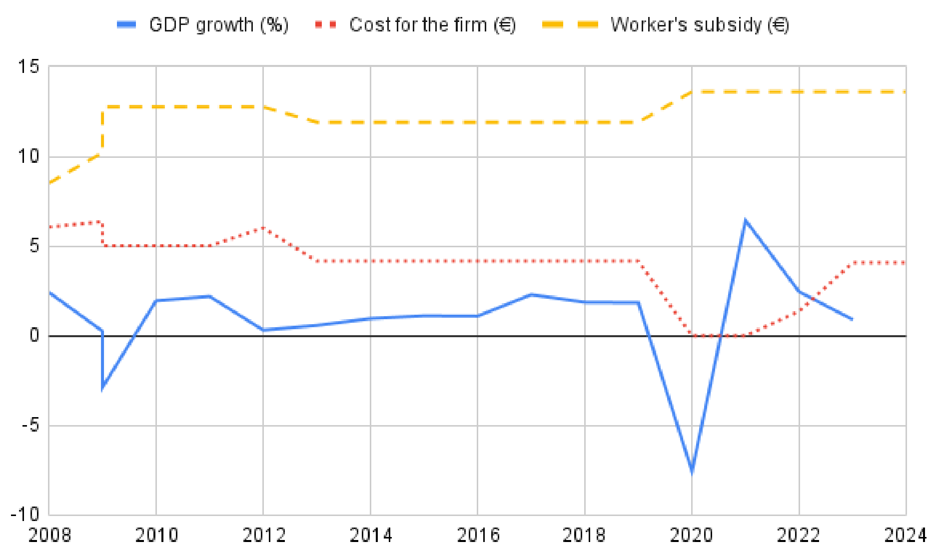

Figure 2: STW cost, consumption and GDP growth in France, 2008-2023

Sources: LegiFrance, INSEE. Own computations. The graph plots the STW cost of adding one hour of STW for a firm (red line) and the ST compensation received by the worker (yellow line) for a firm with less than 250 employees and a worker earning the average wage in 2023 (17€ per hour).

2.3 Local Projection Evidence

In this section, I estimate the impact of a change in monetary incentives to use STW on the number of hours consumed per workers (intensive use), the share of workers enrolled (extensive use), and the public expenditure per worker. I use the narrative database created to assess the response of my three outcomes to a change in ST costs and ST compensation.

To empirically evaluate the dynamic effects of STW reforms, I rely on the local projection method of Jordà (2005) to estimate impulse response functions. The baseline specification is:

$$y_{t+k} - y_{t-1} = \tau_t + \beta_k R_t + \theta X_t + \varepsilon_t$$

in which y is the dependent macroeconomic variable of interest; β_k denotes the (cumulative) response of the variable of interest k years after the shock; τ is a time fixed effects included to take account for global shocks; R denotes the reform shock; and X_t is a vector of control variables including two lags of reform shocks, two lags of real GDP growth and two lags of the relevant dependent variable. The equation is estimated using OLS. Impulse response functions (IRFs) are then obtained by plotting the estimated β_k for k = 0, 1, ...6 with 90 percent confidence bands computed using β_k—based on robust standard errors.

I find that increasing the attractiveness of the STW programme increases enrollment in the programme, decreases the number of hours worked which is associated with an increase in public expenditure for a unsignificant effect on unemployment. Figure (3) shows the quarterly responses of unemployment, the share of short-time workers in the labour force, the number of short-time hours consumed per short-time worker and the log of public expenditure. The positive effect on the number of short-time workers and total public expenditure is immediate, lasts for two periods and then disappears. This shows that STW reforms aimed at making the programmes more attractive for firms, despite being more costly for the government, have the expected effect. Increasing the attractiveness of the programme also led to higher average consumption per enrolled worker. The effect is stable over time and shows that increasing the attractiveness of the programme provides incentives to work less.

STW reforms have the opposite effect of output shock (figure B.2). A positive shock on GDP growth decreases the number of short-time worker, the hours per short-time worker consummed and the ammount of public expenditure of the programme. As intended, STW is a counter-cyclical programmes and is more used during recessions periods even without STW reforms. Local-projection have been advocated by Auerbach and Gorodnichenko (2012) and Romer and Romer (2019) as a flexible alternative to VAR, better suited to estimating a dynamic response such as interactions between shocks and macroeconomic conditions. I also explore whether initial economic conditions at the time of the shock influence its effect on macroeconomic outcomes (equation B.1). As discussed in Auerbach and Gorodnichenko (2012), in this setting the local projection approach to estimating non-linear effects is equivalent to the smooth transition autoregressive (STAR) model developed by Granger (1993). I find that the effect of STW reforms on the labour market is only observed during recession periods (figure B.7). Finally, I test to find different results if I limit the variable to change in ST cost or ST compensation only. The coefficients captured by the local projection does not change (not included in the paper). As STW cost and ST compensation changes arise simulteneaously in most cases, I do not have sufficient variations to distinguish the effect of the two variables.

To run magnitude analysis, I test to include the cost per hour for the State as a proxy for STW reforms. Reforms in the narative database capture the change in cost of using the programms for firms, benefits of entering the programme for the worker, and cost per hour consummed for the State. This ultimate variable can be observed. I divide the total amount of public expenditure by the number of hours consumed to measure the average cost per hour consummed for the state. I then re-run the local-projection estimation with this variable instead of the reform variable. I find similar results but obviously with a difference in magnitude (B.4). The pic of effect of a decrease in ST cost happens after one quarter. An in increase in the generosity of the programme by 1 euro leads to a 1 p.p. increase in the share of ST worker in the labour force, an average reduction of 10 hours worked in the quarter, and a 40% increase in public expenditure with no significant effect on unemployment. The coefficient obtained for the share of ST worker and unemployment are of similar size than the one found by Balleer et al. (2016) for Germany. In the first quarter of 2024, they were 89 thousands of workers in STW which represents 0.3% of the labor force, an increase in public spending would then raise the share of STW to 1.3% with new 300 thousands worker enrolled in the programme. Workers in the programme would work 30 hours less which will increase public expenses by 1.5%. In 2024, the French state spent 80 millions on STW (which equals to 0.2% of the unemployment benefit cost for the state). An increase in 1 euro of the generosity of the programme would raise the public expenses by 1.2 millions.

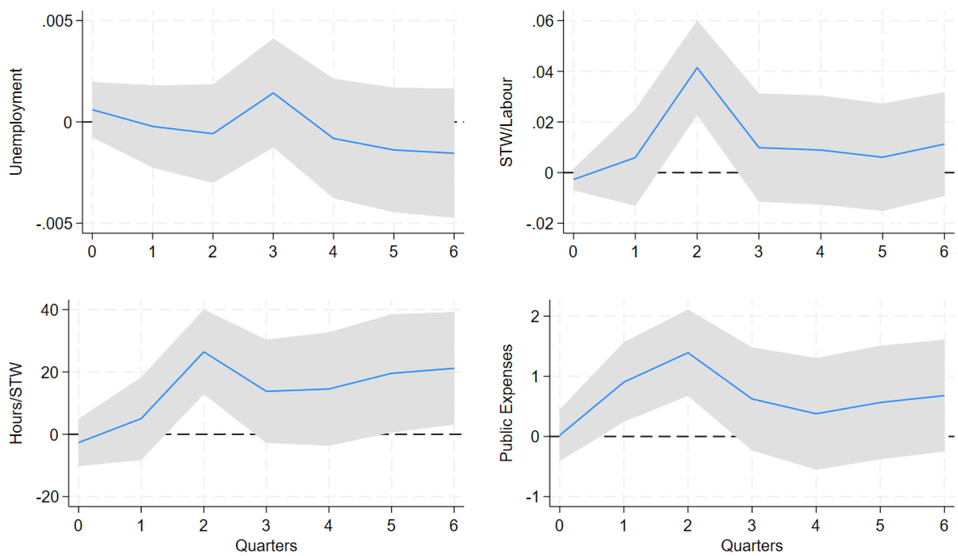

Figure 3: Local-projection: Impulse responses to STW policies

Impulse responses to a STW policy shocks. Local projection estimated with unemployment rate (upper-left graph), log of number of short-time worker (upper-right graph), log hours consummed per workers (bottom right), and log of total public expenditure (bottom left) for 2008Q1 to 2023Q4. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

3 A Labor Market with Hours

This section presents a model that incorporates intensive labour adjustment with STW into the standard search and matching framework of Diamond (1998) and Mortensen and Pissarides (1999). I assume that worker-firm pairs are subject to idiosyncratic shocks to model endogenous job separations and endogenous STW consumption.

The economy is populated by a continuum of infinitely lived households, a continuum of firms, and a policy maker. The timeline is as follows: First, unemployed workers search for jobs and firms post vacancies. Second, the matching function establishes contacts between workers and firms. Third, new contracts and incumbents workers draw a productivity θ from a random distribution $F(\theta)$, the productivity is i.i.d. across workers and times. Fourth, policymakers can modify STW programmes. Finally, firms make their endogenous separation and STW decisions based on the realisation of the draw.

Adjusting the number of working hours is costly. For every hour not worked, the firm has to pay a ST cost. Firms choose both the number of hours worked and whether or not to keep the worker. Reducing the ST cost increases the probability of retaining a job, but it also reduces the number of hours worked by the worker.

I model a policymaker whose objective is to maximise the aggregate surplus generated by worker-firm pairs. The policymaker can change the cost of short-time work paid by the firm, the short-time compensation received by the worker, and levy a tax to finance the policy.

3.1 Firms

A firm can produce the final consumption goods only if it successfully matches with a worker. If a firm finds a match, it obtains a flow profit in the current period after paying the worker. The flow profit consists of three elements. First, the production function $\frac{1}{\alpha}(\theta_t h_t)^\alpha$, a concave increasing function with respect to the worker's productivity θ_t and the number of hours worked $h_t \in [0;1]$. Second, the linear wage w_t and the labour tax τ_t for the hours worked. Third, the cost of STW, i.e. for every hour not worked $1 - h_t$, the firm has to pay a cost b_t. In STW programmes, b_t is always lower than the wage, so that if the firm reduces the number of hours worked, its total labour costs $(\tau_t + w_t)h_t + b_t(1 - h_t)$ are always lower than if the worker were employed full-time $h_t = 1$.

In the next period, if the match survives the firm continues ; if the match breaks down, the firm posts a new vacancy with vacancy value J^v_{t+1}. The match survives if the post does not become obsolete with probability $(1 - \rho)$ and if the workers is kept with probability $(1 - \phi_t)$. The future value is discounted by the factor σ. The firm's match value thus satisfies the Bellman equation:

$$J_t^f = \frac{1}{\alpha}(\theta_t h_t)^\alpha - (\tau_t + w_t)h_t - b_t(1-h_t) + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right]. \tag{1}$$

Creating new vacancies or posting existing vacancies incurs a per-period fixed cost κ. If the vacancy is filled (with probability q_t^v), the firm obtains the value of a match J_{t+1}^f. If the vacancy remains unfilled, then the firm goes into the next period and obtains the continuation value of the vacancy, provided that the vacancy does not be obsolete. Thus, the value of an open vacancy is given by:

$$J_t^v = -\kappa + \sigma(1-\rho)\mathbb{E}\left[q_t^v J_{t+1}^f + (1-q_t^v)J_{t+1}^v\right]. \tag{2}$$

The FOC with respect to hours is given by

$$h_t = \left(\frac{\theta_t^\alpha}{\tau_t + w_t - b_t}\right)^{\frac{1}{1-\alpha}} = \left(\frac{\theta_t^\alpha}{\ell_t}\right)^{\frac{1}{1-\alpha}} \tag{3}$$

The number of hours worked is a ratio between the productivity of the worker θ_t^α and the cost to the firm of adding one hour of work $\ell_t = \tau_t + w_t - b_t$. I can now describe the firing decision of the firm, which depends on the working time h_t. Workers are fired if the losses they generate are higher than the firing cost:

$$J_t^f - J_t^v < -c_f. \tag{4}$$

This defines a firing threshold χ_t^f at which the firm is indifferent between firing and retaining a worker on STW:

$$\chi_t^f = \left[\frac{\alpha}{1-\alpha}\big(-c_f + b_t + J_t^v + \sigma(1-\rho)\mathbb{E}[(1-\phi_{t+1})J_{t+1}^f + \phi_{t+1}J_{t+1}^v]\big)\right]^{\frac{1-\alpha}{\alpha}} \ell_t. \tag{5}$$

I can now derive the effect of the STW cost on the number of hours worked and the firing threshold.

Lemma 1. *The firing threshold and the hours worked are increasing with respect to the ST cost*

PROOF

See appendix C.1 □

Raising the ST cost has a twofold direct effect, it increases the number of hours worked by the worker by making adjustment at the hours margin less flexible and so the worker will produce more, but it reduces the likelihood that a worker will be retained in a firm by increasing the total labour cost paid by the firm.

3.2 Workers

Employed workers receive a flow revenue minus a disutility from working in the current period. The revenue from working is equal to the wage for the hours worked $w_t h_t$ and a compensation for the reduction in hours worked $a_t(1-h_t)$. The worker faces a quadratic disutility for the hours worked $\beta(h_t)^2$. In the next period, if the match survives (with probability $(1-\rho)(1-\phi_{t+1})$), the worker continues; if the match fails, the worker becomes unemployed. The worker's match value thus satisfies the Bellman equation:

$$W_t = w_t h_t + a_t(1-h_t) - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right]. \tag{6}$$

When unemployed, the worker receives an unemployment benefit u_b. If the worker finds a job (with probability q_t^u), the worker obtains the value of a match W_{t+1}. Otherwise, the worker goes into the next period and obtains the continuation value of the unemployment. Thus, the value of unemployment is given by

$$U_t = u_b + \sigma\mathbb{E}\left[q_t^u W_{t+1} + (1-q_t^u)U_{t+1}\right] \tag{7}$$

A worker leaves the match if his surplus from the match is negative. The leaving threshold is defined as the employment value at which the worker is indifferent between staying or leaving the match, i.e:

$$W_t - U_t = 0 \tag{8}$$

3.3 Nash Bargaining Wage

Firms and workers bargain over wages. The Nash bargaining takes place before the realisation of the idiosyncratic shock, so they bargain over the expected value of their respective surplus from the match. The Nash bargaining problem is given by

$$\max_{(w_t)} \Big(\mathbb{E}[W_t - U_t]\Big)^p \Big(\mathbb{E}[J_t^f - J_t^v]\Big)^{1-p}, \tag{9}$$

where p $\in$ [0,1] represents the bargaining power of workers. The share of short-time worker with respect to the share of full-time worker is low in the model at the steady state as in the economy during expansion periods ($\leq$ 10%). I assume that workers and firms expect the employment to be set a full-time job $\mathbb{E}[h_t] = 1$. The first condition implies that

$$(1-p)(J_t^f - J_t^v)\frac{\partial J_t^f - J_t^v}{\partial w_t} = p(W_t - U_t)\frac{\partial W_t - U_t}{\partial w_t}, \tag{10}$$

where, from the worker's value I have $\frac{\partial W_t - U_t}{\partial w_t} = 1$ and from the firm's value function, I have $\frac{\partial J_t^f - J_t^v}{\partial w_t} = -1$. I define the surplus from a match as

$$S_t^f = W_t - U_t + J_t^f - J_t^v = \frac{1}{\alpha}(\theta h_t)^\theta - \tau_t h_t + (a_t - b_t)(1-h_t) - u_b + \kappa + \sigma\mathbb{E}[\cdot], \tag{11}$$

with the continuation value

$$\sigma\mathbb{E}[\cdot] = \sigma\mathbb{E}\Big[\left((1-\rho)(1-\phi_{t+1}) + q_t^v\right)W_{t+1} + \left((1-q_t^u) + \rho\phi_{t+1}\right)U_{t+1}$$
$$+ \left((1-\rho)(1-\phi_{t+1}) + q_t^v\right)J_{t+1}^f + \left((\rho\phi_t) + (1-q_t^v)\right)J_{t+1}^v\Big].$$

The bargaining solution and the expression for employment surplus together imply that the Nash bargaining wage w_t satisfies the Bellman equation

$$w_t = p\mathbb{E}[S_t^f] + \beta + \mathbb{E}[U_t] - \sigma(1-\rho)\mathbb{E}\left[(1-\phi_t)W_{t+1} + \phi_t U_{t+1}\right]. \tag{12}$$

Definition 1. *A labour-contract is an allocation (h_t; ℓ_t) such that: first, the hours worked in equation (3) hold; second, the labour cost satisfies the firing constraint (5), the leaving constraint (8), and the Nash-bargaining program (9);*

The labour-contract defined above is inneficient since it is derived from incomplete information on the productivity draw. This generate wage rigidities that makes labour hoarding inefficiently low without STW as shown by Giupponi and Landais (2023). This also aims at capturing the little fluctuations in real wages observed during recessions (Bewley and Bewley, 2009; Fallick et al., 2016). In addition, in this labour-contract, only the cost of labour is bargained and not the quantity. The efficient labour-contract that would be implemented under complete information is a key benchmark in the model of incomplete information. This optimal labour-contract is defined as follows.

