

Inequality, Educational Choice and Public School Quality in Income Mixing Communities

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Abstract

Why, in some urban communities, do rich and poor households cohabit while, in others, we observe sorting by income? To answer this question I develop a two-community general equilibrium framework of school quality, residential choice and tax decision with probabilistic voting. The model predicts that in highly unequal societies in which households segregate by schooling, low- and high-income households choose to live in the same community. When there is less inequality, we observe the typical sorting by income across communities. The theoretical model suggests that the effect of inequality on the quality of public schooling is ambiguous and depends on the relative endowments of housing in the two communities. When inequality increases, if housing in the community where rich and poor households cohabit is affordable, then an inflow of high-income middle class households towards this community emerges. As a consequence, inequality negatively impacts the quality of public schooling due to an ends-against-the-middle coalition that pushes tax rates down.

Key Words: Inequality; Probabilistic Voting; Segregation; Income Mixing Equilibrium.

JEL Codes: D72, I24, I28, R21.

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1 Introduction

A striking characteristic of U.S. metropolitan areas is the concentration of poverty close to inner cities. Data from the U.S. Census show that 17.6% of the population in the inner cities of all U.S. metropolitan areas had an income below poverty level in 1999, as against only 8.4% of the suburban population¹. According to the 2012 American Community Survey (ACS), in 2012 the share of the inner city population living in poverty was as high as 19.7%². However, in some urban areas, the city center is also home to high-income households who, not surprisingly, may co-reside in the same area with low-income households.

The cohabitation of heterogeneous income groups in the same community, in particular in the same school district, can have important implications for income redistribution, access to high quality education, public policies, political decisions and socio-economic opportunities. A central issue is to understand the consequences of the emergence of mixed-income communities, since the presence of different income groups within the same community may create segregation in terms of access to high quality schooling, thereby hindering the upward mobility of poor households. Two sets of questions arise when mixed-income communities emerge. First, why do some urban areas show sorting by income at community level while, in others, rich and poor households cohabit? How does this difference relate to school districts' quality of public schooling and enrollment in private schools? Are public schools in communities composed mainly of middle-income households qualitatively better than those in mixed-income districts? Second, does income inequality impact residential choices and community segregation?

¹Source: U.S. Census Bureau, 2000 Census, Summary File 3. The data also point to declining poverty in inner cities since 1990.

²In contrast, only 11.2% of the suburban population had an income below the poverty threshold during the same year. See Gabe (2013) for a report on poverty in the United States, Partridge and Rickman (2006) for a through analysis of poverty trends in America. See also Berube and Frey (2002) for an analysis of poverty rates in the 102 largest U.S. metropolitan areas based on the 2000 Census, and Berube and Kneebone (2006) for a similar study based on the 2000 Census and 2005 ACS.

In this paper I develop a general equilibrium model of private/public school choice, political decisions and endogenous residential choice to address these questions. I focus on a two-community economy in which housing market and fiscal policies interact with school and residential location choices, and therefore with the quality of public education. The framework I provide involves an economy composed of two communities with homogeneous land and fixed boundaries, which can be interpreted as two different school districts characterized by the same level of housing quality. Parents have to decide which district to live in and which type of school to send their children to, choosing between a tax-financed public school or a private school financed by tuition fees. The quality of public education in each district is determined by the amount of spending per student financed through property taxes on housing value³. Moreover, a probabilistic voting process in each community determines the local tax rate and, therefore, public education spending.

The theoretical model developed in this paper is aimed at identifying the analytical conditions under which an inter-community political equilibrium with segregation and income mixing exists. In the income mixing equilibrium, poor and rich households cohabit in the same community and send their children respectively to public and private schools. The other community is composed of middle-income households who choose the local public school for their children⁴. An income mixing equilibrium is found to exist if and only if income dispersion is sufficiently high. If the conditions for this particular type of equilibrium are not satisfied, then the model with probabilistic voting predicts a perfect stratification across communities according to income. In this case, the fully public regime prevails, and the community with lower (higher) quality public education

³While I assume proportional income tax rather than property value tax, the model predicts the same qualitative results.

⁴From a political perspective, the equilibrium that features this particular configuration supports the result provided in Epple and Romano (1996) in a single community model: a coalition of rich and poor households will be opposed by a coalition of middle-income households. This outcome, namely *'the ends against the middle'*, implies that high- and low-income households vote for low taxation and public school spending, while middle-income households vote for a high level of redistribution.

is populated by households with low (high) income.

One of the main predictions of the model concentrates on the effect of inequality on endogenous residential location choices in a model that includes simultaneously: public versus private education choice, two-community structure with independent local government, a competitive housing market within each community, a property tax rate based on housing value, a probabilistic voting mechanism rather than majority choice. Since the early 70's, U.S. metropolitan areas have experienced an increase in income inequality, driven largely by income growth in the top half of the income distribution⁵. Increase in inequality has been accompanied by a change in the population composition of some inner cities, with poor households replaced by high-income middle class households⁶. Similarly, the theoretical analysis in this paper suggests an endogenous population reallocation as a consequence of inequality.

The main theoretical contributions of the paper are the following. First, while the majority choice model cannot generate an equilibrium with income mixing if indifference curves have a slope which is non-increasing in income, the probabilistic voting alternative can, under particular conditions. Second, the effect of inequality on residential location decisions and, therefore, on public spending per student, is ambiguous and depends on the relative endowments of housing and prices in the two communities. An increase in inequality leads to an influx of middle-income households in the mixed-income district when the relative supply of housing is sufficiently large. As a consequence, the theoretical model predicts that inequality negatively impacts the quality of public schooling due to an ends-against-the-middle coalition that pushes tax rates down. By contrast, when the relative supply of housing in the community with income mixing is sufficiently low, inequality decreases the tax rate but increases public spending per pupil. In this case,

⁵See Piketty and Saez (2003), Corocan and Evans (2010). According to the U.S. Census Bureau, inequality in the U.S. as measured by the pre-tax Gini coefficient at country level increased from 0.394 in 1970 to 0.469 in 2010.

⁶See Brueckner and Rosenthal (2009). The authors show how high income and low income households locate within a city, providing a new model of the gentrification process in some U.S. cities.

high-income households do not move towards the mixed-income community and the model ends up with one of the results in de la Croix and Doepke (2009): higher inequality is positively associated with public spending per student⁷.

The paper is organized as follows: section 2 summarizes the relevant literature and highlights the contribution of this paper. Section 3 develops the theoretical model, also discussing the conditions necessary for an equilibrium with income mixing as well as for a perfect income stratification equilibrium. Section 4 focuses on the effect of inequality on the equilibrium vector of fiscal policies and public education spending. Section 5 presents an empirical analysis on the relationship between the existence of mixed/middle income school districts and public school quality in the U.S. States of Arizona and Illinois. Section 6 concludes.

2 Related Literature

This paper builds on two strands of the literature. The first one deals with the emergence of mixed-income communities in presence of private school alternatives. Barse et al. (2001), Martinez-Mora (2006), Hanushek et al. (2011) have developed general equilibrium models of community choice and school competition. As in the present paper, these studies analyze the impact of private education options on residential location decisions, as well as the political economy of public education provisions.

Barse et al. (2001) study central versus local school finance in a dynamic Tiebout economy with private alternatives and majority voting. In contrast to the standard Tiebout model, the authors find that communities are not perfectly stratified by in-

⁷The literature studying the effect of inequality on income redistribution has produced ambiguous results. As also observed by Arclean and Schiopu (2016), in Meltzer and Richard (1981) and Persson and Tabellini (1994), higher inequality is associated to more redistribution. By contrast, other papers find that redistribution is weaker in more unequal societies. See, for instance, Alesina et al. (1999), Luttmer (2001). Along these lines, Di Gioacchino and Sabani (2009) show that the greater the wealth inequality compared to initial income inequality, the less egalitarian the incidence of public education expenditure will be.

come, but that rich and poor households choose to cohabit in one community, while middle-income households live in another location. This particular segregated equilibrium is found assuming a CES utility function with elasticity of substitution between consumption and children's education lower than one⁸.

Along these lines, Martinez-Mora (2006) provides a general equilibrium model with two communities in which local public schools coexist with competing private schools. These communities are interpreted as the urban area and the suburbs of a city. Each community finances public education by imposing a proportional property tax on the value of housing. Assuming that an urban area offers lower quality public schooling, the author uses a computational version of the model to construct examples where multiple equilibria may exist. More precisely, the resulting equilibria will be of one of two types: urban trap or urban mixing equilibrium. In an urban trap equilibrium, middle-income households live in the urban area and opt out of the public system by enrolling their children in low quality private schools, while higher income households prefer to use the local high quality public schools⁹. An urban mixing equilibrium is defined as an equilibrium in which poor and rich households cohabit in the urban area and segregate by schooling, while middle class households live in the suburbs and send their children to the local public school¹⁰.

Hanushek et al. (2011), study how the existence of a private school option affects the demand for public school quality. The authors develop a framework that merges the Tiebout and urban residential location models. In this particular location model,

⁸This is equivalent to assume a preference parameter σ larger than one implying a slope of indifference curves strictly increasing in income.

⁹In a urban trap equilibrium households from the top income classes may also acquire elite private schooling in the urban area rather than use the suburban public school. See Martinez-Mora (2006).

¹⁰While the main contribution of the paper by Martinez-Mora is the analysis of the urban trap equilibrium, my work concentrates on the emergence of an urban mixing equilibrium in which middle class households live in the more expensive suburb and send their children to high quality public schools, while poor and rich households cohabit in the urban area and segregate by schooling. In section 3 I will show why an urban trap equilibrium with non-elite private schools cannot be an equilibrium in my model.

households differ by their earnings, as well as in their preferences on education. Taxes are determined by majority vote and income mixing emerges in equilibrium. However, the equilibrium of this model can only be calculated numerically. The computational model shows how the link between spending and quality of public schools is significantly modified in the presence of private schools. The mechanism is primarily based on to the peer composition in both the public and private schools.

On the other strand, the education financial literature, suggests that mechanisms other than majority voting, such as probabilistic voting, might offer a different description of the aggregation of preferences¹¹. De la Croix and Doepke (2009), Bernasconi and Profeta (2012), Dottori et al. (2013), Arcalean and Schiopu (2016) introduce probabilistic voting in the context of education and school choice. In their pioneering work, de la Croix and Doepke (2009) develop a single district economy in which the tax rate and the quality of public education are determined via probabilistic voting. They show that households perfectly segregate by education. This segregation pattern is driven by the fact that parents prefer to enroll their children in private schools when these schools provide higher quality education than what is provided in the public system¹².

My paper builds on these two different approaches. The objective is to construct a 'hybrid' approach able to merge the community choice model, in which private education options are available, with a political mechanism that does not depend solely on the preferences of the median voter, but rather on the whole distribution of voters' preferences. This approach allows me to extend the theoretical contribution of de la Croix and Doepke (2009), developing a fully tractable two-community model able to predict

¹¹Probabilistic voting models were first introduced by Lindbeck and Weibull (1993). Glomm et al. (2011) note that even though these models could ease existence problems, whether they provide a better representation of reality remains an open question.

¹²More precisely, they examine how the quality of public education is affected by the presence of private schools. The choice between public and private education has already been studied by many authors: Stiglitz (1974), de la Croix and Doepke (2004), in a single district economy, Gutierrez and Tanaka (2009), Estevan (2013) in a setting where parents can send children to work instead of educate them.

an equilibrium with income mixing. In particular, my paper provides an important theoretical extension of their work that captures the interaction between housing demand, school quality and tax decisions in a context in which alternative locations are available. Indeed, I assume a two-community model in which the housing market interacts with both school and residential location decisions¹³.

In this set-up I find analytical solutions for a segregated equilibrium in which poor and rich households cohabit in one community while middle-income households live in another community. In particular, my model predicts that this segregated equilibrium with income mixing exists only if income distribution is not sufficiently compressed. However, if the preferred education level varies little in the population and income distribution is sufficiently compressed, there is perfect income stratification across communities and a fully public regime prevails.

3 The Model

In this section I develop a general equilibrium model of two communities with fixed boundaries in which housing market and fiscal policies interact with the quality of public education, school choices and residential decisions.

3.1 Theoretical Assumptions

The economy is populated by a continuum of households of measure one. Each household consists of one adult and one school-aged child. Adults are differentiated by their income endowment x , where x is the wage that an adult can obtain in the labor market. I focus on a uniform distribution of income over the interval $[\mu - \sigma; \mu + \sigma]$ for positive $\mu > \sigma > 0$ ¹⁴.

¹³Moreover, in my paper, public education is financed through property taxes on housing value rather than through income tax.

