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Abstract

In this paper, we examine the possibility for a regulator to reduce policy costs by substituting a voluntary policy based on a legislative threat to an active harvest control. Specifically, we focus on fisheries where the regulator aims to maintain an optimal level of conservation through a voluntary agreement. To achieve this, we identify a mandatory regulation that can serve as a threat to ensure voluntary compliance and avoid regulation costs. However, threats differ from effective policies. To be enforceable, they must be validated through a legislative process, the outcome of which is uncertain and subject to objections. Consequently, we introduce of a random delay in its application and address issues of social acceptability. This threat rests upon two pillars: a moratorium with financial compensation followed by an Individual Transferable Quota (ITQ) mechanism, and a suitably chosen tax on harvesting capacity to deter deviations. We use data from the scallop fishery in the Bay of Saint-Brieuc (France) to illustrate this voluntary mechanism.

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1. Introduction

The UN Fish Stocks Agreement of 1995 and the command and control tools available to Regional Fisheries Organizations are instrumental in making marine resource sustainability possible. Several types of regulatory mechanisms have been implemented in the last twenty years. These range from standard command and control measures (such as restrictions on targeted species, fish size, gears, and areas or seasons of extraction,) to concession-managed fisheries, with or without limited tenure (Costello and Kaffine [17], Costello *et al.* [19], Costello and Grainger [16]). Other examples include quotas versus landing taxes (Hannesson and Kennedy [27]), market-based instruments like Individual Transferable Quotas (Costello *et al.* [15]), and the development of marine protected areas (Smith and Willem [52]). All of these regulatory mechanisms clearly prevent the fishery from collapse and, in some cases, even contribute to the development of new practices. But are these policies themselves sustainable? This undoubtedly raises the old question of "the cost of fishery management" (Arnason *et al* [5], Schrank *et al.*[46]). The risk of fishery collapse or the argument for population regeneration can, of course, be used to validate the short-term social cost of these policies. However, once the fish stock has been restored to a sustainable level, the key factors underlying the Tragedy of the Commons (Hardin [28]) continue to operate, prompting the pursuit of costly policies to maintain the stock at its optimal conservation level. The main purpose of this paper is to demonstrate that an active restoration policy can be replaced by a voluntary regulatory scheme built on the threat of a new active policy, with this threat serving as a credible deterrent against deviations from the optimal extraction trajectory.

Supported by case studies[1], a growing body of the theoretical literature, following Ostrom [39], has examined cooperative equilibria for Common Pool Resources that implement socially desirable outcomes. This topic has been explored in several directions. For instance, strategic games (see Bailey et al. [6] for a review) help to delineate situations in which a cooperative outcome can be obtained as a competitive subgame perfect equilibrium (Polasky *et al.* [40], Tarui *et al.* [54]). Cooperative games (see Pintassilgo *et al.* [41] for a review) have also been introduced to study bargaining solutions, standard coalitional games (see Lindroos *et al.* [36] for a review), and even coalition formation (Ansink-Weikard [3], Ansink *et al.* [4]). Other papers have investigated the consequences of introducing context-dependent behaviors, such as conditional cooperation motivations (Richter and Grasman [42]) or status-seeking behaviors (Long and McWhinnie [37]). Other contributions have highlighted the specific social norms and/or institutional backgrounds necessary for these equilibria to emerge (see Vincent [55], Gutiérrez et al. [25], Basurto and Coleman [7], and Schaap and Richter [44]). Regarding fisheries, the system of territorial use rights has been emphasized (see, for instance, Costello and Kaffine [18]).

However, most of these approaches primarily focus on self-regulation mechanisms that provide incentives for cooperation within fisheries. In voluntary approaches (Segerson and

[1]For recent case studies, see, for example, Haynie et al [30], Cavalcanti et al. [11], and Sarker et al.[43].

Miceli [47],[48]), the regulator participates in the game. As described by Segerson and Miceli [47], the regulator uses either a carrot or a stick to stimulate voluntary agreement on an environmental objective. In the case of the carrot, incentives are provided within the framework of an existing policy[2]. In the case of the stick, they rely on the threat of an ex-post sanction in the event of non-compliance with the agreement. Our paper focuses on the voluntary threat (V-T) approach. In this case, the announcement of a possible future policy provides an incentive to cooperate and, as it is designed to never be enforced, saves on regulatory costs. Of course, one can always argue that with a "hard enough stick," most outcomes can be implemented. However, unlike traditional regulatory mechanisms, the mandatory policy that acts as a threat requires a legislative process before being enforced. In other words, if the voluntary agreement is broken, the announced regulation may likely not be enforced (see, for example, Glachant [22]) or at least be renegotiated, as in Fleckinger-Glachant [20]. In any case, the strength of the stick must be defined to ensure that it becomes an acceptable mechanism (as in Suter *et al.* [53], Chiambretto and Stahn [12]).

Such collective V-T approaches have primarily been studied as *de facto* arrangements in standard contexts of pollution regulation (e.g., Segerson and Wu [49], Dawson and Segerson [23], Ahmed and Segerson [1] or Brau and Carraro [8]). Our objective is to extend this type of regulation to dynamic externalities specific to common property resource (CPR) management issues. Specifically, we consider the case of renewable fish stocks and the regulation of their persistent over-exploitation. To our knowledge, only a few papers have explicitly mentioned voluntary conservation agreements based on a legislative threat. This issue has been advocated for fisheries management based on collective rights (e.g., Zhou and Segerson [56] and Holland [31]). In this context, rights are allocated to groups of vessels, with a penalty applied when the group exceeds its allowable limit (for a description of these penalties, see Bellanger *et al. [9]).* Langpap and Wu [33] and Langpap [32] addressed the conservation of endangered species, but their approaches mainly focused on terrestrial species living on private lands. Specifically, they analyzed when such agreements arise in a two-period model with uncertainty about the species' survival and irreversible investment in conservation. In contrast, our analysis considers a continuous-time setting without uncertainty and examines a V-T mechanism designed to deter deviations from the conservation path. A similar method was used by Mukherjee [38] to regulate a stochastic, non-marketable by-catch. This paper presents the background threat of a permanent tax to incentivize fishermen to voluntarily make an effective avoidance choice. This tax, which is activated above a certain by-catch threshold, is chosen to enforce compliance. More recently, Selles *et al.* [50] adopted an experimental approach and examined whether the threat of economic sanctions guarantees the sustainability of fisheries.

The main finding of this paper is that an active fish stock conservation policy can be replaced by a well-designed V-T mechanism that avoids a large portion of the regulatory

[2]e.g., when used in a policy mix coupled with command and control, see Borkey et al. [10])

costs. To illustrate this argument, we introduce a standard bio-economic model in the Gordon-Schaefer tradition ([24], [45]). Assuming that the optimal level of conservation is achieved through a mandatory regulatory policy, we ask whether, in a conservation-oriented context, such a costly mandatory policy can be replaced by a V-T mechanism that only requires the monitoring of the fish stock. This mechanism relies on the announcement of a new mandatory policy if a decline in fish stocks is observed. In other words, the regulator simply wields a stick to stimulate compliance. However, moving from an announcement to an effective policy is not straightforward. Before a policy can be enacted, it must be adopted by a legislative body through a political process. In static models, this process is often represented by the probability that the policy will not be adopted. In our dynamic framework, we replace this assumption with the existence of a random delay between the detection of the deviation and the activation of the announced policy. This delay corresponds to the time required to negotiate and adopt the policy. This initial announcement must therefore be acceptable in order to be turned into a legislative action, and it must also deter any deviation from the optimal conservation path. For this reason, our V-T mechanism is based on two main pillars. The first simply restores and preserves the conservation level. It takes the form of a moratorium, coupled with financial compensation to ensure acceptability, and is followed by a standard system of individual transferable quotas. The second pillar is designed to discourage any deviation from the optimal conservation path. It consists of a tax on fishing capacity, which reduces the expected gain from any deviation that occurs during the period between the detection and the implementation of the policy. This tax is, of course, set at the lowest possible level. Consequently, the simple announcement of the policy helps maintain the optimal conservation path. However, the question arises as to whether this capacity tax is excessive, or to what extent it is sensitive to delays. Little can be said about these points from a theoretical perspective. This is why, in the final section, we introduce the case of scallop harvesting in the Bay of Saint-Brieuc (France). In this example, we explicitly derive the tax rate and study its relationship with the average delay.

The remainder of this paper is organized as follows. Section 2 presents the bio-economic model and the optimal conservation level. Section 3 presents the V-T mechanism. Moving backward in time, Section 4 discusses the outcome of the fishing game after the policy's implementation. These results contribute to Section 5's analysis of the expected gain of a deviation from the optimal conservation level. Section 6 characterizes the tax rate that deters deviation and studies its relationship with the average delay. The case of the scallop fisheries is presented in Section 7, and Section 8 provides some concluding remarks. Technical proofs are presented in the appendix.

2. Optimal conservation in a bio-economic model

This section briefly presents the main assumptions that characterize single-species fisheries and introduces the optimal conservation target as the solution to the "sole owner" problem. These elements will be used to define our V-T conservation mechanism in the next section.

2.1. A simple bio-economic model

We consider $N > 1$ symmetrical agents, indexed by i, who jointly harvest a common renewable resource over an infinite horizon, $[0, +\infty[$. The resource stock at time t, measured in units of biomass, is denoted by $S(t)$. Our analysis builds upon the widely used Gordon-Schaefer model of fisheries (see, among others, Clark-Munro [14] and Clark [13]). Accordingly, $S(t)$ evolves over time due to natural growth and harvest. This stock is constrained by K, the carrying capacity of the resource, so that $S(t) \in [0, K]$. The resource grows at a natural rate $r(S)$, which decreases with the size of the biomass, $r'(S) < 0$, and stops growing when K is reached, $r(K) = 0$. We further assume that the maximal growth rate $r(0)$ is bounded and that $r"(S) \leq 0$, meaning that the rate of growth decreases over $[0, K]$ at an increasing rate. The maximal harvest per capita at time t is given by $qS(t)$, where q is a coefficient of catchability that reflects the maximal extraction capacity of each agent. The effort variable, $e_i(t) \in [0, 1]$, captures the combined flow of labor and capital services exerted by i for the purpose of extraction. Yields are linear in effort up to the maximal harvest $qS(t)$. The dynamics of the biomass are therefore given by:

$$\tfrac{\dot{S}(t)}{S(t)} = r(S(t)) - q\left(\textstyle\sum_{i=1}^{N} e_i(t)\right), \text{ with } S(0) = \bar{S} \text{ the initial state.} \tag{1}$$

Still in the Gordon-Schaefer ([24], [45]) tradition, we assume that the yields are sold in a competitive market and that the instantaneous profit of each agent is proportional to his harvest. Specifically, $\pi(S)$ denotes the profit per unit of harvest $qS(t)e_i(t) \in [0, qS(t)]$ for a given effort $e_i(t) \in [0, 1]$. We also assume the existence of a minimal biomass stock, $S_{\min}$, for which this profit becomes positive. Moreover, it should be noted that the same effort, $e_i(t)$, leads to higher captures if applied to a larger biomass and therefore reduces the cost of effort per unit of harvest. This implies — at least under pure competition — that the profit per unit of harvest increases with the biomass, i.e., $\pi'(S) > 0$. We nevertheless assume that the rate of increase $(\pi'(S)/S)$ of this profit function is a decreasing function. This is equivalent to assuming that the elasticity of $\pi'(S)$ is smaller than -1, i.e., $e_{\pi'}(S) = \frac{\pi"(S)S}{\pi'(S)} < -1$. [3] Each agent seeks to maximize the present value of its instantaneous returns. If $\rho > 0$ denotes the discount rate, this quantity is given by:

$$\mathcal{R}_i(S(\cdot), e_i(\cdot)) = \int_0^{+\infty} e^{-\rho t} \left[\pi\left(S(t)\right) qS(t)e_i(t)\right] dt. \tag{2}$$

Finally, note that the catchability coefficient q can also be viewed as the individual maximal rate of depletion of the resource. Since this paper focuses on endangered resources, we assume that the total rate of depletion exceeds the growth rate of the biomass when harvesting becomes profitable, i.e., $nq > r(S_{\min})$.

