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Directed Search with Phantom Vacancies

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Abstract

When vacancies are filled, the ads that were posted are generally not withdrawn, creating phantom vacancies. The existence of phantoms implies that older job listings are less likely to represent true vacancies than are younger ones. We assume that job seekers direct their search based on the listing age for otherwise identical listings and so equalize the probability of matching across listing age. Forming a match with a vacancy of age a creates a phantom of age a and thus creates a negative informational externality that affects all vacancies of age a or older. The magnitude of this externality decreases with a. The directed search behavior of job seekers leads them to over-apply to younger listings. We calibrate the model using US labor market data. The contribution of phantoms to overall frictions is large, but, conditional on the existence of phantoms, the social planner cannot improve much on the directed search allocation.

JEL codes: J60, D83

Keywords: Directed search; information persistence; vacancy age

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1 Introduction

> I am currently on the job hunt and I had a question about applying to jobs online. You know how most websites will tell you the job has been posted 1 day ago, 28 days ago, etc. For some reason, I have concluded that I need to apply to a job the first week they post the position to have the best chances of being hired. Although I heard that it can take up to a month for the company to hire anyone for the position, I feel that applying to a job that was posted 3 weeks ago isn't that promising. What is your take on this situation?
>
> AskaManager.com

This quote illustrates two important points about job search. First, some of the information available to job seekers is out of date. Some advertised jobs have already been filled. These ads are for job openings that no longer exist; they are ads for phantom vacancies. Second, job seekers know that some advertised jobs are phantoms, and they adjust their application behavior accordingly. As jobs advertised in older listings are more likely to be phantoms, workers may decide to apply to more recently posted listings, but at the same time responding to older ads is also an option since fewer applicants are likely to be pursuing those jobs. Workers thus face a problem of directed search, namely, how to allocate their applications optimally across job listings of different ages.

Are phantoms a significant problem in real-world labor markets? Do matches fail to form because workers can't locate appropriate job listings or because the jobs that workers apply for are already taken? To the extent workers apply for jobs that are already filled, and casual empiricism suggests this is often the case, phantoms are important.[1] More formally, there is evidence from online job sites. Using Craigslist data, Chéron and Decreuse (2017) show that the distribution of job listings by age over one month (the time at which Craigslist destroys ads) is uniform by week. This implies that ads are not withdrawn as soon as the corresponding jobs are filled; instead, job listings persist for some time as phantoms.[2]

There is also evidence that job seekers target their search towards recently posted job listings.

[1] Of course, phantoms are not solely a feature of online job markets. Word-of-mouth information can also be obsolete. Phantoms are also important in other markets with search frictions, for example, the market for rental housing.

[2] An alternative interpretation is that ads that are not withdrawn are listings by employers with multiple positions to fill. In principle, if not necessarily in practice, Craigslist does not allow ads for multiple positions. (See their FAQ section.) Also, in the Craiglist data, roughly 50% of new listings are for jobs that were not advertised in the previous month. Among the remaining ads, many ads are simply reposted because the previous listing did not generate a match.

The DHI Vacancy and Flow Applications Database links 60 million applications to nearly 7 million vacancy postings, dating from January 2012. These data show that "job seekers exhibit a striking propensity to target new and recently posted vacancies: 39 percent of applications flow to vacancies posted in the last 48 hours, and 59% go to those posted in the last 96 hours. Older postings attract relatively few applications." (DHI Hiring Indicators, October 2016). We explain this pattern by arguing that job seekers try to avoid sending applications for jobs that have already been filled. Another potential explanation is given by stock-flow matching, e.g., Coles and Smith (1998), in which job seekers initially search through the stock of job listings but thereafter only look at the flow of new postings. However, using data from SnagAJob, Faberman and Kudlyak (2014) find (p.4) that "The fraction of applicants to a newly-posted vacancy rises with duration, consistent with a stock-flow model, but it does so only slightly, representing only 17% of applications during a job seeker's six month of search." This suggests that while stock-flow matching may take place, it cannot explain all of the patterns that we see in the data.