Definition 2. *The optimal contract is an allocation (h_t^*, ℓ_t^*) such that: first, the hours worked in equation (3) hold; second, the labour cost satisfies the firing constraint (5), the leaving constraint (8), and the Nash-bargaining program (13)*

$$\max_{(w_t,h_t)} \left(W_t - U_t\right)^p \left(J_t^f - J_t^v\right)^{1-p} \tag{13}$$

3.4 Labour Market

New job matches are formed based on the matching function

$$m_t = \mu u_t^{\alpha_m} v_t^{1-\alpha_m}, \tag{14}$$

where the parameter μ is the scale of matching efficiency and the parameter $\alpha_m \in [0,1]$ is the elasticity of job matching with respect to efficiency units of seeking workers, u_t is the number of unemployed workers and v_t is the number of vacancies. The number of unemployed is equal to the total labour force minus the number of employed:

$$u_t = 1 - N_t. \tag{15}$$

Newly formed matches add to the employment pool, while job separations and obsolescence subtract from it. Thus, aggregate employment evolves according to the law of motion

$$N_t = (1-\rho)(1-\phi_t)(N_{t-1} + m_{t-1}). \tag{16}$$

The total job destruction ϕ_t depends on the endogenous job destruction rate ϕ_t^e and the exogenous job destruction rate ϕ^x:

$$\phi_t = \phi_t^e + \phi^x. \tag{17}$$

The endogenous rate of job destruction is equal to the share of workers with productivity below the firing threshold:

$$\phi_t^e = \int_0^{\chi_t^f} dF(\theta) \tag{18}$$

The stock of vacancies v_t evolves according to the law of motion:

$$v_t = (1 - q_t^v)(1-\rho)v_t + n_t \tag{19}$$

Following Coles and Kelishomi (2011), I assume that vacancy creation entails a non-negative entry cost of x, drawn from an i.i.d. distribution $\psi(\cdot)$. A new vacancy is created if and only if $x \leq J_t^v \equiv x_t^*$, or equivalently, if and only if its net value is non-negative. Thus the number of new vacancies n_t is equal to $\psi(J_t^v)$ - the cumulative density of entry costs evaluated at the vacancy value. As in Leduc and Liu (2020), I assume that the functional form of the distribution function $\psi(\cdot)$ is

$$n_t = \eta (J_t^v)^{\xi}, \tag{20}$$

where η is a scale parameter and ξ measures the elasticity of new vacancies with respect to the value of the vacancy. The special case with $\xi \to +\infty$ corresponds to the standard DMP model with free entry (i.e. J_t^v = 0). The probability of filling the vacancy q_t^v and the probability of finding a job q_t^u are given by

$$q_t^u = \frac{m_t}{u_t} \tag{21}$$

$$q_t^v = \frac{m_t}{v_t}. \tag{22}$$

3.5 Government Policy and Aggregation

The government has a balanced budget and finances the expenditure on short-time work through the labour tax. The expenditure on short-time work is equal to the difference between the short-time compensation a_t minus the short-time costs paid by the firm b_t times the consumption of short-time work $(1 - h_t)$ aggregated for all workers. The policy maker may face a time deficit at time t, but it will impact its future taxes. I model the usual decision of the policy-maker to run a deficit in recessionary periods with the aim of paying it off in subsequent expansionary periods. I allow the policy maker to levy a different labour tax on short-time workers τ_t^{stw} and on full-time workers τ_t^f.

$$G_t = \int_{\chi_t^f}^{\chi^{stw}} (a_t - b_t)(1 - h_t) dF(\theta) - \int_{\chi_t^f}^{\chi^{stw}} \tau_t^{stw} h_t dF(\theta) - \int_{\chi^{stw}}^{1} \tau_t^f dF(\theta) + \sigma^s \mathbb{E}[G_{t+1}] \tag{23}$$

where $\chi_t^{stw} \geq \chi_t^f$ is the exogenous eligibility criterion for STW, which defines the productivity threshold below which a firm-worker pair has access to STW. Above this threshold, the working time is set to the maximum $h_t = 1$. The STW policy seeks to maximise the aggregate surplus of matches S_t^f under the budget constraint.

The labour market equilibrium is defined by equations (1), (2), (6), (7), (12), (14)-(23). Aggregate output Y is defined as

$$
\begin{aligned}
Y_t = {} & N_t \int_{\chi_t^f}^{\chi^{stw}} \left\{ \frac{1}{\alpha} (\theta h_t)^\alpha - \tau_t^{stw} h_t \right\} dF(\theta) \\
& + N_t \int_{\chi^{stw}}^{1} \frac{1}{\alpha} \theta^\alpha - \tau_t^f \, dF(\theta) \\
& - N_t \phi_t^e c_f - u_t u_b - v_t \kappa
\end{aligned} \tag{24}
$$

Aggregate output is equal to output minus resource costs. Note that N_t is the number of all workers employed in period t, i.e. after taking into account the separation rate. The level of output takes into account idiosyncratic productivity, i.e. the number of hours worked as well as the number of full-time employees. Resource costs include tax rates derived from the policy maker's budget, vacancy advertising costs, firing costs and unemployment benefits.

4 Optimal and Suboptimal STW Programmes

The policy maker wants to maximise the surplus generated by the matches along the distribution of productivity $F(\theta)$. In the model, firms are hold by households, by maximizing the surplus the policy-maker optimize the simultaneously the surplus of firms owners and workers. The utility of unemployed worker is beyond the scope of this paper since STW is primarily directed toward employed worker. Still, with the unemployment benefit u_b and the finding probability q_t^v taking as given, I show that this programme maximize the utility of employed and unemployed workers. To do this, it can modify the cost of short-time work paid by the firm b_t, the ST compensation received by the worker a_t and the labour tax τ_t. The social welfare function is written as

$$
SWF = \int S_t^f(\theta) \, dF(\theta). \tag{25}
$$

Definition 3. *An optimal STW policy is an allocation $(\tau_t(\theta), b_t(\theta), a_t(\theta))$ that maximize the social welfare function (25) under the public budget constraint (23).*

In the next section, I derive the optimal STW programme by solving the policy maker's problem. I then use it as a benchmark to define the deadweight loss of the programme and study how the short-run costs affect it.

4.1 Optimal STW Policy

I derive the optimal policy and show that it consists of: first, setting the number of hours worked h_t for each worker-firm pair enrolled in the programme so as to maximise their surplus S_t^f, and second, setting the ST costs b_t and ST compensation a_t so as to share the surplus produced between the worker and the firm.

The Hamiltonian of the policy-maker program for worker-firm pairs on STW is as follows

$$\mathcal{H}(\theta) = S_t^f(\theta)f(\theta) + \lambda(\theta)\left[\tau_t^{stw}h_t(\theta) - (a_t - b_t)(1 - h_t(\theta)\right]f(\theta) \tag{26}$$

with $S_t^f(\theta)$ the surplus of a match with productivity (θ) in equation (11) and $\lambda(\theta)$ the costate variable. The FOC w.r.t. the ST compensation a_t is:

$$1 - h_t(\theta_t) + \lambda(\theta)(-(1 - h_t(\theta_t))) = 0.$$

The costate variable $\lambda(\theta)$ captures the weight of the public budget constraint along the skill distribution and is equal to 1. This is a consequence of the linear SWF and the fact that the policymaker spends all the budget collected through taxes without loss, every money in his pocket has the same social value as a money in the pocket of a worker. It follows that the Hamiltonian equals the surplus from a match (11) with the ST compensation paid by the firm $a_t = b_t$ and without labour tax $\tau_t^{stw} = 0$:

$$\mathcal{H}(\theta) = S_t^f|_{(a=b,\tau=0)} = \frac{1}{\alpha}(\theta h_t)^{\theta} - u_b + \kappa + \sigma\mathbb{E}[\cdot].$$

Proposition 1 *The optimal STW policy solution consists in setting the hours worked by each worker-firm pair such that it maximizes the surplus generated by their match*

$$h_t^* = \underset{h_t}{\arg\max}\left\{S_t^f \mid a_t = b_t,\ \tau_t = 0\right\} = \left\{\left(\frac{\theta_t^{\alpha}}{\ell_t^*}\right)^{\frac{1}{1-\alpha}} \mid \ell_t^* = \tau_t^* + w_t - b_t^*,\ (\tau_t^*, b_t^*) = \underset{\tau_t, b_t}{\arg\max}\{\mathcal{H}\}\right\} \tag{27}$$

PROOF

See appendix C.2 □

In the appendix C.2, I derive the optimal labour contract for each worker-firm pair S_t^f without any intervention by the policy maker ($\tau_t = 0$ and $a_t = b_t$) and show that it admits the hours worked solution h_t^* as a Hamiltonian (26). This shows that the policy maker's problem is to determine the optimal hours worked for each worker-firm pair in STW. To do this, she modifies the labour cost paid by the firm $\ell_t =$

$\tau_t + w_t - b_t$ to provide the incentive for the firm to set the number of hours worked at the optimum. This determines the optimal intensive incentive to use the programme for each worker-firm type θ.

Corollary 1 *The optimal labour contract* (h_t^*, ℓ_t^*)

$$h_t^*(\theta_t) = \left(\frac{\theta_t^\alpha}{2\beta}\right)^{\frac{1}{2-\alpha}} \ \forall\ \theta > 0 \tag{28}$$

$$\ell^*(\theta_t) = w_t + \tau_t - b_t = \left[(2\beta)\theta_t^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{2-\alpha}} \ \forall\ \theta > 0 \tag{29}$$

PROOF

See appendix C.3

Corollary 1 gives the optimal labour contract (h_t^*, ℓ_t^*) for each worker-firm pair along the productivity distribution. Both elements are non-linearly increasing functions of productivity and equal 0 when productivity θ is 0. The optimal hours worked are a ration between marginal productivity and the marginal disutility of working. The optimal labour contract provides the incentive for firms to choose the optimal hours worked. According to proposition 1, the first-best STW programme differs for each productivity draw θ so that more productive workers work more to maximise the surplus generated by the match. The optimal surplus from a match (S_t^{f*}) is define from the optimal labour contract (h_t^*, ℓ_t^*).

Proposition 2. *The optimal STW policy shares the surplus from a match between the firm and the worker*

$$W_t - U_t = k^1 S_t^{f*} \ ; \ J_t^f - J_t^v = k^2 S_t^{f*} \text{ with } k^1, k^2 > 0 \tag{30}$$

PROOF

See appendix C.4

The SWF is maximised if propositions 1 and 2 are satisfied. Propositions 1 and 2 solve the extensive-intensive incentive trade-off highlighted in lemma 1. Proposition 1 ensures that each worker-firm pair generates the largest possible surplus, and Proposition 2 ensures that each worker-firm pair generating a positive surplus are preserved. In other words, proposition 1 gives the optimal intensive incentives to use the scheme (how many hours should be worked), while proposition 2 gives the optimal extensive incentives to use the scheme (which job should be retained). Any deviation from propositions 1 and 2 results in a sub-optimal SWF.

From propositions 1, 2 and the budget constraint (23) I can derive the optimal value of the short-time cost b_t, short-time compensation a_t and labour taxe τ_t^{stw}:

$$a_t^*(\theta) := W_t - U_t = k^1 S_t^{f*} \tag{31}$$

$$b_t^*(\theta) := J_t^f - J_t^v = k^2 S_t^{f*} \tag{32}$$

$$\tau_t^{stw*} = \ell_t^* + b_t^* - w_t. \tag{33}$$

In the system, the ST compensation (31) and the ST cost (32) are set such that extensive incentives to use the programme are optimal according to Proposition 2. The system allows for a range of solutions where the exact value of a_t and b_t and the distance between the two $a_t - b_t$ depends on the weight the policy maker places on firms k^2, workers k^1. The issue of wealth redistribution is beyond the scope of this paper, which focuses on surplus maximisation. Nevertheless, their respective maxima, which satisfy the system, decrease with productivity θ. At low productivity, a small surplus is generated, which reduces the labour cost that the firm can afford and the value created that can be distributed to the worker. The equilibrium defined in the system is a Nash-equilibrium.

If the policy maker weights on workers k^1 and firms k^2 are set to worker bargaining power p and firm bargaining power $(1-p)$ respectively, then the worker surplus from a match $W_t - U_t$ and the firm surplus from a match $J_t^f - J_v^t$ reach the same level as if the optimal Nash bargaining problem (13). In other words, the optimal centralised equilibrium defined by the policy maker programme accepts the same solution of the optimal decentralised equilibrium.

Lemma 2. *The optimal STW policy with weights $k^1 = p$ and $k^2 = (1-p)$ and the optimal labour-contract (13) is such that:*

$$(h_t^*, \ell_t^*) := \max_{(h_t, \ell_t)} \left(W_t - U_t\right)^p \left(J_t^f - J_t^v\right)^{1-p}$$
$$W_t - U_t = pS_t^{f*} = \frac{p}{1-p}\left(J_t^f - J_t^v\right)$$

PROOF

See appendix C.5 □

Lemma 2 shows that the optimal STW policy consists in correcting the labour cost observed in the market. The worker-firm pairs eligible for the STW programme have a productivity θ lower than the average productivity $\mathbb{E}[\theta]$. As a result, the labour cost set by the labour bargaining programme (9) is higher than the optimal one. The optimal STW program lowers the labour cost so that the surplus generated by the match is maximised and then redistributed to the worker and the firm. Lemma 2 implies, first, that no STW policy is needed if firms and workers are able to predict the productivity draw θ (perfect information). Second, if worker-firm pairs were able to re-bargain over the wage and short-time compensation after observing the draw, the policymaker would not need to implement a policy. Third, layoff workers in the optimal equilibrium prefers to be unemployed.

The optimal labour tax for short-time worker (33) is set so that the intensive incentive to use the programme is optimal according to Proposition 1. This labor tax is negative since $w_t \geq \ell_t^* + b_t^*$ and therefore is a wage subsidy. It corrects for the wage sets too high compare to the actual productivity so that the wage paid by the firm is the optimal wage w_t^* derived from the Nash-bargaining programme with perfect expectations (13).

Corollary 2. *The optimal labour tax on short-time worker with weight $k^1 = p$ and $k^2 = (1-p)$ is a wage subsidy equals to the difference between the optimal wage defined in (13) and the observed wage defines in (9)*

$$\tau_t^{stw*} = w_t^* - w_t < 0$$

PROOF

From lemma 2, I get that $w_t^* = \ell_t^* + b_t^*$. From equation (33), $\tau_t^{stw*} = \ell_t^* + b_t^* - w_t$. Combining two equations ends in corollary 2. The difference between $w_t^* - w_t$ is negative since w_t^* is set for a productivity θ and w_t for a productivity $\mathbb{E}[\theta]$ and for short-time worker $\mathbb{E}[\theta] > \theta$. □

Finally, the labour tax on full-time workers solves the public budget constraint (23). As such, its level depends on the deficit generated by the STW programme. Again, its value depends on the preference for redistribution from the state along the productivity distribution which is beyond the scope of this paper. Here, I evidentiate cutoffs for the labour tax on full time worker. When the sum of weight for the worker and the firm in a match is lower or equal to 1 $k^1 + k^2 \leq 1$, then the optimal surplus generated is redistributed by the state without generating a deficit. However, when the sum of weights is bigger than 1 $(k^1 + k^2 > 1)$ there is more surplus redistributed than generated which implies a deficit for the public budget. This public deficit is then financed by full-time workers.