¹⁴The uniform distribution is chosen for simplicity. Accordingly, the associated density function is given by $f(x) = 0$ for $x < \mu - \sigma$ and $x > \mu + \sigma$ and $f(x) = \frac{1}{2\sigma}$ for $0 < \mu - \sigma \leq x \leq \mu + \sigma$. In a recent paper, Arcalean and Schioppa (2016) assume that income is distributed according to a Pareto

Households' preferences are represented by a utility function $U(h, z)$. I follow de la Croix and Doepke (2009) in assuming a logarithmic utility function. Households have identical preferences on the quality of their children's education, z , and on the private good, h , taken here as housing consumption. The assumption that utility depends only on housing consumption and children's schooling quality simplifies the analysis and allows me to obtain analytical solutions¹⁵. Education can be provided by public and private schools that are mutually exclusive and use the same technology to offer educational services¹⁶. Parents may choose to educate their offspring either in a public school, $z = q$, where q denotes the schooling quality, or in a private school, $z = e$, where e represents education spending in the private market.

Public schools follow a residence-based admission policy: households living in a community use the local public school. Each community imposes an *ad valorem* tax on housing to finance the public education system. The tax rate, τ , and therefore the amount of public spending on education, are determined by a political vote by residents of the community. In addition to property taxes on housing value, parents have to pay tuition fees covering the full cost of private education if they decide to opt out of the public system. A household can consume either public or private school services, but not both.

I assume that communities impose a proportional property tax, τ , on the value of housing rather than an income tax because in the U.S., property taxes finance most local distribution rather than a uniform distribution in order to obtain a more flexible parametrization of the income distribution.

¹⁵If we add a numeraire to the logarithmic utility function, that is $U(c, h, z)$, the qualitative results of the analysis do not change but the model loses its analytical tractability. For this reason, and as it is the theoretical purpose of this paper to study the link between school choice and residential location decisions, this assumption seems reasonable. Another interpretation of this simplifying assumption is to consider the variables x , q and e as efficient labor quantities, with labor as the numeraire. In other words, the numeraire is the final good which is used for education spending only. Therefore, if e and q are interpreted as labor, in the model there are two goods: a numeraire (labor) and houses (with price p).

¹⁶For expositional reasons, quality units are normalized such that the price per unit of private schooling is equal to 1.

government education spending. Actually, local governments find funding for education through a combination of property and income taxes. Public school systems in the U.S. are supported by a combination of state, federal and local taxes. However, a substantial proportion of expenditure on public education is financed at local level by property taxes. In order to avoid an additional source of taxation that would complicate the analysis, in this paper I only consider housing value taxation. The ability to opt out of the public system by choosing private education, a residence-based admission policy and the hypothesis that public schools are financed through local property taxes are all assumptions that strongly tie this model to the U.S. school system¹⁷.

A household can choose which community to reside in. To keep the analysis simple, I assume that each community has a fixed amount of homogeneous land from which housing services are produced through the same constant returns to scale production function. Land, and therefore housing, are owned by a competitive *absentee landlord* to whom households have to pay rent at the market price¹⁸. The two communities may differ in the amount of land within their fixed boundaries. Each community contains a set of public schools that provide education of identical quality and can be thought of as school districts¹⁹.

Communities are politically independent but economically integrated. As already observed in Hansen and Kessler (2001), the assumption of political independence between communities implies that each local government can choose fiscal policies autonomously. Economic integration excludes barriers to migration or trade, and allows households to be perfectly mobile among communities at no direct cost. Moreover, I exclude peer group

¹⁷As observed by de Bartolome and Ross (2008), an important difference between the U.S. and Europe is that in Europe there is less variation in the public service level across jurisdictions and less reliance on property tax.

¹⁸Alternatively, I could assume that households are owners and buy land at the market price. See Hansen and Kessler (2001) for further discussion on the absentee landlord assumption.

¹⁹Otherwise consider that there is only one public school per district. It should also be noted that considering an economy composed of two communities with the same amount of land represents a particular case of the structure analyzed in this paper.

effects so that the quality of public education in each school district is only determined by the amount of spending per pupil financed through property taxes on housing values²⁰.

Public policies are voted through probabilistic voting. Parents have perfect foresight over the outcome of the political process and, consequently, over the policies adopted by the local government of each community. Put differently, expectations about school quality will be realized. Taking fiscal policies as given, households have to choose where to reside and which type of school to send their children to. Households actually face four residential/school choices: *(i)* district 1 and public school, *(ii)* district 1 and private school, *(iii)* district 2 and public school and *(iv)* district 2 and private school.

Finally, the timing of events follows two stages. In the first stage, each adult simultaneously settles in a community, assuming that housing prices endogenously adjust to equate housing demand and supply, and chooses between free-of-charge public school and fee-paying private school for his/her child. In the second stage, the adult residents of each community vote on the property tax rate and public schooling expenditure. All households have to pay taxes even if they decide to opt out of the public system. The outcome of the voting process determines the quality of public education in the two school districts. This timing structure can be justified by observing that public education policy can frequently be adjusted through a yearly budget vote, while residential decisions cannot ²¹.

3.1.1 Households' Problem

Households have to make three decisions: they have to choose which community to live in, they have to decide whether to educate their children in public or in private schools

²⁰See Benabou (1993) and Hanushek et al. (2011) for models with peer group effects.

²¹The same argument can be found in de la Croix and Doepke (2009). Similarly, they observe that the choice between public and private education entails substantial switching costs, especially when education segregation is linked to residential segregation. Notice that results do not change if decision are taken simultaneously since rational agents have expectations about the quality of public schools. Results would change if households could decide to enroll their children in private schools after voting, which would imply an additional derivative in the voting process discussed in the next pages.

and they have to vote for the level of public funding for education. The problem of the representative adult agent can be written as follows²²:

$$(1) \quad \begin{cases} \max_{h,e} U[h, z] = \ln[h] + \ln[\max\{q, e\}] \\ s.t. \quad ph(1 + \tau) = x - e \end{cases}$$

where p is the net-of-tax housing price and is determined in the competitive housing market of each community and τ the property tax rate. Substituting the budget constraint into the objective function, I can rewrite the utility of the representative household as follows:

$$(2) \quad u[q, e, x, \tau, p] = \ln \left[\frac{x - e}{(1 + \tau)p} \right] + \ln[\max\{q, e\}]$$

Parents preferring public education will choose $e = 0$. Let us define as $u^q[q, 0, x, \tau, p]$ and $u^e[0, e, x, \tau, p]$ the utility of a household respectively choosing public or private schooling for his/her child. The problem can be written as:

$$(3) \quad \begin{cases} \max_h u^q[q, 0, x, \tau, p] & \text{if public education} \\ \max_{h,e} u^e[0, e, x, \tau, p] & \text{if private education} \end{cases}$$

The solution to this problem is given by:

$$(4) \quad \begin{cases} e = 0, & h^q = \frac{x}{(1+\tau)p} & \text{if public education} \\ e = \frac{x}{2}, & h^e = \frac{x}{2p(1+\tau)} & \text{if private education} \end{cases}$$

with h^q (h^e) as the housing demand under public (private) education choice²³. As we

²²For simplicity I adopt a logarithmic utility function, but the same results can be obtained with any utility function representing homothetic preference. It should be noted that in general homotheticity implies that a fixed share of income is devoted to each good.

²³Given Cobb-Douglas preferences, it is clear that when private education is chosen, income is spent half on education and half on housing.

can expect, an increase in property value tax rate or in net-of-tax housing prices will reduce the consumption of housing.

Substituting the optimal households' choices into the maximization problem allows us to derive the indirect utility functions of adults choosing public, $V^q[x, q, p, \tau]$, or private, $V^e[x, p, \tau]$ education for their children:

$$(5) \quad \begin{cases} V^q[x, q, p, \tau] = \ln \left[\frac{x}{(1+\tau)p} \right] + \ln[q] \\ V^e[x, p, \tau] = \ln \left[\frac{x}{2(1+\tau)p} \right] + \ln \left[\frac{x}{2} \right] \end{cases}$$

Lemma 1. *Households strictly prefer private education if and only if $x > \tilde{x}[q] = 4q$.*

Proof: Using the indirect functions defined in (5), it is easy to verify that $V^e[x, p, \tau] > V^q[x, q, p, \tau]$ when $x > 4q$.

The expected quality of public education will determine the position of the threshold $\tilde{x}(q)$ in the income distribution and the share of children participating in the public school system. Notice that education quality is a normal good because parents with higher income demand more of it.

3.1.2 The Political Mechanism

While the theoretical literature on community choice has mainly examined the conditions for income mixing under majority choice²⁴, in this paper I focus on the case in which public policies are decided through probabilistic voting. In particular, the analysis follows the general structure of the most recent literature by assuming that the tax rate and the quality of public education are determined via probabilistic voting in

²⁴See for instance Nechiba (2000), Barse et al. (2001), Martinez-Mora (2006), Hanushek et al. (2011).

which each individual carries the same political weight in the political process²⁵. Following Barse et al. (2001), I concentrate on the scenario in which both communities are occupied. Without loss of generality, the authors restrict their analysis to the case $\tau_1 < \tau_2$ and $q_1 < q_2$. These conditions respectively guarantee that households living in community 2 never choose private education for their children and that community 2 is not empty. Similarly, my paper aims to analyze the analytical conditions under which a segregated equilibrium with income mixing might appear when both communities are occupied. I focus on the case in which gross housing prices $p_i^g = (1 + \tau_i)p_i$ with $i \in (1, 2)$ are such that $p_1^g < p_2^g$, and the quality of public schooling in community 2 is sufficiently high relative to community 1²⁶.

My objective is to characterize an “inter-community equilibrium” (see Section 3.2 for a formal definition) with income-mixing. In this equilibrium, poor individuals, with revenue below a certain threshold \tilde{x}_1 and rich individuals, with revenue above \tilde{x}_2 , decide to cohabit in the same community, while households with income between \tilde{x}_1 and \tilde{x}_2 live in another community. Before giving the analytical details of the analysis with probabilistic voting, I briefly present the equilibrium results under majority choice. A majority voting equilibrium exists in the model. Given the timing of events and the assumption of Cobb-Douglas preferences, it is straightforward to demonstrate that under the majority voting rule, all agents choosing public education in one community prefer the same property tax $\tau = 1$, while agents choosing private education prefer a tax rate $\tau = 0$. In other words, the majority of agents choosing private or public school for their children determines the majority choice equilibrium in any community.

Assume, for instance, that in community 1 the majority of households chooses public

²⁵See de la Croix and Doepke (2009), Dottori et al. (2013), Arcalean and Schiopu (2016).

²⁶Non-emptiness requires that $q_2 > \hat{q}_2$, with $\hat{q}_2 \equiv q_1 + \frac{x}{p_1^g} - \frac{x}{p_2^g}$. Notice that if gross housing prices are the same, i.e. $p_1^g = p_2^g$, then the condition for non-emptiness reduces to $q_2 > q_1$. By contrast, regardless of housing prices, whenever $q_1 \geq q_2$ all households reside in community 1. Otherwise, it could be assumed that each school district gives its population an amenity to be shared equally among residents to guarantee that both communities are occupied.

schooling and thus a tax rate of 1. In this scenario, richer households will be relatively reluctant to live in this community and invest in private education for their children. At the same time, if community 2 contains a majority of households consuming private education, the preferred tax rate will be $\tau = 0$. In this scenario, the majority voting equilibrium predicts perfect schooling segregation, as well as perfect income stratification, across communities. Moreover, note that if households choose the same tax rate in both communities, the sorting equilibrium with majority choice could still exist because the tax-inclusive housing price could be lower in one of the two communities.

Compared to a majority voting mechanism, probabilistic voting might promote income mixing because it gives different weight to different groups of electors. Indeed, probabilistic voting might lead to income mixing without any source of heterogeneity other than income being introduced. This is because probabilistic voting enfranchises agents that choose private education for their children, hence leading to a tax rate lower than 1²⁷.

It can be shown that the equilibrium choice under probabilistic voting is equivalent to maximizing a weighted sum of the indirect utilities of individuals²⁸. The social welfare functions maximized in the two communities by the political mechanism are respectively given by:

$$(6) \quad W_1[\tau_1, q_1] = \int_{\mu-\sigma}^{\tilde{x}_1} u[q_1, 0, x, \tau_1, p_1]f(x)dx + \int_{\tilde{x}_2}^{\mu+\sigma} u[0, e_1, x, \tau_1, p_1]f(x)dx$$

$$(7) \quad W_2[\tau_2, q_2] = \int_{\tilde{x}_1}^{\tilde{x}_2} u[q_2, 0, x, \tau_2, p_2]f(x)dx$$

Welfare maximization is constrained to the local government budget rule of the com-

²⁷Note that any model of tax determination that puts weight on households' utility would tend to have a similar effect.