In this paper, we build a model of directed search with phantoms and explore their implications for equilibrium and efficiency. Job listings last for a fixed period of time and get renewed afterwards. In equilibrium, worker directed search satisfies an indifference condition – the expected payoffs associated with applying to ads of different ages must be equalized. However, even though workers individually direct their search optimally, we show that equilibrium is not constrained efficient. From the collective perspective, workers over-apply to younger listings. When a match forms, a phantom is added to the market, and the cost of the phantom is greater the younger is the ad. That is, the social cost of a young phantom is greater than that of an old phantom. To see this, consider a match formed with a vacancy that was posted one month ago. This generates a one-month old phantom. This phantom can only affect the job seekers who pursue listings that are older than one month. However, a match formed with a newly listed vacancy generates a phantom that can affect all job seekers during its lifetime.

We calibrate our model to US data for the period 2000 - 2008. Our model predicts that job seekers disproportionately apply for recent vacancies. Quantitatively, this effect is large and suggests that the bulk of activity takes place in the first days of a listing. In our benchmark calibration, close to 70% of applications go to listings that are less than 48 hours old, and the job queue, the number of applicants per listing, falls by a factor of more than five between the first and the second day. We show that the gains achieved by the constrained efficient allocation are modest. This is because, like job seekers, the social planner cannot distinguish between real vacancies and phantoms, and search strategies that produce more matches also produce more phantoms. Nonetheless, we find that the contribution of phantoms to overall labor market frictions and unemployment is large. In our baseline calibration, phantoms account for about 85% of frictional unemployment and more than one fourth of overall unemployment. This reflects the fact that applying for jobs that have already been filled is a quantitatively important driver of unemployment.

We consider four extensions to our model. We first compute the gains expected from phantom destruction. If employers withdrew their obsolete listings in one week on average, the unemployment rate would decrease by a bit less than one percentage point.

Second, we examine what happens when job listings are renewed at random intervals of time. Vacancy renewal means that a listing for an unfilled vacancy is replaced by a new one of age zero. This concentrates the distribution of vacancies at younger ages. The qualitative conclusion stays the same: directed search is not constrained efficient. Our calibration strategy gives slightly less importance to phantoms, but the model still predicts that 50% of applications go to listings that are less than 48 hours old.

Third, we consider a complementary reason for why job seekers are concerned with the age of job listings. Some jobs may be lemons, i.e., jobs that no one wants to accept. Because no one accepts these jobs, they stay in the distribution of job listings until they naturally disappear. The proportion of job listings that are lemons increases with age, and this composition effect pushes job seekers to direct their search towards younger listings. However, a key difference between lemons and phantoms is that matching does not produce additional lemons. As a result, the quantitative contribution of lemons to overall unemployment is small unless the fraction of lemons in new listings is very large.

Finally, we note that fixed-wage contracts cannot internalize the age-dependent informational externality. One could imagine an unrealistic scenario in which firms post sophisticated contracts advertising wages that vary with the time it takes to fill the vacancy. Alternatively, decentralized bargaining may lead to age-dependent wages. In this extension, we consider Nash bargaining over the wage. We show that the Nash-bargained wage varies with the age of the vacancy. Nonetheless, Nash bargaining fails to produce the age-wage locus that efficiently allocates the job seekers over the listings of different ages.

Our paper is related to three strands of literature. First, the paper closest to ours is Chéron and Decreuse (2017). Their paper introduces the concept of phantom vacancies and shows how phantoms lead to an aggregate matching function even in the absence of other coordination frictions in the market. The authors then embed this matching function in the Diamond-Mortensen-Pissarides equilibrium search model and examine how the dynamics associated with phantom creation and destruction affect the business cycle properties of that model. One thing they do not do, however, is allow workers to direct their search by job listing age. Our paper is thus a natural complement to Chéron and Decreuse (2017).

Second, there is a literature on inefficiencies in search equilibrium that come from composition externalities, e.g., Albrecht, Navarro and Vroman (2010) and Chéron, Hairault and Langot (2011). These are models with worker heterogeneity in which individual decisions (to participate in the labor market, to form or dissolve matches) affect the distribution of worker types across the pool of unemployed. A related compositional effect obtains in our model. A worker who successfully targets a job listing of a particular age changes the distributions by age of both vacancies and phantoms but does not take the effect of these changes on other job seekers into account. Our paper differs from earlier work on composition externalities in that individual job search decisions have dynamic effects. A worker who directs his or her search towards ads of age a affects not only the composition of listings (real vacancies versus phantoms) at that age but also, with a lag, the composition of listings at all ages greater than a. In this sense, ours is a dynamic compositional externality. A related dynamic compositional externality is present in Board et al. (2017) albeit in a frictionless matching environment.

