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Abstract

This paper introduces variable mark-ups in a horizontal-differentiation growth model by considering a larger class of preferences that nests the classic "CES" specification usually present in the workhorse love-for-variety models. Our first result is to obtain a generalized characterization of the Euler condition for this broader class of utility functions: in our model, the Euler rule features a supplementary term aiming at compensating the consumer for variations in the preference for variety along the consumption level. We are then also able to demonstrate that in our generalized framework, the economy's balanced growth path displays both endogenous markups and a strictly positive growth rate of the number of available varieties (being the engine of growth). Finally, we show that under endogenous markups, the economy's growth rate and firms' market power can display a negative correlation, as opposed to the standard result obtained in the CES framework.

Keywords: endogenous growth, variable markups, indirectly additive preferences

JEL classification: D4, L1, O3, O4

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1 Introduction

The relationship between market power and innovation-driven economic growth has been under scrutiny during the last decade. In seminal contributions, Aghion et al (2005, 2009) have theoretically and empirically emphasized the role of product market competition (PMC) as a spur to step-by-step innovation. In the existing literature though, variations in the degree of firms' market power are systematically modeled as variations in a "measure of competition" either linked to product substitutability or to the size of the innovation steps, i.e. as shocks to the consumer's preference specifications or to the economy's technology. The impact of "natural" determinants of the degree of competition within an economy (i.e. market size, number of competing firms, etc.) on an economy's growth rate operating through endogenous markups hence remains to be investigated. An appealing dynamic framework for the implementation of endogenous markups is the model of horizontal differentiation a la Romer, featuring endogenous firm entry in a monopolistic competition set-up. Two important caveats need however to be overcome. (i) First, as noted by Bilbiie et al (2012), variable markups seem hard to reconcile with the Kaldor fact of constant profit shares along the long-run balanced growth path. (ii) Second, as outlined by Acemoglu (2008), when endogenizing firms' market power it would seem reasonable to allow markups to be a decreasing function of the number of firms N as in the industrial organization literature (both empirical and theoretical). Such a feature is however problematic in innovation-led growth models, where the number of firms increases steadily along the balanced growth path: if the markups converge to zero for a large N, then sustained growth is no longer possible.

In this paper we propose a framework allowing to overcome both those hurdles. We do so by considering a larger class of preferences than the classic "constant elasticity of substitution" (CES) preference specification used in the workhorse love-for-variety model. More precisely, we consider the wider class of indirectly additive preferences (which nest CES preferences), and introduce them in an otherwise completely standard expanding product-variety model (Grossman and Helpman, 1993).

Our first result is to generalize the characterization of the Euler condition to our broader class of utility functions: compared to the "classic" Euler rule obtained under CES preferences, the intertemporal law of motion of exenditures obtained in our framework features a further term, aiming at compensating the consumer for variations in the preference for variety along the expenditure level. We are then also able to demonstrate that in such a framework, the economy's balanced growth path displays both constant (and yet endogenous) markups *and* a strictly positive growth rate of the number of available varieties (being the engine of growth). The intuition behind the reconciliation of those two features is that under indirectly additive preferences, firms' market power solely depends on the size of individuals' *overall* consumption. While the steady increase in the number of firms along the BGP implies a steady decrease of the expenditure share devoted to a particular variety, the overall individual consumption remains constant along the BGP, guaranteeing constant markups in line with the Kaldor facts. Any shock affecting consumers' overall consumption however entails a variation in firms' market power: in our model, markups hence depend on market size and R&D productivity. To illustrate the benefits of our approach, we then show that when the productivity in the R&D sector/the market size rises, the firms' entry rate increases while their markups decrease. In our model, the economy's growth rate and firms' market power hence display a negative correlation. On the contrary, the standard model using CES preferences yields an increase in the growth rate when imposing a positive shock on the exogenous markup size, a positive correlation that has been widely criticized in the literature. We show that endogenizing firms' market power can lead to reverse this prediction.

Contribution to the literature

Aghion et al (2005, 2009) as well as Acemoglu and Akcigit (2012) have studied the impact of PMC on long-run growth in Schumpeterian frameworks of step-by-step innovation. However, as specified above, variations in the degree of competition are modeled as shocks on consumers' preference specifications/the economy's technology rather than on economy-driven determinants of the degree of competition (entry rate of firms, market size). Also, the number of actors within an industry is systematically fixed in those Schumpeterian models, automatically precluding any investigation of the impact of firm entry on the long-run growth rate. Endogenous firm entry and the resulting varying markups have recently been shown to significantly impact the propagation of short-run business cycle fluctuations (Bilbiie et al, 2012); the authors however leave the long-run implications of such a model to be explored.