²⁸See Persson and Tabellini (2000) for further discussion.

munity, that is:

$$(8) \quad \tau_1 \int_{\mu-\sigma}^{\tilde{x}_1} p_1 h_1^q f(x) dx + \tau_1 \int_{\tilde{x}_2}^{\mu+\sigma} p_1 h_1^e f(x) dx = \int_{\mu-\sigma}^{\tilde{x}_1} q_1 f(x) dx$$

$$(9) \quad \tau_2 \int_{\tilde{x}_1}^{\tilde{x}_2} p_2 h_2^q f(x) dx = \int_{\tilde{x}_1}^{\tilde{x}_2} q_2 f(x) dx$$

The left-hand side of these two constraints represents total revenues from the taxation on housing values. The right-hand sides give the amount of total spending on public education. Replacing households' housing demands (4) in the balanced budget rules (8) and (9), allows me to express the property tax rates as an increasing function of the quality of public education:

$$(10) \quad \tau_1[q_1] = \frac{4q_1(\tilde{x}_1 - \mu + \sigma)}{2\tilde{x}_1^2 - \tilde{x}_2^2 - 4q_1(\tilde{x}_1 - \mu + \sigma) + 6\mu\sigma - (\mu^2 + \sigma^2)}$$

$$(11) \quad \tau_2[q_2] = \frac{2q_2}{\tilde{x}_1 + \tilde{x}_2 - 2q_2}$$

Notice that housing prices do not directly influence the policies voted by adult residents. Moreover, the timing of events requires residential and educational choices to be predetermined when voting occurs simultaneously in the two communities. Maximizing the welfare functions (6) and (7) with respect to the corresponding local budget constraint (10) and (11), taking the first order conditions for a maximum and solving for education quality allows the voting outcomes to be defined:

$$(12) \quad q_1^* = \frac{2\tilde{x}_1^2 - \tilde{x}_2^2 - (\mu^2 + \sigma^2) + 6\mu\sigma}{8\tilde{x}_1 - 4(\tilde{x}_2 + \mu - 3\sigma)}$$

$$(13) \quad q_2^* = \frac{\tilde{x}_1 + \tilde{x}_2}{4}$$

3.1.3 Housing Market

In each community, there is a local housing market in which prices are determined competitively. Each community has a fixed amount of homogeneous land from which housing stock is produced through the same constant return to scale production function. Communities can differ only in the amount of land contained within their boundaries. I assume the existence of an *absentee landlord* who resides outside the economy and owns the land²⁹.

The aggregate housing demand in each community is obtained by integrating households' housing demand over income interval:

$$(14) \quad h_1^d(x, \tau_1, p_1) = \int_{\mu-\sigma}^{\tilde{x}_1} h_1^q f(x) dx + \int_{\tilde{x}_2}^{\mu+\sigma} h_1^e f(x) dx$$

$$(15) \quad h_2^d(x, \tau_2, p_2) = \int_{\tilde{x}_1}^{\tilde{x}_2} h_2^q f(x) dx$$

In equilibrium aggregate community housing demand equals community housing supply, $h_i^d(x, \tau_i, p_i) = k_i$, where $i = \{1, 2\}$. For simplicity, I assume that k_i represents the total housing units in each community. Since agents have perfect foresight, they take into account their expectations concerning the outcome of the voting process when they formulate their housing demand. Using the equilibrium tax rates, I can derive housing prices in the two communities as a function of income thresholds and parameters:

$$(16) \quad p_1^* = \frac{(\tilde{x}_1 - \tilde{x}_2 + 2\sigma)[2\tilde{x}_1^2 - \tilde{x}_2^2 + 6\mu\sigma - (\mu^2 + \sigma^2)]}{4k_1(2\tilde{x}_1 - \tilde{x}_2 + 3\sigma - \mu)}$$

²⁹The assumption of a fixed amount of homogeneous land, and therefore constant housing supply, is clearly an unrealistic assumption. However, in a static model with exogenous fertility it seems reasonable to consider the housing supply as perfectly inelastic.

$$(17) \quad p_2^* = \frac{\tilde{x}_2^2 - \tilde{x}_1^2}{4k_2}$$

Given \tilde{x}_i , housing prices p_i are decreasing in the amount of land supplied by the Absentee Landlord in community i : the larger the supply, the lower the corresponding housing price.

3.2 Residential Location Decisions in a Segregated Equilibrium with Income Mixing

Before proving the existence of the income mixing equilibrium, it should be noted that a consequence of the assumption of logarithmic preferences is that the slope of the indifference curves of households is invariant to income. Therefore, the standard single-crossing condition for indirect utility functions fails and the uniqueness of a widely defined equilibrium is not guaranteed. Since the theoretical model developed in this paper is an extension of de la Croix and Doepke (2009), my first theoretical contribution is to show that their theoretical results hold even allowing for geographical mobility of households. For this reason, I follow their theoretical assumptions on preferences, uniform distribution and voting. Moreover, since this paper aims principally to analyze the impact of income inequality on public policies when households segregate by schooling, I concentrate on the analytical conditions that guarantee the existence of an equilibrium with school segregation and income mixing³⁰.

Recall that, when taking the decision on which community to live in, households forecast public policies and that, in equilibrium, these are consistent with the policies realized. Given economic integration across communities, each household is free to move from one district to the other at no direct cost. As a consequence of these households' choices, housing prices adjust endogenously. In equilibrium, no household has an in-

³⁰Another reason why I concentrate on this type of equilibrium is that the scenario with income segregation seems to be empirically relevant to U.S. school districts.

centive to move, since the residential choice is the decision that maximizes the family's expected utility. Therefore, I concentrate my attention on the inter-community equilibrium, defined as follows:

Definition (Inter-community Equilibrium) *An inter-community equilibrium is a distribution of households across communities and schools, a pair of income thresholds $(\tilde{x}_1; \tilde{x}_2)$ a vector of policies $(\tau_1^*, \tau_2^*, q_1^*, q_2^*)$ and housing prices (p_1^*, p_2^*) such that:*

(i) households maximize utility with respect to housing consumption (h) and children education (z);

(ii) the units of housing are inelastically supplied and the housing markets clear;

(iii) the regional budgets are balanced;

(iv) public education spending and property taxes are decided by a simultaneous probabilistic voting mechanism in both regions;

(v) no agent wishes to move from one community to another community.

An *income mixing equilibrium* is an inter-community equilibrium satisfying the following system of equations:

$$(18) \quad u[q_1^*, 0, \tilde{x}_1, \tau_1^*, p_1^*] = u[q_2^*, 0, \tilde{x}_1, \tau_2^*, p_2^*]$$

$$(19) \quad u[q_2^*, 0, \tilde{x}_2, \tau_2^*, p_2^*] = u[0, e^*[\tilde{x}_2], \tilde{x}_2, \tau_1^*, p_1^*]$$

Equation (18) states that in equilibrium a household with income \tilde{x}_1 is indifferent between living in community 1 and sending her child to a public school of quality q_1^* , or living in community 2 and sending her child to a public school of quality q_2^* . Similarly, equation (19) implies that a household with income \tilde{x}_2 is indifferent between residing in community 1 and opting out of the public school system, or living in the other community and sending her child to the local public school.

Solving the system composed of equations (18) and (19) and using the equilibrium variables (12), (13), (16), (17) allows us to determine analytically the income thresholds \tilde{x}_1 and \tilde{x}_2 as a function of parameters: $\tilde{x}_1 = f(k_1, k_2, \mu, \sigma)$ and $\tilde{x}_2 = g(k_1, k_2, \mu, \sigma)$ ³¹.

Assumption 1. $\sigma > \bar{\sigma}$, with $\bar{\sigma} \equiv \frac{(k_1+k_2)\mu}{k_1+3k_2}$.

Proposition 1. *Suppose that Assumption 1 is verified. Then there exists an interior equilibrium pair of income thresholds $(\tilde{x}_1; \tilde{x}_2) \in [\mu - \sigma, \mu + \sigma]$ such that the system composed of equations (18) and (19) is satisfied and the expected quality of public education is such that $\tilde{x}_1 < \tilde{x}(q_1) \leq \tilde{x}_2$ and $\tilde{x}_2 < \tilde{x}(q_2)$.*

Proof: See Appendix A.1

Proposition 1 states that if the income dispersion is sufficiently high, then there exists an equilibrium characterized by a distribution of households across communities and types of school in which high-income parents send their children to private schools, while low- and middle-income parents enroll their offspring in public schools of varying quality. In particular, all households with income $x < \tilde{x}_2$ perfectly stratify across communities: poor households live in community 1 while middle class households live in community 2. Moreover, in community 1 poor and rich households cohabit and segregate themselves by schooling.

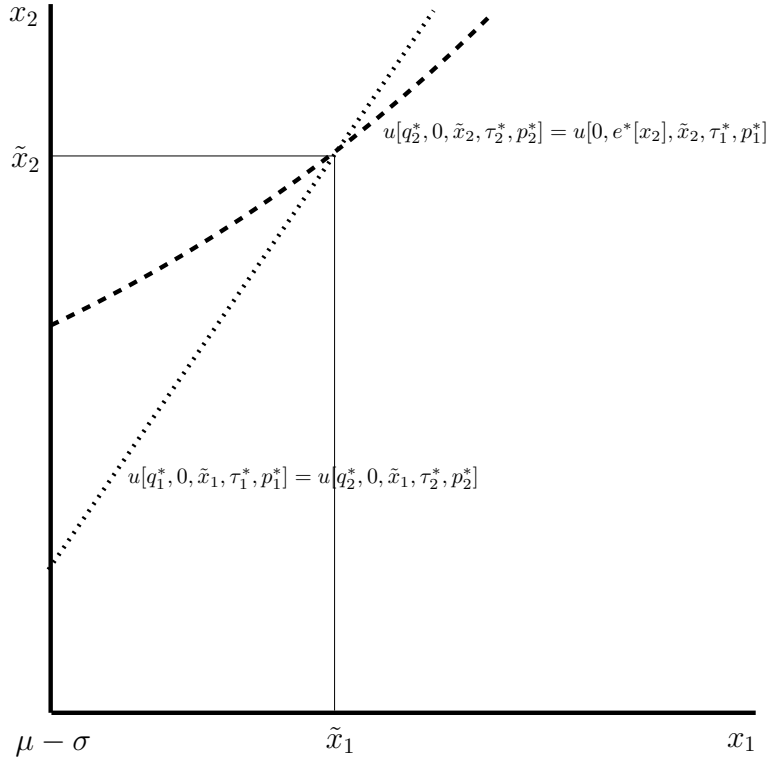
A static model of residential location requires that no agent has an incentive to move, that moving from one district to another cannot increase the household's utility. Figure 1 gives a representation of the income mixing equilibrium in the space (x_1, x_2) ³². The dotted (dashed) curve is the set of x_1 and x_2 satisfying respectively equation 18 and 19. The intersection between the two curves determines the equilibrium values \tilde{x}_1 and \tilde{x}_2 .

When an equilibrium with income mixing exists, it will be characterized by a population distribution in which households with income $x \in [\mu - \sigma, \tilde{x}_1] \cup [\tilde{x}_2, \mu + \sigma]$ settle in

³¹See Appendix A.1 for the analytical expression of these thresholds.

³²As an example, I set the following parameters' values: $k_1 = k_2 = 1$, $\mu = 4$, $\sigma = 2.5$, implying $\bar{\sigma} = 2$.

Figure 1: Inter-community Equilibrium with Income Mixing



community 1 and households with income $x \in]\tilde{x}_1, \tilde{x}_2[$ in community 2. In this equilibrium communities are not perfectly stratified by income³³:

- (i) all households with income $x \leq \tilde{x}_1$ live in community 1 and send their children to a public school of quality q_1 ;
- (ii) all households with income $\tilde{x}_1 < x < \tilde{x}_2$ live in community 2 and send their children to a public school of quality q_2 ;
- (iii) all households with income $x \geq \tilde{x}_2$ live in community 1 and send their children to private school.

Lemma 2. *Define community 1 as the income mixing district and community 2 as the pure public school district. In an income mixing equilibrium taxation and quality of public education are higher in the pure public schooling district.*

³³The same result can be found in Bearse et al (2001) in a model with majority voting and indifference curves with slope rising in income.

Proof: Looking at the corresponding optimal tax rates it can be seen that $\tau_1^* = \frac{\tilde{x}_1 - \mu + \sigma}{\tilde{x}_1 - \tilde{x}_2 + 2\sigma}$ and $\tau_2^* = 1$, with $\tau_1^* \in]0, 1[< \tau_2^*$. Since in an income mixing equilibrium $\mu + \sigma > \tilde{x}_2 > \tilde{x}_1 > \mu - \sigma > 0$, it follows that $q_2^* > q_1^*$.

Middle-income households are not able to enroll their children in private school but they are more demanding in terms of school quality than low-income households. As education is a normal good, they vote for a higher level of taxation and redistribution, and settle in the pure public school district, where the resulting tax rate is the same as that of the majority choice model. In other words, in this community probabilistic voting generates the same result as the majority choice model because this region will be populated only by households choosing public education for their children. By contrast, in community 1, parents can decide to opt out of the public system and enroll their children in private schools. Therefore, with respect to the majority voting model, probabilistic voting might generate income mixing and, as a result, an optimal tax rate $\tau \in]0, 1[$.

