Choice with Time

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Abstract

We propose a framework for the analysis of choice behavior when the later explicitly depends upon time. We relate this framework to the traditional setting from which time is absent. We illustrate the usefulness of the introduction of time by proposing three possible models of choice behavior in such a framework: (i) changing preferences, (ii) preference formation by trial and error, and (iii) choice with endogenous *status-quo* bias. We provide a full characterization of each of these three choice models by means of revealed preference-like axioms that could not be formulated in a timeless setting.

Keywords: Choice, behavior; Time; Revealed preferences; Changing preferences; Learning by trial-and-error; Inertia bias.

1   Introduction

An important accomplishment of modern economic theory is the precise identification of its behavioral implications. A rich - and now classical - tradition of research, initiated by Samuelson (1938), and pursued by Houthakker (1950), Chernoff (1954), Arrow (1959), Richter (1966), Sen (1971), among many others, has formulated these implications in terms of a *choice function*, sometimes generalized to a *choice correspondence*. While a choice function assigns to every set of alternatives - or menu - in some universe a unique element of it, interpreted as the chosen alternative in the menu, a choice correspondence assigns to every menu in the universe a *subset* of this menu, interpreted as containing all alternatives that *could have* been chosen by the agent. A choice correspondence is not directly observable because we do not in practice observe simultaneous multiple choices For this reason, we focus mainly on choice functions in what follows.

The behavioral implications of a significant variety of theories have been examined through the formalism of choice functions. The most well-known of

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them posits that the choice results from the maximization of a single preference defined on the set of all conceivable alternatives. The behavioral implications of this theory on abstract choice functions are the Chernoff (1954) condition (called property α by Sen 1971), Arrow's (1959) condition, Houthakker's (1950) axiom of revealed preference or the Richter's (1966) congruence axioms. This one-rationale explanation of choice has also been applied to the specific context of classical consumer theory where the alternatives are consumption bundles and where the menus are budget sets. In this setting, where additional properties of preferences such as local non-satiation can be defined, the most well-known empirical implication of this one-rationale choice theory is Afriat (1967) Generalized Axiom of Revealed Preferences (GARP), very clearly analyzed in Varian (1982) and discussed in Varian (2006).

The findings of psychology and behavioral economics suggest, however, that the implications of the maximization of a single preference are often rejected by actual choice behavior (see e.g. Fudenberg 2006, Fehr and Hoff 2011, and Hoff and Stiglitz 2016 for reviews). This has led several authors to propose alternative theories of choice and to look for the implications of these on the choice function or correspondence. For example, Masatlioglu, Nakajima, and Ozbay (2012) have identified the properties of a choice function that selects the preferred alternative from a *consideration set* in each menu, rather than from the whole menu itself. This consideration set is interpreted as reflecting what the decision maker pays attention to in her choice process. This consideration set may not coincide with the whole menu of feasible alternatives if, for example, the decision maker is "inattentive" to some of the alternative that are available. Barberà and Neme (2016) have also used a choice function to characterize a model in which the decision maker chooses one of the r-best alternatives according to a preference, rather than the 1st-best as assumed in conventional theory. Sprumont (2000) has used a choice correspondence to identify the implications that a collection of individual agents choose an alternative that could be a Nash equilibrium of a game for some preferences. Others, such as Manzini and Mariotti (2007, 2012) and Apesteguia and Ballester (2013), have identified the observable properties of a choice function that are necessary and sufficient for its rationalization by a sequential lexicographic application of a collection of preferences that ends up selecting a unique alternative from each menu.

Flexible and amenable to formulations of testable implications of many (behavioral) choice models as it is, a choice function (or correspondence) may still be considered unduly abstract for many applications. One of the important and easily observable feature of the reality that it neglects is the *time* at which the menu is made available to the decision maker. Indeed, as used in the literature just described, a choice function describes a timeless process that only provides the chosen alternatives in every admissible menu. It does not record (nor use information on) the *periods* at which the menus are available. Yet, in most data on choice observations that we could think of, the menus of choice will present to the decision maker one after the other and this information is known. For instance, economic experiments often record (or can record) the time sequence of choices. More generally, dynamic choice theory provides several examples

where the time sequence of choices plays a key role. Change of habits, learning, and similar phenomena in which the preferences appear to be endogenous to the *experience* of the decision maker seem to require an explicit integration of time in the description of the choice process.

In this paper, we therefore extend the traditional setting and propose to analyze the choice behavior as an explicit function of *both* the time at which the choice takes place *and* the menu available at that time. We then show that this information on the time period at which the choice takes place enables one to identify the behavioral implications of theories of choice that could not be analyzed without an explicit integration of time. Three examples of such theories are examined and characterized in this paper.

The *first* one concerns the possibility, for the decision maker, to experience a *change in preferences* at some period. In such a model, the decision maker chooses in a way that maximizes a given preference up to some time period and, after this period, switches to another preference and makes its subsequent choices based on this preference. We provide a simple "revealed preference test" for this particular theory of choice, that relates to the literature on changing tastes (see e.g. Gul and Pesendorfer 2005). While we characterize a choice model in which a decision maker changes preferences only once, the generalization to any finite number of changes in preferences that is smaller than the total number of time periods would be straigthtforward.[1]

The *second* theory of choice that we characterize with a very simple revealed preference axiom is that of *learning by trial and error*: In such a theory, the decision maker "tries out" the alternatives before forming her preference over them. Hence, when facing a menu at a given period, the decision maker either tries out one alternative *or* chooses the "best" option according to a single preference relation among the alternatives she has previously chosen at least once. This model provides a rationalization of "inconsistent" behavior at the beginning of some sequence of choices. It describes a plausible process of "trial and error" for discovering what a person "really prefers" that has been widely documented in the literature. It relates to Cooke (2016) and Piermont, Takeoka, and Teper (2016) who characterize similar learning models over uncertain prospects and to Young (2009) who characterizes a similar learning model in a game theoretical environment.

The *third* theory of choice characterized herein is what could be called *choice with inertia bias*. In such a theory, the decision maker takes her last choice as a *default* for her current choice. For each choice situation, the decision maker chooses the best option according to a single preference *or* the alternative she has chosen in the previous period. We interpret this resort to the choice made in the immediately previous period as an "imprisonment in the habit". There has been many contributions in the literature that have examined behavior exhibiting *status-quo* bias (for example Tversky and Kahneman 1991). In a related vein, several authors have provided axiomatic characterizations of choice models with

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[1] Trivially, any choice behavior that depends upon time can be seen as resulting from a decision maker who changes preferences at every period. See Kalai, Rubinstein, and Spiegler (2002) for a similar observation on the standard timeless setting.

a default-option in which this option is exogenously given (e.g. Bossert and Sprumont 2003; Masatlioglu and Ok 2005). In this theory of choice with inertia bias, the default-option is rather endogenous and evolves over time.

These examples are, of course, extremely specific representations of a much wider and richer class of choice models involving non-standard considerations such as preference reversals, learning, and cognitive reference-dependent bias. Yet, we believe that these examples serve rather well their purpose of showing how the introduction of time in the formal description of choice behavior is necessary for identifying the empirical implications of several theories of choice, and how it eases the identification of these implications.