A second strand of the literature to which our paper is related are static models of monopolistic competition featuring variables markups. Foellmi and Zweimüller (2004), Behrens and Murata (2007) as well as Zhelobodko et al. (2012) focus on additively separable preferences, while Mélitz and Ottaviano (2008) embed a non separable quadratic utility into a quasi-linear framework. Last, both Parenti et al. (2014) and Bertoletti and Etro (2014) characterize the equilibrium without requiring any specific functional form. While these models have proven to be tractable in a static framework, our analysis shows that extending them to a dynamic setting is however often impossible. The reason is as simple as it is intuitive, and is linked to the caveat identified by Acemoglu and cited above: most models of monopolistic competition with endogenous markups imply that the markup is a variable function of the number of firms N. Since long-run growth in an expanding product variety model requires a steady increase in N, these models can not generate long-run growth. This is not true however for the class of indirectly additive preferences we use in our model. Bertoletti and Etro (2015) emphasize that indirect preferences allow to restore income effects in static models of monopolistic competition; we show below how this class of preferences also allows to reconcile long-run growth with endogenous markups.

The rest of the paper is structured as follows. In Section 2 we present the model; section 3 defines the balanced growth path and carries out comparative statics. Section 4 concludes.

2 The model

The aim of this section is to introduce endogenous markups in a horizontal-differentiation model by modifying the preference structure of an otherwise standard expanding product-variety model (Grossman and Helpman, 1991).

2.1 Consumers

The economy is populated by a fixed number of L consumers, who live infinitely and supply one unit of labor each period, paid at a wage $w(t)$. At a given time t, individuals enjoy the consumption of $N(t)$ different varieties available within the economy. More precisely, a representative consumer derives from her consumption an instantaneous indirect utility $V(t) = V(\mathcal{P}(t), E(t))$, where $\mathcal{P}(t)$ is the price vector of the $N(t)$ available varieties and $E(t) = \int_0^{N(t)} p_i(t)x_i(t)di$ is the overall individual expenditure at time t, with $p_i(t)$ and $x_i(t)$ denoting respectively the price and consumed quantity of a given variety i. Further assuming that the instantaneous indirect utility is **additively separable** (and using the degree-zero homogeneity property met by any indirect utility function), we are able to further characterize $V(t)$ in the following way:

$$V(t) = \int_0^{N(t)} v\left(p_i(t), E(t)\right) di = \int_0^{N(t)} v\left(\frac{p_i(t)}{E(t)}\right) di \tag{1}$$

Assumption 1 $v(.)$ is thrice differentiable, decreasing and convex (as will be seen below, the convexity of v is necessary for the demand to be a decreasing function of the price).

Several well-known and widely used preference functions meet this assumption. We hereby provide two examples.

Example 1: the CES preferences generally used in the expanding product variety models belong to this class of preferences. In this particular sub-case, we then have $v\left(\frac{p(t)}{E(t)}\right) = \left(\frac{p(t)}{E(t)}\right)^{1-\sigma}$, with σ being the constant elasticity of substitution between any pair of varieties.

Example 2: in the case of "addilog" functions (Bertoletti and Etro, 2015), we have $v\left(\frac{p(t)}{E(t)}\right) = \frac{\left(\alpha - \frac{p(t)}{E(t)}\right)^{1+\gamma}}{1+\gamma}$ with $\alpha > 0$ and $\gamma > 0$.

The intertemporal objective function of the consumer is then given by

$$\mathcal{V} = \int_0^\infty e^{-\rho t} \log V(t) dt \tag{2}$$

and it is maximized with respect to $E(t)$ under the intertemporal budget constraint

$$\dot{a(t)} = w(t) + r(t)a(t) - E(t)$$

with $a(t)$ being the individual stock of assets at time t, and $r(t)$ the economy's interest rate.

Proposition 1: For any indirectly additive preference specification respecting assumption 1, the intertemporal law of motion of expenditures $\frac{\dot{E(t)}}{E(t)}$ takes the following form:

$$\frac{\dot{E(t)}}{E(t)} + \rho + \frac{\dot{\nu}\left(E/p\right)}{\nu\left(E/p\right)} = r(t) \tag{3}$$

with $\nu(z) := -\frac{v(1/z)}{v'(1/z)} z > 0$.