Assuming that income is distributed uniformly over the interval $[\mu - \sigma; \mu + \sigma]$, I can derive the share of households in each community and in each type of school in an income mixing equilibrium. I define by $\Psi_{P,i}$ and $\Psi_{R,i}$ the fraction of children participating in public (P) and private (R) schools respectively, in community $i = \{1, 2\}$:

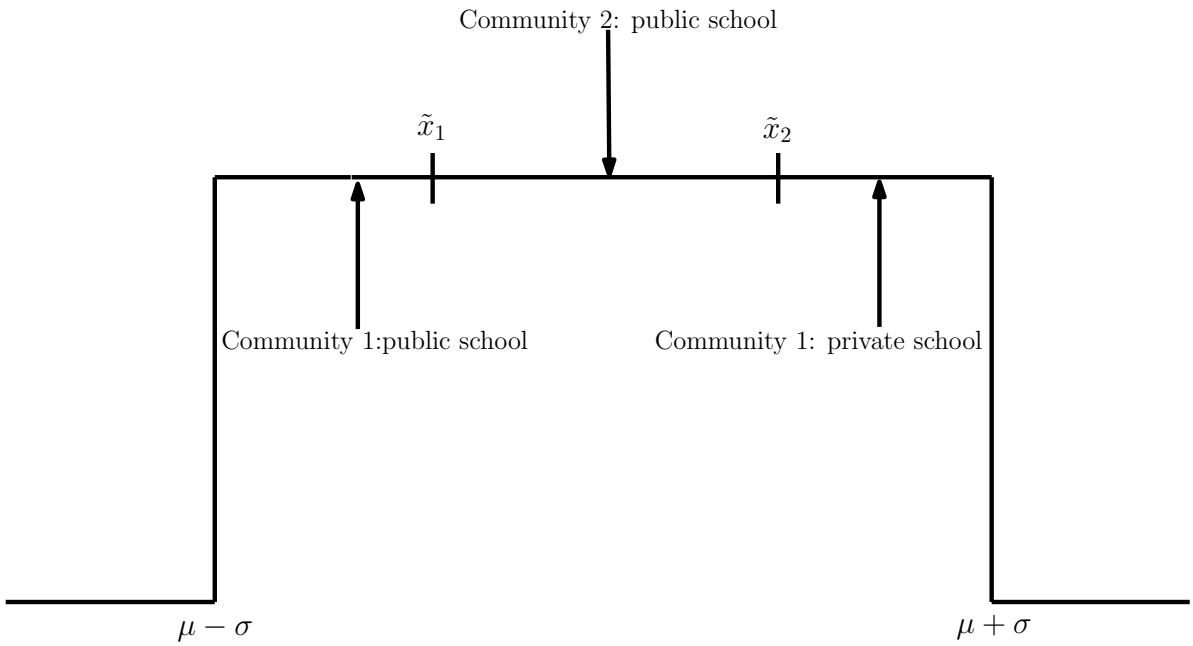
$$(20) \quad \left\{ \begin{array}{l} \Psi_{P,1} = \frac{\tilde{x}_1 - (\mu - \sigma)}{2\sigma} \\ \Psi_{P,2} = \frac{\tilde{x}_2 - \tilde{x}_1}{2\sigma} \\ \Psi_{R,1} = \frac{(\mu + \sigma) - \tilde{x}_2}{2\sigma} \\ \Psi_{R,2} = 0 \end{array} \right.$$

Under perfect foresight we always obtain that the number of households with income $x \leq \tilde{x}_1$ is equal to $\Psi_{P,1}$, those with income $\tilde{x}_1 < x < \tilde{x}_2$ is equal to $\Psi_{P,2}$, and those with

income $x \geq \tilde{x}_2$ is equal to $\Psi_{R,1}$. At given \tilde{x}_i , the population density in the mixed-income community, $\frac{\Psi_{P,1} + \Psi_{R,1}}{k_1} = \frac{\tilde{x}_1 - \tilde{x}_2 + 2\sigma}{2k_1\sigma}$, is positively correlated with σ ³⁴.

Figure 2 graphically describes the households' distribution in this economy, in which rich and poor households cohabit in community 1 and send their children to different types of school, while middle-income households live in community 2 and enroll their children in public schools³⁵.

Figure 2: The Economy Structure with Income Mixing



This particular equilibrium with income mixing has already been obtained in the literature on spatial segregation such as Nechiba (2000), Bearse et al. (2001), de Bartolome and Ross (2003), Hanushek and Yilmaz (2010) and Hanushek et al. (2011). My study differs, in that the slope of the indifference curves is invariant to income; thus

³⁴More precisely, the societies characterized by higher income dispersion exhibit a higher ratio between population and housing in mixed-income communities.

³⁵In my framework, redistribution is interpreted as public education spending. Corocan and Evans (2010) observe that the coalition of rich and poor households votes for low education spending because rich parents prefer to opt out of public education system, while poor parents prefer higher consumption to redistribution.

income mixing is driven by the political mechanism behind residential sorting by school choices and housing market. When private alternatives are available, probabilistic voting might promote income mixing because the outcome of the voting is based on the whole distribution of residents' preferences and not only on the median voter preference. In other words, households have a say in taxation. Rich families would be better off by choosing private education, since education is a normal good. Consequently, they prefer a low level of taxation because they do not enroll their children in public schools. Poor families, while they cannot opt out of the public system, cannot afford high taxes either. Thus, we find high- and low-income households co-residing in the same community and voting for a lower level of taxation than in the community in which there is no income mixing and all residents choose public education.

Interestingly enough, the threshold $\bar{\sigma} \equiv \frac{(k_1+k_2)\mu}{k_1+3k_2}$ is increasing in the mean, μ , of the income distribution. *Ceteris paribus*, this means that economies with a high mean income are less likely to be characterized by mixed-income communities. The threshold $\bar{\sigma}$ is also increasing in the relative dimension of communities 1 and 2, $\frac{k_1}{k_2}$. The more housing units in community 1, the greater $\bar{\sigma}$ needs to be in order to guarantee the existence of this equilibrium³⁶. When there is sufficiently low dispersion of the income distribution, i.e. $\sigma \leq \bar{\sigma}$, households perfectly stratify by income. In this case the fully public regime prevails. The richest group decides to reside in the community that provides the high quality public schooling. The rest of the population lives in the other community, enrolling their offspring in the local public school of lower quality.

Proposition 2. *If $\sigma \leq \bar{\sigma}$, then $\tilde{x}_2 \rightarrow \mu + \sigma$ and $\Psi_{R,1} \rightarrow 0$. The resulting equilibrium is characterized by a fully public regime and perfect income stratification across communities.*

Proof: See Appendix A.2

³⁶Dividing both the denominator and the numerator of $\bar{\sigma}$ by k_2 , it is easy to prove that $\frac{\partial \bar{\sigma}}{\partial (k_1/k_2)} > 0$.

A fully public regime emerges if the income distribution is sufficiently compressed. In this case, the threshold \tilde{x}_2 coincides with the upper bound of the support of the income distribution and no household opts out of public schooling. The reason is that parents have more similar levels of preferred education than when there is highly dispersed income distribution. In the fully public regime, parents do not segregate by schooling but stratify across communities. When the fully public regime does not arise, i.e. $\sigma > \bar{\sigma}$, rich parents are more demanding in terms of education quality. In this case an income mixing district is formed and high-income parents opt out of the public education system, enrolling their children in private schools³⁷.

4 Inequality, Spending on Public Schools and Residential Reallocation Choices

In this section I study the impact of inequality on the equilibrium vector of fiscal policies, housing prices and spending on public schools, when an equilibrium with income mixing exists. This is an interesting exercise because there is an empirical evidence of increasing income inequality in U.S. metropolitan areas over recent decades. As in de la Croix and Doepke (2009), I proxy inequality by higher dispersion of income distribution. Dispersion is increased by extending the entire support of the uniform distribution, both lower and upper bounds, by an increase in parameter σ , so as to create a mean preserving spread. Whatever the model parameters, a mean preserving spread leads to higher private school enrollment, lower enrollment in public schools and higher housing prices

³⁷Differently from Martinez-Mora (2006), an urban-trap equilibrium with non-elite private schools cannot be an equilibrium in my model. Regardless of the value of σ and parameters, the opting-out thresholds, $\tilde{x}[q_i]$ with $i = \{1, 2\}$, are such that $\tilde{x}_1 < \tilde{x}[q_1] \leq \tilde{x}_2$ and $\tilde{x}[q_2] > \tilde{x}_2$. Therefore, private education can be chosen only by households from the top income classes not using the high quality public school in community 2.

in both communities. An increase in inequality generates an effect on income thresholds that modifies residential choices via the housing market and the voting process. However, the effect on school quality in the mixed-income community is ambiguous and depends on the prevailing housing market.

Proposition 3. *When an equilibrium with income mixing exists, the effect of inequality on the quality of public school q_1^* and on the income threshold \tilde{x}_2 is ambiguous and depends on the relative size of the endowment in housing:*

(i) *If $\frac{k_1}{k_2} \geq 1$, then $\frac{\partial q_1^*}{\partial \sigma} < 0$, $\frac{\partial \tilde{x}_2}{\partial \sigma} < 0$, $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0$, $\frac{\partial \tau_1^*}{\partial \sigma} < 0$, $\frac{\partial p_1^*}{\partial \sigma} > 0$, $\frac{\partial q_2^*}{\partial \sigma} < 0$, $\frac{\partial p_2^*}{\partial \sigma} > 0$;*

(ii) *If $\frac{k_1}{k_2} < 1$, then there exists a unique $\bar{k} > 0$ such that:*

(ii.i) *if $\bar{k} > k_2 > k_1$, then the same results as (i) hold;*

(ii.ii) *if $k_2 > \bar{k} > k_1$, then $\frac{\partial q_1^*}{\partial \sigma} > 0$, $\frac{\partial \tilde{x}_2}{\partial \sigma} > 0$, $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0$, $\frac{\partial \tau_1^*}{\partial \sigma} < 0$, $\frac{\partial p_1^*}{\partial \sigma} > 0$, $\frac{\partial q_2^*}{\partial \sigma} < 0$, $\frac{\partial p_2^*}{\partial \sigma} > 0$.*

Proof: See Appendix A.3

When inequality increases, if housing units in community 1 are sufficiently numerous relative to units in community 2, (cases (i) and (ii.i) in Proposition 3), both income thresholds move to the left: households with income level \tilde{x}_1 before the increase in inequality strictly prefer to reside in district 2 and send their children to the local public school of quality q_2 . By contrast, households with income \tilde{x}_2 before the shock on σ strictly prefer to reside in district 1, opt out of the public system and enroll their children in private schools (see Figure 3). More precisely, if the relative size of the housing market is such that the gross-of-tax housing prices in community 1 are sufficiently low, a mean preserving spread might generate an inflow of high-income middle class households towards this community, even though the quality of public schools is lower than in community 2.

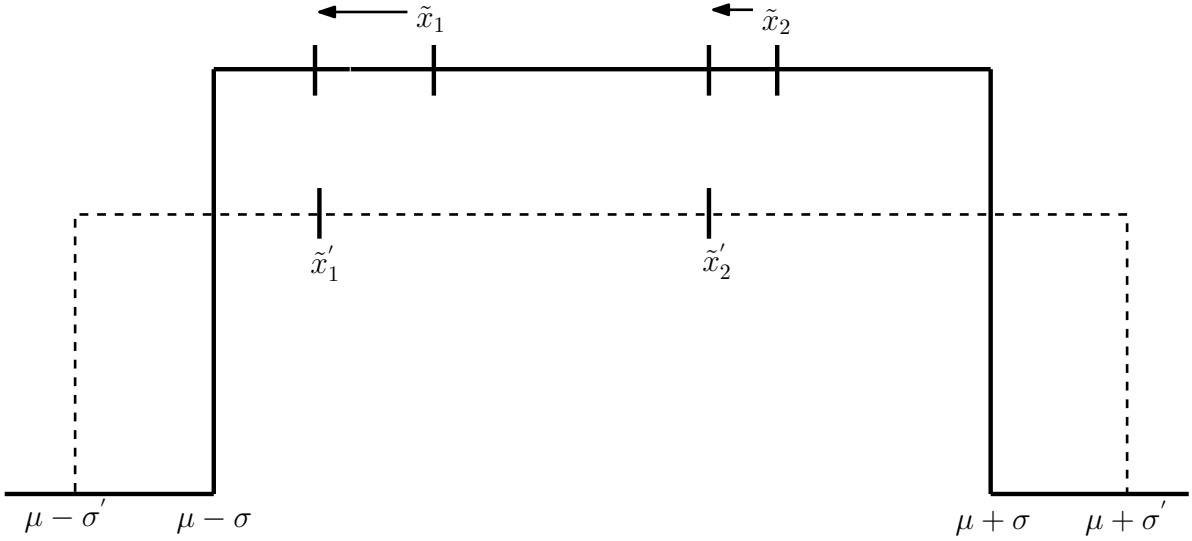
Due to population reallocation, the theoretical model predicts that an increase in

inequality negatively impacts on the quality of public schooling in community 1, because high-income middle class households migrating from community 2 will vote for low taxation. At the same time, the quality of public schooling in community 2 is negatively correlated with inequality. The fact that high-income households migrate to community 1 reduces the tax base, so that spending on public education decreases. Hence, the public spending per student in community 2 will be lower following an increase in inequality.

The reason behind these results is that under the probabilistic voting mechanism, and for sufficiently high income dispersion ($\sigma > \bar{\sigma}$), the tax rate voted is lower in community 1. If following an increase in σ housing prices in community 1 are sufficiently low that the gross-of-tax housing prices are lower than in community 2, then high-income households might decide to reside in this community, even if the quality of its public schooling is lower. In this case, they will opt out of public schooling, enrolling their children in private schools. However, if housing prices in community 2 are low enough, then the opposite applies. The fact that property taxes are lower in community 1 does not guarantee the movement of high-income households towards community 1 as a consequence of increasing inequality.