Corollary 3. *The optimal labour tax for full-time workers is*

$$\tau_t^{f*} \begin{cases} < 0 \text{ if } k^1 + k^2 < 1 \\ = 0 \text{ if } k^1 + k^2 = 1 \\ > 0 \text{ if } k^1 + k^2 > 1 \end{cases} \tag{34}$$

PROOF

See appendix C.6 □

To conclude this section, the optimal policy consists in correcting the worker-firm labour contract obtained when the productivity incentive is not observed under the intensive-extensive trade-off constraint. The main instrument for the intensive incentive is the wage subsidy $\tau_t^{stw*} < 0$, which compensates for the observed excessive wage. Then, ST costs and ST compensation are instruments for the extensive incentive to use the programme and should be set so that the surplus received by workers and firms is positive. The value of short-time compensation and costs depends on the preference for government redistribution to workers, firms and worker-firm pairs along the productivity distribution. No STW policy is needed if the labour contract is renegotiated after the shock is observed for ST workers.

4.2 Second-Best and Deadweight Loss

In the first-best case, ST costs and compensations differ for each worker according to their idiosyncratic productivity and are coupled with wage subsidies. No country has ever implemented such a programme. The value of ST costs and compensations is rigid and common to all worker-firm pairs. Moreover, STW programmes are never coupled with a wage subsidy programme. In this section I derive a policy where the components of the STW programme (b, a) are constants so no longer functions of productivity θ and where no wage subsidy programme is implemented $\tau_t = \tilde{\tau}$. This results in a second-best programme that is closer to the one implemented by policymakers. This setting is similar to assume incomplete information, i.e. the policy maker does not observe the productivity draw θ for each pair, but knows the productivity distribution $F(\theta)$. With this feature, the policy maker's problem is now a Lagrangian one:

$$\mathcal{L} = \int \left(S_t^f(\theta)\right) dF(\theta) + \lambda \left[\int_{\chi_t}^{\chi^{stw}} (a_t - b_t)(1 - h_t) dF(\theta) - \int_{\chi_i}^{\chi^{stw}} \tau_t^{stw} h_t dF(\theta) - \sum_{i=t}^{T-t} \int_{\chi^{stw}}^{1} \tau_i^f dF(\theta)\right]$$

Proposition 3. *At second best, the short-time cost belongs to the interval:*

$$b_t' \in]J_t^f(\chi_t^{f*}) - J_t^v; b_t^*(\tilde{\theta})[\ ;\ \text{with } \tilde{\theta} = \int_{\chi_t^f}^{\chi^{stw}} \theta dF(\theta) \text{ and } b_t^*(\theta) = \ell_t^*(\theta) - w_t - \tilde{\tau}.$$

PROOF

See appendix C.7

The second-best policy is the result of a trade-off between intensive and extensive incentives for worker-firm participation. Since the ST cost is the same for all worker-firm pairs, it cannot simultaneously set the number of hours worked and the firing threshold to their respective optimal values. Then, the ST cost is set at an intermediate value between the one leading to the same firing threshold as in the first best equilibrium $J_t^f(\chi_t^{f*}) - J_t^v$ and the one that would maximize the surplus of short-time workers $b_t^*(\tilde{\theta}$. This is captured by the FOC of the Lagrangian:

$$\underbrace{-S^f(\chi_t^f)\frac{d}{db_t}\chi_t^f}_{<0} + \underbrace{\int \frac{d}{db}\{S_t^f\}dF(\theta)}_{>0} = 0, \tag{35}$$

where the first term equals 0 when the firing threshold reaches its optimal value $S^f(\chi_t^{f*})\frac{d}{db_t}\chi_t^{f*} = 0$, but at this level the second term is positive $\int \frac{d}{db}\{S^f(\theta)\}dF(\theta)$. The second term is zero when the surplus of the medium worker-firm pair in STW reaches its optimum, but at this level the first term is negative. The explicit value depends on the distribution of productivity, i.e. on the ratio between the surplus gain from the saved job and the surplus loss from the lower number of hours worked due to an ST cost reduction. Below the second best $b't$. The number of jobs saved by a ST cost will not outweigh the surplus lost by a reduction in hours worked.

The attentive reader will note that the second and first best solutions satisfy Mirrlees's policy of incentive compatibility Mirrlees (1971). There are no incentives for firms to misreport their productivity draw θ, and the labour costs faced in the first and second best settings are such that firms would receive a lower payoff by choosing a different number of hours θ'.

From Proposition 3, equations (3) and (5), the firing threshold in the second best $\chi_t^{f}{}'$ is higher than in the first best, and the number of hours worked by all workers with productivity above the firing threshold is lower than in the first best. In other words, the social planner cannot achieve the same level of social welfare as in the first best without a different programme for each productivity type θ. To characterise the individual impact of the second best policy on the worker-firm surplus, I define the individual deadweight loss as follows.

Definition 4. *The Individual Deadweight Loss (IDWL) is the difference between the observed worker-firm* *surplus and the first-best surplus.*

$$IDWL_t = S_t^{f*} - S_t^f$$

To characterise the aggregate impact of the second-best policy on the total surplus generated, I define the social deadweight loss as follows.

Definition 5. *The Social Deadweight Loss (SDWL) is the difference between the observed aggregate surplus and the first-best surplus.*

$$SDWL_t = \int S_t^{f*} dF(\theta) - \int S_t^f dF(\theta)$$

In second best, the SDWL is greater than 0 and the firing threshold is greater than in first best. In this setting, each worker-firm pair with productivity $\theta \in]\chi^{f\prime}_t, \chi^{stw}]$ faces a ST cost that is less than its optimal value $b_t' < b_t^*(\theta)$ and thus a labour cost of an additional hour that is greater than the optimal $\ell_t' = w_t - b_t' > \ell_t^*(\theta)$. Figure 4 illustrates the difference in hours worked between the first and second best policies. Workers with productivity above the cutoff $k^0 := b_t' = b^*(k)$ work less than at the optimum, as the cost of short-time is lower for them than in the first-best policy. The difference in hours worked for these workers is captured by area (2). Workers with productivity below k^0 are dismissed, since the firing threshold is binding for them, and so their number of hours worked is lower than in the first-best setting where their matches are preserved. The difference in hours worked for these workers is captured by area (1). Overall, the SDWL in second best is a function of area (1) and (2). Figure 4 illustrates the trade-off between hours worked and employment loss in second best: lowering the level of ST costs increases the number of jobs saved by the programme, but reduces the hours worked by all other worker-firm pairs within the programme.

Lemma 3. *At second-best, a decrease in ST cost b_t increases the current SDWL*

$$-\frac{\partial SDWL_t}{\partial b_t} = \underbrace{-S^f(\chi_t^f)\frac{d}{db_t}\chi_t^f}_{<0} + \int \left\{ \underbrace{-\frac{\partial W_t}{\partial b_t}}_{<0} + \underbrace{-\frac{\partial J_t^f}{\partial b_t}}_{>0} \right\} dF(\theta) < 0$$

PROOF

The result directly follows from proposition 3 □

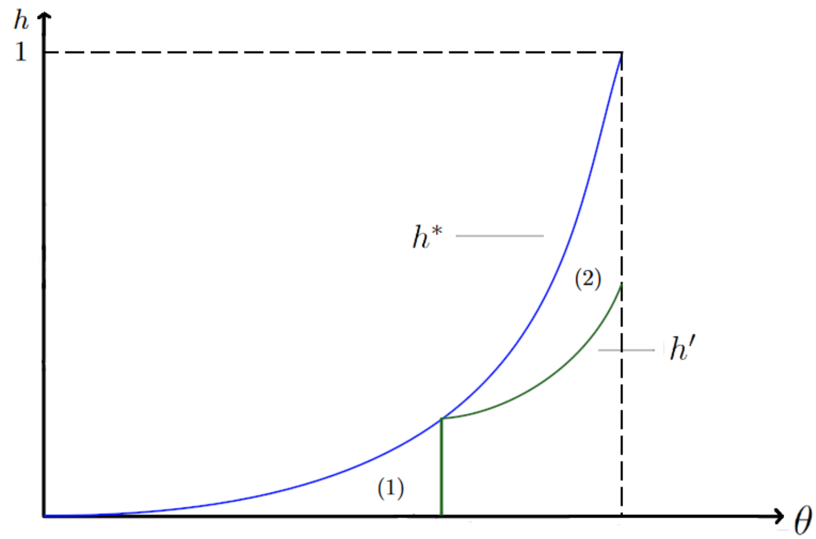

Figure 4: Hours worked with first and second-best policy

The ST cost in the second-best policy is the result of the optimal solution of the trade-off between increasing the IDWL of worker-firms already in STW and reducing the firing threshold. It follows that reducing the ST cost below this level leads to an increase in the IDWL of worker-firms in STW that is not compensated by reducing the firing threshold. Figure 5 illustrates the IDWL for workers above the firing threshold. The pink area captures the IDWL that increases following a reduction in ST cost (with $\ell_t = w_t + \tau_t - b_t$). The difference in the pink area between the two labour costs ℓ in 5 illustrates that the IDWL is an exponential function of the ST cost $\frac{\partial^2 IDWL_t}{\partial b_t^2} > 0$. The ST cost in the second best policy is defined as the point at which reducing the gain in SDWL of the new job saved does not compensate for the reduction in IDWL of the job already saved. If the ST cost is equal to the lowest point defined in the first best $b_t = b_t^*(\chi_t^{f*})$, then reducing b_t reduces the SDWL and the IDWL of workers with productivity below the firing threshold, since their surplus from the match is negative.

Each worker in STW experiences a positive IDWL. Figure 5, illustrates how a change in ST costs affects deadweight loss. In this figure, I plot the labour demand derived from the firms' matching value for two productivity levels, $\theta_1 > \theta_2$. When the worker-firm pairs draw a lower productivity θ_2, the demand for labour D_2 falls and the optimal hours worked and labour costs fall. However, in the second best case, since the ST cost is the same for all workers, the labour cost does not decrease and so the number of hours worked is below the optimum $h' > h_2^*$, generating an IDWL captured by the pink triangle. This shows that the second best policy generates *per se* a IDWL. In this scenario, reducing the ST cost increases the deadweight loss. The reduction in ST costs increases the flow profit received by the firm, but reduces the utility of the workers. This loss for workers is partly compensated by the public deficit.

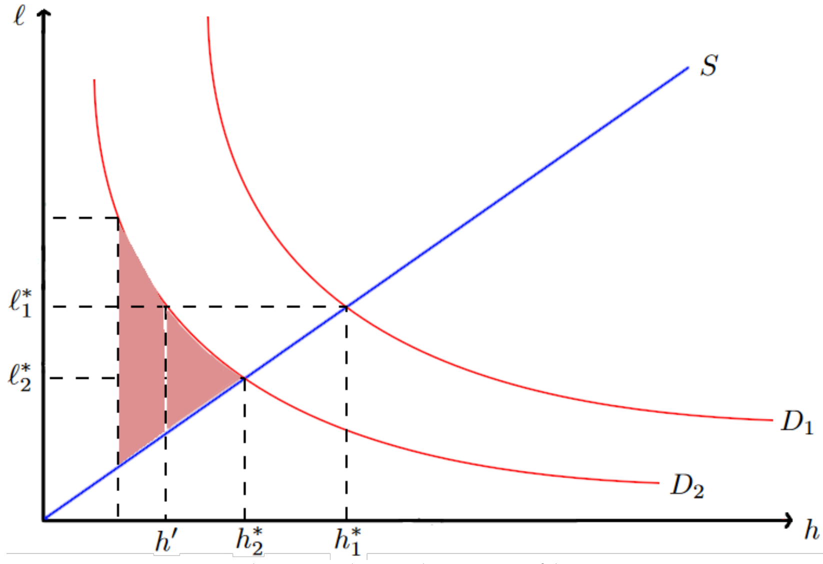

Figure 5: The Deadweight Loss of low STW cost

The graph shows the demand (D) and supply (S) of labour h according to the labour cost ℓ. D_2 is the demand curve after a fall in productivity. h' is the number of hours worked if the labour cost doesn't change. The pink area is the surplus lost due to the labour cost ℓ being too high. Reducing the ST cost b increases the labour cost ℓ and the IDWL.

Lemma 4. *At second-best, a decrease in ST cost b_t increases the future SDWL*

PROOF

The derivative of the SDWL_{t+1} is

$$-\frac{\partial SDWL_{t+1}}{\partial b_t} = \int_{\chi^{stw}}^{1} df(\theta)\frac{\partial \tau_t^{f+1}}{\partial b_t}$$
$$= -\int_{\chi^{stw}}^{1} df(\theta)\frac{1}{\sigma}\left[(a_t - b_t)(1 - h(\chi_t^f))\frac{\partial}{\partial b_t}\chi_t^f + \int_{\chi_t^f}^{\chi^{stw}} \left\{(a_t - b_t)\frac{\partial}{\partial b_t}h_t(\theta) + 1 - h_t(\theta)\right\}dF(\theta)\right] < 0.$$

With the short-time cost b_t set as a constant, then a marginal change in b_t affects the future deadweight loss through the labour tax on full-time workers τ_{t+1}^f. In brackets is the derivative of the public budget according to b_t[3] □

Lemma 4 illustrates the dynamic negative effect of a generous STW programme. In the second best case, STW generates a deficit in the public budget, which has a dynamic effect by raising future taxes. A marginal reduction in the cost of STW has a triple negative effect on public finances. First, it increases

[3]To simplify the labour tax on STW equals to 0 $\tau^{stw} = 0$

public expenditure due to new entrants. Second, it increases public expenditure due to fewer hours worked. Third, it increases public expenditure due to a lower participation of firms in ST compensation. All these effects are additive and lead to an increase in future taxes. A change in ST costs should then be carefully considered.

Lemmas 3 and 4 show the negative effect of a generous STW programme on aggregate surplus. The question now is: where are the real costs of STW relative to the second best? From Proposition 3 I know that current STW programmes are per se sub-efficient and generate a deadweight loss. However, this loss can be mitigated if the observed costs are close to the second best. As shown in section **??**, there is a trend of decreasing ST costs over time, which implies a risk of increasing the SDWL of STW programmes. In the next section, I calibrate the model with French data over different periods to estimate the distance between the second-best and the observed ST cost and the effect on the SDWL.

It should be noted that setting the ST cost to 0 ($b_t = 0$) and the ST compensation equal to or greater than the unemployment benefit ($a_t \geq 0$), as in Germany and other countries (Figure B.2), has some undesirable properties. First, the firing and leaving thresholds are lower than in the first best, which means that even jobs with a negative surplus are preserved at the expense of the public deficit. Second, there is no other ST cost that generates a larger IDWL for worker-firm pairs in STW.

5 Numerical Simulation

In this section, I first describe my calibration strategy. Then I present the results of numerical simulations to compare the effect of the current STW programme with the first-best and second-best policies at steady state and after a productivity shock.

5.1 Calibration

I assume that the idiosyncratic productivity shock $f(\theta)$ follows a cubic distribution and the return to scale is $\alpha = \frac{2}{3}$ to get a closed-form solution. The challenge with calibration is to find functions that can be integrated. To solve this problem I have to define the distribution function and the return to scale together. A return to scale lower than 1 ($\alpha < 1$) implies that the distribution has a positive integer power. In this context, it does not change the results significantly, since STW programmes affect the left tail of the distribution (see figure C.1). Then, reducing the probability of observing high productivity with a distribution function close to a normal distribution does not affect the results derived for short-time workers.