[3] This is, for instance, the case when the fish stock is sold at a competitive price p and the marginal cost c per unit of effort is constant. In this case $\pi(S) = p - \frac{c}{qS}$ and for $S > S_{\min} = \frac{c}{qp}$, we have $\pi'(S) = \frac{c}{qS^2} > 0$ and $\pi"(S) = -\frac{2c}{qS^3} < 0$, so that $e_{\pi'}(S) = -2$.

2.2. The optimal conservation target

The construction of our V-T mechanism requires the introduction of the endogenous optimal conservation level that the policy aims to target. In our setting, over-exploitation is a consequence of open access alone, Therefore, the optimal conservation level can be viewed as the steady state that solves the joint-rent maximization problem. This is given by:

$$\max_{(e_i(t))_{i=1}^N \in [0,1]^N} \int_0^{+\infty} e^{-\rho t} \left[\pi(S(t)) q S(t) \left(\sum_{i=1}^n e_i(t)\right)\right] dt \tag{3}$$

$$\textit{with} \quad \dot{S}(t) = S(t)\left(r(S(t)) - q\left(\sum_{i=1}^n e_i(t)\right)\right), \; S(0) = \bar{S} > 0. \tag{4}$$

Since both the instantaneous returns and the dynamics are linear in the total harvest, the problem can be expressed as the following variational problem:

$$\max_{\dot{S}(t)} \int_0^{+\infty} \underbrace{e^{-\rho} \left[\pi(S(t)) \left(S(t) r(S(t)) - \dot{S}(t)\right)\right]}_{f\left(S(t), \dot{S}(t), t\right)} dt \tag{5}$$

$$\text{with} \quad \dot{S}(t) \in \left[S(t)\left(r(S(t)) - nq\right), S(t) r(S(t))\right]. \tag{6}$$

From the Euler-Lagrange condition, we know that:

$$\frac{d}{dt}\frac{\partial f}{\partial \dot{S}} = \frac{\partial f}{\partial S} \Leftrightarrow \rho\pi(S) = (r(S) S \pi(S))' \tag{7}$$

The (singular) solution $S^{FB} \in (S_{\min}, K)$ to this equation defines the *optimal conservation target* according to Hotelling's rule. Specifically, at the optimal conservation level, the financial return from the profit generated by the last unit of fish caught equals the marginal benefit of leaving that unit in the sea, as expressed by Eq.(7):

$$\rho\pi(S) = \pi'(S) r(S) S + \pi(S) r'(S) S + \pi(S) r(S) \tag{8}$$

This conservation value considers the increase in profit per unit of catch, the gain generated by the increase in the fish population, and the mechanical increase in profit due to applying the same effort to a larger population. By considering the population dynamics given by Eq.(4)), we can now couple the optimal conservation level with symmetrical individual efforts that maintain the resource at S^{FB}. This sustainable effort is:

$$\forall i, \quad e_i^{FB} = e^{FB} = \tfrac{r(S^{FB})}{nq} \in (0, 1) \tag{9}$$

It should also be noted that e^{FB} is strictly less than one, as we are considering the case of endangered species, i.e., $r(S^{FB}) < r(S_{\min}) < nq$. In this case, the total fishing capacity exceeds the reproduction rate of the fish population evaluated at the optimal conservation level.

We can further analyze this case. Following Hartl and Feichtinger [29], Sethi [51], and Anaya *et al.* [2], we know that the optimal solution to this problem is given by the Most Rapid Approach Path (MRAP) to the optimal conservation level S^{FB}. Consequently, if the initial stock S_0 falls below S^{FB}, the optimal approach to the stationary state involves no extraction during a recovery period until S^{FB} is reached at some switching time. Subsequently, the steady state is maintained with regular effort levels, e^{FB}. Conversely, if the trajectory is initiated in $S_0 \geq S^{FB*}$, the maximal harvesting effort is required from each harvester until S^{FB} is reached, after which they must collectively reverse to e^{FB}. More precisely, we can state that:

Proposition 1. *Under our assumptions, the path of the biomass that solves problem (3) is given by $S_{FB}(t, \bar{S})$, the MRAP to the unique optimal conservation level, S^{FB}. This level is reached at a finite time $T(\bar{S})$, and the individual optimal effort $e_{FB}(S(t))$ is either* 0 *or* 1*, depending on whether $S(t) \lesseqgtr S^{FB}$, or switches to e^{FB}, the sustainable effort, when the optimal conservation level is reached, i.e, when $S(t) = S^{FB}$.*

3. A V-T conservation mechanism

Even when an optimal conservation state, $S_0 := S^{FB}$, is reached — for instance, through a standard system of Individual Transferable Quotas (ITQ) — the usual driving forces behind the tragedy of the commons persist and require constant monitoring. The regulator must therefore decide whether to maintain the ITQ system, bearing the associated administrative costs, or to shift to a V-T conservation mechanism that only requires monitoring the biomass $S(t)$. This last conservation policy begins with the ex-ante announcement that a new mandatory policy will be implemented if a change in the level of biomass is observed. The threat of this policy is the core of the conservation mechanism. It rests on two pillars. On one hand, it signals the regulator's readiness to revert to a standard ITQ system to manage optimal conservation. On the other hand, since overfishing is often linked to excess harvesting capacity (see, for instance, Schaap and Richter [44]), this new policy also introduces a threat in the form of a tax on that excess capacity. The primary objective of this announcement is to deter any deviation from the optimal conservation target, thereby encouraging voluntary compliance without the need to implement the policy. To some extent, this threat can thus be viewed as the trigger for a coordination game involving the regulator.

However, implementing such a V-T conservation mechanism is far from straightforward (see Fig.1). In the undesirable event that the threat becomes effective, the regulator faces two challenges: managing an uncertain implementation schedule and addressing the acceptability of the mandatory policy. The *uncertain timetable* arises from the announcement. As no policy is effective at the time of detection, $t = t_d$, when a change in the optimal conservation stock is observed, the new mandatory policy must first be adopted by a legislative body through a political process before it can become effective. In this paper, we do not explicitly model this process (unlike Glachant **[22]**, for example). We

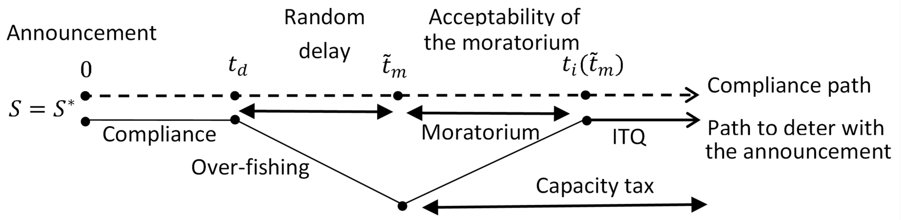

Figure 1: The V-T mechanism

simply state that the process takes time, meaning the policy becomes mandatory at a random time $\tilde{t}_m \geq t_d$, creating a window of potential overfishing. This political bargaining also raises the question of *acceptability*. In fact, the mandatory policy may resemble a collective punishment, especially if a short-term moratorium is imposed, as suggested by the optimal MRAP solution (see Proposition 1). Acceptability requires at least a monetary compensation during the moratorium period. Accordingly, we assume that every fishery receives a subsidy during this period, equal to the profit it would have earned along the optimal conservation path. We also introduce a tax, τ_x, in this announcement on harvesting capacity to discourage deviations. This tax is designed to capture the benefits expected from deviations during the period before the mandatory policy is activated.

It remains to introduce a few additional notations and assumptions to incorporate these various elements. Let us first consider *the random delay*, $\tilde{\Delta}$. Under full uncertainty, we assume that the likelihood of the mandatory regulation becoming effective between t and $t + dt$, given that it has not been enforced before t, is a constant. [4] In this case, the random delay follows an exponential law with cumulative distribution:

$$\forall t \geq t_d, \quad F(t, t_d) = P\left[\tilde{\Delta} \leq t - t_d\right] = 1 - e^{-\delta(t-t_d)}, \tag{10}$$

and the average delay is given by $\mathbb{E}(\tilde{\Delta}) = \frac{1}{\delta}$. Consequently, the probability that the mandatory regulation has not been implemented after a delay of $(t - t_d)$ is $(1 - F(t, t_d)) = e^{-\delta(t-t_d)}$. Moreover, to ensure that the delay meaningfully influences the decision process, we assume that the instantaneous rate of occurrence, δ, of the mandatory regulation is larger than the discounting rate, i.e., $\delta > \rho$.

Let us describe the *announced regulation mechanism*. This mechanism begins at a random date, $\tilde{t}_m$, and at a random fish stock level, $S(\tilde{t}_m) < S^{FB}$. The first stage involves restoring the fish population through a *Moratorium*, which ends at date $t_i(\tilde{t}_m)$. This date depends on the random component $\left(S^{FB} - S(\tilde{t}_m)\right)$ of the stock that needs to be replenished. From the acceptability principle, the harvesters are compensated during this period. They receive a subsidy corresponding to the profit that they would have

[4] Formally, this means that hazard function is constant over time, i.e., $\delta(t) = \lim_{dt \to 0} \frac{P\left[t \leq t_d + \tilde{\Delta} \leq t + dt \;\middle|\; \tilde{\Delta} \geq t_d + t\right]}{dt} = \frac{\dot{F}(t)}{1 - F(t)} = \delta$

obtained on the compliance path. Since the individual sustainable effort at the optimal conservation level S^{FB} is $e^{FB} = \frac{r(S^{FB})}{nq}$ (see Eq.(9)), the compensation provided to each $t \in \left[\tilde{t}_m, t_i(\tilde{t}_m)\right)$ is:

$$\mathcal{C}(S^{FB}) = \pi\left(S^{FB}\right) q S^{FB} e^{FB} = \frac{\pi\left(S^{FB}\right) S^{FB} r(S^{FB})}{n} \tag{11}$$

After the date $t_i(\tilde{t}_m)$, a *standard ITQ system* applies. For $t \in \left[t_i(\tilde{t}_m), +\infty\right)$, $\omega_i(t)$ denotes the harvesting quota allocated to agent i at time t, and $p_\omega(t)$ represents the quota price. Since the conservation stock is maintained for $t \geq t_i(\tilde{t}_m)$, the following condition must hold:

$$\textstyle\sum_{i=1}^{n} \omega_i(t) = nqe^{FB} S^{FB} = r(S^{FB}) S^{FB} \tag{12}$$

i.e, the total number of quotas corresponds to the fish extraction at the sustainable effort level, which equals the growth of the fish population. The second pillar of this announced mechanism is *the fleet capacity charge*. This tax is crucial because it ensures that the announcement deters any deviation from the optimal conservation path in a deterministic manner, even if the policy's start is random. We propose imposing a tax on the profits that could be made by harvesting at full capacity. Specifically, by setting $e = 1$, the catchability coefficient determines an individual maximum harvest of qS^{FB} at the optimal conservation level, while the profit per unit of harvest is $\pi(S^{FB})$. By announcing a tax rate of τ_x, this tax rule becomes:

$$\mathcal{T}\left(S^{FB}, \tau_x\right) = \pi(S^{FB}) q S^{FB} \tau_x, \tag{13}$$

This tax rate is specifically chosen to deter any deviation from the conservation path. The remaining question is how the regulator sets this announced tax. To do so, the regulator must estimate the potential gain a deviator expects when deciding to modify his fishing effort at time t_d.