When inequality increases but the housing supply in community 2 is sufficiently large compared to community 1 (case *(ii.ii)* in Proposition 3), the income thresholds \tilde{x}_2 move to the right, and households with this income level now strictly prefer to live in community 2 and send their children to the local public school of quality q_2 . Since housing prices and, consequently, the tax base are high in community 1, high-income middle class households prefer to live in the other community. A mean preserving spread makes community 1 less attractive for the richest among the middle-income households. However, even if the tax rate goes down due to an increase in the share of rich households, this community's share of high-income households will be smaller than in the scenario with lower inequality (see Figure 4).

Figure 3: Mean-preserving Spread I

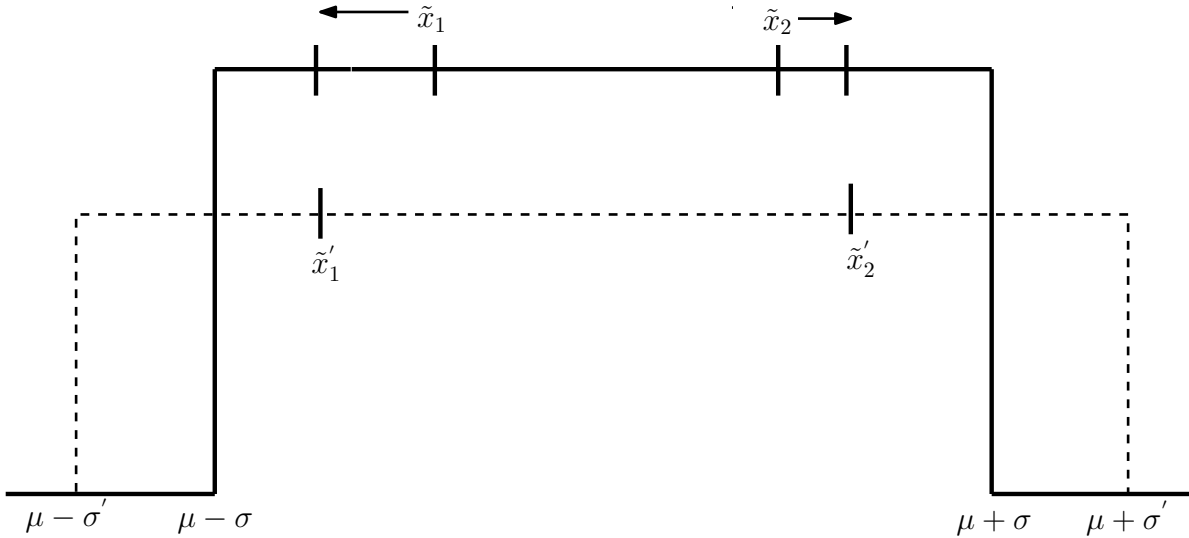


Thanks to this population reallocation, per student spending in public education increases in community 1 as a consequence of inequality. However, here too, inequality negatively impacts the quality of public schooling in community 2. Given the constant tax rate in this community, the shift in threshold \tilde{x}_1 dominates the shift in threshold \tilde{x}_2 , that is, the increasing share of low-income households is greater than the increasing share of high-income households choosing public education, so that public spending per student decreases as a consequence of a mean-preserving spread.

5 Empirical Evidence in Arizona and Illinois

The theoretical model developed in this paper contributes to the theoretical literature by revealing a link between income inequality and the presence of income mixing school districts. In a model where households can move across communities, the theoretical analysis suggests that communities in which rich and poor households cohabit deliver

Figure 4: Mean-preserving Spread II



lower quality public schooling. This is an interesting result, as typically we would expect the quality of public schools to be positively correlated with the local income level. One possible reason for this result is that rich families decide to opt out of the public school system enrolling their children in private schools and moving to another community³⁸. While the economic literature agrees that there is a strong and positive correlation between income level and enrollment in private schools, the relationship between income inequality and public schooling quality is less clear.

In this section, I provide empirical support for two theoretical conclusions of the model developed above. First, I test the relationship between quality of public schooling and type of school district. Second, I concentrate on the evolution of public schooling expenditure over time. The empirical analysis draws on the school district demographic

³⁸It should be noted that the many causes of urban segregation are vary. Using theoretical set-ups rather different from the model developed in this paper, the literature on urban segregation has analyzed other first-order mechanisms behind segregation patterns. For instance, Behrens et al. (2014) developed a model of systems of cities that explains why large cities are more productive than small cities. The model is able to replicate stylized facts about sorting, agglomeration, and selection in cities.

system (SDDS), a web-based source operated by the National Center for Education and Statistics (NCES) of the U.S. Department of Education. The data are based on the American Community Survey (ACS) 2012, 5-year estimates in 2012 inflation-adjusted dollars. The school district funding variables are taken from the Common Core of Data (CCD), a NCES program that annually collects fiscal and non-fiscal data about public school districts in the United States. Details on variables and data sources are given in Appendix B.

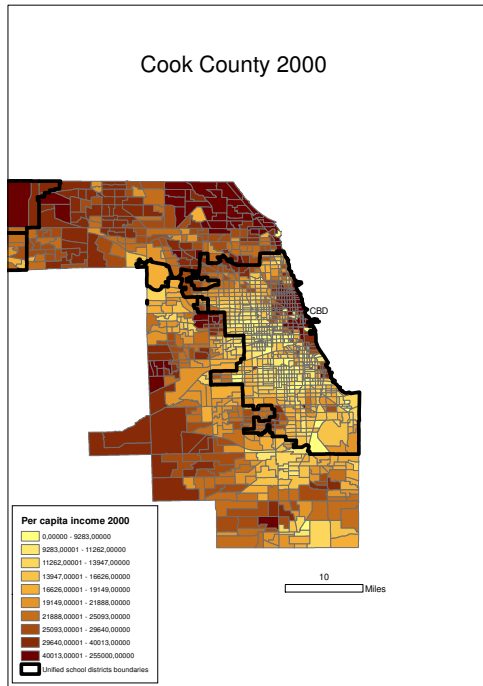
The empirical analysis concentrates on two particular case studies: the American States of Arizona and Illinois. These two States were chosen not only because they differ greatly in terms of year of founding, period of urban development, geographical location, conformation of metropolitan area and other urban characteristics, but also because they differ in terms of correlation between household income and total per pupil spending on public schooling.

First, note that the data strongly support the scenario of inter-community equilibrium described in section 3.2. Figure 5 maps the income deciles for 2000 in Cook (Illinois) and Maricopa (Arizona) Counties, where the cities of Chicago and Phoenix are respectively situated (5.a and 5.c)³⁹. The maps show the existence of mixed-income areas within unified central school districts of both cities, while the suburban school districts seem to have a less heterogeneous income distribution within their boundaries. Although there is a cohabitation of different income groups within central school districts, households tend to be more stratified by income across suburb communities in both Counties. Indeed, some communities only contain households belonging to the same income decile.

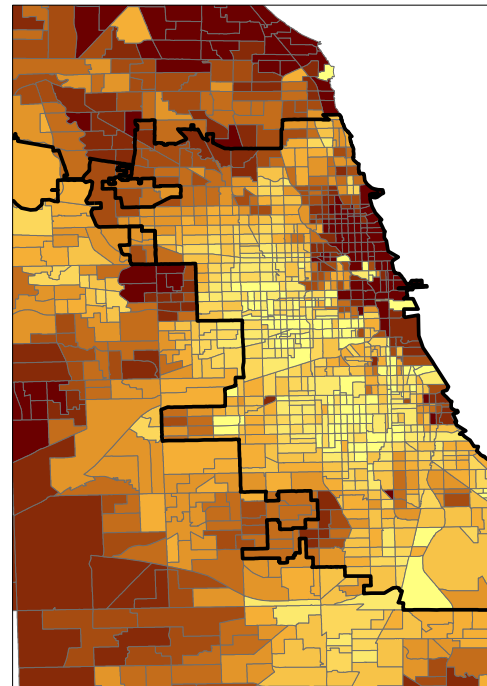
Figure 6 shows the correlation between total public expenditure per student and median household income by school district in Arizona (left panel) and Illinois (right

³⁹Maps are developed with ArcGIS using the 2000 Decennial Census data and the Topologically Integrated Geographic Encoding and Referencing system, TIGER. CBD refers to Central Business District.

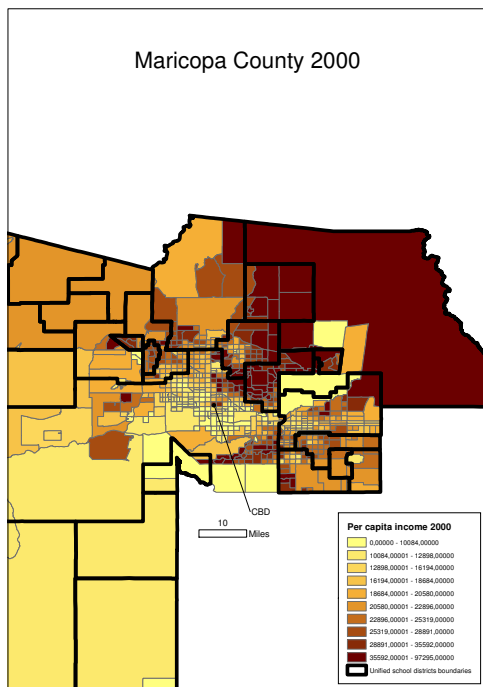
Figure 5: Per Capita Income Distribution within Counties and Cities



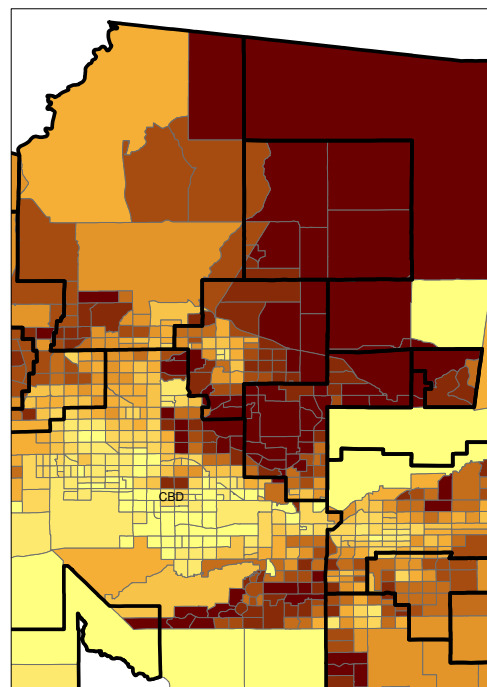
(a) Cook County



(b) Chicago City

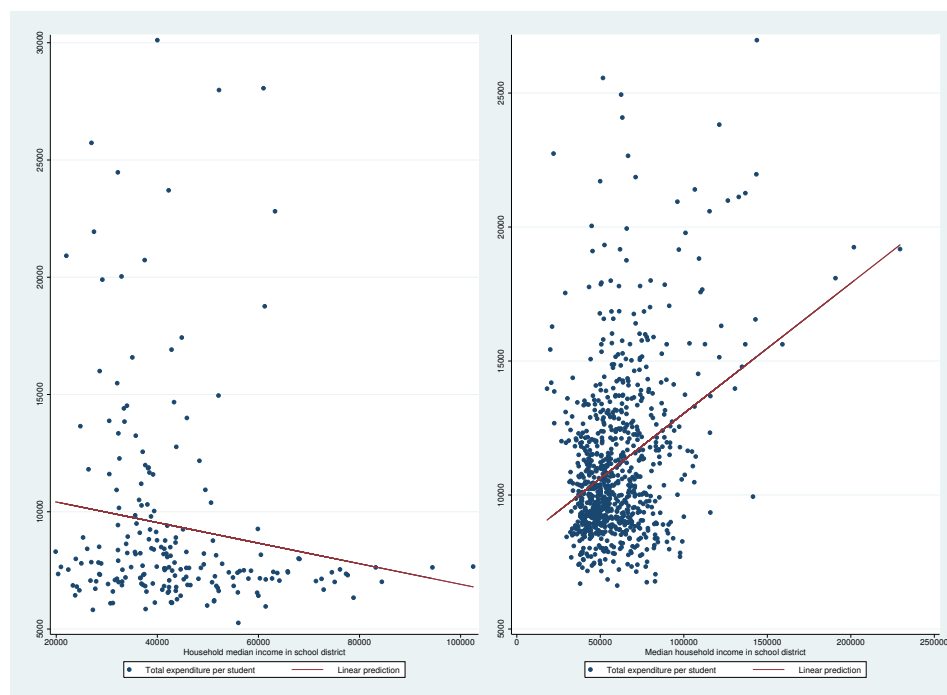


(c) Maricopa County



(d) Phoenix City

Figure 6: Per Pupil Expenditure and Median Household Income by School District in Arizona and Illinois



panel) in 2012. While in Arizona a negative correlation (-0.1431) emerges, in Illinois a positive correlation (0.3560) is observed⁴⁰.