The framework introduced in this paper bears some similarity with that introduced by Bernheim and Rangel (2007; 2009) and Salant and Rubinstein (2008). These authors have analyzed some normative (Bernheim and Rangel 2007; 2009) and positive (Salant and Rubinstein 2008) implications of choice processes in which every menu of alternatives is supplemented with an *ancillary condition* that represents either a "frame" or some other "consequentially-irrelevant" feature of the choice environment. One could, of course, view the time at which the choice is made as a frame or an ancillary condition. Yet time is a somewhat specific feature of the choice environment. One of its specificities is that it leads to an ordering of the menus offered to the decision maker as *per* the time at which they are available. The properties of this ordering (e.g. the fact that one alternative chosen "in the past" is not chosen "in the present") play an important role in the characterization of the choice models that we provide. By contrast, the abstract ancillary conditions and frames examined by Bernheim and Rangel (2007; 2009) and Salant and Rubinstein (2008) do not impose a structure on the set of available menus that is as precise as that of an ordering. Another difference between our approach and those of Bernheim and Rangel (2007; 2009) and Salant and Rubinstein (2008) is that we do not assume the possibility of observing choice behavior that would take place in any conceivable combination of time period and menu at that time period. We only consider, somewhat realistically, that we observe a particular chronology of choices, and we identify the necessary and sufficient conditions that the choice behavior observed in that particular chronology must satisfy in order to result from each of the three theories of choice mentioned above. A third difference between the approach of Bernheim and Rangel (2007; 2009) and Salant and Rubinstein (2008) and ours concerns the interpretation given by the former to the frame and the ancillary condition. Bernheim and Rangel (2007; 2009) define an ancillary condition to be "a feature of the choice environment that may affect behavior, but [that] is not taken as relevant to a social planner's evaluation" (Bernheim and Rangel 2009, 55). As our analysis is more positive than normative, we do not take a position on the issue of whether or not the dependency of the choice on the time period should be "relevant to a social planner's evaluation". Let us simply say that our immediate intuition about this matter is that time should be relevant. After all, preferences (choices) that were revealed (made) very long time ago may have less bearing on our appraisal of the current well-being of the decision maker than those revealed (made) in more recent periods. But nothing

in our analysis depends on this intuition. As for the frame side of the coin *à la* Salant and Rubinstein (2008), we also have difficulty in viewing the time at which a choice is made as an information that is, to take the authors' words, "irrelevant in the rational assessment of the alternatives but nonetheless affects behavior". We suspect that Salant and Rubinstein (2008), who contrary to Bernheim and Rangel (2007; 2009) do not give time as an example of a frame, would agree with us.

In a recent paper, Cerigioni (2016) proposes a choice theoretic framework that explicitly introduces time, but in which the menu available for choice at every time period is supplemented by an abstract vector of (non-time) ancillary conditions that themselves depend upon the time period. He characterizes in this framework a "dual-self" theory of choice. As compared to his, our analysis is therefore closer to the classical choice theory since, except for time, we do not consider any other argument of the choice function than the menu of alternatives to which it applies.

The remainder of the paper is organized as follows. The next Section introduces formally the framework of choice with time, and discuss its connection with the classical timeless choice theoretic setting. In Section 3, we provide the empirical implication of the conventional one preference maximization model in the choice theoretic framework with time. Section 4 identifies the implications, in that framework, of the three time dependant theories of choice discussed above. Section 5 concludes.

2 Framework

2.1 General Notation

In the following, we define a *binary relation* $\succsim$ on any set Ω as a subset of $\Omega \times \Omega$. Following the convention in economics, we write $x \succsim y$ instead of $(x, y) \in \succsim$. Given a binary relation $\succsim$, we define its *symmetric factor* $\sim$ by $x \sim y \iff x \succsim y$ and $y \succsim x$ and its *asymmetric* factor $\succ$ by $x \succ y \iff x \succsim y$ and not $(y \succsim x)$. A binary relation $\succsim$ on Ω is
(*i*) *reflexive* if the statement $x \succsim x$ holds for every x in Ω,
(*ii*) *transitive* if $x \succsim z$ follows $x \succsim y$ and $y \succsim z$ for any x, y, $z \in \Omega$,
(iii) *complete* if $x \succsim y$ or $y \succsim x$ holds for every distinct x and y in Ω and,
(iv) *antisymmetric* if $x \sim y \Rightarrow x = y$.

We call *ordering* a reflexive, transitive, and complete binary relation and *linear ordering* an antisymmetric ordering.

Given two binary relations $\succsim_1$ and $\succsim_2$ we say that $\succsim_2$ is an extension of $\succsim_1$ (or is compatible with $\succsim_1$) if it is the case that, for any x and y in Ω such that $x \succsim_1 y$ one has also $x \succsim_2 y$. Given a binary relation $\succsim$ on a set Ω, we define its *transitive closure* $\widehat{\succsim}$ by: $x \widehat{\succsim} y \iff \exists \{x_t\}_{t=0}^l$ for some integer $l \geq 1$ satisfying $x_t \in \Omega$ for all $t = 0, ..., l$ for which one has $x_0 = x$, $x_l = y$ and $x_j \succsim x_{j+1}$ for all $j = 0, ..., l - 1$. It is well-known that the transitive closure of a binary relation $\succsim$ is the smallest (with respect to set inclusion) transitive binary

relation compatible with $\succsim$.

2.2 Modeling of Choice

Let X be a universe of alternatives of interest for the decision maker, let $\mathcal{P}(X)$ be the set of all non-empty subsets of X, and $\mathcal{F} \subseteq \mathcal{P}(X)$ be a collection of subsets of X, each of which being interpreted as a *choice problem* (using Arrow's 1959 terminology) or a *menu*. A choice function on $\mathcal{F}$ is a mapping $C : \mathcal{F} \longrightarrow X$ that satisfies $C(A) \in A$ for every A in $\mathcal{F}$. The choice-theoretic literature that has emerged in the last sixty years or so has made various assumptions on the domain $\mathcal{F}$ that depend, sometimes, upon the nature of the alternatives in X that are considered. For example, the classical theory introduced by Arrow (1959) has taken X to be an abstract finite set, and $\mathcal{F}$ to coincide with $\mathcal{P}(X)$. This is clearly very demanding from an observational viewpoint, since it is difficult in practice to observe all choices that an agent could make when facing any conceivable menu.

Quite a few years later, Sen (1971) has shown that several results on the rationalization of choice models hold on more restricted domains provided that those domains include all possible pairs and triples of X. While less demanding, this requirement is still quite demanding, as we often do not observe (or do not want to "have" to observe) all possible pairs and triples of X, even when the later is finite. In another attempt to relax the observational demands of the classical theory, Richter (1966; 1971), Hansson (1968), and Suzumura (1976; 1977; 1983) developed the theory of revealed preference for choice functions or correspondences with general domains that do not impose any restriction on the class of menus that may be available. In particular, the authors provided several characterizations of choice functions and/or correspondences defined on any non-empty family of non-empty subsets of X.[2]

In the following, we supplement this general domain with a discrete time horizon $\mathcal{T} = \{1, ..., T\}$. This enables one to define a **chronology of choices** as a function $A : \mathcal{T} \longrightarrow \mathcal{F}$ that assigns to every choice period $t \in \mathcal{T}$ a unique non-empty set $A(t) \in \mathcal{F}$, interpreted as the menu available at *period t*. A **chronological choice function** C for the chronology A is simply a mapping that assigns to every pair $(t, A(t))$ of that chronology a unique element $C(t, A(t)) \in A(t)$.

From a formal point of view, a chronological choice function has two arguments: the choice period at which the choice is made, and the menu available at that period. Because of this, it is possible to have $C(t, A(t)) \neq C(t', A(t'))$ even if $A(t) = A(t')$. That is, a decision maker who faces the same menu at two different time periods may make different choices in this menu. Such a possibility is of course ruled out in the classical timeless choice theoretic framework. On the other hand, since a chronology of choice is taken to be a function from $\mathcal{T}$ to $\mathcal{F}$, one can not have two different menus available at the same period.

Just like in the standard timeless framework, the behavioral implications of the theories that we are looking after will take the form of axioms that are

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[2] See also Bossert, Sprumont, and Suzumura (2005; 2006).

formulated in terms of "revealed preference" relations. For now, two types of such relations shall be considered. The first one is the *direct revealed preference* relation at time t that is defined as follows.