Finally, there is a substantial literature based on data culled from job search engines. Examples include Faberman and Kudlyak (2014) using data from SnagAJob, Marinescu (2016) using data from CareerBuilder, and Banfi and Villena-Roldán (2016) using data from trabajando.com The main focus of these papers is on documenting empirical regularities in these new data sources. Our paper provides a complementary theoretical framework.

The outline of the rest of our paper is as follows. In the next section, we lay out our model of a labor market with phantoms. We then solve for the equilibrium directed search allocation, the random search allocation and the constrained efficient allocation in Section 3. We show that neither the directed search allocation nor the random search allocation is constrained efficient. In Section 4, we present our calibration. Section 5 contains our four extensions and Section 6 concludes.

2 The Model

We focus on the stationary state of a continuous time model. Calendar time is denoted by t, and at each instant, there is a fixed measure of jobs K that can be either vacant or occupied.[3] The measure of vacancies is $\mathsf{v}(t)$. Vacancies differ in age $a \geq 0$. There is also a continuum of workers of size one. Each worker can be either unemployed or employed. The measure of unemployed is $\mathsf{u}(t)$. By construction, we have $\mathsf{v}(t) + 1 - \mathsf{u}(t) = K$.

[3] We assume a fixed measure of jobs to abstract from the well-known congestion and thick-market externalities that result from vacancy creation in order to focus on the externalities associated with phantoms.

Employed workers and jobs separate at exogenous Poisson rate λ. Newly unemployed workers join the pool of unemployed and immediately start job search. Newly destroyed jobs join the pool of vacancies at age zero.

All jobs produce the flow output $y \equiv 1$ and pay the same wage w.[4] The search market is segmented by listing age a. At time t, in each submarket a, $u(a,t)$ unemployed workers try to match with $v(a,t)$ vacancies. The matching process is frictional. On top of the usual search frictions, information persistence about vacancies that have already been filled but are still advertised creates an additional friction. Each time a match is formed and the corresponding ad is not withdrawn, a phantom is created. The flow of new matches in submarket a is

$$M(a,t) = \pi(a,t)m(u(a,t), v(a,t) + p(a,t)), \tag{1}$$

where $p(a,t)$ is the measure of phantoms and $\pi(a,t) = \dfrac{v(a,t)}{v(a,t)+p(a,t)}$ is the nonphantom proportion, i.e., the ratio of vacancies to job listings (vacancies + phantoms) in submarket a. We define $\theta(a,t) \equiv \dfrac{v(a,t)+p(a,t)}{u(a,t)}$ as the tightness in the submarket corresponding to ads of age a. The numerator is composed of the total measure of age-a ads, thus including both phantoms and vacancies.

The contact function $m(u, v+p)$ is strictly concave, has constant returns to scale, and is such that $m(0, v+p) = m(u,0) = 0$, $\lim_{u\to 0} m_1(u, v+p) = \lim_{v+p\to 0} m_2(u, v+p) = \infty$. The elasticity of m with respect to job listings is $\alpha(\theta) = \theta m_2(1,\theta)/m(1,\theta)$. We assume that $\alpha'(\theta) \leq 0$ for all $\theta \geq 0$. Although this assumption is not necessary for our results, it is a sufficient condition for establishing particular results and we indicate this where relevant. The flow of new matches has two components: m, the contact rate, and π, the proportion of contacts where a vacancy rather than a phantom is involved. Phantoms impede the search process: by increasing p at given v and u, the flow of new matches, M, is reduced.

When a vacancy is filled, a phantom is created. However, job listings – both vacancies and phantoms – have finite lifetimes. Once a vacancy reaches age $A > 0$, it is renewed; that is, it is relisted as a new vacancy (age 0). Once a phantom reaches age A, it disappears. The termination age A captures the severity of informational frictions due to phantoms. Having a small A means that most job listings are for unfilled vacancies, so information obsolescence makes a minor contribution to overall labor market frictions, while a large A means that many job listings are for phantoms. Our assumption that job advertisements are posted for a fixed length of time and that they are not removed once the listed job is filled corresponds to what we see on job boards such as Craigslist and Monster.com. We consider alternative assumptions about vacancy renewal and about the death process for phantoms in Section 5.

[4] We discuss the possibility that w may vary with the age of the vacancy in Sections 3.4 and 5.4.