Proof: The Hamiltonian is of the form:

$$\mathcal{H}(t) = e^{-\rho t} \log V(t) + \lambda(t) \left(w(t) + r(t)a(t) - E(t)\right)$$

with $\lambda(t)$ being the shadow price of the state variable $a(t)$. The first-order condition (FOC hereafter) with respect to (w.r.t. hereafter) the control variable $E(t)$ leads to:

$$e^{-\rho t} \frac{\int_0^{N(t)} \left(-\frac{p_i(t)}{E(t)^2}\right) v'\left(\frac{p_i(t)}{E(t)}\right) di}{\int_0^{N(t)} v\left(\frac{p_i(t)}{E(t)}\right) di} = \lambda(t)$$

Under the assumption of firm symmetry and using the operator $\nu(z)$ defined above, the above expression can be reformulated as:

$$\lambda(t) = \frac{e^{-\rho t}}{\nu\left(\frac{E(t)}{p(t)}\right) E(t)} \tag{4}$$

The FOC w.r.t. the state variable $a(t)$ leads to:

$$r(t) = -\frac{\dot{\lambda(t)}}{\lambda(t)} \tag{5}$$

and finally the transversality condition reads as $\lim_{t\to\infty} \lambda(t)a(t) = 0$. Log-differentiating (4) w.r.t. time and using (5), we then obtain the above expression for $\frac{\dot{E(t)}}{E(t)}$.$\square$

This Euler condition is different from the one obtained in the case of the standard textbook expanding-variety model featuring a CES-type instantaneous utility function (Grossman - Helpman 1991, Acemoglu 2008). More precisely, it differs by one term, i.e. $\frac{\dot{\nu}(E/p)}{\nu(E/p)}$. A few remarks are in order here.

First, the economic intuition behind the function ν is that it tells us about the change in expenditures $E(t)$ following a variation in the number of varieties $N(t)$ when the consumer wishes to keep his utility constant. More precisely, if the number of varieties $N(t)$ decreases by 1%, the consumer needs to increase his expenditure level by $\nu > 0$ so as to to keep his utility constant (cf Appendix A). By analogy with Vivès (2001) who deals with *directly* additive preferences, we hence refer to $\nu(.)$ as a measure of consumer's preference for variety (PFV). In the case of a model featuring a CES utility specification with $v\left(\frac{p(t)}{E(t)}\right) = \left(\frac{p(t)}{E(t)}\right)^{1-\sigma}$, we have $\nu(z) = \sigma - 1$, which is constant and does not depend on the expenditure/price levels. In its most general form though, ν might decrease or increase along $E(t)$, which means that the strength of a consumer's preference for variety is a priori not constant along its expenditure level. For instance, in the case of "addilog" functions, we have $\nu(z) = \frac{\alpha z - 1}{1+\gamma}$ so the PFV is increasing along $E(t)$.[1]

We can now move to the interpretation of our "enriched" Euler condition. In dynamical models, this condition ensures that the *costs* of an intertemporal expenditure shift (i.e. increasing a consumer's saving level by one unit at time t) are equal to its benefits $r(t)$. In the traditional expanding-variety model (Grossman - Helpman 1993, Acemoglu 2008), those costs simply amount to the discount rate ρ plus the utility variation resulting from a decrease in current expenditures [2] without any further term stemming from a variation in the PFV. Indeed, as noted above, a

[1]Under this specification, we have $z > 1/\alpha$, which means that the choke price is finite and equal to $\alpha E(t)$.

[2]In its most general form, this utility variation is equal to the variation in current expenditures $\frac{\dot{E}(t)}{E(t)}$ *times* the elasticity of marginal utility; however, in the case of a log-type CIES objective function, the latter is simply equal

consumer's PFV is in this case *constant* along the expenditure level, and hence does not generate any utility variation along the consumer's saving choices. This property ceases to hold in our more general framework.

In our case, the way the consumer enjoys the $N(t)$ available varieties at time t *depends* on his expenditure level at time t (ν varies along E). Depending on whether a consumer's PFV strength increases or decreases along the expenditure level $E(t)$, the utility cost of a variation in his saving level will either be worsened or dampened by the fact that his valuation of the *number* of available varieties will be different at a lower expenditure level. The resulting variation in the PFV is the further term $\frac{\dot{\nu}(E/p)}{\nu(E/p)}$ appearing in our Euler rule.

Having determined the optimal trajectory of the expenditures $E(t)$, it is finally possible to apply Roy's identity so as to determine the demand for each variety i:

$$x_{\mathrm{i}}(\mathcal{P}(t), E(t)) = \frac{v'\left(\frac{p_i(t)}{E(t)}\right)}{\int_0^{N(t)} v'\left(\frac{p_i(t)}{E(t)}\right)\left(\frac{p_i(t)}{E(t)}\right) di} = \frac{v'\left(\frac{p_i(t)}{E(t)}\right)}{\mu\left(\mathcal{P}(t), E(t)\right)} \tag{6}$$

with $-\mu\left(\mathcal{P}(t), E(t)\right) = -\int_0^{N(t)} v'\left(\frac{p_i(t)}{E(t)}\right)\left(\frac{p_i(t)}{E(t)}\right) di > 0$ being the marginal utility of income. The total market demand for one variety is then of the form $q_i(t) = x_i\left(\mathcal{P}(t), E(t)\right) L$.