To assess whether these two major American States feature the characteristics of the theoretical model, I concentrate on the relationship between total expenditure per student and median household income within school districts. The first objective of this empirical analysis is to see whether the theoretical prediction of the emergence of mixed-income communities is observable from school district data. To this end, the analysis concentrates on estimation results for these States with differing school district composition and characteristics. The unit of population considered for the analysis is households with at least one child under 18⁴¹.

⁴⁰Both correlations are significantly different from zero. Using median family income rather than median household income generates a stronger correlation with the total expenditure per student in both U.S. States.

⁴¹Households are defined here as having at least one child under 18 so as to avoid potential income mixing generated by high-income households without children. Taking as unit of reference married

I discriminate between poor, rich and middle-income households by setting a threshold on household income. Since the U.S. Census Bureau does not have an official definition for middle class, I consider middle-income households those with an income higher than \$15000 but lower than \$75000 per year⁴². Therefore, the poor are households with an income lower than \$15000 per year while the rich those with an income higher than \$75000 per year. In terms of percentile, this categorization defines as poor (respectively rich) a household (male or female householder with at least one child under 18) representing approximately the 15th (respectively the 85th) percentile of the U.S. income distribution in 2012.

I then compute the share of the relevant population at State level of aggregation belonging to each category. The percentage of households with at least one child under 18 in each income group for Arizona (Illinois) are the following: poor 13.45% (9.24%), middle income 61.32% (43.64%) and rich 25.23% (47.12%)⁴³. To give empirical support to Proposition 1, I first concentrate on the relationship between population composition in terms of income groups and proportion of private schooling. In particular, I analyze the relationship between the share of the population in the middle class group and the share of enrollment in private schools. The correlation coefficient between the proportion of middle class households and private schooling is negative in both States: -0.0642 for Arizona and -0.2547 for Illinois. Linear regression results indicate significance at the 1% level: $-2.61[0.010]$ for Arizona, and $-7.43[0.001]$ for Illinois⁴⁴. This simple empirical

families with at least one child under 18 does not change the qualitative results of this empirical analysis.

⁴²The Census Bureau provides the poverty thresholds for 2012 by size of family and number of related children under 18. The threshold for a family of two people with a householder under 65 and a child under 18 is \$15825. Thompson and Hickey (2005) claim that the working class income ranges from \$16000 to \$25000 and the lower middle class income from \$35000 to \$75000.

⁴³Considering that in Arizona the median household income derived from the ACS data is \$50256, and in Illinois \$56853, it seems reasonable to use the same threshold for both States. If for Illinois I define as middle class households with income between \$15,000 and \$100,000, I derive the following percentage for middle class and rich: 57.92% and 32.84%, respectively. Notice that if, for Illinois, I consider as rich households whose income is higher than \$100000, the qualitative results of the analysis do not change.

⁴⁴The p-values for the test are reported in square brackets.

exercise sheds light on one of the theoretical implications of the model developed in this paper: the relationship between the share of the population in the middle class group and the share of private schooling is negative.

The next step in providing support for Proposition 1 is to show that the quality of public education is lower in income mixing communities than in communities occupied by middle-income households choosing to enroll their children in public schools. To this end, each school district needs to be characterized within the two relevant categories of the model: mixed-income and middle-income school districts. I therefore compute the ratio between the share of rich, poor and middle-income households within each school district and the share at State level. To distinguish the districts that are relatively homogeneous from those that are heterogeneous in terms of income, I consider a mixed-income school district as a particular district in which the share of poor and rich families is larger than the corresponding share at State level and the ratio between rich and poor, or poor and rich respectively, is at most 1 to 3⁴⁵. Then, I define as a potentially middle-income district any district where the above ratio for middle class is higher than 1. To be consistent with the theoretical model in which a middle-income school district is defined as a pure public school district, I assume another condition before a school district can be considered a middle-income district: the share of enrollment in private primary and secondary education has to be smaller than 0.1%.

From the data set I exclude the districts for which the Census does not provide financial data. This leaves the new data set with 207 observations over 220 public school districts in Arizona, and 857 observations over 881 public school districts in Illinois. Public education quality is measured by current total per pupil spending by the public elementary-secondary school system and total per pupil instruction expenditure.

⁴⁵In order to characterize the residual non-middle-income district, this restriction is a reasonable assumption, even though it has no theoretical foundation in the paper. A sharper restriction, for instance 1 to 2, would not provide categories relevant for Illinois. However, for the state of Arizona is possible to consider sharper conditions. Note that even for a larger restriction, for example 1 to 4, the results will be qualitatively the same.

Table 1: Estimation results: regression of public school expenditure on school district indicators

	Arizona		Illinois	
	(1)	(2)	(1)	(2)
Middle income	2.17** (0.031)	2.73*** (0.007)	-0.11 (0.909)	-0.83 (0.409)
Mixed Income	-2.11** (0.036)	-1.88* (0.062)	0.38 (0.706)	-0.70 (0.481)
Constant	9558.56	5038.28	11077.85	6464.63
Hypothesis test:				
$H_0: Dummymix = Dummymid$	12.71 [0.0005]***	15.35 [0.0001]***	0.13 [0.7187]	0.00 [0.9598]
Observations	207	207	857	857

Notes: The dependent variables are total expenditure per pupil (1) and instruction expenditure per pupil (2). Coefficients for the control variables (described in the text) are not reported. The constants are expressed in dollars. *Dummymix* and *Dummymid* describe the dummy for the hypothesis test for mixed-income and middle-income school districts respectively. The p-values for the test are reported in square brackets. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, *** indicates significance at the 1 percent level.

Table 1 reports the results for estimates of the parameters of the regression of the two measures of quality of public schooling on a constant and two indicators which identify the type of school district, namely $Dummymix = 1$ if the district is defined as a mixed-income school district and $Dummymid = 1$ if the district is defined as a middle-income school district. The regression results for Arizona (left panel), show that the average coefficients for total and instructional expenditure per pupil for the school districts which do not fall within my categorization are \$9558.56 and \$5038.28 respectively. The average coefficients for total and instructional expenditure per pupil in middle-income districts are both above the income of the residual category and are \$11438.31 and \$6286.59 respectively. For the mixed-income districts, both are below the average expenditure of the residual category, respectively \$8154.18 and \$4376.75. All the

coefficients are individually statistically significant at standard confidence levels. Notice that in Arizona, 32.85% of the school districts are defined as mixed income, while 15.46% defined as middle income. Furthermore, I test the joint hypothesis that the coefficients are not statistically different from each other. The results suggest the rejection of the null hypothesis for both measures of public schooling quality and confirm the conclusion of the theoretical model: mixed-income communities have lower public schooling quality, while middle-income communities spend more on public education.

For Illinois (right panel in table 1), results are less clear. First, from my categorization it emerges that the presence of mixed-income and middle-income districts only has a marginal impact. In fact, only 3.15% can be defined as mixed-income districts. Although larger in number, the relatively low share of middle-income districts (3.62%) in Illinois suggests greater stratification across school districts in terms of household income in Illinois than in Arizona. As expected, the results in table 1 do not provide striking evidence either supporting or contradicting the theoretical model. The coefficients for the indicators of the presence of mixed-income or middle-income districts are both individually insignificant at standard confidence level for both schooling quality measures.

In the end, I focus on the evolution of public schooling quality over time in Arizona, the case study providing a good fit with the theoretical set-up developed in the previous section. Seeking theoretical support for Proposition 3, this empirical exercise concentrates on the effect of an increase in inequality on public schooling quality in mixed-income and middle-income school communities. Here, I use CCD data for years 2003/2004 and 2010/2011 with the ELSi application⁴⁶. Income levels over time are compared using the U.S. Government CPI data⁴⁷. Furthermore, the 1-year ACS data on

⁴⁶The table was created using the Elementary/Secondary Information System (ELSi).

⁴⁷Using CPI data it is possible to calculate the inflation rate and the buying power of the dollar over time. To compare income in 2004 with income in 2011, the 2004 data have to be inflation-adjusted by 1.19%.

Table 2: Public Schooling Quality Variation in Arizona

District	Public School Quality 2004 - 2011			
	Δ per pupil	$\Delta\%$ per pupil	Δ instruction	$\Delta\%$ instruction
Mixed	-389.69	-9.24%	-616.60	-20.82%
Middle	-1311.18	-19.61%	-628.21	-21.65%
Average	-636.09	-11.88%	-597.79	-20.41%

Notes: Δ define the mean variation from year 2004 to year 2011. The first and third columns are expressed in dollars. The second and the fourth columns provide mean percentage variations.

household income in Arizona show a Gini index increase from 0.448 in 2004 to 0.460 in 2011⁴⁸.

Using ACS data provided by the NCES, I first derive the ratio between housing units per household with at least one child in the mixed-income and middle-income school districts previously defined. Then, I calculate the ratio between housing units and households in mixed-income and middle-income districts at aggregate level, that is $\frac{k_1}{k_2} = \frac{1.957}{1.311}$. This ratio is 1.493. Proposition 3 predicts a reduction in public schooling quality in both types of school district when inequality increases. To check this, I compare the data on per pupil spending, total and instructional, for the two periods of reference in both mixed-income and middle-income school districts at aggregate level. Table 2 summarizes the main stylized facts emerging for the State of Arizona.

As expected, the data point to a reduction of the inflation-adjusted per pupil and instructional public expenditure on Arizona public schools⁴⁹. In other words, an increase in inequality in the period 2004 - 2011 was accompanied by a reduction in public schooling quality in both mixed-income and middle-income communities.

The case study on Arizona provides a good fit with the theoretical set-up developed

⁴⁸The ACS 2004 data only provide the Gini index for 80 districts. For this reason the variation in the Gini index is considered at State level of aggregation and not by type of school district.

⁴⁹The Joint Legislative Budget Committee Staff Memorandum (2012) estimates a decrease in inflation-adjusted per pupil expenditure in Arizona public schools of 15.1% from 2008 to 2013. Along these lines, Hunting (2013) observes that State funding for K-12 education in Arizona fell by 21% between 2002 and 2011.

in this paper. The data suggest a negative correlation between household median income and quality of public schooling in Arizona. This may be because the geographical mobility of households and the presence of mixed-income communities have an impact on the policy adopted by different school districts. Moreover, the results indicate a negative effect of inequality on public education spending in both mixed- and middle-income school districts. However, deeper empirical analysis would be necessary to determine whether this evidence is primarily due to schooling choices and residential location decisions.

6 Conclusion

In this paper I developed a general equilibrium model on residential sorting which points to mixed-income communities within school districts rather than perfect income stratification across communities. With respect to the previous literature, my model includes simultaneously: public versus private education choice, a two-community structure with independent local government, a competitive housing market within each community, a property tax rate based on housing value rather than income tax, a probabilistic voting mechanism and endogenous residential choices. The theoretical framework can be used to study the relationship between income distribution and residential decisions.

The first prediction of this model with probabilistic voting is that, in highly unequal economies in which households segregate by schooling within communities, poor and rich households cohabit in the same community and send their children respectively to public and private schools. In contrast, middle-income households reside in the other community and choose the local public school for their children. However, households stratify by income across communities in less unequal economies.

More importantly, the model predicts that the effect of inequality on residential location decisions and, therefore, on public spending per student, is ambiguous, and depends on the relative endowments of housing in the two communities. An increase in inequal-

ity leads to an influx of middle-income households to the mixed-income district when the supply of housing is sufficiently large. The housing market is crucial in this framework, since its characteristics affect residential choices and may imply an endogenous population reallocation within the communities.

The theoretical results presented in this paper highlight issues that point to possible directions for further empirical research. First, they suggest that the correlation between quality of public schooling and local income level is not necessarily positive. Second, inequality may have a strong impact on residential decisions and public school investments in cities with differing urban development patterns. Third, school choice could be, among others, one of the key mechanisms driving segregation patterns.

A Proofs of Propositions

A.1 Proof of Proposition 1

Substituting (4), (10), (11), (12), (13), (16) and (17) into the system composed of equations (18) and (19) and solving simultaneously at the equilibrium, allows us to determine the income thresholds that leave households indifferent between living in community 1 or in community 2:

$$\begin{aligned}\tilde{x}_1 &= \frac{\alpha + 2\beta}{2k_2(k_1 + k_2)} \\ \tilde{x}_2 &= \frac{\alpha + \beta}{k_2(2k_1 + k_2)}\end{aligned}$$

with $\alpha = \sqrt{(k_1 + k_2)^2 [k_1^2(\mu - 3\sigma)^2 + 32k_1k_2\sigma^2 + 16k_2^2\sigma^2]} + k_1^2(\mu - 3\sigma) + 3k_1k_2(\mu - 3\sigma)$, and $\beta = k_2^2\mu - 3k_2^2\sigma$. The solution of the system composed of equations (18) and (19) is an interior, $\mu + \sigma > \tilde{x}_2 > \tilde{x}_1 > \mu - \sigma > 0$, if and only if assumption 1 holds $\mu > \sigma > \frac{(k_1+k_2)\mu}{k_1+3k_2} \equiv \bar{\sigma}$. In this income mixing equilibrium, both no-migration conditions (18) and (19) are simultaneously satisfied and no agent has an incentive to move because utilities are maximized and housing markets clear.