Unemployed workers spread themselves over the different submarkets with total unemployment at time t of $\mathsf{u}(t) = \int_0^A u(a,t)da$. Similarly, the total measure of vacancies at time t is $\mathsf{v}(t) = \int_0^A v(a,t)da$ and the total measure of phantoms is $\mathsf{p}(t) = \int_0^A p(a,t)da$. Unemployment obeys the law of motion:

$$d\mathsf{u}/dt = -\int_0^A M(a,t)da + \lambda(1-\mathsf{u}),$$

and the stocks of vacancies and phantoms evolve according to

$$\partial v(a,t)/\partial a + \partial v(a,t)/\partial t = -M(a,t) \tag{2}$$

$$\partial p(a,t)/\partial a + \partial p(a,t)\partial t = M(a,t), \tag{3}$$

with $v(0,t) = \lambda(1-\mathsf{u}(t)) + v(A,t)$ and $p(0,t) = 0$. A given cohort of vacancies decreases across time by the flow number of new matches, while the corresponding cohort of phantoms is increased by this number. That is, $v(a,t) + p(a,t) = v(0,t)$.

The nonphantom proportion, $\pi(a,t)$, evolves according to

$$\partial\pi(a,t)/\partial a + \partial\pi(a,t)/\partial t = -\frac{M(a,t)}{v(a,t)}\pi(a,t), \tag{4}$$

with $\pi(0,t) = 1$.

In steady state, calendar time does not affect any of the variables. That is, $d\mathsf{u}/dt = d\mathsf{v}/dt = 0$, $\partial v(a,t)/\partial t = \partial p(a,t)/\partial t = 0$, $u(a,t) = u(a)$, and $M(a,t) = M(a)$. Hereafter we refer to variables without mentioning calendar time again. A dot over a variable denotes the derivative of the variable with respect to age a, i.e., $\dot{x} \equiv x'(a)$.

For later use, we define the job-finding rate by listing age as $\mu(a) = M(a)/u(a)$, and the job-filling rate by listing age as $\eta(a) = M(a)/v(a)$. Note that $\dot{v} = -\eta(a)v$ and $\dot{p} = \eta(a)v$. Finally, ϕ_v and ϕ_u denote the density functions of vacancies and unemployment by age. By definition, $\phi_v(a) = v(a)/\mathsf{v}$ and $\phi_u(a) = u(a)/\mathsf{u}$ for all $a \geq 0$.

To close the model, we need to specify how job seekers allocate themselves across submarkets.

3 Random search, directed search, and constrained-efficient allocations

We describe three different allocations. Each one is associated with a particular distribution of job seekers over listing ages.

3.1 Random search allocation

Before we derive the directed search allocation, we start, as a baseline, with the allocation obtained when workers do not observe the job listing age. Job seekers then randomly search job listings. The ratio of vacancies and phantoms to job seekers is constant over age.

That is, for all $a \in [0, A]$,

$$\theta(a) = (v(a) + p(a))/u(a) = \theta. \tag{5}$$

Since each filled vacancy is replaced by a phantom, $v(a) + p(a) = v(0)$ and $v(0)/u(a) = \theta$ so that the distribution of the unemployed across vacancy age is uniform. Since $u(a) = v(0)/\theta$ and $\mathsf{u} = \int_0^A u(a)da = Av(0)/\theta$, the density function of unemployment by listing age is $\phi_u(a) = u(a)/\mathsf{u} = 1/A$.

The law of motion of the nonphantom proportion is $\dot{\pi}/\pi = -m(1, \theta)/\theta$, i.e., the nonphantom proportion declines at constant rate. Therefore $\pi(a) = \exp(-am(1, \theta)/\theta)$. Workers who apply to an ad of age a find a job at rate $\mu(a) = m(1, \theta)\pi(a) = m(1, \theta)\exp(-am(1, \theta)/\theta)$. The average job-finding rate is thus

$$\mu = \int_0^A \mu(a)\phi_u(a)da = m(1, \theta) \times \frac{1 - \exp(-Am(1, \theta)/\theta)}{Am(1, \theta)/\theta}. \tag{6}$$

This is the product of two terms: the contact rate $m(1, \theta)$ and the average nonphantom proportion $\frac{1-\exp(-Am(1,\theta)/\theta)}{Am(1,\theta)/\theta}$.