When considering such a deviation, we must account for two time segments which follow two different rationales. For $t \in \left[t_d, \tilde{t}_m\right)$, the deviation follows a standard approach, where the expected payoff from a deviation is based on an optimal effort choice during this period, combined with a conjecture about the opponents' behavior. On the other hand, for $t \in \left[\tilde{t}_m, +\infty\right)$, a collective punishment is applied, which is known to all. This means that the payoff expected after $\tilde{t}_m$ is not the result of a deviation, but is rather the solution of a game that begins once the mandatory policy applies. Once the equilibrium returns, which depend on the capacity tax τ_x, are determined, we can start calculating the payoff of a deviator. Here, the uncertainty regarding the timing of the mandatory policy's implementation becomes significant. Indeed, the deviator will try to plan his efforts over the entire horizon, from t_d to $t \in [t_d, +\infty)$, by considering the probability of shifting to a flow of gains corresponding to the mandatory policy at each instant t. To construct this deviation plan, the deviator forms a belief about the evolution of the fish stock, or, in other words, a conjecture about the opponent behaviors. [5] To this end, we introduce a Nash conjecture, which assumes that the other players remain compliant. After this step, the task is to set the tax τ_x at a level where the expected gain from a deviation beginning at any t_d is less than the gain achieved along the compliance path. In other words, this capacity tax, as part of the regulator's threat, aims to capture the extra profit that the deviator expects to make between the start of his deviation and the eventual activation of the mandatory policy, following a political process. The next three sections address these issues.

4. The returns during the potential mandatory policy

The gains during the mandatory policy (see Fig.1) are the easiest to capture. The deviator knows that after date $\tilde{t}_m$, the collective punishment applies. This means that for $t \in \left[\tilde{t}_m, t_i(\tilde{t}_m)\right)$, the deviator will first be subject to a fishing moratorium, with financial compensation of $\mathcal{C}_M(S^{FB})$, but also with a capacity tax of $\mathcal{T}\left(^{FB}, \tau_x\right)$. The deviator's instantaneous earnings during this period are:

$$v_M(\tau_x) = \mathcal{C}_M(S^{FB}) - \mathcal{T}\left(S^{FB}, \tau_x\right) = \pi(S^{FB})S^{FB}\left(\frac{r(S^{FB})}{n} - q\tau_x\right) \tag{14}$$

After the reconstitution of the fish stock at date $t = t_i(\tilde{t}_m)$, each player can resume fishing, but now under an ITQ regulation system and subject to the same capacity tax. The question, therefore, is to construct the equilibrium gain of each player, especially the deviator. In this scenario, each harvester chooses his optimal effort by maximizing the present value of his profit, considering that he must cover his instantaneous harvest by purchasing rights at a competitive price $p_\omega(t) \geq 0$. Each harvester then solves:

$$\max_{e_i(t)} \int_{t_i(\tilde{t}_m)}^{+\infty} e^{-\rho t}\left(\pi(S(t))S(t)qe_i - p_\omega(t)\left(qe_iS(t) - \omega_i(t)\right) - \mathcal{T}\left(S^{FB}, \tau_x\right)\right)dt \tag{15}$$

$$\text{with} \quad \frac{\dot{S}(t)}{S(t)} = r(S(t)) - q\left(\textstyle\sum_{i=1}^{n} e_i(t)\right)\ , \ S(t_i(\tilde{t}_m)) = S^{FB} > 0 \tag{16}$$

and the tradable quota market satisfies a free disposal equilibrium condition at each instant, i.e.:

$$\forall t \in \left[t_i(\tilde{t}_m), +\infty\right)\ , \quad \begin{cases} p_\omega(t)\left(\sum_{i=1}^{n}\omega_i(t) - \sum_{i=1}^{n} qe_it)S(t)\right) = 0 \\ \left(\sum_{i=1}^{n}\omega_i(t) - \sum_{i=1}^{n} qe_i(t)S(t)\right) \geq 0 \end{cases} \tag{17}$$

This condition simply states that, at a given time t, if there are too many rights on the market relative to fish landings, the price $p_\omega(t)$ must be zero. Conversely, if the price of rights is strictly positive, the quantity of rights is binding and equals the fish landings.

[5] Note that the deviator's belief about the evolution of the fish stock does not need to be consistent with any observed path. This is because, by design of the V-T mechanism, the path is never induced by a deviation.

Now the question arises: does this ITQ mechanism preserve the optimal conservation level S^{FB}? To answer this, we must ensure that, for each $t \in \big[t_i(\tilde{t}_m), +\infty\big)$, there exists a fishing rights price $p_\omega(t)$ and an initial distribution of rights $(\omega_i(t))_{i=1}^n$ such that: (i) the free disposal equilibrium conditions (Eq.(17)) are satisfied; and (ii) for each fishery, the optimal effort path solution to program (15) corresponds, for each $t \in \big[t_i(\tilde{t}_m), +\infty\big)$, to the effort e^{FB} that sustains the optimal conservation level S^{FB} (given by Proposition 1). In a context of complete information, this can be achieved by implementing a no-trade equilibrium. In this case, the regulator distributes quotas corresponding to the fish landings that support the optimal conservation policy. These quotas are given by:

$$\forall i,\, \forall t \in \big[t_i(\tilde{t}_m), +\infty\big)\,,\; \omega_i(t) = \omega = qe^{FB}S^{FB} = \tfrac{r(S^{FB})S^{FB}}{n} \tag{18}$$

If the fisheries are compliant, this automatically induces a no-trade situation. This means that, according to Eq.(17), the price $p_\omega(t)$ remains indeterminate. Therefore, it remains to set the path of the quota price such that it is optimal for each fishery to remain compliant. This price path is fixed as[6] :

$$\forall t \in \big[t_i(\tilde{t}_m), +\infty\big)\,,\; p_\omega(t) = p_\omega = \tfrac{(n-1)\big(e_\pi(S^{FB})+1\big)r(S^{FB})\pi(S^{FB})}{n\rho - r(S^{FB}) - nS^{FB}r'(S^{FB})} > 0 \tag{19}$$

with $e_\pi(S)$ denoting the elasticity of the profit function.

By setting the quota distribution and their price according to Eqs.(18) and (19) and by solving the (singular) program (15), we show that the optimal strategy of every fishery in response to compliant effort levels of their opponents is to choose efforts $e_i(t)$, $t \in \big[t_i(\tilde{t}_m), +\infty\big)$, which induces a MRAP to the optimal conservation stock S^{FB}. As these dynamics only start when the optimal conservation level S^{FB} is reached, we can assert, as expected, that the constant effort functions $e_i(t) = e^{FB}$ for $i = 1, \ldots, n$ constitute a Nash equilibrium of the ITQ game and that the fish stock remains constant and equal to the optimal conservation level S^{FB}. As a result, we can easily estimate an agent's instantaneous payoff during this ITQ stage. According to the non-trading equilibrium on the quota market and the Nash equilibrium described above, this payoff is the same for each player and remains constant over time $t \in \big[t_i(\tilde{t}_m), +\infty\big)$. The payoff is given by:

$$v_{ITQ}(\tau_x) = \pi(S^{FB})S^{FB}qe^{FB} - \mathcal{T}\left(S^{FB}, \tau_x\right) = \pi(S^{FB})S^{FB}\left(\tfrac{r(S^{FB})}{n} - q\tau_x\right) \tag{20}$$

Finally, returning to our original problem of estimating the gain from deviating from the V-T conservation mechanism, we can say that:

Proposition 2. *If a fisherman deviates from a V-T conservation mechanism at time t_d, he knows that:*
(i) After a random delay following his deviation, i.e, at date $\tilde{t}_m$, he is assigned a moratorium until the optimal fish stock is recovered. During this period $\big[\tilde{t}_m, t_i(\tilde{t}_m)\big)$, his instantaneous yield $v_M(\tau_x)$ remains constant and is given by Eq.(14)

[6] The strict positivity of this price is checked in Appendix B

(ii) After date $t_i(\tilde{t}_m)$, the ITQ system is implemented. At the Nash equilibrium of this dynamical game, the fisherman's instantaneous yield $v_{ITQ}(\tau_x)$ is constant over $\left[t_i(\tilde{t}_m), +\infty\right)$ and is given by Eq.(20)
(iii) As $v_M(\tau_x) = v_{ITQ}(\tau_x)$, the instantaneous gain expected from a deviation after the regulation is implemented, remains constant over $t \in \left[\tilde{t}_m, +\infty\right)$ and is given by:

$$v_R(\tau_x) = \pi(S^{FB})S^{FB}\left(\frac{r(S^{FB})}{n} - q\tau_x\right) \tag{21}$$

5. The expected gain from a deviation starting at time t_d

Stepping back (see Fig. 1), we now evaluate the gain of a fishery that decides to leave the compliance path at time t_d. The rationale is somewhat different in this case. At that moment, the deviator defines a new effort plan over the rest of the horizon, considering that, at some future random date $\tilde{t}_m$, his payoff will switch to the one described in Proposition 2. In other words, the deviator maximizes an expected discounted gain. As the probability that the policy will not be implemented before $t > t_d$ is $(1 - F(t, t_d))$, the return starting from t_d, is given by:

$$\mathcal{V}^d(\tau_x, t_d) = \max_{e(t)\in[0,1]} \int_{t_d}^{+\infty} e^{-\rho t}\left[\left(\pi(S^d(t))S^d(t)qe(t)\right)(1 - F(t, t_d)) + v_R(\tau_x)F(t, t_d)\right] dt \tag{22}$$

However, to optimally adjust his effort plan, the deviator needs to predict the evolution of the fish stock $S^d(t)$ or, in other words, form beliefs about the behavior of the other players. In the Nash tradition, we assume that he conjectures that the others remain compliant and select an effort of $e^{FB} = \frac{r(S^{FB})}{nq}$. His belief about the motion of the biomass is thus given by:

$$\frac{\dot{S}^d(t)}{S^d(t)} = r(S^d(t)) - \left(qe(t) + \frac{(n-1)}{n}r(S^{FB})\right) \ , \ S^d(t_d) = S^{FB} \tag{23}$$

Moreover, this belief does not need to be consistent with observations since, by the construction of a V-T policy, we know that this deviation will never be played. It is simply a belief about the motion of the biomass based on a Nash conjecture.

This program, however, remains a singular control problem, as both the instantaneous objective and the dynamics are linear in the control variable. At first glance, it may appear to be a non-autonomous problem due to the introduction of a time-dependent switching probability. This generally implies that the singular stock is time-dependent. However, as the rate of occurrence of the mandatory regulation is constant over time, the singular stock, S^D, remains time-independent and satisfies the following Euler-Lagrange condition:

$$(\rho + \delta)\,\pi(S) = \partial_s\left(\pi(S)S\left(r(S) - \tfrac{n-1}{n}r(S^{FB})\right)\right) \tag{24}$$

$$= \pi'(S)S\left(r(S) - \tfrac{n-1}{n}r(S^{FB})\right) + \pi(S)\left(r(S) - \tfrac{n-1}{n}r(S^{FB})\right) + \pi(S)Sr'(S) \tag{25}$$

This condition is again related to the Hotelling rule introduced in Eq.(7). However, it now considers the random stopping time and only considers the private gains. Specifically, the left-hand side of Eq.(24) represents the financial returns from the profit induced by the last unit of fish caught, albeit at a higher interest rate to reflect the potential shift to a mandatory policy. The right-hand side of Eq.(24) only considers the private marginal gain from leaving that unit at sea, assuming the opponents' behavior is fixed. Now, let us observe that Eq.(24) can also be written as follows:

$$\rho\pi(S) - \partial_s\left(\pi(S)Sr(S)\right) = -\left(\delta\pi(S) + \tfrac{n-1}{n}r(S^{FB})\partial_s\left(\pi(S)S\right)\right) \tag{26}$$

From Eq.(7), the left-hand side of Eq.(26) is zero at $S = S^{FB}$, and by the existence of a MRAP path to S^{FB} (see (2) of Appendix A), we also know that this part of the equation is non-negative for $S \geq S^{FB}$. As the right-hand side of Eq.(26) is negative, the singular stock level, S^d, targeted by the deviator is below the optimal conservation level, S^{FB}, meaning that the deviator plans to over-harvest the resource. However, this does not imply that his harvesting strategy follows an MRAP path, where he initially harvests at capacity and, at some point, switches to an effort that stabilizes the biomass at S^d. This also requires the existence of a long-run effort e^d, which belongs to $[0,1]$ and stabilizes the MRAP path at S^d. According to Eq.(23), a quick calculation yields:

$$r(S^d) - \left(qe^d + \tfrac{(n-1)}{n}r(S^{FB})\right) = 0 \Leftrightarrow e^d = \frac{1}{q}\left(r(S^d) - \tfrac{(n-1)}{n}r(S^{FB})\right) \tag{27}$$