Definition 1 *For any period $t \in \mathcal{T}$ and alternatives x and $y \in X$, we say that x **is directly revealed preferred to** y **at period** t, denoted $x \succsim_C^t y$, if and only if $x = C(t, A(t))$ and $y \in A(t)$.*

In words, the chronological choice function directly reveals a preference for x over y at period t (with x and y distinct) whenever it shows the choice of x at period t in a choice problem where y was available. This direct revealed preference at period t is analogous to the notion formulated by Arrow (1959) in a timeless setting. In the spirit now of Houthakker (1950), one can define the notion of *indirect revealed preference* relation over a time period going from r to s as follows:

Definition 2 *For any periods r and $s \in \mathcal{T}$ such that $r \leq s$ and any alternatives x and $y \in X$, we say that x **is indirectly revealed preferred to** y **between periods** r **and** s, denoted $x \succsim_C^{rs} y$, if and only if there is a sequence $\{t_j\}_{j=1}^k$ of k time periods in the set $\{r, r+1, ..., s-1, s\}$, not necessarily ordered by time, for which one has:*
(i) $x = C(t_1, A(t_1))$,
(ii) $C(t_j, A(t_j)) \succsim_C^{t_j} C(t_{j+1}, A(t_{j+1}))$ for all $j = 1, ..., k-1$, and
(iii) $y \in A(t_k)$.

We observe that, by the very definition of a chronological choice function, the binary relation $\succsim_C^t$ is antisymmetric for every period t. However, for an arbitrary pair of periods r and s satisfying $r \leq s$, the binary relation $\succsim_C^{rs}$ need not be antisymmetric. The fact of having $x \succsim_C^{rs} y$ for two distinct alternatives x and y does not preclude the possibility of having $y \succsim_C^{rs} x$. We emphasize that the sequence of sets involved in the definition of the indirect revealed preference between periods r and s need not be ordered by time. To give just an example, suppose that $r = 1$, $s = 3$, $X = \{a, b, c, d, e\}$, that the chronology of choices offered to the decision maker between period 1 and period 3 is $A(1) = \{a, b, c\}$, $A(2) = \{d, a\}$ and $A(3) = \{b, e\}$, and that the chronological choice function for the periods 1, 2 and 3 is:

$$
\begin{aligned}
C(1, A(1)) &= a, \\
C(2, A(2)) &= d, \text{ and} \\
C(3, A(3)) &= b.
\end{aligned}
$$

If follows from Definition 2 that alternative d is indirectly revealed preferred between periods 1 and 3 to alternative e. In effect, d has been directly revealed preferred to a in period 2 which has been itself directly revealed preferred to b in period 1 which has been directly revealed preferred to e at period 3. The sequence of direct revealed preference statements that connect d to e between periods 1 and 3 is not indexed by time.

3 One preference rational choice and time

Standard choice theory is grounded on the assumption that preferences, and the choices that they induce, are invariant with respect to time. If empirical evidence, casual observation, and introspection suggest that this assumption is not always realistic, it does represent a sensible benchmark for many applications. We therefore find it useful to start our analysis by characterizing a chronological choice function that results from the maximization of a (linear) ordering. The axiom that characterizes this behavior is the following.

Axiom 1 *For any periods r, s and t such that $r \leq s < t$ and some distinct x and $y \in X$, one can not have $x \succsim_C^{rs} y$ and $y \succ_C^t x$.*

We note that this axiom is somewhat simpler to test than GARP. In effect, Axiom 1 requires a consistency between indirect revealed preferences occurring between any two periods r and s, and direct revealed preferences expressed at subsequent time period t. By contrast, the standard timeless GARP test would have ruled inconsistencies also between indirect revealed preferences occurring between any two periods r and s and any direct revealed preference whatsoever, including those observed before r. The later test is then computationally slightly more demanding.

We now define what it means for a chronological choice behavior to result from the maximization of a time-invariant preference.

Definition 3 *A chronological choice function C results from the maximization of a time-invariant preference if and only if there exists a linear ordering $\succsim$ on X such that, for every $t \in \mathcal{T}$, one has $a = C(t, A(t))$ if and only if $a \succsim a'$ for all $a' \in A(t)$.*

We now establish that Axiom 1 is necessary and sufficient for a chronological choice function to result from the maximization of a time-invariant preference.

Theorem 1 *A chronological choice function C satisfies Axiom 1 if and only if it results from the maximization of a time-invariant preference.*

Proof. *We first show that a chronological choice function C for which there exists a linear ordering $\succsim$ on X such that, for every $t \in \mathcal{T}$, one has $a = C(t, A(t))$ if and only if $a \succsim a'$ for all $a' \in A(t)$ satisfies Axiom 1. For this sake, assume the existence of a linear ordering $\succsim$ on X such that, for every $t \in \mathcal{T}$, one has $a = C(t, A(t))$ if and only if $a \succsim a'$ for all $a' \in A(t)$ and consider any periods r, s and t such that $r \leq s < t$ and some distinct x and $y \in X$ for which we have $x \succsim_C^{rs} y$. By Definition 2 , there is a sequence $\{t_j\}_{j=1}^{k}$ of k time periods in the set $\{r, r+1, ..., s-1, s\}$ for which one has $x = C(t_1, A(t_1))$, $C(t_j, A(t_j)) \succsim_C^{t_j} C(t_{j+1}, A(t_{j+1}))$ for all $j = 1, ..., k-1$, and $y \in A(t_k)$. Since the chronological choice function is rationalized by the linear ordering $\succsim$, one has $C(t_j, A(t_j)) \succsim C(t_{j+1}, A(t_{j+1}))$ for all $j = 1, ..., k-1$ and, therefore, $x \succsim y$*

by the transitivity of $\succsim$. Assume contrary to Axiom 1 that $y \succ_C^t x$. By Definition 1, this means that $y = C(t, A(t))$ and $x \in A(t)$. Since $\succsim$ rationalizes the chronological choice function C, this means that $y \succ x$, a contradiction.

To prove the other implication, consider a chronological choice function C that satisfies Axiom 1. Define the binary relation $\succsim_C$ on X by:

$$x \succsim_C y \iff \exists t \in \mathcal{T} \text{ s.t. } x = C(t, A(t)) \text{ and } y \in A(t).$$

Define also the binary relation $\widehat{\succsim}_C$ by:

$$
\begin{aligned}
x \widehat{\succsim}_C y \quad &\iff\quad \exists \{t_j\}_{j=0}^k \text{ with } t_j \in \mathcal{T} \text{ for } j = 0, ..., k \text{ and } l \geq 0) \text{ such that}\\
(i)\ x \quad &=\quad C(t_0, A(t_0))\\
(ii)\ C(t_{j+1}, A(t_{j+1})) \quad &\in\quad A(t_j) \text{ for } j = 0, ..., l-1 \text{ (if any)}\\
(iii)\ y \quad &\in\quad A(t_l)
\end{aligned}
$$

It is immediate to see that $\widehat{\succsim}_C$ is the transitive closure of $\succsim_C$. This means that $\widehat{\succsim}_C$ is transitive by definition. We now show that $\widehat{\succsim}_C$ is antisymmetric if C satisfies Axiom 1. By contradiction, suppose $\widehat{\succsim}_C$ is such that there are two distinct alternatives x and $y \in X$ for which both $x \widehat{\succsim}_C y$ and $y \widehat{\succsim}_C x$ holds. This means that:

$$
\begin{aligned}
\exists \{t_j\}_{j=0}^k \text{ with } t_j \quad &\in\quad \mathcal{T} \text{ for } j = 0, ..., k \text{ (with } k \geq 0) \text{ such that}\\
(i)\ x \quad &=\quad C(t_0, A(t_0))\\
(ii)\ C(t_{j+1}, A(t_{j+1})) \quad &\in\quad A(t_j) \text{ for } j = 0, ..., l-1 \text{ (if any)}\\
(iii)\ y \quad &\in\quad A(t_l)
\end{aligned}
\tag{1}
$$

and

$$
\begin{aligned}
\exists \{t'_j\}_{j=0}^{l'} \text{ with } t'_j \quad &\in\quad \mathcal{T} \text{ for } j = 0, ..., l' \text{ (with } l' \geq 0) \text{ such that}\\
(i)\ y \quad &=\quad C(t'_0, A(t'_0))\\
(ii)\ C(t'_{j+1}, A(t'_{j+1})) \quad &\in\quad A(t'_j) \text{ for } j = 0, ..., l'-1 \text{ (if any)}\\
(iii)\ x \quad &\in\quad A(t_{l'})
\end{aligned}
\tag{2}
$$