The number of vacancies of age a is $v(a) = v(0)\pi(a)$. The stock of vacancies is $\mathsf{v} = \int_0^A v(a)da = \int_0^A v(0)\exp(-am(1, \theta)/\theta)da$, so the density of vacancies by listing age is $\phi_v(a) = v(a)/\mathsf{v} = (m(1, \theta)/\theta)\times \exp[-am(1, \theta)/\theta]/ [1 - \exp(-Am(1, \theta)/\theta)]$. That is, the distribution of vacancies by age follows a truncated exponential law.

We now derive aggregate unemployment. The flow of new vacancies is $v(0) = \lambda(1-\mathsf{u})+v(A)$. Since $v(A) = v(0)\pi(A)$, it follows that $v(0) = \lambda(1 - \mathsf{u})/[1 - \pi(A)]$. Combining this with $v(0) = \theta u(a) = \theta\mathsf{u}/A$ gives an expression for the aggregate unemployment rate:

$$\mathsf{u} = \frac{\lambda}{\lambda + \frac{\theta(1-\pi(A))}{A}} = \frac{\lambda}{\lambda + m(1, \theta) \times \frac{1-\exp(-Am(1,\theta)/\theta)}{Am(1,\theta)/\theta}}. \tag{7}$$

Equation (7) describes the Beveridge curve, a strictly decreasing and convex relationship between the unemployment rate and market tightness. The right-hand side is the ratio of the job loss rate λ to the sum of the job loss rate and average job-finding rate.

To close the model, we use the resource constraint $1 - \mathsf{u} + \mathsf{v} = K$.

Proposition 1 (Random search allocation) The following properties hold for all $a \in [0, A]$:

(i) $\theta(a) = \theta^{\text{rs}}$ where θ^{rs} is uniquely defined by the resource constraint.

(ii) The nonphantom proportion $\pi^{\text{rs}}(a) = \exp(-am(1, \theta^{\text{rs}})/\theta^{\text{rs}})$ declines at constant rate, the job-filling rate $\eta(a) = m(1, \theta^{\text{rs}})/\theta^{\text{rs}}$ is constant and the job-finding rate $\mu(a) = m(1, \theta^{\text{rs}})\pi^{\text{rs}}(a)$ strictly decreases.

(iii) Moreover, aggregate unemployment u and tightness θ^{rs} increase with A.

The proof is given in Appendix A.

There is a unique random search allocation. Taking phantoms into account alters the matching function. However, even though we account for phantoms, the matching function has constant returns to scale. This can be seen by inspecting the mean job-finding rate μ as given by equation (6). This rate only involves the ratio of advertised jobs to unemployed.

In the random search allocation, workers apply at the same rate to listed jobs irrespective of their age. As a result, the job-filling rate $\eta(a)$ is constant over listing age. That is, the chance that a vacancy is filled does not change with age. However, the job-finding rate does vary with listing age. The job-finding rate is $\mu(a) = m(1, \theta)\pi(a)$, and the nonphantom proportion $\pi(a)$ decreases with listing age.

Unemployment increases with the magnitude of obsolete information. An increase in the parameter A reduces the mean nonphantom proportion. This corresponds to a rightward shift of the Beveridge curve (7) in the (θ, u) plane and unemployment increases. When A tends to infinity, workers' search is diluted over an arbitrarily large number of submarkets contaminated by phantoms. This makes the job-finding rate tend to zero and unemployment tend to one. At the other extreme, when A tends to 0 the nonphantom proportion tends to one. Therefore the job-finding rate tends to $m(1, \theta)$ as in the standard matching model without phantoms.

3.2 Directed search allocation

When search is directed, workers observe listing age and choose which submarket to enter. A submarket a is open if and only if $u(a) > 0$. We denote by Ω the set of open submarkets. All open submarkets must be equally attractive to job seekers. That is, the job-finding rate $\mu(a) = M(a)/u(a) = \pi(a)m(1, \theta(a))$ must be the same across open submarkets. Given that $v(a) > 0$ for all $a \in [0, A]$, the properties of the contact function $m(1, \theta(a))$ imply that $u(a) > 0$ for all $a \in [0, A]$. If not, a worker entering a non-open submarket would immediately find a job. Thus all submarkets are open in equilibrium and so $\Omega = [0, A]$.