We can therefore claim that this effort is more important than its counterpart on the first best path, i.e. $e^d > e^{FB}$. In fact, since the population growth rate $r(S)$ decreases and $S^d < S^{FB}$, we have:

$$r(S^d) > r(S^{FB}) \Leftrightarrow r(S^d) - \tfrac{(n-1)}{n}r(S^{FB}) > \tfrac{1}{n}r(S^{FB}) \Leftrightarrow e^d > e^{FB} \tag{28}$$

However, this does not mean that $e^d \leq 1$. Two situations must therefore be considered depending on whether $r(S^d) - \frac{(n-1)}{n}r(S^{FB}) \gtreqless q$. In the first case, the deviator plans to harvest forever at capacity and never reaches the targeted singular stock level, S^D, while in the second, he first harvests at capacity and then switches to e^d when S^d is reached. Since $S^d < S^{FB}$, we can say that his belief about the biomass evolution is given by:

$$S^D(t, t_d) = \max\left\{S^d(t), S^d\right\} \text{ where } S^d(t, t_d) \text{ solves Eq. (23) for } e(t) = 1 \tag{29}$$

If we now denote by $\bar{t}$ the time at which $S^d(\bar{t}) = S^d$ or set $\bar{t} = +\infty$ is $S^d(t)$ never reaches the singular stock, S^d, the effort planned after the deviation is:

$$e^D(t) = \begin{cases} 1 \text{ for } t \in \left[t^d, \bar{t}\right) \\ e^d = \frac{1}{q}\left(r(S^d) - \frac{(n-1)}{n}r(S^{FB})\right) \text{ for } t \in [\bar{t}, +\infty) \end{cases} \tag{30}$$

The next proposition summarizes these results.

Proposition 3. *If an agent deviates at time t_d from the optimal conservation path, we can say that:*
(i) He rearranges his efforts over time to follow the MRAP to a biomass stock of $S^d < S^{FB}$, which he reaches (or not) in finite time, depending on whether $r(S^d) - \frac{(n-1)}{n} r(S^{FB}) < (\geq) q$.
(ii) This planned effort path $e^D(t)$, as given by Eq.(30), is always greater than the optimal conservation effort e^{FB}.
(iii) These new efforts are, according to a Nash conjecture about other players' behavior, sustained by a belief on the biomass dynamics $S^D(t) < S^{FB}$, as given by Eq.(29).
(iv) His expected payoff, which accounts for the random delay of policy enforcement, is given by:

$$\mathcal{V}^d(\tau_x, t_d) = \int_{t_q}^{+\infty} e^{-(\rho+\delta)t+\delta t_d} \left(\pi\left(S^D(t,t_d)\right) S^D(t,t_d)\, q e^D(t)\right) dt + v_R(\tau_x) \tfrac{\delta}{\rho(\rho+\delta)} e^{-\rho t_d} \quad (31)$$

6. Back to the V-T mechanism

Let us recall that a V-T mechanism relies on the announcement of a potential regulation. Its aim is to discourage any deviation from the optimal conservation path, thus preventing the implementation of the announced policy. This is where the choice of the capacity tax rate becomes important. From the previous section, we know from Eq.(31) what a deviator is expected to gain when he deviates at time t_d. As the policy takes time to be activated, this gain is mainly based on two components: the benefits from overfishing until the policy becomes mandatory, and the expected benefits under the mandatory policy. This capacity tax reduces the overall expected gain, effectively neutralizing the gain from overfishing. In other words, a well-chosen tax will discourage any deviation from the conservation trajectory. Let us now proceed to set this tax rate.

On one hand, from Eq.(31), we obtain the expected payoff, $\mathcal{V}^d(\tau_x, t_d)$ of a fishery that decides to deviate at time t_d. This payoff includes the potential tax rate. On the other hand, if this fishery does not plan to deviate at time t_d, it obtains the present value of the compliance payoff, which is given by:

$$Cpl(t_d) = \int_{t_d}^{+\infty} e^{-\rho t}\left[\pi\left(S^{FB}\right) S^{FB} q e^{FB}\right] dt = \tfrac{1}{\rho}\pi\left(S^{FB}\right) S^{FB} q e^{FB} e^{-\rho t_d} \quad (32)$$

As a result, a fishery will never plan to deviate if:

$$\forall t_d \geq 0,\; Cpl(t_d) \geq \mathcal{V}^d(\tau_x, t_d) \quad (33)$$

In other words, the regulator should announce the lowest tax rate, τ_x^*, that satisfies the previous set of inequalities. To estimate τ_x^*, recall that $\mathcal{V}^d(\tau_x, t_d)$ is a linear and decreasing function of τ_x (see Eqs.(21) and (31)). Therefore, the minimal tax rate $\tau_x(t_d)$ that deters deviation at time t_d solves Eq.(33) with equality, and due to the linearity, is given by:

$$\tau_x(t_d) = \tfrac{\rho}{\delta}\left(\int_{t_d}^{+\infty} (\rho+\delta) e^{-(\rho+\delta)(t-t_d)} \left[\tfrac{\pi(S^D(t,t_d)) S^D(t,t_d)}{\pi(S^{FB}) S^{FB}} e^D(t)\right] dt - \tfrac{r(S^{FB})}{qn}\right) \quad (34)$$

As the announced tax rate needs to deter any deviation whenever it occurs, i.e, for all t_d, we can express this as:

$$\tau_x^* = \sup_{t_d \in [0,+\infty[} \tau_x(t_d) \tag{35}$$

The existence of this maximum is not an issue, as we can prove that for all t_d, $\tau_x(t_d) \in [0, \bar{\tau}_x]$, where:

$$\bar{\tau}_x = \frac{\rho}{\delta}\left(1 - \frac{r\left(S^{FB}\right)}{nq}\right) < 1 \tag{36}$$

In the context of a voluntary agreement, we must also ensure that the penalty, τ_x^*, is not too harsh. However, it is difficult to characterize τ_x^* analytically, as it solves a complex optimization problem. Therefore, acceptability can only be assessed by considering the upper bound $\bar{\tau}_x$ of τ_x^*. To understand this bound, recall that effort $e^{FB} = \frac{r\left(S^{FB}\right)}{nq}$, which lies in the range $[0, 1]$ captures the rate of activation of the harvesting capacity. Hence, $\left(1 - \frac{r\left(S^{FB}\right)}{nq}\right)$ represents the rate of excess capacity compared to the optimal conservation level. Moreover, as the hazard rate is constant, $\frac{1}{\delta}$ is the average time needed to implement the policy. This upper bound is, therefore, the product of the discount rate, average delay, and excess capacity. For example, with a discount rate of 5%, an average delay of two years, and an excess capacity of 25%,, the upper bound is 2.5%. To fully understand this tax rate and its relationship to the average policy implementation time, specific cases must be considered. This will be carried out in the next section. But first, let us summarize the main results of this section.

Proposition 4. *There exists a V-T mechanism that guarantees voluntary compliance with the optimal conservation level for fish stocks, without the need for any effective policy. This mechanism is based on the announcement of (i) a moratorium with financial compensation, followed by an ITQ to maintain the stock at the conservation level, and (ii) a tax rate on capacity, given by:*

$$\tau_x^* = \sup_{t_d \in [0,+\infty[} \frac{\rho}{\delta}\left(\int_{t_d}^{+\infty} (\rho+\delta)e^{-(\rho+\delta)(t-t_d)} \left[\frac{\pi(S^D(t,t_d))S^D(t,t_d)}{\pi(S^{FB})S^{FB}} e^D(t)\right] dt - \frac{r(S^{FB})}{qn}\right) \geq 0 \tag{37}$$

Moreover, the capacity tax τ_x^* is bounded from above by: $\frac{\rho}{\delta}\left(1 - \frac{r\left(S^{FB}\right)}{nq}\right) < 1$, which is the product of the discount rate, average delay, and excess capacity at the optimal conservation level.

7. An illustration: the scallop fishery in the bay of Saint-Brieuc

The Great Atlantic scallop (*Pecten maximus*) is emblematic of the Bay of Saint-Brieuc, located on the northern coast of Brittany (France). This bivalve species is typically found on soft, sandy bottoms at depths ranging between 20 and 50 meters below the surface. Often referred to as "white gold," the scallop makes a major contribution to the wealth of traditional coastal fisheries. This fishing activity began in the early 1960s, and while scallops were developing rapidly, the resource was soon over-exploited. This led to an initial round of measures (see Guyader *et al.* [26]) aimed at limiting the fleet's fishing capacity through restrictions on the number and size of boats, as well as on fishing time. However, this was not enough. Additional restrictions, coupled with scientific monitoring of the scallop stock by IFREMER, were subsequently introduced. These restrictions included non-transferable individual quotas per vessel, restrictions on dredge mesh size, and limitations on harvestable shell size. While these restrictions have contributed significantly to restoring the scallop population, they have come with a cost, estimated at 300 k€ per year in 2006 (see Le Gallic *et al.* [34]). This situation closely mirrors the one addressed in our paper, raising the question of whether the introduction of a voluntary conservation mechanism to replace the second set of restrictions could lead to a less costly regulation. It also offers an opportunity to examine the level of the capacity tax that should be announced, and its relationship with the political delay in implementing the proposed measures.

The data and functional forms used in this illustration are mainly borrowed from Frésard and Ropars-Collet [21]. Accordingly, we adopt a logistic growth rate function, $r(S) = r(1 - S/K)$, and a unit profit function per harvest, given by $\pi(S) = (p - \frac{c}{qS})$. The values of the parameters are summarized in Table 1.

Parameter	Description	Value
r	Intrinsic growth rate	0.649
K	Carrying capacity	54 252 tons
p	Ex-vessel unit price	2 000 € per ton
q	Catchability coefficient	2.961×10^{-3}
c	Unit cost of fishing effort	4 746 € per boat
n	Number of vessels	250
ρ	Discount rate	0.05

Table 1: The scallop fishery of the Saint-Brieuc Bay

This dataset includes the first set of restrictions on fishing capacity: (i) the number of boats limited to 250 by a numerus clausus licensing policy, with a maximum boat length of 13 meters; and (ii) fishing is allowed for only 45 minutes, two days a week, during the season. However, unlike Frésard and Ropars-Collet [21], who treated restriction (ii) as a 42-hour upper limit on the yearly individual effort, we multiply their catchability coefficient and their unit effort cost by 42 to ensure that the effort remains within the range $[0, 1]$. Regarding this dataset, we also observe that the minimum biomass stock, $S_{\min} = \frac{c}{pq}$, at which profit becomes positive is approximately $S_{\min} \simeq 801.4$ tons. This suggests that this species remains endangered since $nq \simeq 0.742 > r(S_{\min}) \simeq 0.639$, meaning the total exhaustion rate is greater than the biomass growth rate at which harvesting becomes profitable. This clearly indicates that, in the case of the Bay of St Brieuc, the first round of restrictions on harvesting capacity was not effective. The second set of restrictions, based on quotas, mesh size, and harvestable shell size, has proven far more effective. Consequently, the biomass stock is now largely preserved and close to the optimal conservation situation described in Table 2.

Variable	Description	Value
S^{FB}	Optimal biomass stock	25 503 tons
S_{min}	Minimal stock for positive profit	801.418 tons
$nS^{FB}qe^{FB}$	Total catch per season	8 771 tons/year
e^{FB}	Individual optimal effort	0.4646
$S^{FB}qe^{FB}$	Individual catch per season	35.0834 tons/year
$\pi(S^{FB})S^{FB}qe^{FB}$	Individual stationary profits	67 962 €/year
$\pi(S^{FB})S^{FB}q$	Individual profit at capacity	146 280 €/year

Table 2: The first best stationary conservation target

However, the control costs associated with this conservation policy are much higher. This raises the question of introducing a V-T regulation. Let us now build a V-T mechanism for the scallop fishery and, in doing so, examine the impact of the average delay, $1/\delta$, on this policy instrument.