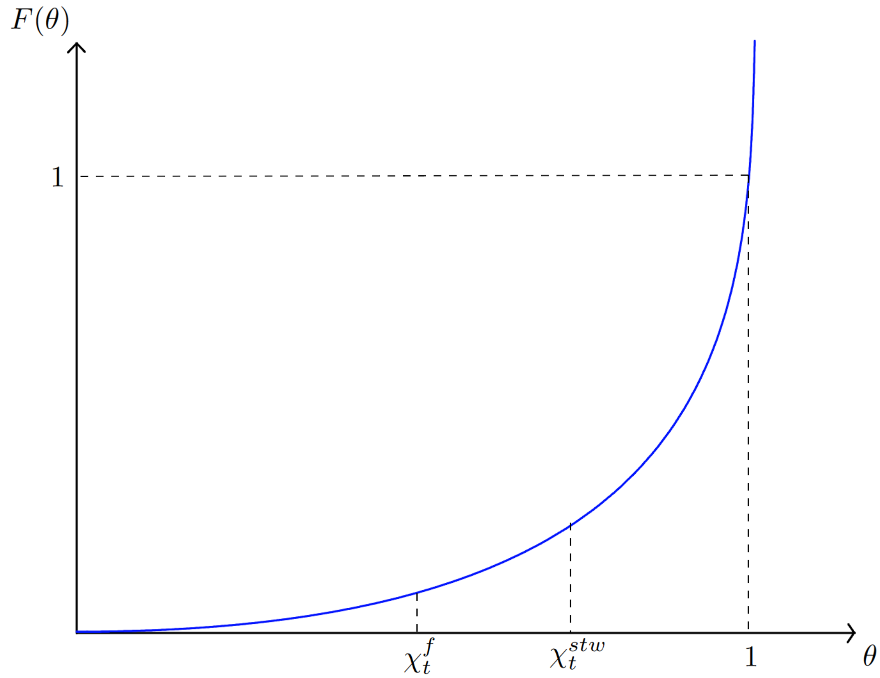

Figure 6: Illustration of the distribution of idiosyncratic productivity to the worker-firm pair and firing thresholds

I calibrate the baseline model to the French economy. Table 1 summarizes my parameters and my calibration targets. The quarterly discount factor σ is 0.99, which matches an annual real interest rate of 4.1%. The matching elasticity α_m is set to 0.6 and the matching efficiency to 0.43. I set the bargaining power to an intermediate value of $p = 0.5$.

Following Christoffel et al. (2009), I target a steady state value for the quarterly separation rate of 3%. As in Krause and Lubik (2007) one third of separations is endogenous, whereas two thirds are exogenously determined. At steady-state, the unemployment rate u_{ss}, the average number of hours worked by ST workers $\tilde{h}_{ss}^{STW}$, short-time cost b_{ss}, a_{ss} compensations and rate ϕ_{ss}^{STW} are set to match French data in the year before the start of the Covid-19. These targets allow me replicate the French STW programme and produce credible counterfactuals.

The ST cost affect the labour market equilibrium through the change in firing threshold (figure C.1) and the number of hours worked by ST worker (5). The second then impact the expected surplus from a match for workers and firms and therefore leads to a change in job creation. In appendix D, I detail the equilibrium conditions, the steady state, and show that lowering the ST cost below the second best level preserve more employments but lower the job-creation since firms will anticipate a raise in future taxation.

Table 1: Calibration

Parameter		Value
σ	Subjective discount factor	0.99
μ	Matching efficiency	0.43
α_u	Elasticity matching function	0.6
p	Nash bargaining weight	0.5
ξ	Elasticity of vacancy creation	1
α	Return to scale	$\frac{2}{3}$
Steady states targets		**Value**
u_{ss}	Unemployment rate	0.08
ϕ_{ss}	Firing rate	0.03
$\frac{1}{3}$ endogenous, $\frac{2}{3}$ exogenous		
b_{ss}	STW cost	0.34×w
a_{ss}	Short-time compensation	0.6×w
ϕ_{ss}^{STW}	STW share	0.0168
$\tilde{h}_{ss}^{STW}$	Hours worked by ST workers	0.77

5.2 Steady States

In this section, I estimate how a change in the short-time work programme affects the steady-states. I first compute the steady-state with the target in table 1, then I substitute the value of the STW policy with the first and second-best solution. For the first-best, I substitute the labor-tax τ_{ss}, the ST cost b_{ss} and ST compensation a_{ss} with the value that solves the first-best programme when the weight of the policy-maker for workers and firms respectively equals their bargaining power in equations (31), (32), and (33).

For the second best solution, I substitute the value of the ST cost b_{ss} with the one solving equation (35). Using Newton's method, I find that the STW cost equals should equal 51% of the wage in the second-best case. Recalling that the ST compensation equal to 60% of the wage, it implies that nearly all the expense on STW is covered by firms reducing drastically public deficit. Overall, an increase of the ST cost from 30% to 58% of the wage would reduce public expenses on STW by 80%. I find that this change would have a small impact on unemployment with an increase of 1.2 p.p but would increase the aggregate surplus of ST worker by 9.4%. The first-best policy would even lead to higher increase in surplus with an increase of 22% with a similar unemployment rate (a decrease of 0.15 p.p.).

Figure 7 shows how the ST cost affects the worker-firm surplus S_{ss}^f, the firm surplus $J_{ss}^f - J_{ss}^v$, the public budget G_{ss}, the hours worked h_{ss} along the productivity θ. The green line is drawn for the productivity of the average worker on STW, the red line for the current ST cost and the blue line for the ST cost in the second best case. A plane is drawn at 0. The top left graph shows how the worker-firm surplus rises

Table 2: The effect of STW programme on the Steady-State

	2^{nd} Best	1^{st} Best
SWF (of ST workers)	+9.4 %	+ 22 %
Average hours worked of ST workers	+ 25 %	+ 83 %
Budget Expenses	- 92 %	- 100 %
Firing rate	+ 0.03 p.p	-0.06 p.p

steadily up to the second-best ST cost. The top right graph illustrates both the firing threshold and how the ST cost affects it, it also illustrates that the firm surplus increases as the ST cost decreases. Combining the two graphs shows that a generous programme benefits the firm at the expense of the aggregate surplus as predicted in Lemma 3. The bottom left-hand corner of the graph shows the public expenditure on the match. Similarly, a low ST cost has a negative impact on the government budget as predicted in lemma 4. On the bottom right of the graph is the effect of ST costs on hours worked, which shows how steeply the number of hours worked decreases with ST costs and productivity, and therefore how strongly ST costs affect the hours worked and output produced by a match as predicted in Lemma 1.

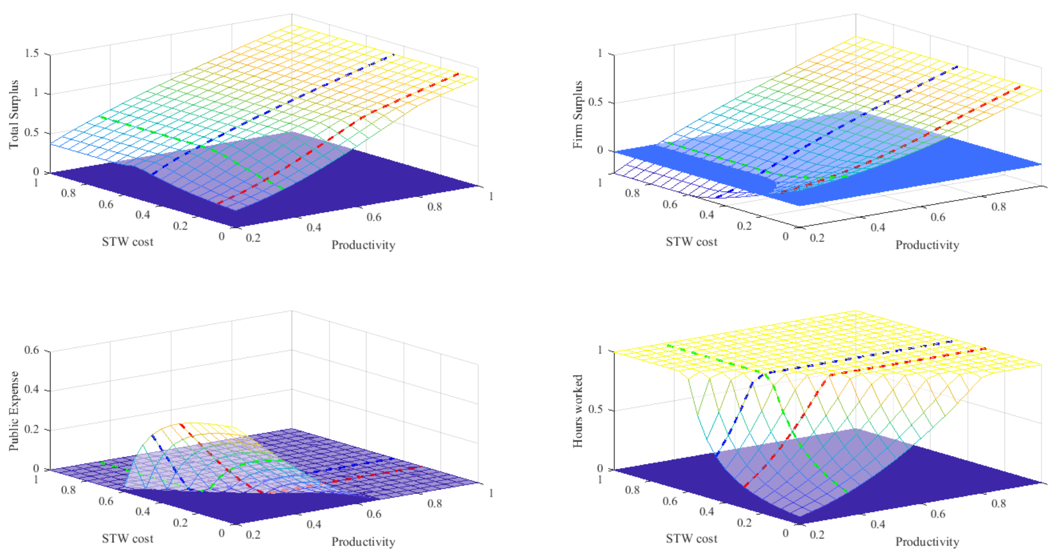

Figure 7: The effect of ST cost along the productivity distribution

5.3 Dynamics

In this section I estimate how short-time work policies affect the model during recessions. STW schemes are generally made more attractive during economic downturn in order to encourage worker-firm pairs to enter and remain in the scheme. I model a half standard negative deviation of the productivity distribution during one period. The aim is to model a one-period recession and the effect of different STW policies on the deviation of the model from steady state. Figure B.8 shows the impulse responses to a fall in productivity during one periods for the three scenarios. The first scenario, which I call ”current programme”, mimics the French policy during the Covid-19 with the ST cost reduced to 0 during the recession and then goes back to the steady state value. In the second scenario, which I call the ”first-best”, I apply the first-best policy described in equations (31), (32), and (33) during the recession with the weights for workers and firms equal to their bargaining power. In the third scenario, which I call ”second-best”, the ST cost is lowered so that the firing rate is the same as in the ”first-best” scenario during the recession. This policy is not the same as the one described in proposition 2 since it does not take into account the deadweight loss generated by a low ST cost. In this setting, all matches that could generate a positive surplus are preserved. I choose this second-best policy because the goal of the policymaker during recessions is often to protect the level of employment rather than to maximise the surplus produced. I find that the ST cost in this setting is 32% of the wage, which was nearly the value of the French ST cost before and after the Covid-19 (34%).

The current programme freezes the labour market during recessions. In this scenario, there is an increase in employment relative to the steady state due to a drop in layoff. The model reproduces that during the Covid-19 crisis, France reduced the ST cost to 0 and observed the lowest job destruction rate and one of the lowest job creation rates of the century. This is due to the fact that each firm can reduce the number of hours worked by its worker to 0 at no cost and the worker receives an ST compensation higher than the unemployment benefit with the insurance of having a job in the next periods. This comes at the public cost of an increase in public expenditure, which will then increase future taxes. The reduction in the mass of unemployed workers plus the future taxes reduce the number of new vacancies created. As described in lemmas 3 and 4, a ST cost set below the second-best level has an immediate and dynamic negative effect on the surplus from a match. The surplus then falls more sharply during recessions and takes longer to return to its steady-state value. The negative immediate effect is compensated by the public expenditure with low ST costs received by firms and high ST compensation received by workers. As a result, the value of the firm falls during the recession at a similar level to the first best scenario and the effect on the value of the worker is small. However, after the recession, the taxes delay the recovery of the firm value and reduce the worker value even more than during the recession. As the STW programme is generous, there is a huge increase in the use of the programme extensively and intensively during the recession, with an associated steep increase in public spending. The negative surplus effect after the shock captures the effect of post-recession austerity, as after the Great Recession (House et al., 2020). In the model, the severity of the austerity depends on the deficit registered during the recession.

The first-best programme maximised the surplus from each match at no public cost. The content of the policy and then its effect on the labour market is very different from the other two policies modelled. In the first-best policy, the recession period has a small effect on employment thanks to a dynamic labour market. A 0.5 deviation in productivity produces a similar deviation in the firing rate (figure **??**), but only a -0.1 deviation in unemployment. The recession has a small effect on unemployment thanks to an increase in the number of vacancies created. In the first-best case, the increase in the mass of unemployed workers and the maximisation of the surplus from a match stimulate the creation of new jobs, which dampens the negative effect of the fall in productivity on employment. The positive effect of the first-best policy on new vacancies is captured by a smaller fall in the value of the firm during the recession and an immediate take-up afterwards. Employee value, on the other hand, falls more sharply because employees are not paid as much as in the first-best scenario. However, it rebounds immediately afterwards with a small increase due to lower taxes in the next period. As productivity falls, the programme is used more extensively and intensively during the recession. The reduction in working hours is small due to the opposite effect of the individual reduction in working hours and the recomposition of the sample of ST workers after the productivity shock. Indeed, during the recession more workers are in ST work, but the average skill of ST workers increases as the mass of workers with skills close to the STW threshold χ^{stw} increases. Even without this effect, each ST worker works more with the first-best policy. Finally, this policy is not costly for the policy maker, as the surplus of each match is fully redistributed without additional spending. This reduces public expenditure relative to the steady state.

The second-best policy is an intermediate point between the first-best and the current programme, i.e. it preserves employment, firm and worker value at a small cost relative to the current programme. The second-best policy lowers the ST cost so that the firing rate is equal to the first-best firing threshold, which is higher than the firing threshold of the current programme. The second-best policy implements the lower bound of Proposition 2, I estimate the second-best ST cost during the recession to be 29% of wages, which is 5 p.p. lower than the steady state value and 29 p.p. higher than the current programme value. In this setting, the level of employment is lower than in the current programme due to the higher firing rate and lower than in the first best due to a less dynamic labour market captured by a small fall in vacancies created. The loss of surplus during the recession is the lowest in the second best scenario because only those matches that generate a positive surplus at a low cost to the firm are retained, while the value of the worker is protected through ST compensation. The proportion of ST workers and the reduction in hours worked are lower than in the current programme because the cost of ST is higher. The rate of ST workers is lower than in the first best because more workers are unemployed. As shown in figure 4, the reduction in hours worked is greater than in the first-best scenario. As a result, the increase in public spending is lower than in the current programme.

Overall, the ST cost was too low during and after the Covid-19 recession in France. This leads to a lower surplus generated by each worker-firm pair, which is partly compensated by public spending. In the steady state, this mainly benefits firms that reduce their labour costs and increase their profits, ceteris paribus. In the dynamic, both workers and firms are losers, as the immediate benefit of low ST costs is offset by future taxation to finance the public deficit. The reduction in ST costs, as in the last two recessions, is necessary but has been too generous. I find that by reducing the ST cost to 0, policymakers are freezing the labour market by maintaining the maximum possible number of matches at the expense of the immediate and future surplus generated by matches. I estimate that a -0.5 deviation in labour productivity should imply a ST cost of at most 0.29% of wages. In France, labour productivity fell by 16% in the first quarter of 2020 (Devulder et al., 2024); in this context, the gap in ST costs and deadweight loss between the second best and the current programme is even larger. In figure B.9, I plot the IRF for a deviation in labour productivity of -0.16, the model reproduces the French labour market features observed during Covid-19 with an 18 p.p. growth in the share of short-time workers in the model vs. a 20 p.p. in the data, and an increase in employment of 0.15 % in the model vs. 0.1% in the data and an increase in hours consumed per worker of 180% in the model vs. 120% in the data.

In figure B.10 I model an autoregressive one standard negative deviation of ST costs. As in the local projection (3), it has a small effect on unemployment, but increases the extensive and intensive use of the programme, resulting in a higher public expenditure effect. The small effect on employment is due to the fact that ST costs are already too low, so the mass of labour-firm pairs below the dismissal threshold is small. In the model, the increase in the share of ST workers is smaller than in the local projection, probably due to a . However, as in the local projection, the reduction in ST costs reduces the number of hours worked and public expenditure. As there is no change in ST compensation, it immediately increases the surplus through the increase in compensation received by workers and the decrease in hours worked. However, the immediate positive effect on the surplus is offset by a larger decrease in the surplus in the next periods due to the increase in taxes. In addition, when ST costs decrease, the number of new vacancies decreases due to a lower number of unemployed workers and a higher future taxation.

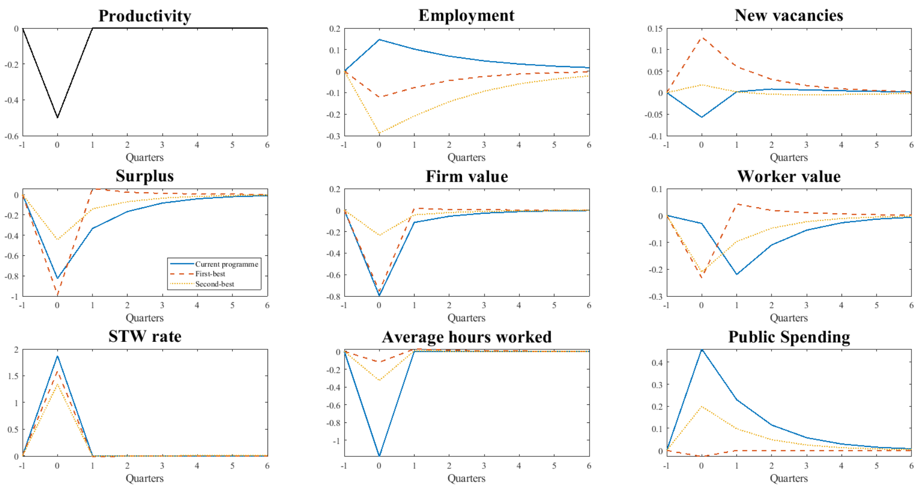

Figure 8: Impulse responses of a negative shock to aggregate productivity

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in aggregate productivity. The blue line corresponds to the scenario where the short-time cost is reduced to 0 during the recession. The yellow line correspond to a scenario where the policy-maker reduce to 30% of the wage during the recession. The red line corresponds to a scenario where the policy-maker implements a first-best policy during the recession. The top left graph shows the productivity shock (common to the 3 scenarios). The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labour force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

6 Conclusion

This paper proposes a theory of optimal STW in a labour market with hours. The optimal STW is defined so that the hours worked by the worker are a ratio between his productivity and his marginal disutility of work which maximises the surplus generated by the worker-firm match. The cost paid by the firm for using the programme and the compensation received by the worker should then be set to share the surplus generated by the match. This optimum implies that the programme should differ between worker-firm pairs according to the productivity loss observed. One way of achieving this equilibrium is to decentralise the STW programme and allow firms and workers to negotiate the costs and compensation of short-time working. If the programme is rigid among workers, it can only lead to a second best with a higher unemployment rate, a lower generated surplus and a higher government deficit.