Lemma 1 establishes that the distribution of households across education sectors exhibits perfect income segregation within a particular location. In particular, all households with income $x > \tilde{x}[q]$ strictly prefer private education for their children. To formally prove the existence of an income mixing equilibrium in this economy, we need to check if boundary indifference holds. First, consider the case of an agent with income level x living in community 1 and choosing public schooling. This agent has no incentive to move if her/his utility level is strictly greater than the utility of the agent that is indifferent between living in community 1 or in community 2 when choosing public schooling, i.e. the agent with income \tilde{x}_1 . Using (18), we have to check

if $u[q_1^*, 0, x, \tau_1^*, p_1^*] > u[q_2^*, 0, \tilde{x}_1, \tau_2^*, p_2^*] = u[q_1^*, 0, \tilde{x}_1, \tau_1^*, p_1^*]$. Using the indirect utility function defined by (5), we observe that in equilibrium the above inequality is verified when $\ln\left(-\frac{4k_1x}{\mu^2-6\mu\sigma+\sigma^2-2x^2+\tilde{x}_2^2}\right) - \ln\left(-\frac{4k_1\tilde{x}_1}{\mu^2-6\mu\sigma+\sigma^2-2\tilde{x}_1^2+\tilde{x}_2^2}\right) + \ln\left(-\frac{\mu^2-6\mu\sigma+\sigma^2-2x^2+\tilde{x}_2^2}{8x-4(\mu-3\sigma+\tilde{x}_2)}\right) - \ln\left(-\frac{\mu^2-6\mu\sigma+\sigma^2-2\tilde{x}_1^2+\tilde{x}_2^2}{8\tilde{x}_1-4(\mu-3\sigma+\tilde{x}_2)}\right) > 0$. Taking the exponential function, given the optimal income threshold \tilde{x}_2 and assumption 1, we find that $\frac{x(8\tilde{x}_1-4(\mu-3\sigma+\tilde{x}_2))}{\tilde{x}_1(8x-4(\mu-3\sigma+\tilde{x}_2))} > 1$ for any $x < \tilde{x}_1$. The utility level for an agent with income level x living in community 1 and choosing public schooling is strictly greater than the utility level of the indifferent agent with income \tilde{x}_1 living in community 1 or in community 2 and choosing public schooling when $x < \tilde{x}_1$.

The equilibrium also requires that the utility level of an agent with income x living in community 2 and choosing public schooling be larger than the utility of the agent indifferent between living in community 1 and choosing private education or living in community 2 and choosing public education. Using (19), we want to prove that in equilibrium $u[q_2^*, 0, x, \tau_2^*, p_2^*] > u[0, e^*[\tilde{x}_2], \tilde{x}_2, \tau_1^*, p_1^*] = u[q_2^*, 0, \tilde{x}_2, \tau_2^*, p_2^*]$. This condition is verified when $\ln\left(\frac{2k_2x}{x^2-\tilde{x}_1^2}\right) - \ln\left(\frac{2k_2\tilde{x}_2}{\tilde{x}_2^2-\tilde{x}_1^2}\right) + \ln(x + \tilde{x}_1) - \ln(\tilde{x}_1 + \tilde{x}_2) > 0$. Taking the exponential function, we observe that the above condition holds whenever $\frac{x(\tilde{x}_2-\tilde{x}_1)}{(x-\tilde{x}_1)\tilde{x}_2} > 1$, that is, when $x < \tilde{x}_2$. At the same time, using the previous part of the proof, it is straightforward that the utility of the agent with income x is larger than the utility of the indifferent agent with income \tilde{x}_1 choosing public education and community 1 when $x > \tilde{x}_1$. Therefore, both inequalities simultaneously hold when $\tilde{x}_2 > x > \tilde{x}_1$.

Finally, in equilibrium the utility of an agent with income x living in region 1 and choosing private education must be strictly greater than the utility of the agent with income \tilde{x}_2 indifferent between choosing public education and living in community 2 or choosing private education but living in community 1. Using (19), we want to verify that $u[0, e^*[x], x, \tau_1^*, p_1^*] > u[q_2^*, 0, \tilde{x}_2, \tau_2^*, p_2^*] = u[0, e^*[\tilde{x}_2], \tilde{x}_2, \tau_1^*, p_1^*]$. Proceeding as in the previous step of the proof, an agent choosing private schooling and living in community 1 will never move if $\ln\left(-\frac{2k_1x^2}{\mu^2-6\mu\sigma+\sigma^2+x^2-2\tilde{x}_1^2}\right) - \ln\left(-\frac{2k_1\tilde{x}_2^2}{\mu^2-6\mu\sigma+\sigma^2-2\tilde{x}_1^2+\tilde{x}_2^2}\right) > 0$. Taking the

exponential function, this condition reduces to $\frac{x^2(\mu^2 - 6\mu\sigma + \sigma^2 - 2\tilde{x}_1^2 + \tilde{x}_2^2)}{\tilde{x}_2^2(\mu^2 - 6\mu\sigma + \sigma^2 + x^2 - 2\tilde{x}_1^2)} > 1$ and is verified when $x > \tilde{x}_2$.

The solution of the system composed of equations (18) and (19) is an interior. An income mixing equilibrium emerges and is stable if and only if $\mu > \sigma > \frac{(k_1 + k_2)\mu}{k_1 + 3k_2} \equiv \bar{\sigma}$ so that $\mu + \sigma > \tilde{x}_2 > \tilde{x}_1 > \mu - \sigma > 0$ and no agent wishes to move. The condition reduces to $\frac{\mu}{2} < \sigma < \mu$ if we consider equal numbers of housing units in the two communities, that is $k_1 = k_2 = \bar{k}$.

A.2 Proof of Proposition 2

Use the threshold \tilde{x}_2 defined in proof 1. Equation \tilde{x}_2 is continuous in σ . The limit of \tilde{x}_2 when $\sigma \rightarrow \bar{\sigma} \equiv \frac{(k_1 + k_2)\mu}{k_1 + 3k_2}$ is equal to $-\frac{2k_1\mu(k_1^2 + 3k_1k_2 + k_2^2) + 2(k_1 + 3k_2)}{k_2(2k_1 + k_2)(k_1 + 3k_2)} \sqrt{\frac{\mu^2(k_1 + k_2)^2(k_1^2 + 4k_1k_2 + 2k_2^2)^2}{(k_1 + 3k_2)^2}}$. Given the parameters, this limit belongs to the domain $[\mu - \sigma; \mu + \sigma]$ if and only if $\sigma > \bar{\sigma}$. If $\sigma = \bar{\sigma}$ then $\tilde{x}_2 = \mu + \sigma$. Moreover, the limit of $\Psi_{R,1}$ when $\sigma \rightarrow \bar{\sigma}$ is zero since $\tilde{x}_2 \rightarrow \mu + \sigma$. When $\sigma < \bar{\sigma}$ we can observe that $\tilde{x}_2 > \mu + \sigma$. This scenario is excluded since \tilde{x}_2 is outside the domain of the income distribution.

A.3 Proof of Proposition 3

First, derive the sign of the derivatives of the residential thresholds \tilde{x}_1 and \tilde{x}_2 with respect to σ . Using the income thresholds defined in appendix A.1, we want to show that $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0, \forall k_i > 0$. This inequality holds when $a + 3b\sqrt{c} > 0$, with $a = 3k_1^2(\mu - 3\sigma) - 32k_1k_2\sigma - 16k_2^2\sigma$, $b = k_1 + 2k_2$ and $c = k_1^2(\mu - 3\sigma)^2 + 32k_1k_2\sigma^2 + 16k_2^2\sigma^2$. Notice that $a < 0$, $b > 0$ and $c > 0$. Taking the square of $a + 3b\sqrt{c} > 0$, after some algebra, we find that $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0$ when $4k_2(176k_1k_2^2\sigma^2 + 80k_2^3\sigma^2 + k_1^3(9\mu^2 - 6\mu\sigma + 9\sigma^2) + k_1^2k_2(9\mu^2 - 30\mu\sigma + 77\sigma^2)) > 0$. Since $9\mu^2 - 6\mu\sigma + 9\sigma^2 > 0$ and $9\mu^2 - 30\mu\sigma + 77\sigma^2 > 0$ for $\mu > \sigma > 0$, it follows that $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0 \forall k_i > 0$.

The sign of the derivative $\frac{\partial \tilde{x}_2}{\partial \sigma}$ is not monotonic in σ and depends on the relative size

of the housing endowment in the two communities, k_1 and k_2 .

$$\frac{\partial \tilde{x}_2}{\partial \sigma} = -\frac{e + \frac{ad}{\sqrt{cd}}}{f}$$

where $d = (k_1 + k_2)^2$, $e = 3k_1^2 + 9k_1k_2 + 3k_2^2$ and $f = k_2(2k_1 + k_2)$.

Notice that $d, e, f > 0$. We have to study the sign of the numerator $e + \frac{ad}{\sqrt{cd}}$. After some algebra, the numerator is positive if $\frac{2k_1^2 + 4k_1k_2 - 7k_2^2}{(k_1 + k_2)^2} + \frac{16k_1^2\mu^2}{k_1^2(\mu - 3\sigma^2) + 32k_1k_2\sigma^2 + 16k_2^2\sigma^2} > 0$. When $k_1 > k_2$ the latter is positive. It follows that $\frac{\partial \tilde{x}_2}{\partial \sigma} < 0$.

When $k_2 > k_1$ the algebra becomes more complicated. We can construct the proof using the limit of the derivatives. The limit of $\frac{\partial \tilde{x}_2}{\partial \sigma}$ when $k_2 \rightarrow 0$ is equal to $-\infty$ and the limit of $\frac{\partial \tilde{x}_2}{\partial \sigma}$ when $k_2 \rightarrow \infty$ is equal to $\frac{4\sqrt{\sigma^2}}{\sigma} - 3$. Since this derivative is continuous and increasing in k_2 within the domain, it must cross the x-axis only once, when $k_2 = \bar{k}$.

The sign of the derivative $\frac{\partial q_1}{\partial \sigma}$ reflects the behavior of $\frac{\partial \tilde{x}_2}{\partial \sigma}$. In particular, we can observe that if $\frac{\partial \tilde{x}_2}{\partial \sigma} > 0$ then $\frac{\partial q_1}{\partial \sigma} > 0$ and vice-versa. The derivative $\frac{\partial q_1}{\partial \sigma}$ is equal to

$$\begin{aligned} & \frac{-3\mu\sqrt{(k_2+1)^2((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)-3(k_2(k_2+3)+1)\mu^2+18(k_2(k_2+3)+1)\mu\sigma}}{4k_2(k_2+2)((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)} \\ & + \frac{(16k_2(k_2+2)+9)\sigma\left(\sqrt{(k_2+1)^2((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)-3(k_2(k_2+3)+1)\mu^2+18(k_2(k_2+3)+1)\mu\sigma}\right)}{4k_2(k_2+2)((16k_2(k_2+2)+9)\sigma^2+\mu^2-6\mu\sigma)} \end{aligned}$$

Assume for simplicity that $k_1 = 1$. The limit of $\frac{\partial q_1}{\partial \sigma}$ when $k_2 \rightarrow 0$ is equal to $-\infty$ and the limit of $\frac{\partial q_1}{\partial \sigma}$ when $k_2 \rightarrow \infty$ is equal to $\frac{\sqrt{\sigma^2}}{\sigma} - \frac{3}{4} > 0$. Since this derivative is continuous and increasing in k_2 within the domain, it must cross the x-axis only once, when $k_2 = \bar{k}$. Numerically, \bar{k} is the threshold for both $\frac{\partial \tilde{x}_2}{\partial \sigma} = 0$ and $\frac{\partial q_1}{\partial \sigma} = 0$, and this is true for all possible parameter values. Set for instance $\mu = 3$ and $\sigma = 1$. We get $\frac{\partial \tilde{x}_2}{\partial \sigma} = 0$ iff $k_2 = 1.65181$. At the same time we get $\frac{\partial q_1}{\partial \sigma} = 0$ iff $k_2 = 1.65181$. Set for instance $\mu = 3.5$ and $\sigma = 2$. Both derivatives, $\frac{\partial \tilde{x}_2}{\partial \sigma}$ and $\frac{\partial q_1}{\partial \sigma}$, are zero iff $k_2 = 1.23508$. If k_2 is smaller than this threshold value, then the signs of the derivatives $\frac{\partial \tilde{x}_2}{\partial \sigma}$ and $\frac{\partial q_1}{\partial \sigma}$ are negative, otherwise they are non-negative.