2.2 Production

2.2.1 Product varieties production

Each consumer supplies one unit of labor. The overall labor supply at time t is divided between the production of the $N(t)$ existing product varieties and the production of new blueprints in the R&D sector. Each product variety is produced with the linear technology

$$y_i(t) = l_i(t)$$

with a total of $L_E(t) = N(t)l_i(t)$ units of labor being devoted to the final goods production at time t. Producers maximize the following profits:

$$\pi(p_i(t), E(t)) = \frac{\left(p_i(t) - w(t)\right) v'\left(\frac{p_i(t)}{E(t)}\right) L}{\mu\left(\mathcal{P}(t), E(t)\right)}$$

We define the price-elasticity of demand for variety i as $\varepsilon(z) := -\frac{v''(1/z)}{zv'(1/z)}$, and the convexity of

to 1, and the corresponding cost boils down to $\frac{\dot{E}(t)}{E(t)}$.

demand for variety i as $\zeta(z) := -\frac{v'''(1/z)}{zv''(1/z)}$. In the following, we require two extra conditions to guarantee the existence and uniqueness of a solution to the firm problem.

Assumption 2

$$\varepsilon(z) > 1 \quad \forall z \geq 0 \tag{7}$$

$$2\varepsilon(z) > \zeta(z) \quad \forall z \geq 0 \tag{8}$$

The assumption of a continuum of varieties guarantees that firms do not take into account the impact of their own pricing decisions on the marginal utility of income μ. Profit maximization then yields the following FOC:

$$v'\left(\frac{p_i(t)}{E(t)}\right) + \frac{(p_i(t) - w(t))\, v''\left(\frac{p_i(t)}{E(t)}\right)}{E(t)} = 0$$

This FOC yields the following formula for the relative markup $M(t)$:

$$\frac{p_i(t) - w(t)}{p_i(t)} = -\frac{v'\left(\frac{p_i(t)}{E(t)}\right)}{v''\left(\frac{p_i(t)}{E(t)}\right)\frac{p_i(t)}{E(t)}} = \frac{1}{\varepsilon\left(\frac{E(t)}{p_i(t)}\right)} = M(t) \tag{9}$$

where the second-order condition is given by (8) and states that the demand can not be too convex.

We define $X(t) = N(t)x(t) = \frac{E(t)}{p_i(t)}$ as the individual overall consumption level. We then obtain the following expression for the price $p(t)$ charged by every symmetric firm i at time t:

$$p(t) = \frac{w(t)}{1 - \varepsilon^{-1}(X(t))} = \frac{w(t)}{1 - M(t)} \tag{10}$$

This is the standard Lerner rule where $\varepsilon(X(t))$ is the price-elasticity of demand for an overall consumption $X(t)$. Contrary to a CES model, the price-elasticity of demand (and therefore the markups charged by each firm) hence varies with overall individual consumption in our framework.

The associated profits are:

$$\pi(t) = (p(t) - w(t))\, q(t) = \frac{M(t)}{1 - M(t)} \frac{LX(t)}{N(t)} w(t) \tag{11}$$

2.2.2 R&D (technology sector)

We denote by $L_R(t)$ the number of units of labor being employed in the R&D sector at time t. Under the standard assumption of spillovers in the blueprint production process, the law of motion of the mass of blueprints takes the following classical form:

$$\dot{N(t)} = \eta N(t) L_R(t) \tag{12}$$

Denoting by $B(t)$ the production cost of one blue-print, we have $B(t) = \frac{w(t)L_R(t)}{\dot{N}(t)}$, which when using (12) yields:

$$\eta B(t) N(t) = w(t) \tag{13}$$

Free-entry in the R&D sector implies the classic equality condition between the costs associated to the production of a blueprint and the accruing profits once having innovated. We hence also have $B(t) = \int_t^{+\infty} \left[e^{-\int_t^s r(\tau)d\tau} \right] \pi(s) ds$, which, when log-differentiating w.r.t. time, yields:

$$\frac{\dot{B}(t)}{B(t)} = r(t) - \frac{\pi(t)}{B(t)} \tag{14}$$

2.2.3 Market clearing

(i) Labour-market equilibrium:

$$L(t) = L_R(t) + L_E(t) \tag{15}$$

(ii) Product-market equilibrium - we have that for each variety, the producing firm supplies its demand: $Lq_i(t) = y_i(t)$. At the economy level, we hence obtain:

$$X(t) = \frac{L_E(t)}{L(t)} \tag{16}$$

Finally, in the following section we choose the wage $w(t)$ as the numeraire, which implies that equation *(13)* becomes

$$1 = \eta N(t) B(t) \tag{17}$$

2.3 General equilibrium

Definition: *An equilibrium is defined by a time path* $\{E(t), X(t), L_E(t), L_R(t), M(t), \pi(t), B(t), r(t), N(t)\}_{t=0}^{\infty}$ *that satisfies (3), (12), (9), (10), (11), (15), (16), (13) and (14), and the transversality condition* $\lim_{t\to\infty} \lambda(t)a(t) = 0$ *with* $N(0)$ *given (and thus a given* $B(0)$ *using* (17)*).*

We summarize the equilibrium conditions:

$$\frac{\dot{E(t)}}{E(t)} = r(t) - \rho - \frac{\nu\left(\dot{X}(t)\right)}{\nu\left(X(t)\right)} \tag{18}$$

$$\frac{\dot{N}(t)}{N(t)} = \eta L_R(t) \tag{19}$$

$$M(t) = \frac{1}{\varepsilon\left(X(t)\right)} \tag{20}$$

$$\pi(t) = \frac{M(t)}{1 - M(t)} \frac{LX(t)}{N(t)} \tag{21}$$

$$L = L_R(t) + L_E(t) \tag{22}$$

$$X(t) = \frac{L_E(t)}{L(t)} \tag{23}$$

$$1 = \eta N(t) B(t) \tag{24}$$

$$\frac{\dot{B}(t)}{B(t)} = r(t) - \frac{\pi(t)}{B(t)} \tag{25}$$

$$E(t) = \frac{1}{1 - M\left(t\right)} X(t) \tag{26}$$

It is worth noticing that those conditions collapse into the ones obtained in the standard expanding product-variety model (cf Acemoglu (2008) p. 450-451) when imposing $\nu(X) = \sigma - 1$ (which entails $\varepsilon(X) = \sigma$). Also, those general equilibrium equations feature only two dynamic independent variables $X(t)$ and $N(t)$, since the normalization implies that $B(t)$ and $N(t)$ are one-to-one related, while (20) and (26) imply that $E(t)$ and $X(t)$ are also one-to-one related. We now use the above equilibrium conditions so as to obtain below a characterization of the reduced dynamic system characterizing the trajectory of the economy in our general framework.

2.4 Equilibrium dynamics

Using (22) and (23), (19) yields:

$$\frac{\dot{N(t)}}{N(t)} = \eta L(1 - X(t)) \tag{27}$$

Using (27), (24) then yields $\frac{\dot{B(t)}}{B(t)} = -\eta L(1 - X(t))$. Combining this result with (21) and (24), (25) then yields the following implicit relationship between the interest rate $r(t)$ and the overall demand $X(t)$:

$$r(t) = \eta L \left[\phi(X(t)) - 1\right] \tag{28}$$

where we define ϕ as $\phi(z) := \frac{z}{1-\varepsilon^{-1}(z)}$. From (26) and (20), we also have:

$$\phi(X(t)) = E(t) \tag{29}$$

Log-differentiating (29) w.r.t. time and using (18), we get:

$$\frac{\phi'(X(t))\dot{X}(t)}{\phi(X(t))} = \eta L \left[\phi(X(t)) - 1\right] - \rho - \frac{\nu\,(\dot{X}(t))}{\nu\,(X(t))} \tag{30}$$

This defines implicitly the law of motion of $X(t)$. This expression can be simplified (see appendix B) using (30), (18) and (28)

$$\frac{\dot{X(t)}}{X(t)} = \frac{\left(\varepsilon(X(t)) - 1\right)\left(\eta L \left[\phi(X(t)) - 1\right] - \rho\right)}{\left(\varepsilon(X(t)) - 1\right)\left(1 + \frac{1}{\nu(X(t))} - \varepsilon(X(t))\right) + 2\varepsilon(X(t)) - \zeta(X(t))} \tag{31}$$

Last we substitute $\phi\,(X(t))$ by its expression from (29). The law of motion of $X(t)$ can then finally be rearranged into:

$$\frac{\dot{X(t)}}{X(t)} = L\frac{\eta\varepsilon(X(t))X - \left[\eta + \frac{\rho}{L}\right]\left(\varepsilon(X(t)) - 1\right)}{\left(\varepsilon(X(t)) - 1\right)\left(1 + \frac{1}{\nu(X(t))} - \varepsilon(X(t))\right) + 2\varepsilon(X(t)) - \zeta(X(t))} \tag{32}$$

It is worth noting that in the standard model using a CES preference specification, we have that the preference for variety $\nu(X)$, the demand elasticity $\varepsilon(X)$ and the demand convexity $\zeta(X)$ are not only constant and exogenous, but also governed by a single parameter - the constant elasticity of substitution between any two varieties σ. More precisely, in the case of CES preferences, we have $\nu(X) = \sigma - 1$, $\varepsilon(X) = \sigma$ and $\zeta(X) = \sigma + 1$.