I derive a second-best policy that solves the trade-off between protecting jobs and reducing working hours. The optimal solution depends on the distribution of productivity across workers. By calibrating the model to French unemployment, short-time work consumption and the short-time work programme in the steady state, I derive a numerical approximation for this second-best solution. I find that the second-best cost of short-time work is twice as high as the one currently implemented in France. As a result, short-time workers work less, produce less and generate larger public deficits for a small increase in the employment level. This finding is supported by local projections in which I estimate the effect of reducing short-time costs on employment, hours worked and public expenditure.

The structural budget deficit of STW programmes increases during recessions. As productivity falls, more workers become eligible for short-time work, thus increasing public expenditure. This effect is accentuated by the fact that STW policies reduce the costs paid by firms during recessions. As a result, each short-time worker works less, more worker-firm pairs enter the programme and firms contribute less to ST compensation. During the Covid-19, the cost paid by the firm in France was reduced to 0. At this point, some worker-firm pairs are maintained, even though they generate a negative surplus, only because of public support.

I focus on the French case because of the availability of data and the age of the programme. However, the French programme followed the European trend of a decrease in the contribution paid by the company and an increase in the compensation received by the employee. Therefore, the results obtained with French data are likely to be reproducible with other European countries. Moreover, the theoretical predictions on the deadweight loss of STW are derived from the common features of the programme and would therefore apply to all countries.

Finally, the paper doesn't address the impact of STW on labour market tightness and redistribution. Instead, it focuses on maximising aggregate surplus. However, I still find that by protecting jobs, the labour market programme affects the probability of finding a job and the rate of job creation. Further work should be devoted to extending the optimal policy to account for this effect. Second, the paper remains agnostic about the social weight assigned to agents by the policymaker. The STW programmes affect the distribution of surplus between workers and firms, and the distribution of surplus among the labour force, through the contributions of firms and the compensation of workers. The optimal policy then depends on the social choice made by the policy maker. At present, I find that STW programmes tend to favour the profit of firms, since their contribution is low compared to the optimal and second-best settings.

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Appendix A Tables

Table A.1: Short-Time Work Evolution in Germany, France, Italy, and Spain

Time period		2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	07-17
Unit of measure	Area												
Public Expend.													
Millions	France	19	15	319	282	67	103	186	212	203	171	141	638
Millions	Germany	491	554	5,166	3,837	1,327	829	1,019	613	709	711	738	50
Millions	Italy	1,353	1,485	4,959	5,796	4,914	6,148	6,791	6,112	4,668	3,721	2,329	72
Millions	Spain	34	63	570	453	442	718	718	407	222	175	159	363
Millions	SUM	1,897	2,116	11,014	10,368	6,750	7,798	8,714	7,344	5,802	4,778	3,367	77
Share of GDP	France	0.001	0.001	0.016	0.014	0.003	0.005	0.009	0.010	0.009	0.008	0.006	500
Share of GDP	Germany	0.020	0.022	0.211	0.150	0.049	0.030	0.036	0.021	0.023	0.023	0.023	15
Share of GDP	Italy	0.084	0.091	0.314	0.360	0.298	0.378	0.421	0.376	0.282	0.219	0.134	59
Share of GDP	Spain	0.003	0.006	0.053	0.042	0.042	0.070	0.070	0.039	0.021	0.016	0.014	366
Share of GDP	MEAN	0.027	0.030	0.149	0.142	0.098	0.121	0.134	0.112	0.084	0.067	0.044	63
Share of GDP	OECD	0.007	0.008	0.043	0.035	0.019	0.025	0.026	0.019	0.016	0.015	0.011	57
Participants													
Persons	France	12,000	41,700	227,100	86,100	36,000	61,800	70,400	63,800	61,300	55,500	35,300	194
Persons	Germany	59,470	90,684	1,117,533	474,235	133,786	99,726	110,711	79,725	74,925	115,862	101,281	70
Persons	Italy	90,540	107,863	358,888	319,809	283,929	329,026	345,166	267,650	190,855	119,910	123,892	36
Persons	Spain	1,794	3,323	13,270	11,874	16,230	25,565	29,842	17,795	10,476	7,320	4,662	159
Persons	SUM	163,804	243,570	1,716,791	892,018	469,945	516,117	556,119	428,970	337,556	298,592	265,135	61
Share of L.F.	France	0.042	0.146	0.788	0.297	0.124	0.211	0.240	0.217	0.208	0.188	0.119	183
Share of L.F.	Germany	0.143	0.217	2.670	1.140	0.325	0.241	0.266	0.190	0.178	0.269	0.234	63
Share of L.F.	Italy	0.371	0.436	1.460	1.300	1.150	1.300	1.370	1.050	0.749	0.465	0.478	28
Share of L.F.	Spain	0.008	0.014	0.057	0.051	0.069	0.109	0.129	0.078	0.046	0.032	0.020	150
Share of L.F.	MEAN	0.141	0.203	1.244	0.697	0.417	0.465	0.501	0.384	0.295	0.239	0.213	50
Share of L.F.	OECD	0.130	0.175	0.691	0.439	0.208	0.291	0.330	0.172	0.164	0.152	0.136	4
Expend./Worker													
Thousands	France	2	0	1	3	2	2	3	3	3	3	4	150
Thousands	Germany	8	6	5	8	10	8	9	8	9	6	7	-12
Thousands	Italy	15	14	14	18	17	19	20	23	24	31	19	25
Thousands	Spain	19	19	43	38	27	28	24	23	21	24	34	78
Thousands	MEAN	11	10	16	17	14	14	14	14	15	16	16	46

Source: LMP database, category 8.2. ('Partial unemployment benefits'). **Note**: Expend. stands for expenditure. L.F. stands for labour force.

Table Appendix A.2: Evolution of the law relative to the STW scheme in France from 2008

Effective date	**Main changes**	**Legal reference**
June 14^{th} 1996	Creation of the STW scheme	*Loi n° 96-502 du 11 juin 1996*
	(…)	
May 1^{st} 2008	Redesign of the STW system that will serve as a basis for all the next reforms until today with a fixed rate of allocation and the eligibility conditions (including the economic condition that I explore in this paper)	*Création Décret n°2008-244 du 7 mars 2008*
May 1^{st} 2008	Redesign of the STW system that will serve as a basis for all the next reforms until today with a fixed rate of allocation and the eligibility conditions (including the economic condition that I explore in this paper)	*Création Décret n°2008-244 du 7 mars 2008*
January 1^{st} 2009	Upgrade of STW allocation to workers from 50% to 60% of the wage Raise of the maximum STW consecutive period allowed (up to 6 weeks) Raise of the maximum STW period allowed (up to 1 000 hours per year for some sectors) Firms STW subsidy per hour is 3.84€ (vs 2.44€ before, if $\leq$250 employees) Firms STW subsidy per hour is 3.33€ (vs 3.13€ if $>$250 employees)	*Avenant du 15 décembre 2008* *Décret du 22 décembre 2008* *Arrêté du 30 décembre 2008* *Décret n°2009-110 du 29 janvier 2009*
May 1^{st} 2009	Creation of long-time STW (*APLD*) Min-Max duration: 3-12 months Compensation raised at 75% of gross hourly wage Firms subsidy per hours increased up to 7.74€	*Décret n°2009-478 du 29 avril 2009 et convention Etat-Unédic du 1er mai 2009*
January 1^{st} 2010	The maximum STW period allowed is generalized for all sectors and all workers	*Arrêté du 31 décembre 2009 et ANI du 8 juillet 2008*
March 1^{st} 2012	Firms do not need to ask for a pre-authorization to consume STW Firm subsidy per hour is 4.84€ (6.74€ for ADLP) Long-period STW is now financed by *unédic* The minimum time for a long-period STW take-up is lowered down to 2 months	*Décret n°2012-183 2012 et arrêté du 4 mai 2012* *Décret n°2012-275*
November 1^{st} 2012	Firms not need to wait for a pre-authorization to consume STW	*Décret n°2012-1271*
July 1^{st} 2013	Merge of the "classic" STW program and the long-period STW program Firms STW subsidy per hour is 7.74€ (if $\leq$250 employees) Firms STW subsidy per hour is 7.23€ (if $>$250 employees) Downgrade of STW allocation to workers down to 70% of the wage	*Loi n°2013-504 and décret n°2013-551*

Table Appendix A.1: Evolution of the law relative to the STW scheme in France from 2008 to 2024, cont'd

July 2^{nd} 2014	The application process is now an online procedure	*Décret n° 2014-740*
March 25^{th} 2020	The state finance the ST compensation up to 4,5 times the minimum wage	*Décret n°2020-325*
January 1^{st} 2021	Downgrade of STW allocation to worker down to 60% of the wage	*Décret n°2020-1316*
February 1^{st} 2021	The state finance the STW at 36% of the wage (remaining 34% is paid by the firm)	*Décret n°2020-1319*
May 1^{st} 2022	Firms STW subsidy per hour is 7.73€	*Décret n° 2022-654*
August 1^{st} 2022	Firms STW subsidy per hour is 7.88€	*Décret n° 2022-1072*
January 1^{st} 2024	Firms STW subsidy per hour is 8.30€ STW allocation to worker is 36% of the wage	*Décret n° 2023-1305*

Notes: This table does not display the directives sent by the government during the periods that played a major role in STW take-up. Also, this table does not present the evolution before 2008. French legislation.

Appendix B Figures

B.1 Short-Time Work facts

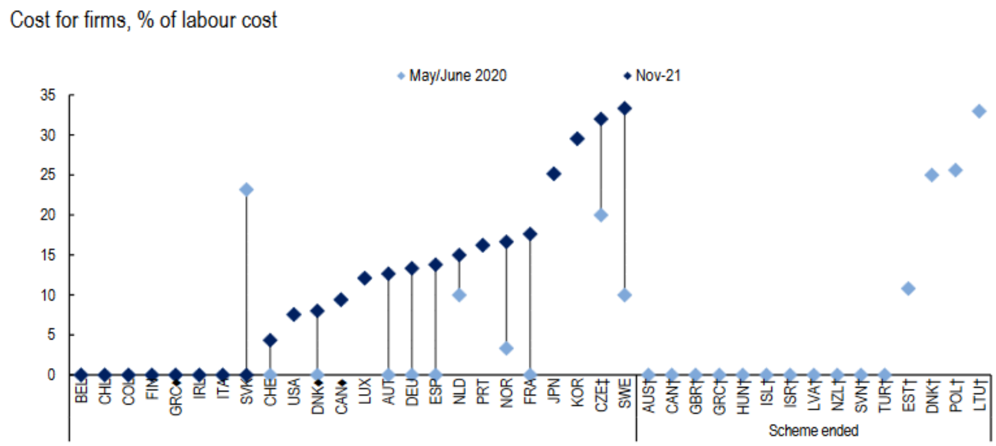


Figure B.1: Short-time cost paid by firms

Source: OECD. Country answers and ad hoc updates to OECD Policy Questionnaire on Working Time Regulation and Short-Time Work Schemes. Note: ‡ Czech Republic: For November 2021, refer to Antivirus, regime B. During the Crisis, Antivirus, regime 3A. Canada: There are two schemes, the Canada Emergency wage subsidy, ended in November 2021, and the Work-sharing programme (indicated as ⧫), ongoing. Denmark: There are two schemes, the system of division of labour (Arbejdsfordeling, indicated as ⧫), that was temporary redesigned and the Wage compensation scheme (Lønkompensation), ended in June 2021. Greece: There are two schemes, the Special purpose compensation, restricted to some specific sectors and Syn-Ergasia, (indicated as ⧫), ongoing. † Schemes no longer operational or not widely available. Mandatory employer contributions for private insurance are not taken into account (consistent with the OECD methodology of Taxing Wages). Norway: for the first 3 months (60 days).

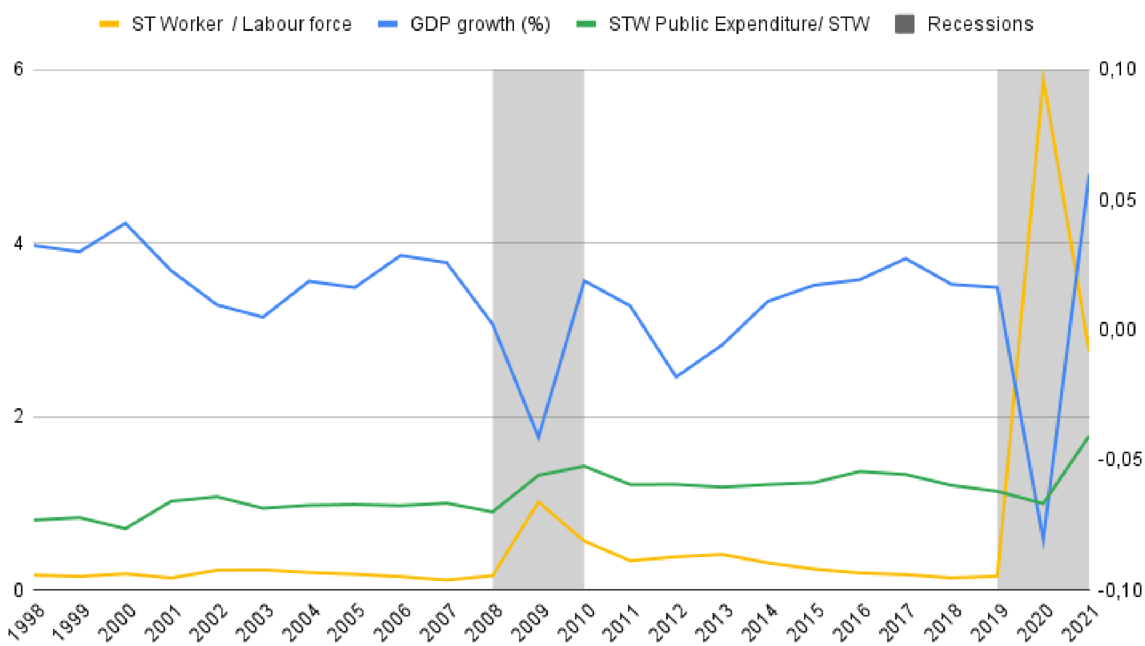
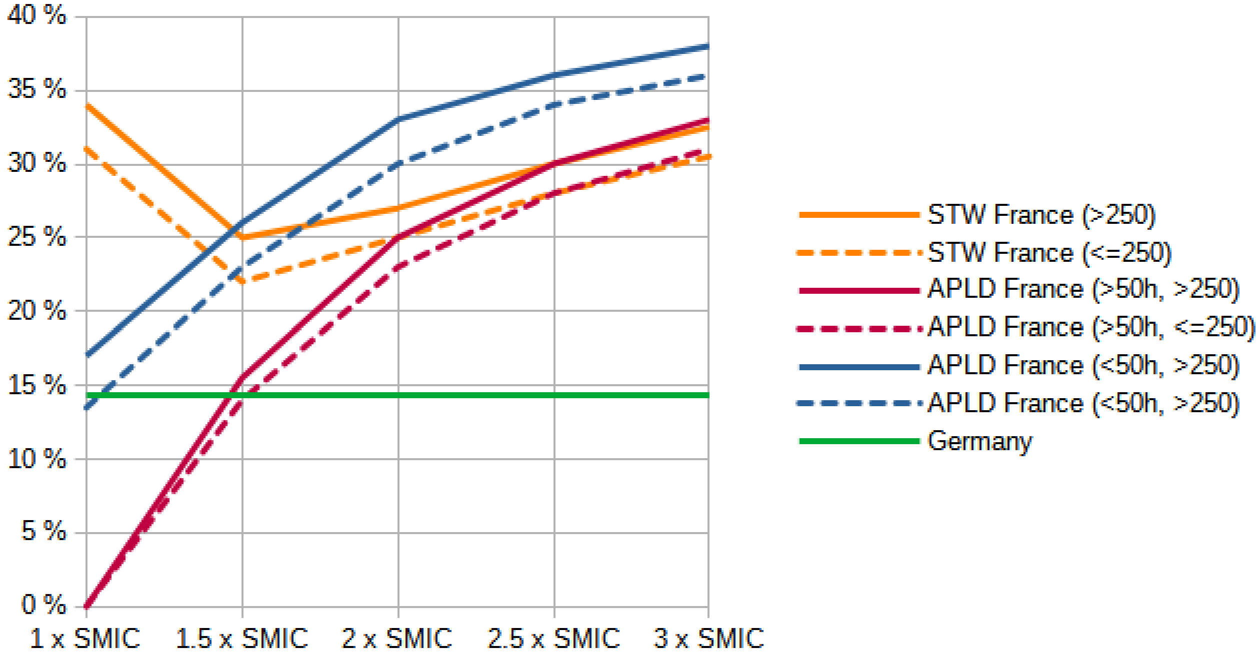


Figure B.2: Evolution of STW use in Europe

Source: OECD. The graph plots the average GDP growth (blue line, right axis), public STW expenditure per worker (green line, right axis), and share of short-time worker in the labour force (yellow line, left axis) for 6 European countries: France, Germany, Italy, Portugal, Spain, and Belgium. Grey areas correspond to recessions.