Consider the sets of time periods $T = \bigcup_{j=0}^l \{t_j\}$ and $T' = \bigcup_{j=0}^{l'} \{t'_j\}$ involved in expressions (1) and (2) respectively. As these two expressions define a cycle of revealed preference relations connecting alternative x to itself, this cycle can be started at any $C(t, A(t))$ (for some $t \in T \cup T'$) that we wish. In particular, t can be the maximal (with respect to the natural ordering of time) such period in $T \cup T'$. We then have $C(t, A(t)) \succsim_C^t C(s, A(s))$ for some $s < t$. By definition of the cycle induced by expressions (1) and (2), there is also a period $r < t$ such that $C(t, A(t)) \in A(r)$. Let (r, s) denote the set of all choice problems between r and s, and let $s' = \max(r, s)$. Using the definition of the cycle and Definition 2, it follows that $C(s, A(s)) \succsim_C^{1s'} C(t, A(t))$ and $C(t, A(t)) \succsim_C^t C(s, A(s))$, a

contradiction of Axiom 1. Hence $\widehat{\succsim}_C$ is an antisymmetric and transitive binary relation. By Spilrajn extension theorem, one can therefore extend $\widehat{\succsim}_C$ into a complete linear ordering $\succsim$. Let us now show that for every $t \in \mathcal{T}$, one has $x = C(t, A(t)) \iff x \succsim a$ for all $a \in A(t)$. Consider therefore any $t \in \mathcal{T}$. Assume first $x = C(t, A(t))$. Then, by definition of $\succsim_C$, one has $x \succsim_C a$ for every $a \in A(t)$ so that the implication $x \succsim a$ for every $a \in A(t)$ follows from the fact that $\succsim$ extends $\widehat{\succsim}_C$ which extends itself $\succsim_C$. Assume now that $x \succsim a$ for every $a \in A(t)$ for some $x \in A(t)$. Suppose by contradiction that $x \neq C(t, A(t))$. Then, there exists some alternative y distinct from x such that $y = C(t, A(t))$. By definition of $\succsim_C$, one has $y \succsim_C x$ and, therefore, $y \widehat{\succsim}_C x$ and $y \succsim x$. But, since $x \succsim a$ for every $a \in A(t)$, this is incompatible with $\succsim$ being antisymmetric. ∎

Theorem 1 shows that Axiom 1 is necessary and sufficient for "rational behavior". At the same time, the proof of Theorem 1 clearly suggests that indexing the choice behavior by time is irrelevant for the possibility of rationalizing that behavior as resulting from the maximization of a linear ordering.

Even though the result of Theorem 1 is simple, we notice that, to the best of our knowledge, it has never been established before for a choice function defined on an arbitrary domain. On an arbitrary domain, Suzumura (1976) shows that SARP (a slight strengthening of GARP) is a necessary and sufficient condition for a *choice correspondence* to be rationalized by a complete and acyclical binary relation.

While the introduction of time does not play any significant role in characterizing one-preference rational behavior, we show in the next section that there are alternative theories of choice whose empirical implications cannot be characterized without an explicit inclusion of time.

4   Examples of time-dependent choice models

There is a growing support to the view that the economic agent's preferences are best represented as *time-dependent* and that we often observe choice behavior for the same set of alternatives to differ across time. Preference (and choice) reversals, learning, and several types of cognitive bias are among the phenomena most studied in behavioral economics (see e.g. Fehr and Hoff 2011; Hoff and Stiglitz 2016).

In what follows, we provide characterizations of three choice models that indicate a dependence of behavior upon time that can not be explained by the standard one-preference maximization model.

4.1   Changing Preferences

The first of these models considers the possibility for the decision maker to behave as if her preferences or tastes were *unpredictably* changing over time. We

analyze the case in which the decision maker preferences may change unpredictably *at most* once. From the observer point of view, this corresponds to the case where there is a single period, unknown *a priori* by the observer, in which the decision maker "switches" from one preference to another. For instance, someone that likes meat could become vegetarian at a given point in time.

To see the reach of this changing preferences model, and the relevance of including time in the analysis of its behavioral implications, consider the following two examples:

Example 1 *Let $\mathcal{T} = \{1, 2, 3\}$ and consider the following chronology of (gastronomic) menus:*
$A(1) = \{chicken, dahl\}$, $A(2) = \{chicken, dahl, tuna\}$, $A(3) = \{chicken, dahl, beef\}$. *The chronological choice function C defined by $C(1, A(1)) = chicken$, $C(2, A(2)) = dahl$, and $C(3, A(3)) = chicken$ is not consistent with one change in preferences. Indeed both the choices at the first and at the last period reveal a (carnivorous) preference for chicken over dahl, while the choice made at the second period reveals a preference for dahl over chicken. In order to generate such a pattern of choice, the decision maker must have changed preferences at every period.*

Example 2 *Let again consider $\mathcal{T} = \{1, 2, 3\}$ and the same chronology:*
$A(1) = \{chicken, dahl\}$, $A(2) = \{chicken, dahl, tuna\}$, $A(3) = \{chicken, dahl, beef\}$. *The chronological choice function C defined by $C(1, A(1)) = C(2, A(2)) = chicken$ and $C(3, A(3)) = dahl$ is consistent with one change in preferences. The decision maker switches once for a vegetarian preference between the second and the third periods.*

Notice that it would not be possible to distinguish between the two choice behaviors without the introduction of time. Indeed, without time, both examples entail a single violation of GARP in the traditional sense.

We now provide an axiom on a chronological choice function that characterizes a decision maker who chooses in every period according to some preference relation, and who experienced at most one preference change in time.

Axiom 2 *If there are periods r, s and t such that $r \leq s < t$ and $x \succ_C^{rs} y$ and $y \succ_C^t x$ for some distinct x and $y \in X$, then, for every distinct w and $z \in X$, one can not have $w \succsim_C^{uv} z$ and $z \succ_C^\tau w$ for periods u, v and τ such that $t \leq u \leq v < \tau$.*

This axiom says that if one observes a violation of Axiom 1 between period 1 and a given period t, then it is not possible to observe a second violation of Axiom 1 between t and T. This axiom is therefore almost as easy to test as Axiom 1. We now define what is meant by a chronological choice behavior to result from one change in preferences.

Definition 4 *A chronological choice function C results from one change in preferences if there exists two (possibly identical) linear orderings $\succsim_1$ and $\succsim_2$ on X and one period $t \in \mathcal{T}$ such that $a_j = C(j, A(j))$ if and only if $a_j \succsim_1 a'_j$ for all $a'_j \in A(j)$ and $j \in \mathcal{T}$ such that $j < t$ and $a_v = C(v, A(v))$ if and only if $a_v \succsim_2 a'_v$ for all $a'_v \in A(v)$ and for all $v \in \mathcal{T}$ such that $v \geq t$.*

The characterization of this choice model is provided in the following theorem.