The dynamic system characterizing the complete trajectory of the economy is hence defined by equations (27) and (32). The boundary conditions then close the system: $N(0)$ is given, and the transversality condition yields $X(t)$.

3 Comparative Statics along the Balanced Growth Path

3.1 Balanced Growth Path

We define a balanced growth path (BGP) as an equilibrium path along which every variable grows at a constant rate, either null or positive. In an expanding *product*-variety model featuring no productivity improvement, the BGP is characterized by constant levels of individual consumption X^* and a strictly positive, constant growth rate of the number of blueprints $g_N = \frac{\dot{N}}{N}$ (consumers become better-off over time due to the expansion of the pool of consumed varieties).

Proposition 2: For any initial number of blueprints $N(0)$ and any arbitrary value of the initial per-capita stock of assets $a(0)$, the considered economy immediately jumps on its Balanced Growth Path.

Proof: In our closed economy, we have an accounting equality between the value of the households' assets and the value of the stock of blueprints: $La(t) = N(t)B(t)$ at any time $t > 0$. Along the normalization condition (24), we furthermore have that the value of the overall stock of blueprints is equal to a constant: $N(t)B(t) = \frac{1}{\eta}\ \forall\ t > 0$. For any arbitrary initial value $a(0)$, the per-capita stock of assets will hence immediately jump to a constant positive value $a(t) = a^* = \frac{1}{L\eta}\ \forall\ t > 0$, also entailing constant values of consumers' expenditures and of the interest rate: $E(t) = E^*$ and $r(t) = r^*\ \forall\ t > 0.\Box$

We now move to characterizing the properties of this Balanced Growth Path.

In our general framework, since the numerator of the law of motion (31) is increasing in $X(t)$, there is only one candidate X^* to the BGP, defined implicitly at $\frac{\dot{X(t)}}{X(t)} = 0$ by the following condition:

$$\frac{\rho}{\eta L} + 1 = \frac{X^*}{1 - \varepsilon^{-1}(X^*)} \tag{33}$$

Proposition 3: (33) admits a unique solution satisfying for any ρ, η and L.

Proof: We first prove that the RHS is increasing if (8) is satisfied. Taking the derivative of $\phi(z) := \frac{z}{1-\varepsilon^{-1}(z)}$ leads to

$$\frac{d\phi}{dz} = \frac{(1 - \varepsilon(z)^{-1}) - z\left(-\left(-\frac{\varepsilon'(z)}{\varepsilon(z)^2}\right)\right)}{(1 - \varepsilon(z)^{-1})^2}$$

In order to simplify the expression above, we use the following identity (see appendix C):

$$\frac{\varepsilon'(z)z}{\varepsilon(z)} = -\left(1 + \varepsilon(z) - \zeta(z)\right)$$

so that the numerator boils down to

$$\left(1 - \varepsilon(z)^{-1}\right) + \varepsilon(z)^{-1} + 1 - \varepsilon(z)^{-1}\zeta(z) = \varepsilon(z)^{-1}\left(2\varepsilon(z) - \zeta(z)\right) > 0$$

which is always true from (8). Given (7), the right hand side term increases from 0 to $+\infty$. □

Along (29) and (28) we then have $E^* = \phi(X^*)$ and $r^* = \eta L(E^* - 1)$, while the steady-state growth rate g_N of the economy is given by (27): $g_N = \eta L(1 - X^*)$. So as to simplify comparisons with the CES benchmark, it is possible to reformulate the latter as:

$$g_N = \left(\frac{\eta L}{\varepsilon(X^*)} - \rho\frac{\varepsilon(X^*) - 1}{\varepsilon(X^*)}\right) \tag{34}$$

This coincides with the expression of the canonical model presented in Acemoglu (2008) p. 451 for which the price-elasticity of demand is constant with $\varepsilon(X^*) = \sigma$. This also shows that, while $\nu(X)$, $\varepsilon(X)$ and $\zeta(X)$ appear in the law of motion (32), only the price-elasticity of demand matters for comparative statics along the BGP.[3]

3.2 Comparative statics

We consider again the implicit definition of X^* along the BGP, given by (33):

$$\frac{\rho}{\eta L} + 1 = \phi(X^*)$$

Since ϕ is increasing (cf demonstration of Proposition 3), we can conclude that a larger market size L, a lower discount rate ρ or higher R&D spillovers ν decrease the total consumption of the differentiated good X^*. It is worth noting that in the limit case where $\rho = 0$, market size has no impact on X^*. This is consistent with the static model presented by Bertoletti and Etro (2015). Compared to their framework, the dynamics present in our model restore the impact of market size on X^*, and thereby the markups.