Figure B.3: Share of ST subsidy paid by the firm in France and Germany in 2009



The y-axis is the share of STW subsidy received by the worker that is paid by the firm. The x-axis is the level of wage of the worker measured according to the French minimum wage (SMIC). The solid blue line corresponds to the *classic* French STW program for firms with more than 250 employees. The dashed blue line corresponds to the *classic* French STW program for firms with less than 250 employees. The solid orange line corresponds to the French long-term STW (ADLP) for the 50-first hours for firms with more than 250 employees. The dashed orange line corresponds to the French long-term STW (ADLP) for the 50-first hours for firms with less than 250 employees. The solid magenta line corresponds to the long-term STW (ADLP) after the 50-first hours for firms with more than 250 employees. The dashed magenta line corresponds to the French long-term STW (ADLP) after the 50-first hours for firms with more than 250 employees. The green line corresponds to the German STW program (after the 07.09)

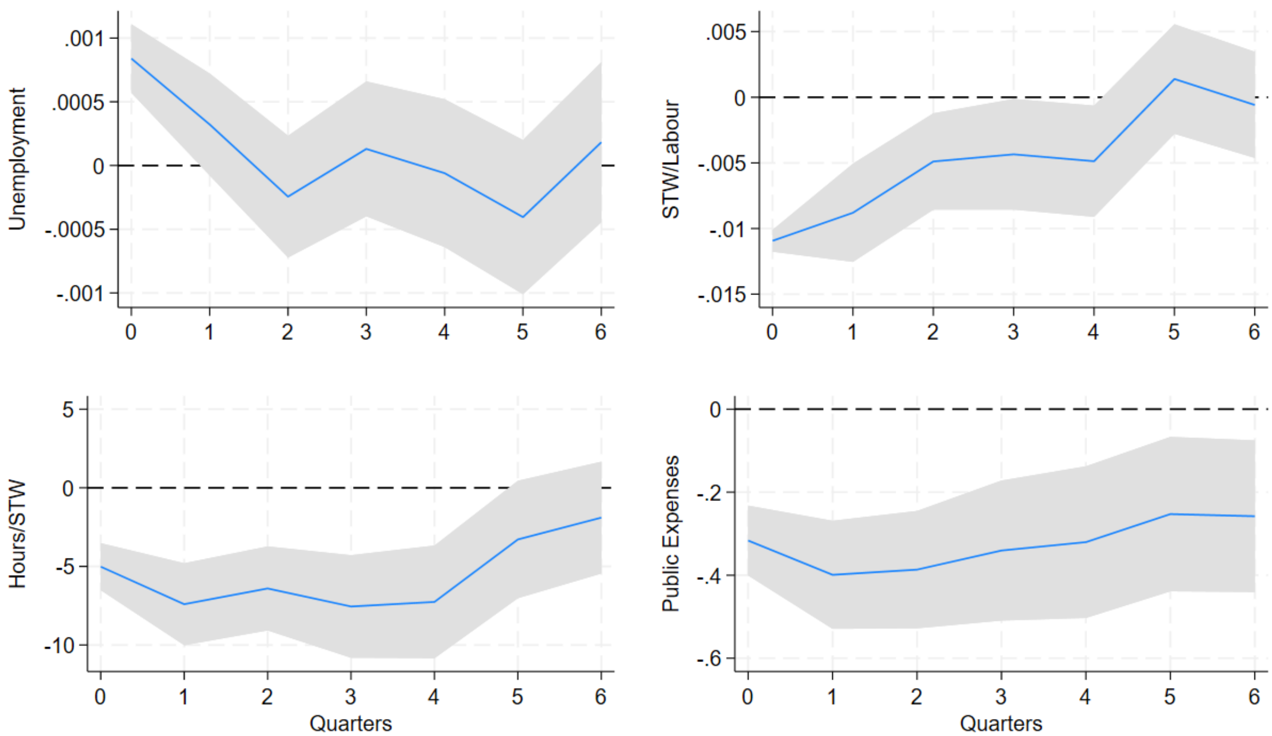


Figure B.4: Local-projection: Impulse responses to output shock

Impulse responses to a GDP growth shock. Local projection estimated with unemployment rate, number short-time worker divided by the labour force, the number of hours consummed per workers, and the log of total public expenditure for 2008Q1 to 2024Q1. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

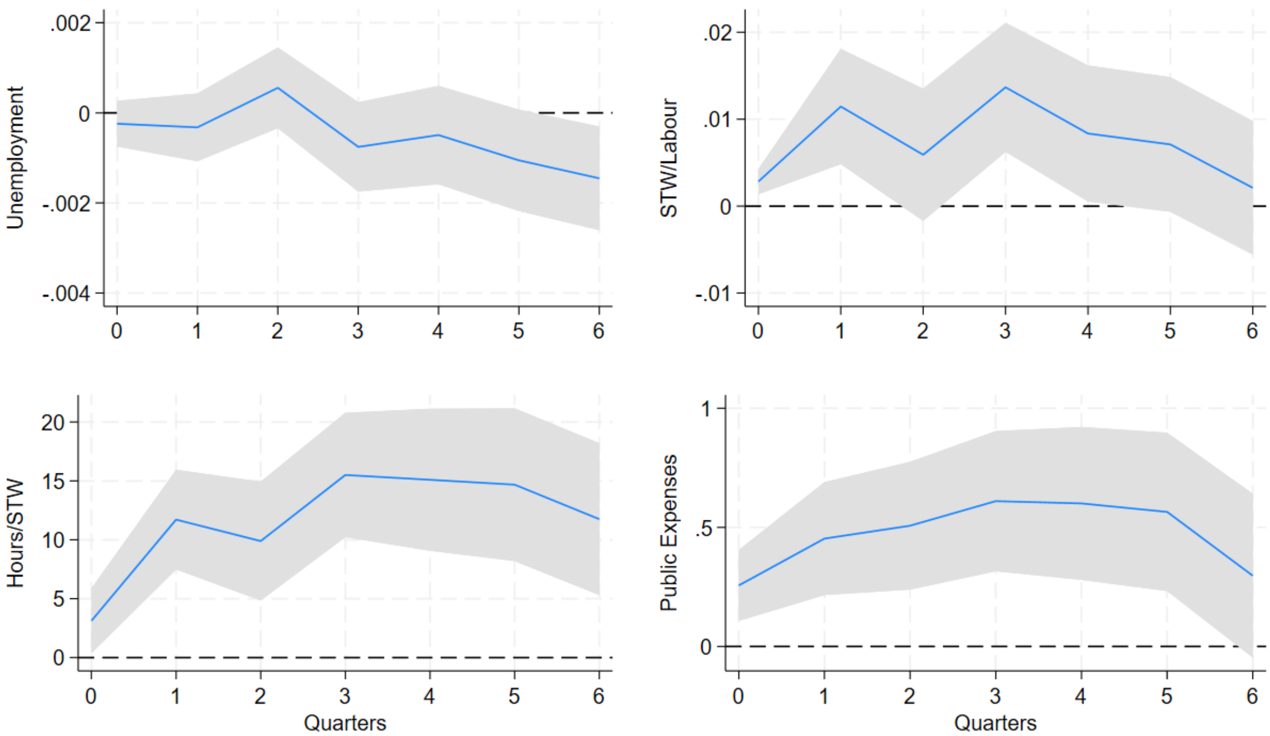


Figure B.5: Local-projection: Impulse responses to STW policies

Impulse responses to a public expenditure per hour of STW shock. Local projection estimated with unemployment rate, number short-time worker divided by the labour force, the number of hours consummed per workers, and the log of total public expenditure for 2008Q1 to 2024Q1. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

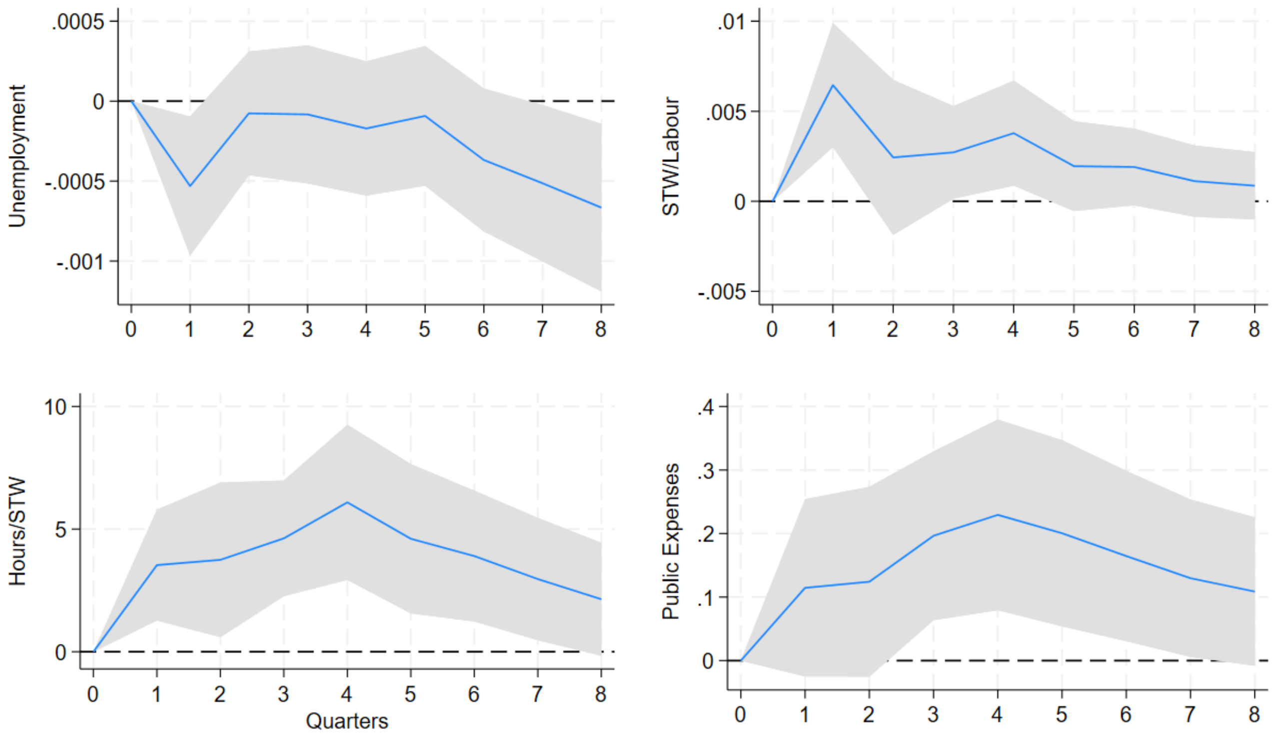


Figure B.6: VAR: Impulse responses to STW policies

Impulse responses to a public expenditure per hour of STW shock. VAR estimated with unemployment rate, number short-time worker divided by the labour force, the number of hours consummed per workers, and the log of total public expenditure for 2008Q1 to 2024Q1. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

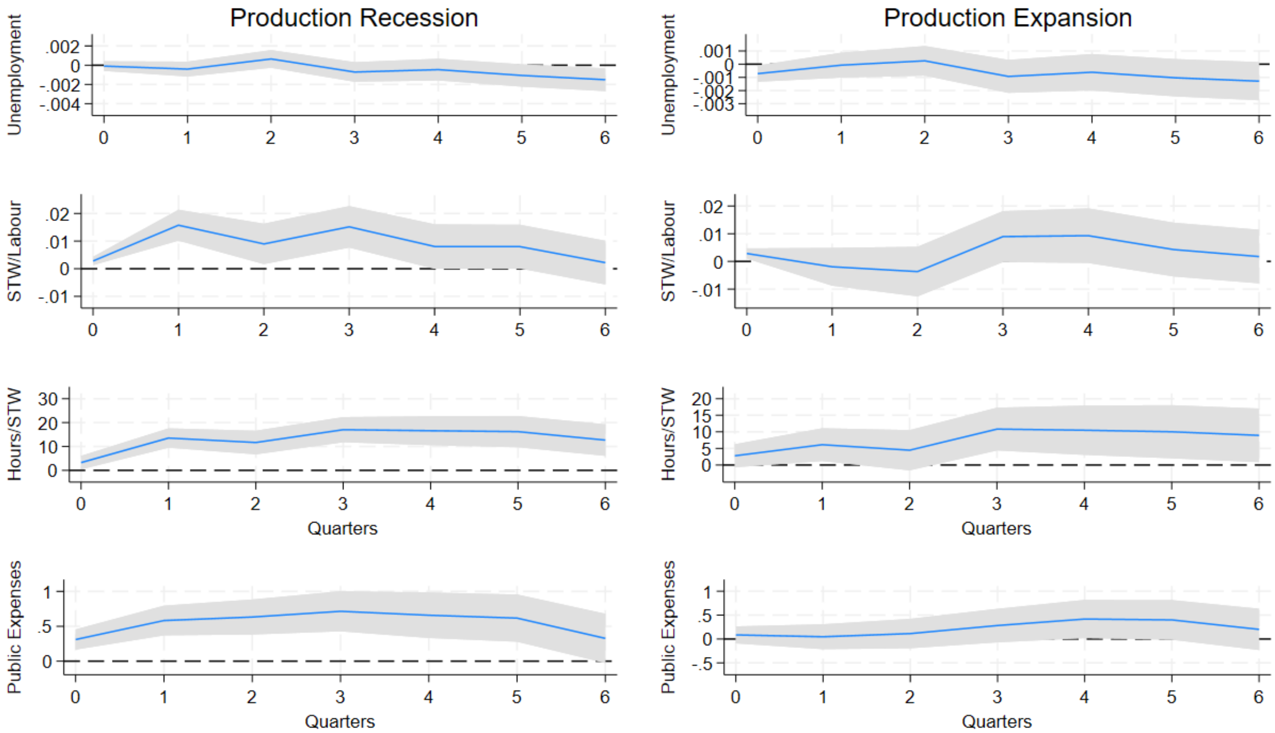


Figure B.7: Local-projection: Impulse responses to STW public expenditure per hour

Impulse responses to a STW policy shock. Local projection estimated with unemployment rate, log of number of short-time worker, log hours consummed per workers, and log of total public expenditure for 2008Q1 to 2023Q4. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

I explore whether initial economic conditions at the time of the shock influence its effect on macroeconomic outcomes. I implement this by allowing the response to vary as follows:

$$Y_{t+k} = \tau_k + \sum_{i}^{k} \beta_i^L F(z_i) R_{t-1-i} + \sum_{i}^{k} \beta_i^H (1 - F(z_i)) R_{t-1-i} + \sum_{i}^{k} \beta_i Y_{t-1-i} + \varepsilon_t \qquad \text{(B.1)}$$

with $F(z_i) = \frac{exp(-\gamma z_i)}{1+exp(-\gamma z_i)}$

in which z_i is an indicator of economic activity (proxied by GDP growth) normalized to have zero mean and unit variance. R_{t-1-i} denotes the reform shock. The coefficients β_i^L and β_i^H capture the trade impact of reform shocks at each horizon k in cases of recessions ($F(z_i) \approx 1$ when z goes to minus infinity) and expansions ($1 - F(z_i) \approx 1$ when z goes to plus infinity), respectively. I follow Duval et al. (2020) and choose $\gamma = 1.5$.