Theorem 2 *A chronological choice function C satisfies Axiom 2 if and only if it results from one change in preferences.*

Proof. *For the necessity of the condition, assume that C is a chronological choice function that results from one change in preferences as per Definition 4. This means that there exists two (possibly identical) linear orderings $\succsim_1$ and $\succsim_2$ on X and one period $t \in \mathcal{T}$ such that $a_j = C(j, A(j))$ if and only if $a_j \succsim_1 a'_j$ for all $a'_j \in A(j)$ and $j \in \mathcal{T}$ such that $j < t$ and $a_v = C(v, A(v))$ if and only if $a_v \succsim_2 a'_v$ for all $a'_v \in A(v)$ and for all $v \in \mathcal{T}$ such that $v \geq t$. Assume by contradiction that this chronological choice function violates Axiom 2. That is, assume that there are periods r, s and t' satisfying $r \leq s < t'$ for which one has $x \succ_C^{rs} y$ and $y \succ_C^{t'} x$ for some distinct x and $y \in X$, and that there are also some distinct w and $z \in X$ for which one observes $w \succsim_C^{uv} z$ and $z \succ_C^{\tau} w$ for some periods u, v and τ such that $t' \leq u \leq v < \tau$. We first show that having both $x \succ_C^{rs} y$ and $y \succ_C^{t'} x$ implies that $r < t \leq t'$. By contradiction, suppose first that $t \leq r < t'$. Since one has $a_v = C(v, A(v))$ if and only if $a_v \succsim_2 a'_v$ for all $a'_v \in A(v)$ and all v such that $v \geq t$, the fact of observing both $x \succ_C^{rs} y$ and $y \succ_C^{t'} x$ would imply, given the definition of $\succ_C^{rs}$ and $\succ_C^{t'}$ and the transitivity of $\succsim_2$, that both $x \succ_2 y$ and $y \succ_2 x$ holds, which is a contradiction. Similarly, if $r < t' < t$, and given the fact that $a_j = C(j, A(j))$ if and only if $a_j \succsim_1 a'_j$ for all $a'_j \in A(j)$ and all j such that $j < t$, observing both $x \succ_C^{rs} y$ and $y \succ_C^{t'} x$ would imply, given the definition of $\succ_C^{rs}$ and $\succ_C^{t'}$ and the transitivity of $\succsim_1$, that both $x \succ_1 y$ and $y \succ_1 x$ holds, which is also a contradiction. Since $r < t \leq t'$, one has that $a_v = C(v, A(v))$ if and only if $a_v \succsim_2 a'_v$ for all $a'_v \in A(v)$ and all v such that $v \geq t$. But then, the assumed existence of w and $z \in X$ for which one has $w \succsim_C^{uv} z$ and $z \succ_C^{\tau} w$ for some periods u, v and τ such that $t' \leq u \leq v < \tau$ leads to the conclusion that both $w \succ_2 z$ and $z \succ_2 w$ holds, which is a contradiction. Hence a chronological choice function that results from one change in preferences satisfies Axiom 2.*

In order to prove the converse implication, consider a chronological choice function that satisfies Axiom 2. If there exists no r, s, $t \in \mathcal{T}$ such that $1 \leq r < s < t$ for which one has $x \succ_C^{rs} y$ and $y \succ_C^{t} x$ for some distinct $x, y \in X$, then this means that the chronological choice function satisfies Axiom 1. In that case, set $\succsim_1 = \succsim$ where $\succsim$ is the linear ordering whose existence was established in Theorem 1 and let $\succsim_2$ be any linear ordering whatsoever. As shown in Theorem 1, the linear ordering $\succsim$ will rationalize the behavior of the chronological choice function C from 1 up to T.

Suppose now that there exists some $r, s, t \in \mathcal{T}$ such that $1 \leq r < s < t$ for which one has $x \succ_C^{rs} y$ and $y \succ_C^{t} x$ for some distinct $x, y \in X$. Define then $\widehat{t}$ to be the smallest such t. By Axiom 2, the chronological choice function satisfies Axiom 1 on the time horizon $\{1, ..., \widehat{t} - 1\}$ and it also satisfies Axiom 1 on the (non-empty) time horizon $\{\widehat{t}, ..., T\}$. The result then follows from applying Theorem

1 to the time horizons $\{1,...,\widehat{t}-1\}$ and $\{\widehat{t},...,T\}$ sequentially. ∎

Theorem 2 hence provides an easy way to test if the behavior of an agent is consistent with changing preferences at most once, and choosing at each period as per the preference of this period. Remark that Theorem 2 easily extends to the case with more than one change in preferences. This could be done by just rewriting Axiom 2 for k-changes, and applying it for $k+1$ partitions of the time horizon in the proof. Of course, if the number of changes in preferences is equal to the number of periods, then any choice behavior can be rationalized (see Kalai, Rubinstein, and Spiegler 2002 for a similar observation in the timeless setting).

The changing preferences choice model examined in this section is somewhat different from the *revealed preference theory of changing tastes* analyzed by Gul and Pesendorfer (2005). The authors characterize a model of consistent planning, that rationalize changing tastes due to temptation and self-control. On the one hand, the changing preferences model examined in this section is more general than theirs since it allows for any source of change in preferences. On the other hand, and contrary to Gul and Pesendorfer (2005), our model is silent on the effect of current choices on the shape of the future menus of available alternatives. Indeed, in our approach, the chronology of choices is exogenously given and it is not affected by the choices made by the agent. It would be interesting to allow the chronology of choices to be affected by the chronological choice function.

4.2 Learning by Trial and Error

We now consider the possibility for a decision maker to behave as if she was forming her preference between two alternatives only after the two alternatives have been previously "tried" at least once. This choice model is consistent with "rational behavior", but accommodates some learning that may lead to some initial "contradictions" in choices. Hence, we require the decision maker to be consistent in her choices in the sense of Axiom 1 *only* when those choices concern alternatives that have been tried at least once in the past.

To illustrate the model we have in mind, we find again useful to consider the following two examples.

Example 3 *Let $\mathcal{T} = \{1,2,3,4\}$ and consider again a chronology of (gastronomic) menus:*
$A(1) = \{chicken, dahl\}$, $A(2) = \{beef, dahl\}$, $A(3) = \{beef, chicken, dahl\}$ and $A(4) = \{beef, chicken\}$.
The chronological choice function C defined by $C(1, A(1)) = chicken$, $C(2, A(2)) = beef$, $C(3, A(3)) = chicken$, and $C(4, A(4)) = beef$ is not consistent with a learning by trial and error model. In the first period, without any information about her preferences for food, the decision maker goes for chicken and experienced the taste. In the second period she goes for beef and tries out its taste. In the third period, where she has the choice between chicken, beef, and dahl,

she reveals a preference for chicken over beef. Given that she knows the tastes (because she has tried both in the past), the choice in the third period reveals a "definite" preference for chicken over beef. But then the choice at the fourth period - beef over chicken - is inconsistent with this preference.

Example 4 *Let $\mathcal{T} = \{1, 2, 3, 4\}$ and consider the following chronology:*
$A(1) = \{beef, chicken, dahl\}$, $A(2) = \{beef, chicken\}$, $A(3) = \{chicken, dahl\}$ and $A(4) = \{beef, dahl\}$.
The chronological choice function C defined by $C(1, A(1)) = chicken$, $C(2, A(2)) = beef$, $C(3, A(3)) = chicken$, and $C(4, A(4)) = beef$ describes a behavior consistent with a learning by trial and error model. Indeed, albeit one observes revealed preferences "inconsistencies" (in the traditional sense) between the choices made at the two first periods, these inconsistencies may be interpreted as the results of trial and error. Indeed, the decision maker may be trying out chicken in the first period and trying out beef at the second. After these trials, the decision maker reveals a "definite" preference for chicken over beef (in period 3), and in this example she is consistent with it in the following period.

We emphasize the crucial importance of introducing time for characterizing a behavior resulting from preference formation by trial and error. Indeed, the only difference between the two examples is the time order at which the menus - identical in both examples - appear. Hence, without time, the two choice behaviors could not be distinguished and, as a result, it would not be possible to identify those violations of standard revealed preference that are compatible with a process of preference formation through trial and error and those violations that are not so.

In order to characterize the behavioral implications of this model, we find it useful to define the following "revealed definitely preferred" binary relation.