To set up this mechanism, we begin by analyzing the expected gain from a potential deviation at time t_d. The yearly profit obtained after the enforcement of the threat is straightforward to calculate. Using Eq.(21) and Tables 1 and 2, we obtain :

$$v_R(\tau_x) = \pi(S^{FB})S^{FB}\frac{r(S^{FB})}{n} - \pi(S^{FB})S^{FB}q\tau_x = 67962 - 146280\tau_x \tag{38}$$

If we now move to the program of a deviator (Eq.(22), we know from Proposition 3 that the deviator aims to target, for a given δ, a new biomass stock $S^d(\delta)$ that solves Eq.(24). This proposition also indicates that this target can be reached (or not) in finite time if $r(S^d(\delta)) - \frac{(n-1)}{n}r(S^{FB}) < (\geq)q$. In our example, we show (see Appendix E) that this will never be the case. This means that a fishery that deviates from the compliance path at time t_d plans to fish at full capacity, i.e., $e^D(t) = 1$ for $t \geq t^d$, until, at some random date, the announced regulation comes into effect. This choice is supported by the anticipation of the fish stock's evolution (see Eq.(23)), which solves:

$$\frac{\dot{S}^D(t)}{S^D(t)} = -1.1963 \cdot 10^{-5}S^D(t) + 0.3035 \ , \ S^D(t_d) = 25503 \tag{39}$$

and, under our logistic growth rate assumption, we also obtain an explicit solution given by:

$$S^D(t, t_d) \simeq \left(-2.0482 \cdot 10^{-7}e^{-0.3035(t-t_d)} + 3.9417 \cdot 10^{-5}\right) -1 \tag{40}$$

At this point, we can now compute the expected gain from a deviation. This quantity is given by: [7]:

$$\mathcal{V}^d(\tau_x, t_d, \delta) \simeq e^{-0.05t_d}\left(\int_0^{+\infty} \frac{5.922e^{-(0.05+\delta)x}\cdot 10^5}{-2.0482\cdot 10^{-2}e^{-0.3035x}+3.9417}dx + \frac{\delta(67962-146280\tau_x)-237.3}{0.05(0.05+\delta)}\right) \quad (41)$$

As this function is decreasing in t_d, we can conclude that if a deviation occurs in the case of this scallop fishery, it must necessarily take place immediately, i.e. at $t_d = 0$

From this last remark, the goal is to prevent any deviation at time $t_d = 0$. Since the other option is to remain compliant, let us first observe from Eq.(32) that:

$$Cpl(0) \simeq \frac{1}{\rho}\pi\left(S^{FB}\right)S^{FB}qe^{FB} = 1.3592\cdot 10^6 \quad (42)$$

By setting $\mathcal{V}^d(\tau_x, 0, \delta) = Cpl(0)$, we finally obtain a relation between the optimal tax choice, τ_x^*, and the hazard rate, δ, which is given by:

$$\tau^*(\delta) \simeq \frac{(5+100\delta)}{\delta}\int_0^{+\infty} \frac{e^{-(0.05+\delta)x}}{-10.1188e^{-0.3035x}+1.9473\cdot 10^3}dx - \frac{0.0249}{\delta} \quad (43)$$

Additionally, in our model, the average time required to implement the threat can be identified as $1/\delta$, the inverse of the hazard rate. We can therefore provide a useful table for policymakers that matches a tax to announce with their estimation of the time required by the political process to implement the threat. These results are summarized in Table 3.

Average delay (in years)	.5	1	1.5	2	2.5	3	3.5	4
Deterrent capacity tax	.013	.027	.040	.053	.067	.080	.093	.106

Table 3: Average policy delay and deterrent capacity tax

To conclude this illustration of the St Brieuc scallop fishery, we can state that the regulator can replace the costly regulations based on quotas, mesh size control, and harvestable shell size with a V-T mechanism, while still preserving the optimal conservation stock. This V-T mechanism relies primarily on the announcement of a new set of restrictions if the scallop stock deviates from the optimal level. These potential restrictions include, on one hand, a moratorium followed by ITQ regulations and, on the other hand, a tax on the annual profit obtained from harvesting at full capacity. The tax rate, as outlined in Table 3, is dependent on the average time the regulator needs to implement the policy. This announcement dissuades any deviation from the optimal conservation target, ensuring voluntary compliance without the need to enforce the policy.

[7] The reader may be surprised by the absence of t_d in the integral; in fact, we have made a change of variable given by $x = t - t_d$ (see Appendix E)

8. Concluding remarks

In this article, we have shown that an active fish stock conservation policy can be replaced by a V-T mechanism that avoids a significant portion of the regulatory costs. This argument has been developed using a standard bio-economic model in the tradition of Gordon-Schaefer. By assuming that the optimal level of conservation is achieved through a mandatory regulatory policy, we have proposed replacing this costly policy with a V-T mechanism, which is based on the announcement of a new mandatory policy if a decline in fish stocks is observed. The primary goal of this announcement is to discourage any deviation from the harvesting effort trajectory that is compatible with optimal conservation. This announced threat rests on two main pillars. The first restores and preserves the conservation level. It takes the form of a moratorium, coupled with financial compensation to ensure acceptability, and is followed by a standard system of individual transferable quotas. The second is designed to discourage any deviation from the optimal conservation path. It consists of a tax on fishing capacity, which reduces the expected gain from any deviation occurring during the period between detection and the policy's implementation. We have shown how this mechanism works and, in particular, how to set the capacity tax. Finally, we have illustrated our argument using the case of scallop harvesting in the Bay of Saint-Brieuc (France). In this example, we explicitly derive the tax rate and study its relationship with the average delay in the policy's implementation.

Nevertheless, several points in the argument are open to debate. The first concerns the dynamic context of the game. By design, V-T mechanisms are intended to deter any deviation from a chosen outcome. However, this requires knowledge of the deviator's expected payoff and, therefore, his conjecture about the behavior of opponents during this period. In this paper, we made a specific choice. From the date of deviation until the implementation of the mandatory policy, we introduce a Nash conjecture, which assumes the deviator believes that the other players will remain compliant. However, once the policy is implemented and becomes a collective punishment, we have assumed that the deviator's expected outcome aligns with the one obtained in a Nash equilibrium from that date onward. This modeling choice clearly influences the setting of the capacity tax rate and raises the question of what might happen if the deviator formulates more complex conjectures. For example, the deviator could imagine that the other players will also deviate, or, within the context of a sub-game perfect approach, he may only consider the equilibrium gain obtained in the sub-game starting from that date. This question should certainly be investigated, but perhaps not within the context of a Gordon-Schaefer model. This leads to a second limitation of this paper. In essence, the Gordon-Schaefer model is a competitive supply-side model with MRAP solutions due to the linearity of both the instantaneous payoff and the dynamics with respect to effort. This raises the question of whether and how a V-T mechanism can be implemented in models that incorporate the demand side and/or imperfectly competitive behavior. For instance, this calls for the use of "great fish war" models in the Levhari and Mirman style [35], or their equivalent in continuous time. Finally, it is also worth noting that most conservation models are based on a single species. In fisheries, the primary objective is to preserve the resource to improve the welfare of its single predator: humans. However, species such as scallops are also part of a global food chain that must be considered, for instance, within a prey-predator approach.

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Appendix A. Proof of Proposition 1

The proof is based on Hartl and Feichtinger's sufficient conditions [29], precisely, on Theorem 3.1 and on Remark 4.3 for infinite horizon and catching up optimality. To adapt to their notations, we start by writing the program (5) as follows:

$$\max_{\dot{S}} \int_0^{+\infty} e^{-\rho t} \left(M_{FB}(S) + N_{FB}(S)\dot{S} \right) dt \tag{A.1}$$

$$\text{s.t. } \dot{S}(t) \in \left[S(t) \left(r(S(t)) - nq \right), S(t) r(S(t)) \right] \tag{A.2}$$

with $M_{FB}(S) = \pi(S)Sr(S)$ and $N_{FB}(S) = -\pi(S)$ and introduce:

$$I_{FB}(S) = -\rho N_{FB}(S) - M'_{FB}(S) = \left(\rho - r'(S)S - r(S)\right) \pi(S) - \pi'(S)Sr(S) \tag{A.3}$$

(1) $I_{FB}(S) = 0$ admits a unique solution $S^{FB} \in]S_{\min}, K[$

Concerning *existence*, observe that:
(i) $I_{FB}(S_{\min}) = -\pi'(S_{\min})S_{\min}r(S_{\min}) < 0$ since $\pi(S_{\min}) = 0$, $r(S_{\min}) > 0$ and $\pi'(S_{\min}) > 0$
(ii) $I_{FB}(K) = \left(\rho - r'(K)K\right) \pi(K) > 0$ since $\pi(K) > 0$, $r(K) = 0$ and $r'(K) < 0$.
The continuity of $I_{FB}(S)$ then induces the existence of a solution, $S^{FB} \in [S_{\min}, K]$ to $I_{FB}(S) = 0$. We also observe that $S^{FB} > S_{\min}$ so that $\pi\left(S^{FB}\right) > 0$
Uniqueness is achieved if $\left(I_{FB}\right)'(S)$ maintains its sign at each solution S_0. To verify this point, observe first that $I_{FB}(S)$ is also given by:

$$I_{FB}(S) = \pi(S) \underbrace{\left(\rho - r'(S)S - \left(\frac{\pi'(S)S}{\pi(S)} + 1 \right) r(S) \right)}_{=f(s)} \tag{A.4}$$

Accordingly $\left(I_{FB}\right)'(S) = \pi'(S)f(S) + \pi(S)f'(S)$. As this function is evaluated at a solution S_0 for which $\pi(S_0) > 0$, it follows that $f(S_0) = 0$ and that $sign\left(\left(I_{FB}\right)'(S_0)\right) = sign\left(f'(S_0)\right)$. Moreover:

$$f'(S_0) = -r"(S_0)S_0 - r'(S_0)\left(\frac{\pi'(S_0)S_0}{\pi(S_0)} + 2\right) - r(S_0)\frac{\pi'(S_0)}{\pi(S_0)}\left(\left(\frac{\pi"(S_0)S_0}{\pi'(S_0)} + 1\right) - \frac{\pi'(S_0)S_0}{\pi(S_0)}\right) \tag{A.5}$$

and, from our assumptions, we know that for all $S \in [S_{\min}, K]$, $r(S) \geq 0$, $r'(S) < 0$, $r"(S) < 0$, $\pi(S) \geq 0, \pi'(S) > 0$ and $e_{\pi'}(S) = \frac{\pi"(S)S}{\pi'(S)} \leq -1$. So for each solution S_0, $\left(I_{FB}\right)'(S_0) > 0$. This proves uniqueness.

(2) $\forall S \in [S_{\min}, K]$ if $S < S^{FB}$ (resp. $>$) then $I_{FB}(S) < 0$ (resp. $>$).

The result follows directly from (i), (ii) and uniqueness.

(3) $\liminf_{t\to+\infty} e^{-\rho t} \int_{S(t)}^{S^{FB}} N_{FB}(s)ds \geq 0$ for a every feasible path (catching up optimality)

Using the non-negativity condition of the instantaneous profit, we can restrict the set of feasible paths to those belonging to $[S_{\min}, K]$. On this compact set, the quantity $-\pi(S)$ is, by continuity, bounded. Accordingly, any path $S(t) \in [S_{\min}, K]$ has the property that for any $t > 0$, the quantity $\int_{S(t)}^{S^{FB}} (-\pi(s))\, ds$ is also bounded. We can therefore claim that:

$$\lim_{t\to+\infty} e^{-\rho t} \int_{S(t)}^{S^{FB}} (-\pi(s))\, ds = 0 \tag{A.6}$$

(4) Conclusion

Since (1), (2) and (3) are satisfied, it only remains to verify that the long-term path given by $S^{FB}(t) = S^{FB}$ and the related effort $e^{FB} = \frac{r(S^{FB})}{nq} \in (0, 1)$ are feasible, i.e, that it satisfies Eq.(A.2). As $\dot{S}^{FB}(t) = 0$, this requires that $0 \in \left[S^{FB}\left(r(S^{FB}) - nq\right), S^{FB} r(S^{FB})\right]$. Clearly, $S^{FB} r(S^{FB}) > 0$. Moreover, since $S^{FB} > S_{\min}$ and $r(S)$ decreases, we have $r(S^{FB}) < r(S_{\min})$. Finally, assuming endangered species, $r(S_{\min}) < nq$, we can assert that $S^{FB}\left(r(S^{FB}) - nq\right) < 0$.