B.2 Numerical simulation

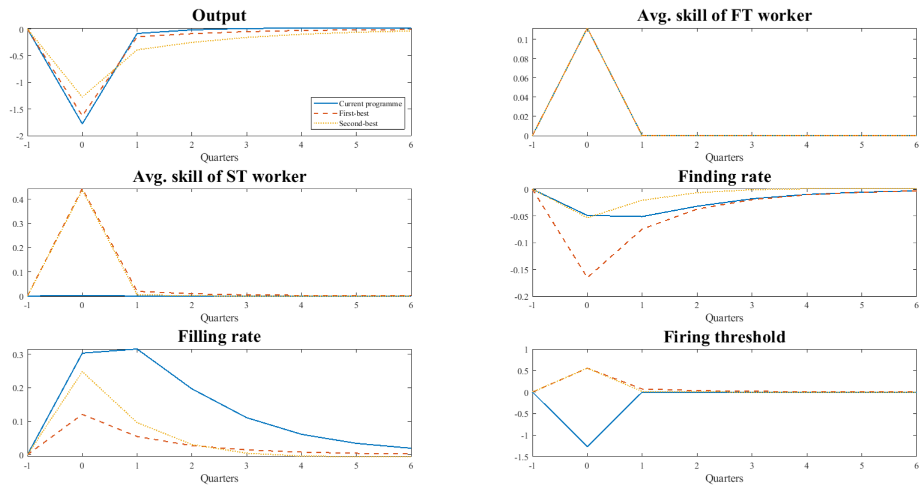


Figure B.8: Impulse responses of a negative productivity shock

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in aggregate productivity. The blue line corresponds to the scenario where the short-time cost is reduced to 0 during the recession. The yellow line correspond to a scenario where the policy-maker reduce to 30% of the wage during the recession. The red line corresponds to a scenario where the policy-maker implements a first-best policy during the recession. The top left graph shows the productivity shock (common to the 3 scenarios). The top left graph shows the short-time cost shock. The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labour force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

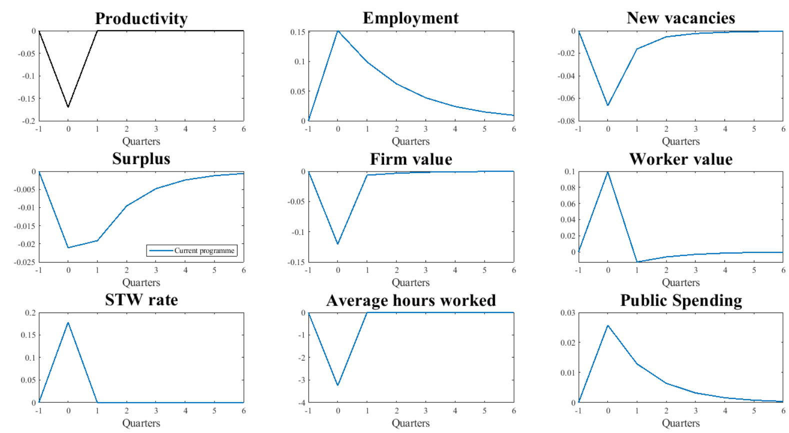


Figure B.9: Impulse responses of a negative shock to productivity shock

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in aggregate productivity. The top left graph shows the output response function. The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labour force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

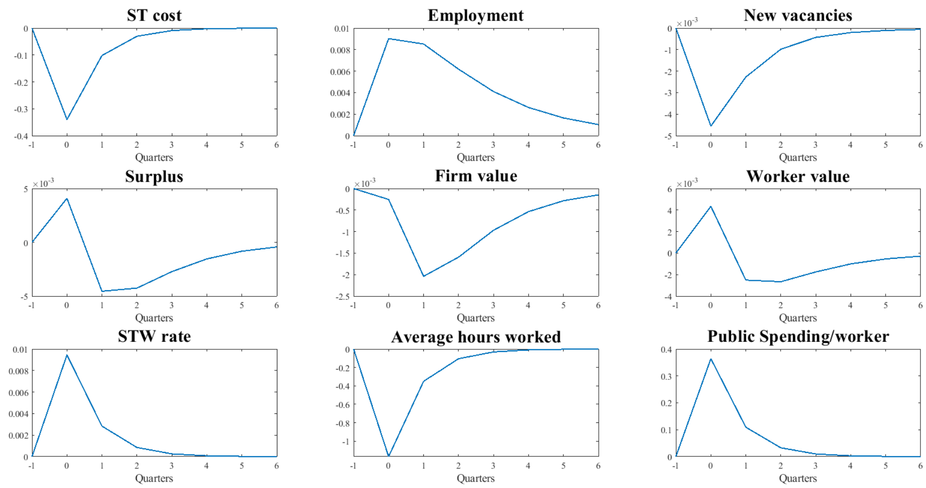


Figure B.10: Impulse responses of a negative shock to short-time cost

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in short-time cost. The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labour force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

Appendix C Proof

C.1 Lemma 1

Recalling the firing threshold:

$$J_t^f - J_t^v = -c_f \Leftrightarrow$$

$$\chi_t^f = \left[\frac{\alpha}{1-\alpha}\big(-c_f + b_t + J_t^v + \sigma(1-\rho)\mathbb{E}[(1-\phi_{t+1})J_{t+1}^f + \phi_{t+1}J_{t+1}^v]\big)\right]^{\frac{1-\alpha}{\alpha}} \ell_t$$

Where χ_t^f is the lowest level of productivity θ at which, all else being equal, the worker is retained. I use the implicit function theorem to find the sign of the first derivative with respect to b_t. I write $F(\theta, b_t) = J_t^f - J_t^v = -c_f$ and examine the sign of the first derivative at the point described by the equation above.

$$\begin{aligned}\frac{\partial J_t^f - J_t^v}{\partial \theta}|_{(\chi_t, b_t)} &= -\theta^{\frac{\alpha}{1-\alpha}-1}\ell_t^{-\frac{\alpha}{1-\alpha}} \\ &= -\chi_t^{f\,\frac{\alpha}{1-\alpha}-1}\ell_t^{-\frac{\alpha}{1-\alpha}}\end{aligned}$$

I assume $\chi_t^f > 0$ and $\ell_t > 0$. With $\chi_t^f > 0$ implying that there is endogenous firing in the model and $\ell_t > 0$ implying that the ST cost is lower than the wage paid by the firm $\tau_t + w_t$, the reverse would imply that there is no monetary incentive for the firm to participate in the STW programme (to the best of my knowledge, no country has ever set the ST cost equal to or greater than the wage). Then $\frac{\partial J_t^f - J_t^v}{\partial \theta}|_{(\chi_t, b_t)} < 0$. Now the first derivative with respect to b_t is

$$\begin{aligned}\frac{\partial J_t^f - J_t^v}{\partial b_t}|_{(\chi_t, b_t)} &= \theta^{\frac{\alpha}{1-\alpha}}\ell_t^{-\frac{\alpha}{1-\alpha}-1} - 1 \\ &= h_t - 1\end{aligned}$$

By definition the number of hours worked is lower or equal to one so $\frac{\partial J_t^f - J_t^v}{\partial b_t}|_{(\chi_t, b_t)} < 0$. Thus

$$\frac{\partial \chi_t^f}{\partial b_t} = -\frac{\frac{\partial J_t^f - J_t^v}{\partial b_t}|_{(\chi_t, b_t)}}{\frac{\partial J_t^f - J_t^v}{\partial \theta}|_{(\chi_t, b_t)}} \leq 0 \; \square$$

C.2 Proposition 1

The Hamiltonian of the policy-maker program for worker-firm pairs on STW is as follows

$$\mathcal{H} = S_t^f(\theta)f(\theta) + \lambda(\theta)\left[\tau h(\theta) + (b_t - a_t)(1 - h(\theta)\right] f(\theta) \tag{C.1}$$

With $S_t^f(\theta)$ the surplus of a match with productivity (θ) the the sum of the worker's and the firm's surplus $W_t - U_t + J_t^f - J_t^v$ defines in equations (1), (2) (6), (7), and $h_t(\theta)$ the number of hours worked defines in equation (3). With lemma 2, the costate variable equals to one $\lambda(\theta) = 1$, the Hamiltonian becomes

$$H = \frac{1}{\alpha}(\theta h_t)^\alpha - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}[J_{t+1}^f + W_{t+1}] - U_t - J_t^v$$

The Hamiltoninian is know equal to the maximization of the surplus from a match S_t^f with the ST compensation sets to equal the ST cost $a_t = b_t$ and the labor tax sets to 0 τ_t.

$$\begin{aligned} S_t^f|_{(a=b,\tau=0)} &= \frac{1}{\alpha}(\theta h_t)^\alpha - (w_t + \tau_t)h_t - b_t(1-h_t) + w_t h_t + a_t(1-h_t) - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}[J_{t+1}^f + W_{t+1}] - U_t - J_t^v \\ &= \frac{1}{\alpha}(\theta h_t)^\alpha - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}[J_{t+1}^f + W_{t+1}] - U_t - J_t^v \end{aligned}$$

Then, the FOC with respect to the ST cost b_t is

$$\frac{\partial \mathcal{H}}{\partial b_t} = \frac{\partial h_t}{\partial b_t}(\theta^\alpha h_t^{\alpha-2} - 2\beta) = 0$$

Noticing that the derivative with respect to the hours worked is different from 0

$$\frac{\partial h_t}{\partial b_t} = \frac{1}{1-\alpha}(\theta)^{\frac{\alpha}{1-\alpha}} \ell_t^{\frac{-1}{1-\alpha}-1} \neq 0$$

Then, the FOC solution with respect to the ST cost b_t solves

$$\frac{\partial \mathcal{H}}{\partial b_t} = (\theta^\alpha h_t^{\alpha-2} - 2\beta) = 0$$

which is the same problem of the FOC with respect to the hours worked

$$\frac{\partial \mathcal{H}}{\partial h_t} = (\theta^{\alpha} h_t^{\alpha-2} - 2\beta) = 0$$

Proposition 1 is proved □

C.3 Corollary 1

From proposition 1, the optimal number of hours worked is $h_t^* = \arg\max_{h_t} \left\{ S_t^f \mid a_t = b_t,\ \tau_t = 0 \right\}$, from the surplus equation (11), the surplus from a match without intervention of the policy-maker is $S_t^f|_{\tau=0,a=b} = \frac{1}{\alpha}(\theta h_t)^{\alpha} - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}[J_{t+1}^f + W_{t+1}] - U_t - J_t^v$. Then, from the FOC with respect to hours worked h_t I find the optimal number of hours worked (28). Finally, from the equation of hours worked as set by the firm (3) $h_t = \left(\frac{\theta_t^{\alpha}}{\ell_t^*}\right)^{\frac{1}{1-\alpha}}$ and the solution (28), I find the optimal labor-cost (29). □

C.4 Proposition 2

The SWF (25) reaches its maximum level if and only if every and only worker-firm pairs for which the surplus is positive $S_t^f(\theta) > 0$ are preserved. A match is preserved if the leaving threshold (8) $W_t - U_t \geq 0$ and the firing threshold is satisfied (5) $J_t^f - J_t^v \geq 0$. With equation (30) every and only match generating a positive surplus are preserved □

C.5 Lemma 2

The optimal Nash bargaining program (13) after log-linearisation:

$$\max_{(w_t,a_t)} p \ln(W_t - U_t) + (1-p)\ln(J_t^f - J_t^v)$$

The F.O.C. is:

$$\frac{\partial}{\partial w_t} = 0 \Leftrightarrow$$

$$p\frac{W'_w}{W_t - U_t} + (1-p)\frac{{J^f}'_w}{J_t^f - J_t^v} = 0$$

$$-\frac{W'_w}{(W_t - U_t)}\frac{J_t^f - J_t^v}{{J^f}'_w} = \frac{1-p}{p}$$

With $W'_w = \frac{\partial W_t}{\partial w_t}$ and $J^f{}'_w = \frac{\partial J_t^f}{\partial w_t}$

$$\frac{\partial}{\partial a_t} = 0 \Leftrightarrow$$

$$\begin{aligned}
p\frac{W'_a}{U} + (1-p)\frac{J^f{}'_a}{J_t^f - J_t^v} &= 0 \\
-\frac{W'_a}{(W_t - U_t)}\frac{J_t^f - J_t^v}{J^f{}'_a} &= \frac{1-p}{p} \\
-\frac{W'_a}{(W_t - U_t)}\frac{J_t^f - J_t^v}{J^f{}'_a} &= -\frac{W'_w}{(W_t - U_t)}\frac{J_t^f - J_t^v}{J^f{}'_w} \\
-\frac{W'_a}{J^f{}'_a} &= -\frac{W'_w}{J^f_{'w}} \\
W'_a J^f{}'_w - J^f{}'_a W'_w &= 0
\end{aligned}$$

Recalling that:

$$W_t = w_t h_t + a_t(1-h_t) - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right]$$

and

$$J_t^f = \frac{1}{\alpha}(\theta_t h_t)^\alpha - (\tau_t + w_t)h_t - b_t(1-h_t) + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right]$$

and

$$h_t = \left(\frac{\theta^\alpha}{\tau_t + w_t - b_t}\right)^{\frac{1}{1-\alpha}}$$

Then with $\ell_t = w_t - b_t$ and $a_t = b_t$ I have

$$W_t = \left(\frac{\theta}{\ell_t}\right)^{\frac{\alpha}{1-\alpha}} + a_t - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right]$$

and

$$J_t^f = \frac{1-\alpha}{\alpha}\left(\frac{\theta}{\ell_t}\right)^{\frac{\alpha}{1-\alpha}} - a_t + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right]$$

The derivatives with respect to w_t and a_t are:

$$\frac{\partial h_t}{\partial w_t} = h'_w = -\frac{1}{1-\alpha} h_t \ell_t^1$$
$$\frac{\partial h_t}{\partial a_t} = h'_a = -h'_w$$
$$\frac{\partial h_t^2}{\partial w_t} == h'_{2w} = -\frac{2}{1-\alpha} \beta h_t^2 \ell_t^{-1}$$
$$\frac{\partial h_t^2}{\partial a_t} = h'_{2a} = \frac{2}{1-\alpha} \beta h_t^2 \ell_t^{-1}$$
$$\frac{\partial W_t}{\partial w_t} = h'_w \ell_t + h_t + \frac{2}{1-\alpha} \beta h_t^2 \ell_t^{-1} = -\frac{\alpha}{1-\alpha} h_t + \frac{2}{1-\alpha} \beta h_t^2 \ell_t^{-1}$$
$$\frac{\partial W_t}{\partial a_t} = h'_a \ell_t + 1 - h_t - \frac{2}{1-\alpha} \beta h_t^2 \ell_t^{-1} = \frac{\alpha}{1-\alpha} h_t - 1 - \frac{2}{1-\alpha} \beta h_t^2 \ell_t^{-1}$$
$$\frac{\partial J_t^f}{\partial w_t} = -h_t$$
$$\frac{\partial J_t^f}{\partial a_t} = h_t - 1$$

Thus, the F.O.C. w.r.t. a_t becomes

$$\frac{\partial}{\partial a_t} = 0 \Leftrightarrow$$

$$\begin{aligned}
W'_a {J^f}'_w - {J^f}'_a W'_w &= 0 \\
\left(\frac{\alpha}{1-\alpha} h_t + 1 + \beta h'_{2w}\right)(h_t) - \left(\frac{\alpha}{1-\alpha} h_t + \beta h'_{2w}\right)(h_t - 1) &= 0 \\
\frac{\alpha}{1-\alpha} h_t + 1 + \beta h'_{2w} - \frac{\alpha}{1-\alpha} h_t - \beta h'_{2w} - \frac{\alpha}{1-\alpha} - \frac{\beta h'_{2w}}{h_t} &= 0 \\
1 - \frac{\alpha}{1-\alpha} + \beta \frac{h'_{2w}}{h_t} &= 0 \\
\frac{1}{1-\alpha} + \beta \frac{2}{1-\alpha} (\theta)^{\frac{2\alpha}{1-\alpha} - \frac{\alpha}{1-\alpha}} \ell_t^{-\frac{2}{1-\alpha} - 1 + \frac{1}{1-\alpha}} &= 0 \\
\ell_t^{\frac{2-\alpha}{1-\alpha}} &= 2\beta\theta^{\frac{\alpha}{1-\alpha}} \\
\ell_t^* &= \left(2\beta\theta^{\frac{\alpha}{1-\alpha}}\right)^{\frac{1-\alpha}{2-\alpha}}
\end{aligned}$$