Definition 5 *For any period $t \in \mathcal{T}$ and some x and $y \in X$, we say that x **is directly revealed definitely preferred to** y **at period** t, denoted $x \succsim_C^{Dt} y$, if and only if there are periods r, s, and t in $\mathcal{T}$ satisfying $r < t$ and $s < t$ such that $x = C(r, A(r)) = C(t, A(t))$, $y \in A(t)$ and $y = C(s, A(s))$.*

In words, the chronological choice function directly reveals a *definite preference* for x over y at period t (with x and y distinct) whenever it reveals (by choice) a preference for x over y at a period t that follows periods where x and y have been tried. Since both x and y have been tried before t, one can interpret the choice of x over y in period t as revealing a "definite" preference between the two alternatives.

Given this "direct revealed definite preference at period t" relation, one defines the revealed definite preference relation over a sequence of periods going from r up to s as follows.

Definition 6 *For any periods r and s such that $r \leq s$ and some x and $y \in X$, we say that x **is indirectly revealed definitely preferred to** y **between**

***periods* r *and* s**, *denoted* $x \succsim_C^{Drs} y$, *if and only if there is a sequence* $\{t_j\}_{j=1}^k$ *of k time periods in the set* $\{r, r+1, ..., s-1, s\}$, *not necessarily ordered by time, for which one has:*
(i) $x = C(t_1, A(t_1))$,
(ii) $C(t_j, A(t_j)) \succsim_C^{Dt_j} C(t_{j+1}, A(t_{j+1}))$ *for all* $j = 1, ..., k-1$, *and*
(iii) $y \in A(t_k)$ *and* $y = C(t_h, A(t_h))$ *for some* $t_h < r$.

The following axiom will be shown to be necessary and sufficient for a chronological choice function to be rationalized by a preference formation procedure through trial and error.

Axiom 3 *For any periods r, s and t such that* $r \leq s < t$ *and some distinct x and* $y \in X$, *one can not have* $x \succsim_C^{Drs} y$ *and* $y \succ_C^{Dt} x$.

In plain English, this axiom says that we should never observe a violation of Axiom 1 for two alternatives that have been previously chosen at least once in the past. We now define what is meant by a chronological choice behavior to result from the maximization of a preference formed by trial and error.

Definition 7 *A chronological choice function C results from the maximization of a preference formed by trial and error if there exists a linear ordering* $\succsim$ *on X such that, for all* $t \in \mathcal{T}$, *either* $a_t = C(t, A(t))$ *if and only if* $a_t \succsim a_t'$ *for all* $a_t' \in A(t)$ *for which* $a_t' = C(s, A(s))$ *for some* $s < t$, *or there is no* $s' \in \mathcal{T}$ *such that* $C(t, A(t)) = C(s', A(s'))$ *and* $s' < t$.

That is, a chronological choice behavior results from the maximization of a preference formed by trial and error if there exists a linear preference such that the choice made by the decision maker at every period is either the "best" option for that preference among all alternatives that have been previously tried or, if this is not the case, it is because the chosen option has never been tried before. Note that the "best" option for that preference among all alternatives that have been previously tried is not necessarily the maximal option for that preference. It is possible that the maximal option has itself never been tried before. Of course, in this case, the decision maker "does not know" yet that this option is maximal.

It is easy to show that Axiom 3 is a necessary and sufficient condition for a chronological choice function to result from the maximization of a preference formed by a trial and error process.

Theorem 3 *A chronological choice function C satisfies Axiom 3 if and only if it results from the maximization of a preference formed by trial and error.*

Proof. *For the "if" part of the theorem, assume by contradiction that a chronological choice function C results from the maximization of a preference formed by trial and error but that it violates Axiom 3 Hence, there are periods r, s and t in* $\mathcal{T}$ *such that* $r \leq s < t$ *and some distinct x and* $y \in X$, *for which one have*

15

$x \succsim_C^{Drs} y$ and $y \succ_C^{Dt} x$. By definition of $x \succsim_C^{Drs} y$, there is a sequence $\{t_j\}_{j=1}^{k}$ of k time periods in the set $\{r, r+1, ..., s-1, s\}$ such that $x = C(t_1, A(t_1))$, $C(t_j, A(t_j)) \succsim_C^{Dt_j} C(t_{j+1}, A(t_{j+1}))$ for all $j = 1, ..., k-1$, and $y \in A(t_k)$ and $y = C(t_h, A(t_h))$ for some $t_h < r$. By definition of $C(t_j, A(t_j)) \succsim_C^{Dt_j}$ $C(t_{j+1}, A(t_{j+1}))$ for all $j = 1, ..., k-1$, there are, for any such j, periods r_j, and s_j in $\mathcal{T}$ satisfying $r_j < t_j$ and $s_j < t_j$ such that $C(r_j, A(r)) = C(t_j, A(t_j))$, $C(t_{j+1}, A(t_{j+1})) \in A(t_j)$ and $C(t_{j+1}, A(t_{j+1})) = C(s_j, A(s_j))$. Since C results from the maximization of a preference formed by trial and error, there exists a linear ordering $\succsim$ on X such that $x = C(t_1, A(t_1)) \succsim C(t_2, A(t_2)) \succsim$ $... \succsim C(t_k, A(t_k))$ for all $j = 1, ..., k-1$. Since $y \in A(t_k)$ holds and C results from the maximization of a preference formed by trial and error, one has $C(t_k, A(t_k)) \succsim y$. By the transitivity and the linearity of $\succsim$ (as x and y are distinct) one has $x \succ y$. But then, assuming $y \succ_C^{Dt} x$ for $t > s \geq r$ implies, under the assumption that C results from the maximization of a preference formed by trial and error, that $y \succ x$, which is a contradiction.

To prove the other direction of the implication, consider a chronological choice function C that satisfies Axiom 3 and define the following "definite revealed preference" relation $\succsim_C^D$:

$$x \quad \succsim \quad _C^D y \Longleftrightarrow \exists r, s, t \in \mathcal{T} \text{ satisfying } r < t \text{ and } s < t \text{ such that:}$$
$$x \quad = \quad C(t, A(t)), \ y \in A(t), \ x = C(r, A(r)) \text{ and } y = C(s, A(s))$$

Notice that this binary relation can be empty. This would happen, for example, for a chronology in which the same menu is available at every period and a chronological choice function that chooses the same alternative from that same menu at every period. In such a trivial case, the decision maker would never experience anything other than this chosen option, and there would therefore be no pair of alternatives between which the binary relation $\succsim_C^D$ would hold. That is, the decision maker would never be given the opportunity to express a "definite preference". In such a case, the choice behavior can be (trivially) rationalized by any linear ordering $\succsim$ whatsoever. Indeed, take any linear ordering $\succsim$ and consider any period t for which $x \succsim a_t$ for some $x \in A(t)$ and all $a_t \in A(t)$ but for which $x \neq C(t, A(t))$. There may not be any such t, in which case the linear ordering $\succsim$ rationalizes the choice behavior in the usual sense. If however such a t exists, we then know from the emptiness of the binary relation $\succsim_C^D$ that either $C(t, A(t)) \neq C(s, A(s))$ for all $s < t$ or $x \neq C(s, A(s))$ for all $s < t$. Hence the chronological choice behavior is trivially rationalized as resulting from the maximization of preference formed by trial and error when $\succsim_C^D$ is empty. If $\succsim_C^D$ is not empty, one can define its transitive closure $\widehat{\succsim}_C^D$ by:

$$x \ \widehat{\succsim}_C^D \ y \quad \Longleftrightarrow \quad \exists \ \{x_j\}_{j=0}^{l} \text{ for some } l \geq 1 \text{ such that:}$$
$$x_0 \quad = \quad x,$$
$$x_l \quad = \quad y \text{ and,}$$
$$x_j \quad \succsim \quad _C^D x_{j+1} \text{ for all } j = 0, ..., l-1$$