Since $\pi(0) = 1$ the fact that $\mu(a)$ is constant across age implies for all $a \in [0, A]$ that

$$\pi(a)m(1, \theta(a)) = m(1, \theta(0)). \tag{8}$$

This equation defines a strictly decreasing relationship between $\theta(a)$ and $\pi(a)$; the relationship is parameterized by $\theta(0)$. Unlike the random search allocation, tightness is not constant in the directed search allocation. It strictly increases with $\theta(0)$ and strictly decreases with $\pi(a)$.

Differentiating (8) with respect to age gives:[5]

$$-\alpha(\theta)\frac{\dot{\theta}}{\theta} = \frac{\dot{\pi}}{\pi}, \tag{9}$$

where $\alpha(\theta)$ is the elasticity of the contact rate with respect to job listings, as defined above. Using (4) and (9), we can characterize the derivatives of π and θ as

$$\dot{\pi} = -\frac{m(1,\theta)}{\theta}\pi, \tag{10}$$

$$\alpha(\theta)\frac{\dot{\theta}}{\theta} = \frac{m(1,\theta)}{\theta}. \tag{11}$$

These two differential equations can be solved given $\pi(0) = 1$ and $\theta(0)$. The latter value can only be found once the equilibrium is solved, i.e., there is a fixed-point problem.

The number of vacancies by age is given by $v(a) = v(0)\pi(a) = v(0)\times$ exp $\left[-\int_0^a \frac{m(1,\theta(b))}{\theta(b)}db\right]$. The stationary number of unemployed balances inflows and outflows. The fact that the exit rate from unemployment does not vary with listing age simplifies the calculation. Outflows are $\int_0^A M(a)da = m(1,\theta(0))\mathsf{u}$ and inflows are $\lambda(1-\mathsf{u})$. Thus $\mathsf{u} = \lambda/[\lambda + m(1,\theta(0))]$.

Finally, the resource constraint is $1 - \mathsf{u} + \int_0^A v(a)da = K$.

Proposition 2 (Directed search allocation) Let $\theta^{\text{ds}} : [0,A] \to \mathbb{R}_+$ and $\pi^{\text{ds}} : [0,A] \to [0,1]$ be the tightness and nonphantom proportion in a directed search allocation. The following properties hold for all $a \in [0,A]$:

(i) $\pi^{\text{ds}}(a)m(1,\theta^{\text{ds}}(a)) = m(1,\theta^{\text{ds}}(0))$;

(ii) market tightness and the nonphantom proportion evolve according to

$$\alpha\frac{\dot{\theta}^{\text{ds}}}{\theta^{\text{ds}}} = -\frac{\dot{\pi}^{\text{ds}}}{\pi^{\text{ds}}} = \frac{m(1,\theta^{\text{ds}})}{\theta^{\text{ds}}} \tag{12}$$

with initial conditions $\pi^{\text{ds}}(0) = 1$ and $\theta^{\text{ds}}(0) = \theta_0$, where θ_0 is defined implicitly by the resource constraint.

(iii) the job-filling rate $\eta(a)$ and the nonphantom proportion $\pi^{\text{ds}}(a)$ strictly decrease, tightness $\theta^{\text{ds}}(a)$ strictly increases and the job-finding rate $\mu(a)$ is constant in a.

The proof is given in Appendix A.

Part (i) restates the no-arbitrage condition that determines the allocation of the unemployed across submarkets. This allocation must be such that the job-finding rate is the same for all

[5] To economize on notation, we suppress the arguments of $\theta(a)$ and $\pi(a)$ whenever confusion is not likely to result.

listing ages. The resulting differential equation in part (ii) describes tightness by listing age given initial condition $\theta^{\text{ds}}(0)$. The resource constraint then determines $\theta^{\text{ds}}(0)$.

Part (iii) describes the salient features of the directed search allocation. Tightness strictly increases with listing age. In contrast to the random search allocation, workers adjust their search behavior to compensate for the change in the nonphantom proportion. In particular, they disproportionately search for younger job listings. They do this in such a way that the job-finding rate, $\mu(a) = m(1, \theta(0))$ is constant with respect to listing age. Consequently, the job-filling rate $\eta(a) = m(1, \theta(a))/\theta(a)$ decreases with listing age.

In the standard model with no phantoms, the job-finding rate is also constant across age, but in that case, the reason is simply that the ratio of vacancies to unemployed is constant across age. Accounting for phantoms while allowing for directed search gives the same result but for a different reason. Workers who take obsolete information into account apply to younger listings. Thus tightness increases and the job-filling rate decreases with age.