Appendix B. Proof of Proposition 2

On the basis of our discussion in section 4, it still remains to show two points. First, we prove that the equilibrium quota price, given by Eq.(19), is positive and identify the new lower bound, $S_{\min}^{ITQ}$, on the state variable induced by the new condition of non-negativity of profits. In a second step, we show that (i) if the initial quotas are given by Eq.(18) and (ii) if the equilibrium market price is given by Eq.(19), then each fishery, which solves the program (15) and conjectures that the other players are compliant, chooses a MRAP strategy to reach the optimal conservation level. Since the IQT policy starts when the fish stock reaches S^{FB}, this shows that being compliant is a Nash equilibrium of the ITQ game.

Point 1: Quota price and profit non-negativity

(1) Positivity of the quota price

From Eq.(19), the equilibrium quota price is:

$$p_\omega = \frac{(n-1)\left(e_\pi(S^{FB})+1\right) r(S^{FB})\pi(S^{FB})}{n\rho - r(S^{FB}) - nS^{FB} r'(S^{FB})} \tag{B.1}$$

The numerator of this expression is strictly positive since $r(S^{FB}) > 0$, $\pi(S^{FB}) > 0$ and $e_\pi(S^{FB}) = \frac{\pi'(S^{FB})S^{FB}}{\pi(S^{FB})} > 0$. Concerning the denominator, observe that:

$$D = n\rho - r\left(S^{FB}\right) - nS^{FB} r'(S^{FB}) > n\left(\rho - r\left(S^{FB}\right) - S^{FB} r'(S^{FB})\right) \tag{B.2}$$

As $I_{FB}(S^{FB}) = 0$, we know from Eq.(A.4) that:

$$\left(\rho - r(S^{FB}) - r'(S^{FB})S^{FB}\right) = \frac{\pi'(S^{FB})S^{FB}}{\pi(S^{FB})} r(S^{FB}) \tag{B.3}$$

so that $D > n\frac{\pi'(S^{FB})S^{FB}}{\pi(S^{FB})} r(S^{FB}) > 0$.

(2) Introduction of a new lower bound $S_{\min}^{ITQ} \in \left[S_{\min}, S^{FB}\right]$

A look at the program (15) shows that a positive effort now requires $\pi(S) - p_w > 0$. This leads to a new lower bound, $S_{\min}^{ITQ}$, on the potential states targeted by a MRAP strategy. Since $pi'(S) > 0$ and $p_\omega > 0$, it's obvious that $S_{\min}^{ITQ} > S_{\min}$. To state that $S_{\min}^{ITQ} < S^{FB}$, we only have to check that $\pi(S^{FB}) - p_\omega > 0$. If $D > 0$ denotes the denominator of p_ω (see Eq.(B.2)), we have:

$$
\begin{aligned}
\pi(S^{FB}) - p_\omega &= \tfrac{1}{D}\left(D\pi(S^{FB}) - (n-1)\left(\pi'(S^{FB})S^{FB} + \pi(S^{FB})\right) r(S^{FB})\right) \\
&= \tfrac{1}{D}\left(n\left(\rho - r\left(S^{FB}\right) - S^{FB} r'(S^{FB})\right)\pi(S^{FB}) - (n-1)\,\pi'(S^{FB})S^{FB} r(S^{FB})\right) \\
&> \tfrac{n}{D}\left(\left(\rho - r\left(S^{FB}\right) - S^{FB} r'(S^{FB})\right)\pi(S^{FB}) - \pi'(S^{FB})S^{FB} r(S^{FB})\right) \\
&= 0 \text{ since } I_{FB}(S^{FB}) = 0 \text{ (see Eq.(A.3)} \qquad \text{(B.4)}
\end{aligned}
$$

Point 2: the MRAP result

We again use Hartl and Feichtinger's sufficient conditions [29]. So let us write program (15) as:

$$
\max_{\dot{S}} \int_{t_i(\tilde{t}_m)}^{+\infty} e^{-\rho t}\left(M_{ITQ}(S) + N_{ITQ}(S)\dot{S}\right) dt \qquad \text{(B.5)}
$$

$$
\text{s.t. } \dot{S}(t) \in \left[S(t)\left(r(S(t)) - q - \tfrac{n-1}{n} r(S^{FB})\right), S(t)\left(r(S(t)) - \tfrac{n-1}{n} r(S^{FB})\right)\right] \qquad \text{(B.6)}
$$

with:

$$
\begin{cases} M_{ITQ}(S) = (\pi(S) - p_\omega)\, S\left(r(S) - r(S^{FB})\tfrac{n-1}{n}\right) + p_\omega \omega - \mathcal{T}\left(S^{FB}, \tau_x\right) \\ N_{ITQ}(S) = -\left(\pi(S) - p_\omega\right) \end{cases} \qquad \text{(B.7)}
$$

and let us introduce:

$$
\begin{aligned}
I_{ITQ}(S) &= -\rho N_{ITQ}(S) - M'_{ITQ}(S) \\
&= (\pi(S) - p_\omega)\left(\rho - \left(r(S) - \tfrac{n-1}{n} r(S^{FB})\right) - S r'(S)\right) - \pi'(S) S\left(r(S) - \tfrac{n-1}{n} r(S^{FB})\right) \qquad \text{(B.8)}
\end{aligned}
$$

(1) <u>$I_{ITQ}(S) = 0$ admits a unique solution $S^{ITQ} = S^{FB} \in \left]S^{ITQ}_{\min}, K\right[$</u>

The existence of a solution is not really a problem, since the price p_{omega} is designed to implement the optimal conservation level S^{FB}. To verify this point, observe that $I_{ITQ}(S)$ can be written as follows:

$$
I_{ITQ}(S) = \left(\begin{array}{l} \underbrace{\left(\pi(S)\left(\rho - r(S) - S r'(S)\right) - \pi'(S) S r(S)\right)}_{= I_{FB}(S)} \dots \\ -\frac{1}{n}\left(p_\omega\left(n\rho - n r(S) + r(S^{FB})(n-1) - n S r'(S)\right) - (n-1)\left(e_\pi(S) + 1\right)\pi(S) r(S^{FB})\right) \end{array}\right) \qquad \text{(B.9)}
$$

Recall that $p_\omega = \frac{(n-1)\left(e_\pi(S^{FB})+1\right) r(S^{FB})\pi(S^{FB})}{n\rho - r(S^{FB}) - n S^{FB} r'(S^{FB})}$ and observe by computation that $I_{ITQ}(S^{FB}) = I_{FB}(S^{FB})$. It follows that S^{FB} is a solution of $I_{ITQ}(S) = 0$.

For *uniqueness,* we check again that $I^{prime}_{ITQ}(S)$ has the same sign at each solution S_0. From Eq.(B.8), we see that $I_{ITQ}(S)$ can be expressed as follows:

$$
I_{ITQ}(S) = (\pi(S) - p_\omega)\underbrace{\left(\rho - (tfrac\pi'(S) S \pi(S) - p_\omega + 1)\left(r(S) - \tfrac{n-1}{n} r(S^{FB})\right) - S r'(S)\right)}_{= g(S)} \qquad \text{(B.10)}
$$

Since the solutions, S_0, are such that $\pi(S_0) - p_\omega > 0$, we can assert, with the same argument as in point (1) of Appendix A, that $sign\left(\left(I_{ITQ}\right)'(S_0)\right) = sign\left(g'(S_0)\right)$. Furthermore, by calculation, we obtain :

$$g'(S_0) = \underbrace{-r"(S_0)S_0}_{>0} \underbrace{- \left(\frac{\pi'(S_0)S_0}{\pi(S_0)-p_\omega} + 2\right) r'(S_0)}_{>0}$$

$$- \left(r(S_0) - \frac{n-1}{n} r(S^{FB})\right) \underbrace{\frac{\pi'(S_0)}{\pi(S_0)-p_\omega} \left((e_{\pi'}(S_0) + 1) - \frac{\pi'(S_0)S_0}{(\pi(S_0)-p_\omega)}\right)}_{<0} \tag{B.11}$$

Since we have assumed that $r'(S) < 0$, $r"(S) < 0$, $pi'(S) > 0$, and $e_{\pi'}(S) < -1$, and we know that $\pi(S_0) - p_\omega > 0$, the terms above the various braces in Eq.(B.11) are signed. If $r(S_0) - \frac{n-1}{n} r(S^{FB}) > 0$, we can say that $g'(S_0) > 0$, hence $(I_{ITQ})'(S_0) > 0$. To verify this last point, let us return to Eq.(B.10) and observe that :

$$I_{ITQ}(S_0) = 0 \Leftrightarrow r(S_0) - \frac{n-1}{n} r^*(S^{FB}) = \left(\frac{\pi'(S_0)S_0}{\pi(S_0)-p_\omega} + 1\right)^{-1} (\rho - S_0 r'(S_0)) \tag{B.12}$$

Under our assumptions, the right-hand side of Eq.(B.12) is positive, and therefore $r(S_0) - \frac{n-1}{n} r(S^{FB}) > 0$.

(2) $\forall S \in \left[S_{\min}^{ITQ}, K\right]$ if $S < S^{FB}$ (resp. $>$) then $I_{ITQ}(S) < 0$ (resp. $>$).

Since S^{FB} is the only solution of the continuous function $I_{ITQ}(S) = 0$, we just need to evaluate this function at a point smaller (resp. larger) than S^{FB} to know its sign. From Eq.(B.8), it remains to observe that :

(i) $I_{ITQ}\left(S_{\min}^{ITQ}\right) = -\pi'\left(S_{\min}^{ITQ}\right) S_{\min}^{ITQ} \left(r\left(S_{\min}^{ITQ}\right) - r(S^{FB})\frac{n-1}{n}\right)$ because $\pi(S_{\min}^{ITQ}) - p_\omega = 0$. Moreover since $r'(S) < 0$ and $S_{\min}^{ITQ} < S^{FB}$, we have $r\left(S_{\min}^{ITQ}\right) > r(S^{FB})$. Finally using $r'(S) < 0$, we conclude that $I_{ITQ}\left(S_{\min}^{ITQ}\right) < 0$

(ii) $I_{ITQ}(K) = (\pi(K) - p_\omega)\left(\rho + r^*(S^{FB})\frac{n-1}{n} - Kr'(K)\right) + \pi'(K)Kr(S^{FB})\frac{n-1}{n}$ because $r(K) = 0$. Moreover by construction $(\pi(K) - p_\omega) > 0$. Finally since $r\prime(K) < 0$ and $\pi\prime(K) < 0$, we conclude that $I_{ITQ}(K) > 0$.

(3) $\liminf_{t\to+\infty} e^{-\rho t} \int_{S(t)}^{S^{FB}} N_{ITQ}(s)ds \geq 0$ for a every feasible path (catching up optimality)

Let us first observe that we can, by the the non-negativity condition of the instantaneous profit, restrict the set a feasible paths to those belonging to $\left[S_{\min}^{ITQ}, K\right]$. On this compact set, the quantity $-\pi(S)$ is, by continuity, bounded. It follows that any path $S(t) \in \left[S_{\min}^{ITQ}, K\right]$ has the property that for any $t > 0$, the quantity $\int_{S(t)}^{S^{FB}} (-\pi(s))\, ds$ is also bounded. We can therefore claim that $\lim_{t\to+\infty} e^{-\rho t} \int_{S(t)}^{S^{FB}} (-\pi(s))\, ds = 0$.