Let us now show that the (transitive) binary relation $\widehat{\succsim}_C^D$ is also antisymmetric. By contradiction, suppose there are two distinct x and $y \in X$ such that $x \widehat{\succsim}_C^D y$ and $y \widehat{\succsim}_C^D x$. By definition of $\widehat{\succsim}_C^D$ and $\succsim_C^D$, there are two sequences of triples of periods $\{r_j, s_j, t_j\}_{j=0}^l$ and $\{r_j', s_j', t_j'\}_{j=0}^{l'}$ (for some l and $l' \geq 1$) satisfying, for every j, $r_j < t_j$, $s_j < t_j$, $r_j' < t_j'$ and $s_j' < t_j'$ for which one has:

$$x_j = C(r_j, A(r_j)) = C(t_j, A(t_j)), \ x_{j+1} = C(s_j, A(s_j)) \ and \ x_{j+1} \in C(t_j, A(t_j))$$

as well as :

$$x_j' = C(r_j', A(r_j')) = C(t_j', A(t_j')), \ x_{j+1}' = C(s_j', A(s_j')) \ and \ x_{j+1}' \in C(t_j', A(t_j'))$$

for two sequences of alternatives $\{x_j\}_{j=0}^l$ and $\{x_j'\}_{j=0}^{l'}$ satisfying $x_j \in X$, $x_j' \in X$ for all j, $x_0 = x_{l'}' = x$ and $x_l = x_0' = y$. This generates a cycle of revealed definite preference connecting alternatives in X that can be initiated at every period of the sets of periods $\{t_j\}_{j=0}^l$ and $\{t_j'\}_{j=0}^{l'}$ defined above. In particular, one can take the maximal (with respect to the natural ordering of time) of this period, and apply the reasoning of the proof of Theorem 1 to obtain the required violation of Axiom 3. Since $\widehat{\succsim}_C^D$ is antisymmetric and transitive, it can be extended to a linear ordering $\succsim$ using Spilrajn extension theorem. Let us now prove that the chronological choice function C results from the maximization of $\succsim$ formed by a trial and error process. Consider any $t \in \mathcal{T}$. Either $C(t, A(t)) \succsim a_t$ for all $a_t \in A(t)$ or $\exists x \in A(t)$ such that $x \neq C(t, A(t))$ and $x \succsim C(t, A(t))$. In the first case, $\succsim$ rationalizes the choice made in the choice problem at t and there is nothing to prove. In the second case, take without loss of generality the alternative $x \in A(t)$ to be such that $x \succsim a_t$ for all $a_t \in A(t)$. By assumption $x \neq C(t, A(t))$. Suppose that, contrary to the requirement that the chronological choice function C results from the maximization of $\succsim$ formed by a trial and error process, it is neither the case that $C(t, A(t)) \neq C(s, A(s))$ for all $s < t$ nor $x \neq C(s, A(s))$ for all $s < t$. This means that there exists a period $r < t$ such that $x = C(r, A(r))$ and there exists a period $s < t$ such that $C(t, A(t)) = C(s, A(s))$. It then follows from the definition of $\succsim_C^D$ that $C(t, A(t)) \succsim_C^D x$ and, since $\succsim$ is an extension of $\succsim_C^D$, that $C(t, A(t)) \succsim x$. This means that we have both $x \succsim C(t, A(t))$ and $C(t, A(t)) \succsim x$, a contradiction of $\succsim$ being antisymmetric. ∎

The trial and error method of learning seems quite plausible as a way to discover one's preference. For example, children learn that they prefer apples over bananas by trying both at different times, and by "discovering" that they indeed prefer apples to bananas. Once this discovery is made - and provided that no subsequent change in preferences take place - children will stick to this preference and never choose a banana when an apple is also available. In economics, the trial and error method of learning has been studied for organizational learning such as within-firm experimentation (see e.g Nelson 2008 and Callander 2011). Young (2009) has also examined a trial and error learning rule in a game theoretical environment while Cooke (2016) and Piermont, Takeoka, and Teper (2016) have examined models of preference formation through experimentation over uncertain prospects. To the best of our knowledge, Theorem

3 is the only available characterization of a learning process by trial and error over an abstract set of alternatives.

4.3   Choice with Inertia Bias

Another theory of choice behavior whose behavioral implications can be characterized by means of a chronological choice function is that of a decision maker who has a bias towards her (immediately) last choice. One interpretation of such behavior is that the decision maker has inertia in her preferences and sees $C(t-1, A(t-1))$ as a default option when making a choice at period t. Another interpretation is that the decision maker has an "imperfect recall" of the choices that took place earlier in the time horizon, and takes the previous choice as a *status-quo* option. In our setting, this means that for each choice problem, the decision maker either chooses the best option according to a time invariant preference *or* chooses the option that she has chosen in the previous choice problem.

The following examples illustrate the behavioral implications of this theory of choice.

Example 5 *Let $\mathcal{T} = \{1, 2, 3\}$ and consider the following chronology:*
$A(1) = \{chicken, beef\}$, $A_2 = \{beef, dahl\}$, $A_3 = \{chicken, beef, dahl\}$.
The chronological choice function C defined by $C(1, A(1)) = C(2, A(2)) = beef$ and $C(3, A(3)) = chicken$ is not consistent with a model of choice with inertia bias. Note that the decision maker has chosen to eat chicken in the last period, while her immediately preceding choice - beef - was available. Hence our decision maker has "broken" her inertia by choosing something else than her last choice. If this "break" is motivated by a desire to obtain a preferable alternative for a well-defined time-invariant preference, as assumed in this model, then the ranking of alternatives provided by this preference must be the same at every period at which the preference is expressed. When can we be (more) confident that such preference is expressed? When the choice made at some period is different from the choice made at the immediately preceding period or, trivially, at the beginning of the history (when there is no past and, therefore, no source of inertia). Yet, here, in the first period, the decision maker has revealed a preference for beef over chicken, that is inconsistent with the "active" preference (as opposed to inertia) revealed by her choice in the last period.

Example 6 *Let again $\mathcal{T} = \{1, 2, 3\}$ and consider the same chronology as before:*
$A(1) = \{chicken, beef\}$, $A_2 = \{beef, dahl\}$, $A_3 = \{chicken, beef, dahl\}$.
The chronological choice function C defined by $C(1, A(1)) = chicken$, $C(2, A(2)) = C(3, A(3)) = beef$, is consistent with a model of choice with inertia bias. Notice that the chronological choice behavior violates Axiom 1. Indeed, the preference for beef over chicken revealed, in the traditional sense, in the last choice period is inconsistent with the preference for chicken over beef revealed in the first period. However, the alternative chosen in the third period is also the one that

was chosen in the second period. Hence, the choice of the third period can not be interpreted as revealing an active preference. It may also be the result of an inertia bias.

Again, these two examples could not be distinguished without the introduction of time. As in the previous subsection, we find convenient to redefine the revealed preference relations in a way that is suitable for identifying an inertia bias explanation of choices. As discussed in the two examples, when the default option is present and chosen, the observer of the choice does not know if it reveals a active preference for the chosen option over the non chosen one or if it results from an inertia bias. We accordingly define the notions of direct and indirect active preferences as follows.

Definition 8 *For any period t and some x and $y \in X$, we say that x **is directly revealed actively preferred to** y **at period** t, denoted $x \succsim_C^{At} y$, if and only if $x = C(t, A(t))$, $y \in A(t)$ and either $t = 1$ or $x \neq C(t - 1, A(t - 1))$.*

Definition 9 *For any periods r and s such that $r \leq s$ and some x and $y \in X$, we say that x **is indirectly revealed actively preferred to** y **between periods** r **and** s, denoted $x \succsim_C^{Ars} y$, if and only if there is a sequence $\{t_j\}_{j=1}^k$ of k time periods in the set $\{r, r + 1, ..., s - 1, s\}$, not necessarily ordered by time, for which one has:*
(i) $x = C(t_1, A(t_1))$
(ii) $C(t_j, A(t_j)) \succsim_C^{At_j} C(t_{j+1}, A(t_{j+1}))$ for all $j = 1, ..., k - 1$, and
(iii) $y \in A(t_k)$.

We now non-surprisingly formulate the axiom which characterizes a behavior described by a chronological choice function which results from inertia bias.