Unlike the random search model, the directed search model predicts that workers disproportionately search young job listings. This prediction is in line with the evidence, reported in the Introduction, from the DHI Vacancy and Flow Applications Database. It is also in line with Faberman and Menzio (2017). In their Figure 1, they show the cross-sectional distribution of vacancy duration across all hires in 1982. They find that 35% of all hires are from vacancies aged one week at most, 20% from vacancies aged between one and two weeks, 5% between two and three weeks. They also report the number of applications per vacancy age. Unconditional estimates reveal that the number of applications per week decreases from 22.3 in the first week to 1.5 after one month. Conditional estimates show a more modest decline, from 17.3 to 5.7.

3.3 Social Planner Problem

The planner observes the age of job listings and decides on the allocation of the unemployed across the different submarkets. Like job seekers, the planner cannot distinguish between vacancies and phantoms.

Given our focus on steady-state allocations, we assume the discount rate is equal to zero. Moreover, we restrict our attention to time-independent policies. Thus the constrained efficient allocation minimizes the steady-state unemployment rate.

To set up the planner's optimal control problem, it is convenient to note that, given the assumption that there is a fixed measure of jobs in the market, minimizing the steady-state level of vacancies is equivalent to minimizing the steady-state unemployment rate. Also, in order to facilitate the comparison of the social planner allocation with the random search and directed search allocations, we note that the choice of an allocation of job seekers across listing age can equivalently be expressed as a choice of market tightness across submarkets.

The constrained efficient allocation is the solution to the following optimal control problem:

$$\max_{\theta(.)} - \int_0^A v(a)da \tag{$*$}$$

subject to

$$\begin{aligned}
\dot{v} &= -\eta(\theta)v && (c1)\\
v(0) &= \lambda\left(K - \int_0^A v(a)da\right) + v(A) && (c2)\\
\dot{p} &= \eta(\theta)v && (c3)\\
p(0) &= 0, && (c4)\\
1 - \int_0^A \left[(v(a) + p(a))/\theta(a)\right] da + \int_0^A v(a)da &= K. && (c5)
\end{aligned}$$

The planner is constrained by the evolution of vacancies over age ($c1$)–($c2$), the evolution of phantoms over age ($c3$)–($c4$), and the resource constraint ($c5$).

Proposition 3 (Constrained efficient allocation) Let $\theta^{\text{eff}} : [0, A] \to \mathbb{R}_+$, $\pi^{\text{eff}} : [0, A] \to [0, 1]$ and $u^{\text{eff}} : [0, A] \to \mathbb{R}_+$ be, respectively, the efficient tightness, nonphantom proportion and job seekers by listing age. The following properties hold for all $a \in [0, A]$:

(i) $(s_2 - s_1)(1 - \alpha(\theta^{\text{eff}}))\pi^{\text{eff}} m(1, \theta^{\text{eff}}) = 1$, where the functions $s_1 : [0, A] \to \mathbb{R}$ and $s_2 : [0, A] \to \mathbb{R}$ are such that

$$\dot{s}_1 = B + \frac{1}{\theta^{\text{eff}}} - \frac{1}{(1 - \alpha(\theta^{\text{eff}}))\pi^{\text{eff}}\theta^{\text{eff}}}, \tag{13}$$

$$\dot{s}_2 = \frac{1}{\theta^{\text{eff}}}, \tag{14}$$

with $s_1(0) = s_1(A) = -[(1 - \alpha(\theta^{\text{eff}}(A)))\pi^{\text{ds}}(A)m(1, \theta^{\text{ds}}(A))]^{-1}$, $s_1(a) < s_2(a) \leq s_2(A) = 0$, and

$$B = \frac{\frac{u^{\text{eff}}(0)\alpha(\theta^{\text{eff}}(0))}{1-\alpha(\theta^{\text{eff}}(0))} - \frac{u^{\text{eff}}(A)\alpha(\theta^{\text{eff}}(A))}{1-\alpha(\theta^{\text{eff}}(A))}}{\lambda(1 - \mathsf{u}^{\text{eff}})} > 0.$$