We now move to the impact of market size and R&D productivity on the growth rate.

[3] Note however that this is conditional upon the existence of a BGP, which itself requires that the demand is not too convex (condition (8)).

Proposition 4: The long-run growth rate increases with market size and R&D productivity.

Proof: log-differentiating (27) w.r.t. ηL leads to

$$\frac{\eta L}{g_N}\frac{dg_N}{d\left[\eta L\right]} = 1 - \frac{1}{g_N}\frac{dX(t)}{d\left[\eta L\right]}$$

Since market size and productivity unambiguously decrease individual overall consumption, $\frac{dX(t)}{d[\eta L]} < 0$, which proves proposition 4. $\square$

Now, equation (34) emphasizes that variable markups may magnify or dampen these impacts. Specifically, it is crucial to know how ε varies with X, which we discuss below.

How does the price-elasticty of demand vary with X? The literature, at least since Krugman (1979) has assumed that the price-elasticity of demand is an increasing function of the price. Assuming so leads to the prediction that trade has a pro-competitive effect as it reduces firms' markups (Mélitz and Ottaviano 2008). Empirically this is backed up by the data (Feenstra and Weinstein, 2010). Theoretically, Mrazova and Neary (2013) have also found out that this assumption can be traced back to Marshall and is known as the second Marshall law of demand. Last, it is worth mentioning that under indirect additivity, it amounts to assuming that consumers are less sensitive to variation in prices when they have a higher income (Bertoletti and Etro, 2015), which is quite natural. *We hence focus below on the case where the price-elasticity of demand ε is a decreasing function of $\frac{E}{p} \equiv X$.* It should be clear however that the opposite case can be dealt with as easily.

The impact of market size and R&D productivity Since X^* decreases with market size and higher R&D productivity, proposition 4 implies that those two types of shocks lead to lower markups (the latter being equal to $\varepsilon(X^*)^{-1}$). It immediately follows from (34) that, compared to the CES case, the positive impact of market size and R&D productivity on the growth rate is *dampened* by variable markups, everything else being equal. Interestingly enough, the variable markups can hence lead to reducing the scale effect usually present in standard expanding-variety models.

Last and most importantly, a variation of L or η generates a negative correlation between markups and the growth rate. In contrast, drawing comparative statics w.r.t. to ε in (34) as if it was exogenous would systematically predict a positive correlation. This is the result usually obtained in the CES case, a feature that has been extensively criticized in the literature as being counterfactual. Our framework hence shows that endogeneizing firms' market power allows us to revisit the interaction between product market competition and growth in expanding-variety growth models. It also opens the floor for similar investigations in quality-ladder models: introducing endogenous mark-ups in a Schumpeterian model might be a way to theoretically replicate the empirical relationship identified by Aghion et al (2009) in a framework allowing for free-entry of firms.

4 Conclusion

Using the general class of indirectly additive preferences in an otherwise totally standard expanding product-variety model, we provide a framework featuring both endogenous markups and a strictly positive growth rate along the BGP. We then investigate the impact of "natural" determinants of firms' market power such as R&D productivity and market size on the economy's growth rate. We show in particular that while an increase in the market size impacts positively the entry rate of firms, it is detrimental for the size of markups, hence establishing a negative correlation between endogenous firms' market power and long-run growth. Such a result reverses the predictions of a positive impact of exogenous markups on growth usually obtained in the standard models.

Further lines of research can be sketched. First, traditional models where markup measures are linked to an exogenous preference parameter preclude any welfare analysis of modifications in the degree of product market competition. Our model makes it possible to investigate this venue. Second, as we emphasized at the end of section 2.4, several demand characteristics impacting the economy's dynamic system outside the long-run BGP are governed by a single parameter in the case of CES preferences: namely, the preference for variety, price-elasticity and convexity of demand are all pinned down by the value of the elasticity of substitution σ. An enriched version of our more general framework would allow us to precisely investigate which elasticity drives the variations in both firms' market power and the economy's growth rate during the transition to the BGP.