(4) Conclusion

As for point (4) of Appendix A, it remains to check that the long-term constant path is feasible, i.e, fulfills Eq.(B.6). This condition is now given by $0 \in \left[S^{FB}\left(\frac{r(S^{FB})}{n} - q\right), S^{FB}\frac{r(S^{FB})}{n}\right]$ and is satisfied since $e^{FB} = \frac{r(S^{FB})}{nq} \in (0,1)$ (see Appendix A) which implies that $\left(\frac{r(S^{FB})}{n} - q\right) < 0$.

Appendix C. Proof of proposition 3

To study the program (22), let us start by writing it in the manner of Hartl and Feichtinger [29]. We obtain :

$$\max_{\dot{S}} \int_{t_d}^{+\infty} e^{-\rho t} \left(M_d(S(t),t) + N_d(S(t),t)\dot{S}(t) \right) dt \tag{C.1}$$

$$\text{s.t. } \dot{S}(t) \in \left[S(t) \left(r(S(t)) - q - \tfrac{(n-1)}{n} r(S^{FB}) \right), S(t) \left(r(S(t)) - \tfrac{(n-1)}{n} r(S^{FB}) \right) \right] \tag{C.2}$$

with

$$\begin{cases} M_d(S,t) = \pi(S)S \left(r(S) - \frac{n-1}{n} r(S^{FB}) \right) (1 - F(t,t_d)) + v_r(\tau_x, t_d) F(t,t_d) \\ N_d(S,t) = -\pi(S) (1 - F(t,t_d)) \end{cases} \tag{C.3}$$

It follows that:

$$I_d(S,t) = -\rho N_d(S,t) + \partial_t N_d(S,t) - \partial_S M_d(S,t) \tag{C.4}$$

Since $F(t,t_d) = 1 - e^{-\delta(t-t_d)}$, an exercise of computation leads to:

$$I_d(S,t) = e^{-\delta(t-t_d)} \underbrace{\left(\left(\rho + \delta - r(S) + \tfrac{n-1}{n} r(S^{FB}) - Sr'(S) \right) \pi(S) - \pi'(S) S \left(r(S) - \tfrac{n-1}{n} r(S^{FB}) \right) \right)}_{=I_d(S)} \tag{C.5}$$

This last equation clearly suggests that even if the problem is not homogeneous, the singular stock remains time-independent.Moreover, as in Appendix A and Appendix B, points (1) to (3) can be proved to be valid. However, we will be unable to conclude because the path induced by the singular stock may be infeasible, i.e, does not verify Eq.(C.2). To overcome this problem, we first identify a necessary and sufficient condition for feasibility and apply Hartl and Feichtinger's result. In the case in which this condition is not fulfilled, we make a guess about the solution and verify that this one satisfies the sufficiency conditions for optimality. But let us first verify points (1) to (3).

(1) $I_d(S) = 0$ admits a unique solution $S^d \in \left]S_{\min}, S^{FB}\right[$

Concerning *existence*, notice that:

(i) $I_d(S_{\min}) = -\pi'(S_{\min})S_{\min} \left(r(S_{\min}) - \frac{n-1}{n} r(S^{FB}) \right)$ since $\pi(S_{\min}) = 0$. From point (1) of Appendix A, we also know that $S_{\min} < S^{FB}$, thus $r(S_{\min}) > r(S^{FB})$ because $r'(S) < 0$. It follows that $I_d(S_{\min}) < 0$.

(ii) By the definition of S^{FB} (see Eq.(B.3)), $I_d(S^{FB})$ becomes:

$$I_d(S^{FB}) = \left(\delta + \tfrac{n-1}{n} r(S^{FB}) \right) \pi(S^{FB}) + \pi'(S^{FB}) S^{FB} \tfrac{n-1}{n} r(S^{FB}) \tag{C.6}$$

Since $\pi(S^{FB}) > 0$ and $\pi'(S) > 0$, it follows that $I_d(S^{FB}) > 0$
Finally continuity of $I_d(S)$ ensure the existence of $S^d \in \left]S_{\min}, S^{FB}\right[$ such that $I_d(S^d) = 0$.

Let us now move to the *uniqueness* part. As in point (1) of Appendix A, we will show that $I'_d(S)$ maintains its sign at each solution. But let us first notice that $I_d(S)$ can be written as follows:

$$I_d(S) = \pi(S) \underbrace{\left(\rho + \delta - (e_\pi(S) + 1) \left(r(S) - \tfrac{n-1}{n} r(S^{FB}) \right) - Sr'(S) \right)}_{h(S)} \tag{C.7}$$

where $e_\pi(S)$ denotes the elasticity of $\pi(S)$. It follows that for any solution of $I_d(S_0) = 0$, $sign\left(I'_d(S_0)\right) = sign\left(h'(S_0)\right)$. Moreover by computation, we get :

$$h'(S_0) = -S_0 r''(S_0) - r'(S_0) \left(e_\pi(S_0) + 2 \right) - \left(e_\pi(S_0) \right)' \left(r(S_0) - \tfrac{n-1}{n} r(S^{FB}) \right) \tag{C.8}$$

Since $r'(S), r''(S) < 0$ and $\pi'(S) > 0$, the two first terms of this expression are positive. Concerning the last one, let us first observe, since $e_{\pi'}(S) < -1$, that:

$$\left(e_\pi(S_0) \right)' = \tfrac{\pi'(S_0)}{\pi(S_0)} \left(\left(e_{\pi'}(S_0) + 1 \right) - \tfrac{\pi'(S_0) S_0}{\pi(S_0)} \right) < 0 \tag{C.9}$$

Since the solution $S_0 < S^{FB}$ and $r'(S) < 0$, $\left(r(S_0) - \frac{n-1}{n} r(S^{FB})\right) > 0$. It follows that $I'_d(S_0) > 0$ at each solution.

(2) $\forall S \in [S_{\min}, K]$ if $S < S^d$ (resp. $>$) then $I_d(S) < 0$ (resp. $>$).

From the existence part of (1), it is immediate that this property holds on $\left[S_{\min}, S^{FB}\right]$. The open question is its extension on $\left]S^{FB}, K\right]$. So let us observe by Eqs.(A.3) and (C.5) that:

$$I_d(S) = I_{FB}(S) + \underbrace{\left(\delta + \tfrac{n-1}{n} r(S^{FB})\pi(S) + \pi'(S)\tfrac{n-1}{n} r(S^{FB})\right)}_{>0} \tag{C.10}$$

Moreover from point (2) of Appendix A, we know that $\forall S \in \left]S^{FB}, K\right]$, $I_{FB}(S) > 0$, hence $I_d(S) > 0$.

(3) $\liminf_{t\to+\infty} e^{-\rho t} \int_{S(t)}^{S^{FB}} N_d(s,t)ds \geq 0$ for a every feasible path (catching up optimality)

The proof is similar to point (3) of Appendix Appendix A. The only difference is that $N_d(s,t) = -\pi(S)\left(1 - F(t, t_d)\right)$. But $\left(1 - F(t, t_d)\right) \in [0,1]$ and is therefore bounded.

We now reach the question of the feasibility of a MRAP. This one is given by a harvest at capacity followed by a switch to e^d given by Eq.(27) when the stock S^d is met. Since the stock remains constant after this switching time, the feasibility condition (Eq.(C.2)) requires that:

$$0 \in \left[S^d\left(r(S^d) - q - \tfrac{(n-1)}{n} r(S^{FB})\right), S^d\left(r(S^d) - \tfrac{(n-1)}{n} r(S^{FB})\right)\right] \tag{C.11}$$

Since $S^d < S^{FB}$ by point (1) and $r'(S) > 0$, it is immediate that the upper bound of this interval is always positive. Nothing can however be said about the lower bound. Two cases must therefore be considerate.

Case 1 : $r\left(S^d\right) - \frac{(n-1)}{n} r(S^{FB}) \leq q$

In this case, the lower bound is negative and we can use Hartl and Feichtinger's result to conclude that the MRAP is the optimal solution.

Case 2 : $r\left(S^d\right) - \frac{(n-1)}{n} r(S^{FB}) > q$

To study this case, let us consider the path $S^d(t)$ corresponding to a harvest at capacity, $e(t) = 1$, over the whole horizon starting at t_d. This one solves:

$$\dot{S}(t) = S(t)\left(r(S(t)) - q - \tfrac{(n-1)}{n} r(S^{FB})\right) \text{ with } S(t_d) = S^{FB} \tag{C.12}$$

Let us now verify that this path is a solution to program (C.1). To set up the problem, let us denote by $S(t)$ any feasible path satisfying Eq.(C.2) and let us write by $\mathcal{J}(S(\cdot), T)$, the value reached by the objective until time $T > t_d$. If $\oint_S$ stands for the line integral along the curve S, $\mathcal{J}(S(\cdot), T)$ is given by:

$$\mathcal{J}(S(\cdot), T) = \oint_S e^{-\rho t}\left(M_d(S,t)dt + N_d(S,t)dS\right) \tag{C.13}$$

If we prove that:

$$\lim_{T\to+\infty} \inf \Delta(T) = \lim_{T\to+\infty} \inf \left(\mathcal{J}\left(S^d(\cdot), T\right) - \mathcal{J}(S(\cdot), T)\right) \geq 0 \tag{C.14}$$

we can conclude that $S^d(t)$ is an optimal path for the infinite horizon problem (in the sense of catching up optimality).

From Anaya et al. ([2] remark 2.1), we know that $S^d(t)$ is the lowest bound of all feasible paths starting at $S(t_d) = S^{FB}$. This means that if we consider the restriction of a feasible path $S(\cdot) \neq S^d(\cdot)$ on $[t_d, T]$, either $S(t) > S^d(t)$ on an open sub-interval of $[t_{dev}, T]$ or $S(t) > \bar{S}^{dev}(t)$ for $(t', T]$ (see Fig.C.2). So even if the configuration ABA can occur several time, we can say, from the properties of the line integral, that $\Delta(T)$ is typically of the form of:

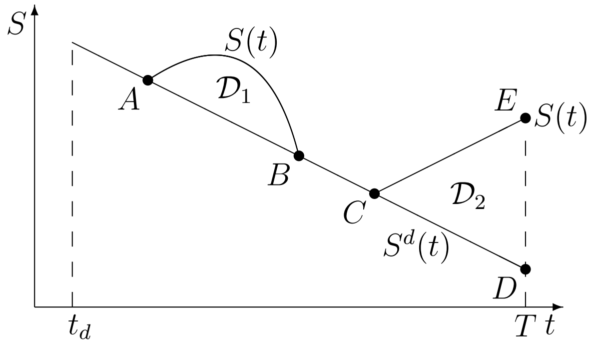

Figure C.2: Paths of $S^d(t)$ and $S(t)$

$$\Delta(T) = \left(\oint_{ABA} e^{-\rho t}\left(M_d(S,t)dt + N_d(S,t)dS\right)\right) + \left(\oint_{CDEC} e^{-\rho t}\left(M_d(S,t)dt + N_d(S,t)\right)dS\right) - \left(\oint_{DE} e^{-\rho t}\left(M(S,t)dt + N(S,t)dS\right)\right) \tag{C.15}$$

Moreover by Green's theorem, we know that:

$$\begin{cases} \oint_{ABA} e^{-\rho t}\left(M_d(S,t)dt + N_d(S,t)dS\right) = \iint_{\mathcal{D}_1}\left(\frac{\partial}{\partial t}\left(e^{-\rho t}N_d(S,t)\right) - \frac{\partial}{\partial S}\left(e^{-\rho t}M_d(S,t)\right)\right)dSdt \\ \oint_{CDEC} e^{-\rho t}\left(M_d(S,t)dt + N_d(S,t)\right)dS = \iint_{\mathcal{D}_2}\left(\frac{\partial}{\partial t}\left(e^{-\rho t}N_d(S,t)\right) - \frac{\partial}{\partial S}\left(e^{-\rho t}M_d(S,t)\right)\right)dSdt \end{cases} \tag{C.16}$$

Now observe that:

$$\begin{aligned} \frac{\partial}{\partial t}\left(e^{-\rho t}N_d(S,t)\right) - \frac{\partial}{\partial S}\left(e^{-\rho t}M_d(S,t)\right) &= e^{-\rho t}\left(-\rho N_d(S,t) + \partial_t N_d(S,t) - \partial_S M_d(S,t)\right) \\ &= e^{-\rho t} I^d\left(S,t\right) \text{ (see Eq.(C.4))} \end{aligned} \tag{C.17}$$

Since any feasible path, never reaches S^d and, by the choice of the initial condition, lies above S^d, we can say from point (2) of this appendix that:

$$\frac{\partial}{\partial t}\left(e^{-\rho t}N_d(S,t)\right) - \frac{\partial}{\partial S}\left(e^{-\rho t}M_d(S,t)\right) \geq 0 \tag{C.18}$$

As a consequence, the two line integrals given in Eq.(C.16) are non-negative. If we now move to Eq.(C.15) and observe (see fig.C.2) that in the third line integral $dt = 0$, we can say that:

$$\Delta(T) \geq -\left(e^{-\rho T}\int_{S^d(T)}^{S(T)} N(S,T)dS\right) \tag{C.19}$$

It follows from point (3) of this appendix that $\lim_{T\to+\infty}\inf \Delta(T) \geq 0$. We can therefore conclude that $S^d(t)$ is a solution of the infinite horizon problem (C.1) in the sense of catching up optimality.