Axiom 4 *For any periods r, s and t such that $r \leq s < t$ and some distinct x and $y \in X$, one can not have $x \succsim_C^{Ars} y$ and $y \succ_C^{At} x$.*

We can also define what is meant by a chronological choice behavior to result from a choice model with inertia bias.

Definition 10 *A chronological choice function results from choice with inertia bias if there exists a linear ordering $\succsim$ on X such that, for all $t \in \mathcal{T}$, either $x = C(t, A(t))$ if and only if $x \succsim a_t$ for all $a_t \in A(t)$ or $t > 1$ and $C(t, A(t)) = C(t - 1, A(t - 1))$.*

Then, one can establish the following:

Theorem 4 *A chronological choice function C satisfies Axiom 4 if and only it it results from choice with inertia bias.*
Proof. *As the argument is very similar to those of Theorems 2 and 3, we only sketch the proof, and leave to the reader the task of verifying that a chronological choice function that results from choice with inertia bias satisfies Axiom 4. As for the converse implication, consider a chronological choice function C that*

satisfies Axiom 4 and define the following "active revealed preference" relation $\succsim_C^A$:

$x \succsim_C^A y \iff \exists t \in \mathcal{T}$ *s. t.* $x = C(t, A(t))$, $y \in A(t)$ *and either* $C(t-1, A(t-1)) \neq C(t, A(t))$ *or* $t = 1$:

This binary relation is not empty [because $C(1, A(1)) \succsim_C^G y$ *for every* $y \in A(1)$*]. One can then define its transitive closure* $\widehat{\succsim}_C^A$ *by:*

$$
\begin{aligned}
x \;\widehat{\succsim}_C^A\; y &\iff \exists \{t_j\}_{j=0}^l \text{ for some } l \geq 1, \text{ with } t_j \in \mathcal{T} \text{ for all } j \text{ such that:} \\
x &= C(t_0, A(t_0)), \\
y &\in A(t_l) \text{ and,} \\
C(t_{j+1}, A(t_{j+1})) &\in A(t_j) \text{ for all } j = 0, ..., l-1 \text{ and} \\
\text{either } C(t_j - 1, A(t_j - 1)) &\neq C(t_j, A(t_j)) \text{ or } t_j = 1 \text{ for all } j = 0, ..., l
\end{aligned}
$$

Let us now show that the (transitive) binary relation $\widehat{\succsim}_C^A$ *is antisymmetric. By contradiction, suppose there are two distinct x and $y \in X$ such that $x \, \widehat{\succsim}_C^A \, y$ and $y \, \widehat{\succsim}_C^A \, x$. Using an analogous reasoning as in Theorems 2 and 3, this would generate a cycle of revealed preferences that would be inconsistent with Axiom 4. Hence $\widehat{\succsim}^A$ must be antisymmetric and transitive. It can therefore be extended to a linear ordering $\succsim$ using Spilrajn extension theorem just as before. We just need to prove that C is such that, for every period $t \in \mathcal{T}$, either $x = C(t, A(t))$ if and only if $x \succsim a_t$ for all $a_t \in A(t)$ or $t > 1$ and $C(t, A(t)) = C(t-1, A(t-1))$. Consider first $t = 1$. By definition of $\succsim_C^A$, one has $C(1, A(1)) \succsim_C^A a_1$ for all $a_1 \in A(1)$ and, since $\succsim$ is an extension of $\succsim_C^A$, one has $C(1, A(1)) \succsim a_1$ for all $a_1 \in A(1)$ as well. Moreover the antisymmetry of $\succsim$ prevents any alternative x of $A(1)$ distinct from $C(1, A(1))$ to be such that $x \succsim a_1$ for all $a_1 \in A(1)$. Hence one has $x = C(1, A(1)) \iff x \succsim a_1$ for all $a_1 \in A(1)$. Consider now any period $t > 1$. Assume $x = C(t, A(t))$. Either $C(t, A(t)) = C(t-1, A(t-1))$ or $C(t, A(t)) \neq C(t-1, A(t-1))$. There is nothing to be proved in the first case. In the second case, one has $C(t, A(t)) \succsim_C^A a_t$ for all a_t by definition of $\succsim_C^A$ and $C(t, A(t)) \succsim a_t$ for all a_t in $A(t)$ by definition of the linear ordering $\succsim$ to be an extension of $\widehat{\succsim}_C^A$. Since, as just established, $C(t, A(t)) \succsim a_t$ for all a_t in $A(t)$ and $\succsim$ is linear, there can not be a $z \in A(t)$ distinct from $C(t, A(t))$ such that $z \succsim a_t$ for all a_t in $A(t)$. Hence one has $x = C(t, A(t))$ if and only if $x \succsim a_t$ for all a_t in $A(t)$, as required.* ∎

The model of choice characterized by Theorem 4 can be connected with the numerous models of choice with *reference-dependent* preferences and *status-quo* bias discussed in the literature (e.g. Tversky and Kahneman 1991) and characterized axiomatically (see e..g. Bossert and Sprumont 2003; Masatlioglu and Ok 2005). However, all previous characterizations that we are aware of have considered a default alternative that is fixed and exogenous. We depart here from the literature by characterizing a model with an endogenous process for the formation of a *status-quo* bias that could be interpreted a coming from an

"imprisonment in the habits" phenomenon. We have given two potential interpretations for this phenomenon, either as inertia or imperfect recall of (remote) past choices. For example, inertia in preferences has been documented in management and economics literature (see e.g. Dubé, Hitsch, and Rossi 2010 for inertia in brand choice). Finally, it can be easily verify that the choice model characterized by Theorem 4 is not observationally equivalent to those involving exogenous *status-quo* bias and could therefore be tested against these models in experimental contexts.

5   Conclusion

In this paper, we have argued in favor of explicitly introducing time in the description of choice behavior provided by a choice function. We have used our setting to characterize the behavioral implications of three alternative theories of choice. We end this paper with a brief discussion of some potential limitations of the approach and possible extensions.

First, we have limited our attention to choice functions, but one could wish to extend this analysis for choice correspondences. As noted in Section 3, Suzumura (1976) has shown that SARP is a necessary and sufficient condition for a choice correspondence to be rationalized by a complete and acyclical binary relation. But a choice correspondence could be used to find similar characterizations for other one-rationale or multiple-rationale choice theories. A related - but distinct - possibility would be to use a chronological choice function to induce a standard timeless choice correspondence as done in Bernheim and Rangel (2007; 2009) or Salant and Rubinstein (2008). In our framework, if the menus $A(t)$ and $A(t')$ offered to the decision maker at distinct time periods t and t' where the same (and, say, equal to the set A) and if $C(t, A(t)) \neq C(t'A(t'))$, then the timeless choice correspondence C_c induced by the chronological choice function C would yield $C_c(A) = \{C(t, A(t)), C(t'A(t'))\}$.

A second limitation of our approach lies in the fact that we have restricted it to deterministic choices. Yet, it would be quite possible - if not complex - to use a chronological choice function to describe stochastic choice or choice behavior that evolves stochastically over time (see e.g. Becker, DeGroot, and Marschak 1963; Barberà and Pattanaik 1986; McFadden and Richter 1990; Loomes and Sugden 1995; Gul and Pesendorfer 2006; Apesteguia, Ballester, and Lu 2017).

Third, we note that while the identification of the behavioral implications of the chronological choice functions characterized in this paper could lead to interesting empirical or experimental applications, these implications are formulated in terms of indirect revealed preference relations. While this is quite standard in the choice theoretic literature, we emphasize that the empirical tests of such revealed preference axioms may be computationally demanding if the universe of alternatives is large.

Finally, we find worth pointing out the ease by which the characterization of the choice behavior exhibited in the three examples examined herein was obtained. Hence the simple fact of introducing time in the description of choice

behavior seems to have the significant payoff of alleviating what Rubinstein (2012) calls "the burden on researchers" of finding the observable properties of the behavioral decision making models that they are interested in.

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