Let us now study the derivative of $\frac{\partial \tau_1}{\partial \sigma} = 0$, that is, $-\frac{g(\sqrt{cd}+k_1^2(\mu-3\sigma)+k_1k_2(\mu-3\sigma))}{k_2\sqrt{cd}(h+\sqrt{cd})^2} = 0$, with $g = 8k_1^2(k_1+k_2)^2(2k_1+k_2)\mu$ and $h = k_1k_2(\mu-15\sigma) + k_1^2(\mu-11\sigma) - 4k_2^2\sigma$. As the denominator is always positive, in order to prove that the optimal tax rate in community 1 is negatively correlated with σ , it is sufficient to show that $\sqrt{cd} + k_1^2(\mu-3\sigma) + k_1k_2(\mu-3\sigma) > 0$ since $g > 0$. After some algebraical manipulations, we can derive that $\sqrt{cd} + k_1^2(\mu-3\sigma) + k_1k_2(\mu-3\sigma) = 6k_2(k_1+k_2)^2(2k_1+k_2)\sigma^2 > 0$.

To study the sign of the derivative $\frac{\partial q_2}{\partial \sigma} = \frac{\frac{\partial \tilde{x}_1}{\partial \sigma} + \frac{\partial \tilde{x}_2}{\partial \sigma}}{4}$, we have first to derive the intensity of a variation in σ on the thresholds when σ increases. Using the previous results we can observe that the effect of σ on threshold \tilde{x}_1 is always greater than the effect on threshold \tilde{x}_2 . Under assumption 1 we always observe that $|\frac{\partial \tilde{x}_1}{\partial \sigma}| > |\frac{\partial \tilde{x}_2}{\partial \sigma}|$. Therefore, $\frac{\partial q_2}{\partial \sigma} < 0$ also when threshold \tilde{x}_2 moves to the right.

Now we have to show that inequality is positively correlated with housing prices in both communities. From equation (17) we know that $p_2 = \frac{\tilde{x}_2^2 - \tilde{x}_1^2}{4k_2}$, so that the derivative $\frac{\partial p_2}{\partial \sigma} = \frac{\tilde{x}_2 \frac{\partial \tilde{x}_2}{\partial \sigma} - \tilde{x}_1 \frac{\partial \tilde{x}_1}{\partial \sigma}}{2k_2}$. Since $\frac{\partial \tilde{x}_1}{\partial \sigma} < 0 \forall k_i$, if $\bar{k} > k_2 > k_1$, then $\frac{\partial \tilde{x}_2}{\partial \sigma} > 0$. It follows directly that $\tilde{x}_2 \frac{\partial \tilde{x}_2}{\partial \sigma} - \tilde{x}_1 \frac{\partial \tilde{x}_1}{\partial \sigma} > 0$ and consequently $\frac{\partial p_2}{\partial \sigma} > 0$. When $\frac{\partial \tilde{x}_2}{\partial \sigma} < 0$ we can observe that $\frac{\tilde{x}_2}{\tilde{x}_1} = \frac{\frac{\partial \tilde{x}_1}{\partial \sigma}}{\frac{\partial \tilde{x}_2}{\partial \sigma}}$ for $\sigma = \bar{\sigma}$. Since $|\frac{\partial \tilde{x}_1}{\partial \sigma}| > |\frac{\partial \tilde{x}_2}{\partial \sigma}|$, it follows from continuity that $\tilde{x}_2 \frac{\partial \tilde{x}_2}{\partial \sigma} > \tilde{x}_1 \frac{\partial \tilde{x}_1}{\partial \sigma}$ if $\sigma > \bar{\sigma}$. Under Assumption 1 we observe that $\frac{\partial p_2^*}{\partial \sigma} > 0$. The same argument can be used to prove that $\frac{\partial p_1^*}{\partial \sigma} > 0, \forall \sigma > \bar{\sigma}$.

B Data Appendix

The data by school district on household income, public and private enrollment, housing units, Gini index are taken from the 2012 American Community Survey (ACS) 5-year estimates released in September 2013. Compared to the 1-year and 3-year estimates, the 5-year estimates contain data on small geographies. Data are collected using the school district demographic system (SDDS), a web-based source operated by the National Center for Education Statistics (NCES) of the U.S. Department of Education.

Districts are divided into elementary, high school and unified (for Arizona) or unit (for Illinois) school districts. The SDDS collects data for all children under 18 years of age and not high school graduates (ages 18-19). Household income data are divided into 16 income brackets and family type is determined by presence of children under 18 years. Income for the past 12 months is calculated in 2012 inflation-adjusted dollars. Per pupil current spending for the public elementary-secondary system and instructional expenditure are taken from the financial survey of school system finances. Although the Census also provides data for funding variables for 2010, since the income levels are defined in 2012 inflation-adjusted dollars, I use 2012 data for funding variables. The data for the analysis in table 2 were obtained with the Elsi application from the Common Core of Data (CCD) and Private School Survey (PSS) for years 2003/2004 and 2010/2011.

C Robustness Check: Bounded Pareto Distribution

In this appendix, I assume that income is distributed according to a bounded Pareto distribution with probability density function (p.d.f.), $f(x) = \frac{\alpha x_{min}^\alpha x^{-\alpha-1}}{1 - (\frac{x_{min}}{x_{max}})^\alpha}$, cumulative density function (c.d.f.), $F(x) = \frac{1 - x_{min}^\alpha x^{-\alpha}}{1 - (\frac{x_{min}}{x_{max}})^\alpha}$, support $0 < x_{min} \leq x \leq x_{max}$ and parameter $\alpha > 2$. The mean is given by: $\mu = \frac{x_{min}^\alpha}{1 - (\frac{x_{min}}{x_{max}})^\alpha} \left(\frac{\alpha}{\alpha-1} \right) \left(\frac{1}{x_{min}^{\alpha-1}} - \frac{1}{x_{max}^{\alpha-1}} \right)$. The bounded Pareto distribution is chosen for two reasons. First, analytical tractability: unlike other similar income distributions, such as the unbounded Pareto distribution or log-normal distribution, this income distribution allows me to derive an analytical formulation for both voted policies τ_1^* and τ_2^* and, therefore, public education spending q_1^* and q_2^* . Moreover, using this particular income distribution, I am able to illustrate the main mechanism of the theoretical model as well as to compare the main results with the predictions of the model with a uniform distribution. Second, the Pareto distribution is a relatively good approximation of the actual US income distribution, in particular for the upper tail of the income distribution.

It is straightforward to demonstrate that, in this scenario too, the opting-out threshold is the one defined by Lemma 1. Moreover, given the timing of events, private consumption decisions would not be affected by the type of income distribution. What would be affected are the policies adopted and, therefore, public education spending. Since the objective of this appendix is to show the existence of an equilibrium with income mixing, we concentrate on the particular case in which both districts are occupied and private education is provided in community 1.

Assume for simplicity that $p_1 = p_2 = 1$ and that $k_1 = k_2 = 1$. The government budget constraints are balanced when in both regions the collected tax revenues are equal to the total public education spending. In region 2:

$$\frac{\tau_2}{1 + \tau_2} \int_{\tilde{x}_1}^{\tilde{x}_2} x f(x) dx = q_2 \int_{\tilde{x}_1}^{\tilde{x}_2} f(x) dx$$

Observe that $\int_{\tilde{x}_1}^{\tilde{x}_2} f(x) dx = \frac{(\tilde{x}_1^{-\alpha} - \tilde{x}_2^{-\alpha}) x_{min}^\alpha}{1 - (\frac{x_{min}}{x_{max}})^\alpha}$ and $\int_{\tilde{x}_1}^{\tilde{x}_2} x f(x) dx = \frac{\alpha(\tilde{x}_2^{1-\alpha} - \tilde{x}_1^{1-\alpha}) x_{min}^\alpha}{(1-\alpha)(1 - (\frac{x_{min}}{x_{max}})^\alpha)}$. It follows that:

$$q_2[\tau_2] = \frac{\tau_2}{1 + \tau_2} \frac{\alpha(\tilde{x}_1^\alpha \tilde{x}_2 - \tilde{x}_1 \tilde{x}_2^\alpha)}{(1 - \alpha)(\tilde{x}_2^\alpha - \tilde{x}_1^\alpha)}$$

Using the social welfare function defined by (7) and the government budget constraint defined above, we can derive the tax rate voted in region 2 when income is distributed according to a bounded Pareto distribution: $\tau_2^* = 1$. Not surprisingly, the optimal tax rate is the same as in the benchmark case with uniform distribution of income. Unlike the benchmark case, public education spending is given by:

$$q_2^* = \frac{\alpha(\tilde{x}_1^\alpha \tilde{x}_2 - \tilde{x}_1 \tilde{x}_2^\alpha)}{2(1 - \alpha)(\tilde{x}_2^\alpha - \tilde{x}_1^\alpha)}$$

In region 1, parents can decide to opt out of the public school system by enrolling their children in private schools. The local government budget rule for this community

is given by the following equation:

$$\frac{\tau_1}{1 + \tau_1} \left(\int_{x_{min}}^{\tilde{x}_1} x f(x) dx + \frac{1}{2} \int_{\tilde{x}_2}^{x_{max}} x f(x) dx \right) = q_1 \int_{x_{min}}^{\tilde{x}_1} f(x) dx$$

Observe that: $\int_{x_{min}}^{\tilde{x}_1} f(x) dx = \frac{1 - \tilde{x}_1^{-\alpha} x_{min}^\alpha}{1 - (\frac{x_{min}}{x_{max}})^\alpha}$, $\int_{x_{min}}^{\tilde{x}_1} x f(x) dx = \frac{\alpha \tilde{x}_1^{-\alpha} (x_{min} \tilde{x}_1^\alpha - x_{min}^\alpha \tilde{x}_1)}{(\alpha-1)(1 - (\frac{x_{min}}{x_{max}})^\alpha)}$ and $\int_{\tilde{x}_2}^{x_{max}} x f(x) dx = \frac{\alpha x_{min}^\alpha (x_{max}^{1-\alpha} - \tilde{x}_2^{1-\alpha})}{(\alpha-1)(1 - (\frac{x_{min}}{x_{max}})^\alpha)}$. Proceeding as for region 2, we define the local government budget constraint as follows:

$$q_1[\tau_1] = \frac{\alpha \tau_1 \tilde{x}_2^{-\alpha} x_{max}^{-\alpha} (\tilde{x}_1^\alpha (\tilde{x}_2^\alpha (2x_{min} x_{max}^\alpha - x_{max} x_{min}^\alpha) + \tilde{x}_2 x_{max}^\alpha x_{min}^\alpha) - 2\tilde{x}_1 \tilde{x}_2^\alpha x_{max}^\alpha x_{min}^\alpha)}{2(\alpha-1)(1+\tau_1)(\tilde{x}_1^\alpha - x_{min}^\alpha)}$$

Adapting the social welfare function (6) to the case of a bounded Pareto distribution with support $0 < x_{min} \leq x \leq x_{max}$ and using the government budget constraint defined above, we can derive the tax rate voted in region 1:

$$\tau_1^* = \frac{\tilde{x}_1 - x_{min}}{\tilde{x}_1 - \tilde{x}_2 + x_{max} - x_{min}}$$

First of all, notice that $0 < \tau_1^* < \tau_2^* = 1$. Both districts are occupied if and only if $q_2^* > q_1^*$. Therefore, when an equilibrium with income mixing exists, it will be characterized by a distribution of households across communities and types of school as described by proposition 1. Unlike the scenario with uniform distribution, using a bounded Pareto distribution means we cannot obtain analytical solutions for the no-migration thresholds \tilde{x}_1 and \tilde{x}_2 .

A numerical exercise shed light on the properties of this particular equilibrium when income is distributed according to a bounded Pareto distribution. Assume that $x_{min} = 0.1$, $x_{max} = 1$ and $\alpha = 20$ so that $\mu = 0.105263$. Using the no-migration conditions defined by (18) and (19) and the voted policies derived in this appendix, we derive: $\tilde{x}_1 = 0.351329$ and $\tilde{x}_2 = 0.549127$. Substituting these thresholds in the voted policies

and in public education spending allows us to derive: $\tau_1^* = 0.357915$, $q_1^* = 0.0277449$ and $q_2^* = 0.184896$. A mean preserving spread can be obtained by assuming a negative variation of parameter α and the lower bound x_{min} , such that the mean of the income distribution, μ , does not change. As in the main text, we approximate an increase in inequality with a mean-preserving spread. Assume, for instance, that α decreases by 0.5%, that is, from $\alpha = 20$ to $\alpha = 19.9$. To maintain a constant mean at $\mu = 0.105263$, the lower bound x_{min} has to decrease to $x_{min} = 0.099973$. Solving the no-migration conditions using these parameters, we observe that both thresholds move to the left: $\tilde{x}_1 = 0.312263$ and $\tilde{x}_2 = 0.38112853$. Since we have assumed $p_1 = p_2 = 1$ and that $k_1 = k_2 = 1$, the expected effect from increased inequality on public education spending would be negative in both regions, as suggested by proposition 3, part *i*. Using these no-migration conditions: $\tau_1^* = 0.25538$, $q_1^* = 0.0214135$ and $q_2^* = 0.163674$. As expected, inequality negatively impacts the quality of public schooling in both communities.

Even though this numerical exercise has no general validity, it indicates that an equilibrium with income mixing can also exist when assuming a bounded Pareto distribution. Moreover, it suggests that inequality can have a negative impact on public education quality, as shown in the paper assuming a standard uniform income distribution.

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