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Appendix

A Preference for variety

At a symmetric price profile p, the overall indirect utility reads as

$$\mathcal{V} = Nv\left(\frac{p}{E}\right)$$

$$E = \frac{p}{v^{-1}\left(\frac{\mathcal{V}}{N}\right)}$$

Now we can express the elasticity of the expenditure E w.r.t. the number of varieties

$$\mathcal{E}_{E,N} := -\frac{d\ln\left(v^{-1}\left(\frac{\mathcal{V}}{N}\right)\right)}{d\ln N}$$

which can be rearranged as follows

$$\mathcal{E}_{E,n} = -\frac{1}{v'(v^{-1}\left(\frac{\mathcal{V}}{n}\right))}\frac{1}{v^{-1}\left(\frac{\mathcal{V}}{n}\right)^2}\left(-\frac{\mathcal{V}}{n^2}n\right) \equiv -\nu\left(\frac{E}{p}\right) < 0$$

So ν is the change in expenditure which offsets the gain in variety.

$$\frac{d\mathcal{V}}{\mathcal{V}} = \frac{dN}{N} + \frac{d[p/E]v'\left(\frac{p}{E}\right)}{v\left(\frac{p}{E}\right)}$$

Utility changes because of a variety effect and because of an income effect holding the prices of varieties fixed.

B The law of motion of $X(t)$

Log-differentiating the LHS term in (30) leads to

$$\frac{\phi'(X(t))}{\phi(X(t))}\dot{X(t)} = \dot{X} + \frac{\dot{M}}{1-M}$$

Using the formula in Appendix C, we have

$$\dot{M} = \frac{\dot{X(t)}}{\varepsilon X(t)}\left(1+\varepsilon(X(t))-\zeta(X(t))\right)$$

Plugging it in the above expression leads to

$$\frac{\phi'(X(t))}{\phi(X(t))}\dot{X(t)} = \dot{X(t)} + \frac{\varepsilon(X(t))}{\varepsilon(X(t))-1}\left(\frac{1+\varepsilon(X(t))-\zeta(X(t))}{\varepsilon(X(t))}\right)\frac{\dot{X(t)}}{X(t)}$$

which can be rearranged as follows:

$$\frac{\phi'(X(t))}{\phi(X(t))}\dot{X(t)} = \frac{\varepsilon(X(t))}{\varepsilon(X(t))-1}\left(\frac{2\varepsilon(X(t))-\zeta(X(t))}{\varepsilon(X(t))}\right)\frac{\dot{X(t)}}{X(t)}$$

Now, the RHS term in (30) reads as

$$RHS := \eta L\left[\phi(X(t))-1\right] - \rho - \frac{\dot{X}}{X(t)}\frac{X(t)\nu'(X(t))}{\nu(X(t))}$$

Using again appendix C, can rearrange this expression as follows

$$RHS = \eta L\left[\phi(X(t))-1\right] - \rho - \frac{\dot{X}}{X}\left(1+\frac{1}{\nu(X(t))} - \varepsilon(X(t))\right)$$

Equalizing the RHS and the LHS terms leads to the law of motion

$$\frac{\dot{X(t)}}{X(t)} = \frac{\left(\varepsilon(X(t))-1\right)\left(\eta L\left[\phi(X(t))-1\right]-\rho\right)}{\left(\varepsilon(X(t))-1\right)\left(1+\frac{1}{\nu(X(t))}-\varepsilon(X(t))\right)+2\varepsilon(X(t))-\zeta(X(t))}$$

C Preference for variety, price-elasticity of demand and demand convexity.

We start by differentiating $\varepsilon(z) = -\frac{v''(1/z)}{z\cdot v'(1/z)}$:

$$\frac{\partial\varepsilon}{\partial z} = -\frac{-\left(1/z^2\right)\cdot v'''(1/z)\cdot z\cdot v(1/z) - v''(1/z)\left(v'\left(1/z\right) - 1/z^2\cdot v''\left(1/z\right)\cdot z\right)}{\left(zv'\left(1/z\right)\right)^2}$$

Expressing the elasticity yields

$$\frac{z}{\varepsilon(z)}\frac{\partial\varepsilon}{\partial z} = -\frac{v'''(1/z)}{v''(1/z)\cdot z} - 1 + \frac{v''(1/z)}{v'(1/z)\cdot z}$$

which boils down to the desired formula

$$\frac{z}{\varepsilon(z)}\frac{\partial\varepsilon}{\partial z} = \zeta(z) - 1 - \varepsilon(z)$$

Similarly, the preference for variety can be related to the price elasticity of demand as

$$\frac{z}{\nu(z)}\frac{\partial\nu}{\partial z} = 1 + \frac{1}{\nu(z)} - \varepsilon\left(z\right)$$

Further implications of these formulae are examined in Mrazova and Neary (2014).