Re-inserting this into h_t, I have:

$$h_t^* = \left(\frac{\theta^\alpha}{2\beta}\right)^{\frac{1}{2-\alpha}}$$

With h_t^*, the F.O.C. w.r.t. w_t is now:

$$\frac{\partial}{\partial w_t} = 0 \Leftrightarrow$$

$$\begin{aligned}
p\frac{W'_w}{W_t - U_t} + (1-p)\frac{{J^f}'_w}{J_t^f - J_t^v} &= 0 \\
p\frac{h_t}{W_t - U_t} + (1-p)\frac{-h_t}{J_t^f - J_t^v} &= 0 \\
p(J_t^f - J_t^v) &= (1-p)(W_t - U_t) \\
W_t - U_t &= pS_t^f \\
J_f^t - J_t^v &= S_t^f
\end{aligned}$$

Lemma 2 is proved □

C.6 Corollary 3

From proposition 2:

$$\begin{aligned}
a_t^*(\theta) &:= W_t - U_t = k^1 S_t^f(\theta) \\
b_t^*(\theta) &:= J_t^f - J_t^v = k^2 S_t^f(\theta) \\
\tau_t^{stw*} &= \ell_t^* + b_t^* - w_t
\end{aligned}$$

With the firm's and worker's match value (1) (6):

$$\begin{aligned}
a_t^*(\theta) &= \frac{1}{1-h_t}\left(k^1 S_t^f(\theta) - w_t h_t + \beta(h_t)^2 - \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right] + U_t\right) \\
-b_t^*(\theta) &= k^2 S_t^f(\theta) - \frac{1}{\alpha}(\theta_t h_t)^\alpha + \ell_t h_t + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right] + J_t^v \\
\tau_t^{stw*} &= \ell_t^* + b_t^* - w_t
\end{aligned}$$

Then the deficit generated for a worker with productivity draw θ:

$$
\begin{aligned}
&\tau_t^{stw} h_t - (a_t - b_t)(1 - h_t) \\
&= (\ell_t^* + b_t^* - w_t)h_t - a_t - b_t + b_t h_t \\
&= (\ell_t^* - w_t)h_t - \left(k^1 S_t^f(\theta) + w_t h_t - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right] + U_t\right) \\
&- \left(k^2 S_t^f(\theta) - \frac{1}{\alpha}(\theta_t h_t)^\alpha + \ell_t h_t + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right] + J_t^v\right) \\
&= \frac{1}{\alpha}(\theta h_t)^\alpha - \beta(h_t)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right] - U_t \\
&+ \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right] - J_t^v - k^1 S_t^f(\theta) - k^2 S_t^f(\theta) \\
&= S_t^f(\theta) - (k^1 + k^2) S_t^f(\theta)
\end{aligned}
$$

Then it follows that

$$
\tau_t^{stw} h_t - (a_t - b_t)(1 - h_t) = S_t^f(\theta) - (k^1 + k^2) S_t^f(\theta) \begin{cases} < 0 \text{ if } k^1 + k^2 < 1 \\ = 0 \text{ if } k^1 + k^2 = 1 \\ > 0 \text{ if } k^1 + k^2 > 1 \end{cases} \tag{C.2}
$$

Thus

$$
\tau_t^{f*} = \frac{1}{T-t}\left[\int_{\chi_t}^{\chi^{stw}} (a_t^* - b_t^*)(1 - h_t^*) dF(\theta) - \int_{\chi_i}^{\chi^{stw}} \tau_t^{stw*} h_t^* dF(\theta)\right] \begin{cases} < 0 \text{ if } k^1 + k^2 < 1 \\ = 0 \text{ if } k^1 + k^2 = 1 \\ > 0 \text{ if } k^1 + k^2 > 1 \end{cases} \tag{C.3}
$$

Corollary 3 is proved □

C.7 Proposition 3

I derive the Lagrangian according to the ST compensation a_t and find that at the optimum the Lagrange multiplier $\lambda = 1$ as in Lemma 2. I rearrange the Lagrangian with the value of λ and derive it according to the short-time cost b_t using the Leibniz rule to find the FOC:

$$
-S^f(\chi_t^f)\frac{d}{db_t}\chi_t^f + \int \frac{d}{db}\{S^f(\theta)\} dF(\theta) = 0 \tag{C.4}
$$

From lemma 1, the first term is negative and the second term is positive for $\theta \in [\chi_t^{f*}; 1]$. The first term is 0 when $b_t = b_t^*(\chi_t^{f*})$. I note that the second term is equal to

$$(\chi^{stw} - \chi_t^f)\tilde{s}_t^f(\theta)$$

with $s_t^f(\theta) = \int \frac{\partial S_t^f}{\partial b} dF(\theta)$, and $\tilde{s}_t^f = \frac{1}{\chi^{stw} - \chi_t^f} \int_{\chi_t^f}^{\chi^{stw}} s_t^f dF(\theta)$. I define $k := s(k) = \frac{1}{\chi^{stw} - \chi_t^f} \int_{\chi_t^f}^{\chi^{stw}} s_t^f(\theta) dF(\theta)$. It follows that the second term is 0 for $b_t = b_t^*(k)$. The exact value of k depends on the shape of the distribution function $F(\theta)$. Still, I find that $\frac{1}{\chi^{stw} - \chi_t^f} \int_{\chi_t^f}^{\chi^{stw}} s_t^f dF(\theta) < \int_{\chi_t^f}^{\chi^{stw}} s_t^f(\theta) dF(\theta)$, which implies that the second term is 0 for $b_t = b_t^*(k) < b_t^*(\tilde{\theta})$. It is easy to see that then $b_t^*(k) > b_t^*(\chi_t^{f*})$, as the first term is 0 for $b_t = b_t^*(\chi_t^{f*}) < b_t^*(k)$ and decreases with b_t, while the second term is 0 for $b_t = b_t^*(k) < b_t^*(\tilde{\theta})$ and increases with b_t, If both terms are continuous in b_t, this implies that the value of b_t that solves the second best problem belongs to the interval $[b_t^*(\chi_f^*);\ b_t^*(\tilde{\theta})]$ □

D Numerical Simulation

D.1 Equilibrium Condition

The search equilibrium is a system of 17 equations for 17 variables summarized in the vector:

$$\left[J_t^f, J_t^v, \chi_t^f, W_t, U_t, w_t, m_t, u_t, N_t, \phi, \phi_t^e, b_t, n_t, q_t^u, q_t^v, G_t, Y_t\right]$$

The average filling value

$$J_t^f = \int_{\chi_t^f}^{\chi_t^{stw}} \left\{\frac{1}{\alpha}(\theta_t h_t)^\alpha - (\tau_t + w_t)h_t - b_t(1-h_t) - f_c\right\} dF(\theta) + \int_{\chi_t^{stw}}^{1} \left\{\frac{1}{\alpha}\theta_t^\alpha - (\tau_t + w_t + f_c)\right\} dF(\theta) \quad \text{(D.1)}$$
$$- (1 - \phi_t^{stw} - \phi_t^f)l_c + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right]$$

The vacancy value

$$J_t^v = -\kappa + \sigma(1-\rho)\mathbb{E}\left[q_t^v J_{t+1}^f + (1-q_t^v)J_{t+1}^v\right] \quad \text{(D.2)}$$

The firing threshold

$$\chi_t^f = \left[\frac{\alpha}{1-\alpha}\big(-l_c + b_t + f_c + J_t^v + \sigma(1-\rho)\mathbb{E}[(1-\phi_{t+1})J_{t+1}^f + \phi_{t+1}J_{t+1}^v]\big)\right]^{\frac{1-\alpha}{\alpha}} \ell_t \quad \text{(D.3)}$$

The average employment value

$$W_t = \int_{\chi_t^f}^{\chi_t^{stw}} \left\{w_t h_t + a_t(1-h_t) - \beta(h_t)^2\right\} dF(\theta) + \int_{\chi_t^{stw}}^{\chi_t^1} \{w_t - \beta\} dF(\theta) \quad \text{(D.4)}$$
$$+ \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})W_{t+1} + \phi_{t+1}U_{t+1}\right]$$

The unemployment value

$$U_t = u_b + \sigma\mathbb{E}\left[q_t^u W_{t+1} + (1-q_t^u)U_{t+1}\right] \quad \text{(D.5)}$$

The wage

$$w_t = p\left(\frac{1}{\alpha}\theta_t^\alpha - \tau_t - f_c - \beta + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J_{t+1}^f + \phi_t J_{t+1}^v\right] - J_t^v\right) \quad \text{(D.6)}$$
$$+ (1-p)\beta + (1-p)U_t - (1-p)\left(\sigma(1-\rho)\mathbb{E}\left[(1-\phi_t)W_{t+1} + \phi_t U_{t+1}\right]\right)$$

The number of new match

$$m_t = \mu u_t^{\alpha_m} v_t^{1-\alpha_m} \tag{D.7}$$

The unemployment level

$$u_t = 1 - N_t \tag{D.8}$$

The employment level

$$N_t = (1-\rho)(1-\phi_t)(N_{t-1} + m_{t-1}) \tag{D.9}$$

The total job destruction

$$\phi_t = \phi_t^e + \phi^x \tag{D.10}$$

The endogenous rate of job destruction

$$\phi_t^e = \int_0^{\chi_t^f} dF(\theta) \tag{D.11}$$

Number of vacancies

$$v_t = (1 - q_t^v)(1-\rho^o)v_t + n_t \tag{D.12}$$

Number of new vacancies

$$n_t = \eta (J_t^v)^{\xi} \tag{D.13}$$

Finding and filling rate

$$q_t^u = \frac{m_t}{u_t} \tag{D.14}$$

$$q_t^v = \frac{m_t}{v_t} \tag{D.15}$$

The budget constraint

$$G_t = \int_{\chi_t^f}^{\chi^{stw}} (a_t - b_t)(1-h_t)dF(\theta) - \int_{\chi_t^f}^{\chi^{stw}} \tau_t^{stw} h_t dF(\theta) - \int_{\chi^{stw}}^{1} \tau_t^f dF(\theta) + \sigma \mathbb{E}[G_{t+1}] \tag{D.16}$$

The aggregate output

$$
\begin{aligned}
Y_t = N_t \int_{\chi_t^f}^{\chi^{stw}} &\left\{ \frac{1}{\alpha}(\theta h_t)^{\alpha} - \tau_t h_t - f_c \right\} dF(\theta) \\
+ N_t \int_{\chi^{stw}}^{1} &\left\{ \frac{1}{\alpha}\theta^{\alpha} - \tau_t - f_c \right\} dF(\theta) \\
- N_t \phi_t^e l_f - u_t u_b - v_t \kappa &
\end{aligned}
\tag{D.17}
$$

D.2 Steady state

Employment rate

$$N_{ss} = 1 - u_{ss}$$

Number of match

$$m_{ss} = \frac{\phi_{ss}}{1-\phi_{ss}} N_{ss}$$

Number of vacancies

$$v_{ss} = (\frac{m_{ss}}{\mu u_{ss}}^{\alpha_m})^{\frac{1}{1-\alpha_m}}$$

Filling rate

$$q_{ss}^{v} = \frac{m_{ss}}{u_{ss}}$$

Finding rate:

$$q_{ss}^{u} = \frac{m_{ss}}{u_{ss}}$$

Number of new vacancy

$$n_{ss} = v_{ss}(1 - (1 - q_{ss}^{v})(1-\rho))$$

Vacancy post value

$$J_{ss}^{v} = \frac{\epsilon}{n_{ss}}$$

Value of a filled job

$$J_{ss}^{f} = \frac{1}{\sigma q_{ss}^{v}}(q_{ss}^{v} J_{ss}^{v} + \kappa)$$

Firing threshold

$$\chi_{ss}^{f} = (\phi_{ss}^{e})^{\frac{1}{4}}$$

Wage

$$w_{ss} = \frac{1}{0.7} \left[(\tilde{h}_{ss}^{STW})^{\alpha-1} (\tilde{\theta}^{STW})^{\frac{\alpha}{}} \right]$$

Fixed cost

$$f_c = \frac{1}{\alpha}(\tilde{\theta})^{\alpha} - w_{ss} - J_{ss}^{f} + \sigma(\phi_{ss} J_{ss}^{f} + (1-\phi_{ss}) J_{ss}^{v})$$

Working value

$$W_{ss} = \frac{w_{ss} - \beta + \sigma \frac{p}{1-p}(J_{ss}^{f} - J_{ss}^{v}))}{1-\sigma}$$

Unemployment value

$$U_{ss} = Wss - p/(1-p) * (J_{ss}^{f} - J_{ss}^{v})$$

Unemployment benefit

$$u_b = U_{ss} - \sigma q_{ss}^{u} W_{ss} - \sigma(1 - q_{ss}^{u}) U_{ss}$$

Labour cost

$$\ell_{ss} = w_{ss} - b_{ss} w_{ss}$$

Firing cost

$$c_f = \frac{1-\alpha}{\alpha} \frac{\chi_{ss}^{f}}{\ell_{ss}}^{\frac{\alpha}{1-\alpha}} - b_{ss} w_{ss} + \sigma(\phi_{ss} J_{ss}^{f} + (1-\phi_{ss}) J_{ss}^{v}) - J_{ss}^{v}$$

Short-time work threshold

$$\chi_{ss}^{stw} = (\phi_{ss}^{stw} + (\chi_{ss}^{f})^{4})^{\frac{1}{4}}$$

Public budget

$$G_{ss} = \frac{1}{1-\sigma} \left[(a_{ss} - b_{ss}) w_{ss} \ell_{ss}^{\frac{-1}{1-\alpha}} \frac{4}{7}((\chi_{ss}^{stw})^{7} - (\chi_{ss}^{f})^{7})) + (b_{ss} - a_{ss}) w_{ss}((\chi_{ss}^{stw})^{4} - (\chi_{ss}^{f})^{4}) \right]$$

Aggregate surplus

$$\begin{aligned} S_{ss} = \frac{1-\alpha}{\alpha} \ell_{ss}^{\frac{-\alpha}{1-\alpha}} \frac{4}{7}((\chi_{ss}^{stw})^{7} - (\chi_{ss}^{f})^{7}) + \frac{1}{\alpha}(1 - (\chi_{ss}^{stw})^{4}) + (a_{ss} w_{ss} - b_{ss} w_{ss})(\chi_{ss}^{stw4} - \chi_{ss}^{f\,4}) \\ + (w_{ss} - a_{ss} w_{ss}) \ell_{ss}^{\frac{-1}{1-\alpha}} \frac{4}{7}(\chi_{ss}^{stw7} - \chi_{ss}^{f\,7}) - \beta \ell_{ss}^{\frac{-2}{1-\alpha}} \frac{4}{13}(\chi_{ss}^{stw1}3 - \chi_{ss}^{f\,1}3) - \beta(1 - (\chi_{ss}^{stw})^{4}) \end{aligned}$$

Aggregate output

$$
\begin{aligned}
Y_{ss} &= N_{ss}\frac{1-\alpha}{\alpha}\ell_{ss}^{\frac{-\alpha}{1-\alpha}}\frac{4}{7}((\chi_{ss}^{stw})^7-(\chi_{ss}^{f})^7)-\tau_t^{stw}\ell_{ss}^{\frac{-1}{1-\alpha}}\frac{4}{7}((\chi_{ss}^{stw})^7-\chi_{ss}^{f\,7}) \\
&+ N_{ss}\frac{1}{\alpha}(1-(\chi_{ss}^{stw})^4)-N_{ss}\tau_{ss}^{f}\beta*(1-(\chi_{ss}^{stw})^4) \\
&- n_{ss}\phi_{ss}^{e}c_f-u_{ss}u_b-v_{ss}\kappa
\end{aligned}
$$