Appendix D. Proof of proposition 4

Let us verify that $\tau_x^* \leq \frac{\rho}{\delta}\left(1 - \frac{r(S^{FB})}{nq}\right)$. We know that:

$$\forall t_d \geq 0,\ \tau_x(t_d) = \frac{\rho}{\delta}\left(\int_{t_d}^{+\infty}(\rho+\delta)e^{-(\rho+\delta)(t-t_d)}\left[\frac{\pi(S^D(t,t_d))S^D(t,t_d)}{\pi(S^{FB})S^{FB}}e^D(t)\right]dt - \frac{r(S^{FB})}{qn}\right) \tag{D.1}$$

Moreover, under our assumption, $\pi(S)S$ increases and $\forall t_d$ and $\forall t \geq t_d$, $S^D(t,t_d) \leq S^{FB}$. This implies that $\frac{\pi(S^D(t,t_d))S^D(t,t_d)}{\pi(S^{FB})S^{FB}} \leq 1$. As $e^D(t) \in [01]$. W can therefore say that:

$$\forall t_d \geq 0,\ \tau_x(t_d) \leq \frac{\rho}{\delta}\left(\int_{t_d}^{+\infty}(\rho+\delta)e^{-(\rho+\delta)(t-t_d)}dt - \frac{r(S^{FB})}{qn}\right) \tag{D.2}$$

Finally, observe that, $\int_{t_d}^{+\infty}(\rho+\delta)e^{-(\rho+\delta)(t-t_d)}dt = 1$. It follows that:

$$\forall t_d \geq 0,\ \tau_x(t_d) \leq \frac{\rho}{\delta}\left(1 - \frac{r\left(S^{FB}\right)}{nq}\right) \tag{D.3}$$

Appendix E. Some complements on the numerical illustration

In this example the growth rate of the biomass and the unit profit per harvest given by $r(S) = r(1-\frac{S}{K})$ and $\pi(S) = (p - \frac{c}{qS})$ and the values of the parameters are those described in Table 1. The computations are done with Matlab[8]

The optimal conservation level: From Eq.(8), the optimal conservation level solves:

$$\begin{aligned} &\rho\pi(S) = \pi'(S)r(S)S + \pi(S)r'(S)S + \pi(S)r(S) \\ \Leftrightarrow\ &\rho(p - \tfrac{c}{qS}) - \tfrac{c}{qS^2}r(1-\tfrac{S}{K}) + (p - \tfrac{c}{qS})\tfrac{r}{K}S - (p - \tfrac{c}{qS})r(1-\tfrac{S}{K}) = 0 \end{aligned} \tag{E.1}$$

By using the data of table 1, we get:

$$\frac{1}{S}\left(0.0479S^2 - 1217.3S - 80420\right) = 0 \tag{E.2}$$

It follows that the optimal conservation level is of $S^{FB} \simeq 25503$ tons. From this quantity, we deduce the individual optimal effort given by :

$$e^{FB} = \frac{r(S^{FB})}{nq} = \frac{r}{nq}\left(1 - \frac{S^{FB}}{K}\right) \simeq 0.4646 \tag{E.3}$$

As well as growth rate of the biomass and the unit profit per harvest, respectively given by:

$$r(S^{FB}) = 0.649(1 - \frac{S^{FB}}{54252}) \simeq 0.3439 \ \ and \ \ \pi(S^{FB}) = (2000 - \frac{1.6028\cdot 10^6}{S^{FB}}) \simeq 1937.15 \tag{E.4}$$

The other quantities of table 1 are obtained by a combination these quantities.

The behavior of the deviator : targeted stock $S^d(\delta)$ and dynamics of $S^D(t,t_d)$ From Eq.(C.5), we know that the singular stock, $S^d(\delta)$, solves, for each δ, $I_d(S,\delta) = 0$. However to know the harvesting strategy (see proposition 3), we have to check if this stock is or not reachable in finite time. To answer this question, recall, from the uniqueness part of point (1) of Appendix C, that $\partial_S I_d(S,\delta)|_{S=S^d(\delta)} > 0$ and observe that $\partial_\delta I_d(S,\delta) = \pi(S) > 0$ for $S = S^d(\delta)$. It follows that $\left(S^d\right)'(\delta) < 0$. Since $r\prime(S) < 0$, we can say that if $r\left(S^d(0)\right) - \frac{(n-1)}{n}r(S^{FB}) > q$ this inequality holds for all $\delta \geq 0$, in other words if $S^d(0)$ is not reachable, the same remains true for $S^d(\delta)$ independently of δ. This is the case in our example. By solving $I_d(S,0) = 0$, we get $S^d(0) = 11268$. and we observe that:

$$r(S^d(0)) - \frac{(n-1)}{n}r(S^{FB}) = 0.1717 > q = 0.002961 \tag{E.5}$$

[8]Codes are available upon request.

Hence, any fishery which leaves the compliance path at time t_d plans to fish, independently of δ, at full capacity, i.e. $e^D(t) = 1$ for $t \geq t^d$. The dynamics of $S^D(t, t_d)$ induced by Eq.(23) is therefore given by:

$$\frac{\dot{S}^D(t)}{S^D(t)} = r(S^D(t)) - \left(q + \frac{(n-1)}{n} r(S^{FB})\right), S^D(t_d) = S^{FB} \tag{E.6}$$

$$\Leftrightarrow \frac{\dot{S}^D(t)}{S^D(t)} = -\underbrace{\frac{r}{K}}_{=a} S^D(t) + \underbrace{\left(r - q - \frac{(n-1)}{n} r(S^{FB})\right)}_{=b},\ S^D(t_d) = S^{FB} \tag{E.7}$$

By the usual change of variable, given by $u(t) = \frac{1}{S^D(t)}$, we get the linear differential equation $\dot{u}(t) = -bu(t) + a$, with $u(t_d) = \frac{1}{S^{FB}}$, whose solution is:

$$u(t, t_d) = \left(\frac{1}{S^{FB}} - \frac{a}{b}\right) e^{-b(t-t_d)} + \frac{a}{b} \tag{E.8}$$

It follows that $S^D(t, t_d) = (u(t, t_d))^{-1}$ and, by using the data set, we get:

$$S^D(t, t_d) \simeq \left(-2.0482 \cdot 10^{-7} e^{-0.3035(t-t_d)} + 3.9417 \cdot 10^{-5}\right)^{-1} \tag{E.9}$$

The computation of $\mathcal{V}^d(\tau_x, t_d, \delta)$

From Eq.(31), we know that:

$$\begin{aligned}
\mathcal{V}^d(\tau_x, t_d, \delta) &= \int_{t_d}^{+\infty} e^{-(\rho+\delta)t+\delta t_d} \left[\pi(S^D(t, t_d)) S^D(t, t_d) q\right] dt + v_R(\tau_x) \frac{\delta}{\rho(\rho+\delta)} e^{-\rho t_d} \\
&= e^{-\rho t_d} \left(pq \int_{t_d}^{+\infty} e^{-(\rho+\delta)(t-t_d)} S^D(t, t_d) dt - c \int_{t_d}^{+\infty} e^{-(\rho+\delta)(t-t_d)} dt + v_R(\tau_x) \frac{\delta}{\rho(\rho+\delta)}\right) \\
&= e^{-\rho t_d} \left(pq \int_{t_d}^{+\infty} e^{-(\rho+\delta)(t-t_d)} S^D(t, t_d) dt - \frac{c}{\rho+\delta} + v_R(\tau_x) \frac{\delta}{\rho(\rho+\delta)}\right)
\end{aligned} \tag{E.10}$$

We even observe from Eq.E.9 that $S^D(t, t_d)$ is a function of $x = t - t_d$. It follows that:

$$\mathcal{V}^d(\tau_x, t_d, \delta) = e^{-\rho t_d} \left(pq \int_0^{+\infty} e^{-(\rho+\delta)x} S^D(x) dx - \frac{c}{\rho+\delta} + v_R(\tau_x) \frac{\delta}{\rho(\rho+\delta)}\right) \tag{E.11}$$

If we finally use our data set, and the characterization of $v_R(\tau_x)$ and $S^D(x)$ (see respectively Eq.(38) and Eq.(E.9)), we obtain:

$$\mathcal{V}^d(\tau_x, t_d, \delta) \simeq e^{-0.05 t_d} \left(\int_0^{+\infty} \frac{5.922 e^{-(0.05+\delta)x} \cdot 10^5}{-2.0482 \cdot 10^{-2} e^{-0.3035x} + 3.9417} dx + \frac{\delta(67962 - 146280\tau_x) - 237.3}{0.05(0.05+\delta)}\right) \tag{E.12}$$

The choice of the tax rate $\tau_x(\delta)$

Let us first observe from Eq.(E.11), that $\partial_{t_d} \mathcal{V}^d(\tau_x, t_d, \delta) < 0$. So, if a deviation occurs, it takes place at $t_d = 0$. It follows by Eqs.(38), (32), that the optimal tax rate, $\tau_x(\delta)$, as a function of the delay $1/\delta$ solves $Cpl(0) - \mathcal{V}^d(\tau_x, 0, \delta) = 0$.That is:

$$\frac{\pi\big(S^{FB}\big)S^{FB}r(S^{FB})}{\rho n} - \left(pq\int_0^{+\infty} e^{-(\rho+\delta)x}S^D(x)dx - \frac{c}{\rho+\delta} + \left(\frac{r(S^{FB})}{n} - q\tau_x\right) tfrac\pi(S^{FB})S^{FB}\delta\rho(\rho+\delta)\right) = 0 \tag{E.13}$$

$$\Leftrightarrow \frac{\pi\big(S^{FB}\big)S^{FB}r(S^{FB})}{n(\rho+\delta)} - pq\int_0^{+\infty} e^{-(\rho+\delta)x}S^D(x)dx + \frac{c}{(\rho+\delta)} + \frac{\pi(S^{FB})S^{FB}\delta}{\rho(\rho+\delta)}q\tau_x = 0 \tag{E.14}$$

$$\Leftrightarrow \tau_x(\delta) = \frac{p\rho(\rho+\delta)}{\pi(S^{FB})S^{FB}\delta}\int_0^{+\infty} e^{-(\rho+\delta)x}S^D(x)dx - \frac{1}{\delta}\left(\frac{c\rho}{\pi(S^{FB})S^{FB}q} + \frac{r(S^{FB})\rho}{nq}\right) \tag{E.15}$$

If we now use the data set and the definition of $S^D(x)$ (see Eq.(E.9)), we get :

$$\tau^*(\delta) \simeq \frac{(5+100\delta)}{\delta}\int_0^{+\infty} \frac{e^{-(0.05+\delta)x}}{-10.1188e^{-0.3035x}+1.9473\cdot 10^3}dx - \frac{0.0249}{\delta} \tag{E.16}$$

the function which was used to compute results of Table 3.