(ii) Market tightness and the nonphantom proportion evolve according to

$$-\frac{\theta^{\text{eff}}\alpha'(\theta^{\text{eff}})}{1 - \alpha(\theta^{\text{eff}})}\frac{\dot{\theta}^{\text{eff}}}{\theta^{\text{eff}}} + \alpha(\theta^{\text{eff}})\frac{\dot{\theta}^{\text{eff}}}{\theta^{\text{eff}}} + \frac{\dot{\pi}^{\text{eff}}}{\pi^{\text{eff}}} = B(1 - \alpha(\theta^{\text{eff}}))\pi^{\text{eff}}m(1, \theta^{\text{eff}}) - \frac{m(1, \theta^{\text{eff}})}{\theta^{\text{eff}}}, \tag{15}$$

$$\frac{\dot{\pi}^{\text{eff}}}{\pi^{\text{eff}}} = -\frac{m(1, \theta^{\text{eff}})}{\theta^{\text{eff}}}. \tag{16}$$

The initial conditions are $\pi^{\text{eff}}(0) = 1$ and $\theta^{\text{eff}}(0) = \theta_0$, where θ_0 is defined implicitly by the resource constraint.

(iii) Tightness $\theta^{\text{eff}}(a)$ strictly increases, the nonphantom proportion $\pi^{\text{eff}}(a)$ and the job-filling rate $\eta(a)$ strictly decrease, and, provided $\alpha'(\theta) \leq 0$, the job-finding rate is larger at the beginning of a listing's existence than at its end, i.e., $\mu(0) > \mu(A)$.

The proof is given in Appendix A.

Part (i) describes the efficient allocation of job seekers to the different listing ages. The functions s_1 and s_2 are normalizations of the costates associated with the state variables, v and p, in the planner's problem; specifically, the normalizing constant is such that the value of an application to a new listing equals one. The variables $s_1(a)$ and $s_2(a)$ are the shadow values at age a of a vacancy and of a phantom, respectively. Allocating an additional job seeker to a listing of age a increases expected employment by the marginal productivity of an age a listing, $(1-\alpha(\theta))\pi m(1,\theta)$, but this increase in expected employment entails an opportunity cost, namely, the creation of a phantom with shadow value $s_2(a)$, and the elimination of a vacancy with shadow value $s_1(a)$.

Both shadow values are negative and the phantom value is larger (less negative) than the vacancy value. With a fixed number of jobs, creating an additional vacancy entails one fewer filled job. By contrast, forming a phantom involves losing a vacancy, which comes at a lower cost.

The shadow value s_2 quantifies the externality associated with phantoms. Integrating forward equation (14) gives $s_2(a) = -\int_a^A \theta(b)^{-1} db$. At given tightness $\theta = (v+p)/u$, having an additional phantom increases the number of job seekers by $1/\theta$, a social waste because these job seekers could be allocated to a different submarket. As the phantom ages, this impact persists, though its magnitude varies with tightness. The shadow value of a phantom is therefore the cumulative impact from its current age to its death.

The shadow value s_1 accounts for social costs and benefits of a vacancy. Integrating equation (13) forward gives $s_1(a) = s_1(A) - \int_a^A \{B + \theta(b)^{-1} - [(1-\alpha)\pi(b)\theta(b)]^{-1}\} db$. This value is the same at the beginning and the end of possible listing ages because vacancies are renewed after the termination age A. A vacancy of age a generates the fixed opportunity cost B, corresponding to the utilization of a scarce resource, and also attracts $1/\theta$ job seekers, just as phantoms do. However, the vacancy is converted into a job at rate $\eta(\theta(a))$, which creates the change in value $s_2 - s_1$. Since $\eta(\theta) = m(1,\theta)/\theta$, using the fact that $(s_2 - s_1)(1-\alpha)\pi m(1,\theta) = 1$ explains the term $[(1-\alpha)\pi(b)\theta(b)]^{-1}$.

Part (ii) shows the resulting motions of tightness and the nonphantom proportion. By differentiating the allocation rule in (i) and using (13) and (14), we can eliminate the two costates. Equation (15) shows the motion of the marginal productivities of vacancies, whereas equation (16) shows the motion of the nonphantom proportion. Combining the two equations, we obtain the following differential equation characterizing the change in tightness:

$$\left[\frac{-\theta\alpha'(\theta)}{1-\alpha(\theta)}+\alpha(\theta)\right]\frac{\dot{\theta}}{\theta}=B(1-\alpha(\theta))\pi m(1,\theta).$$

Part (iii) describes some of the properties of the efficient allocation. The nonphantom proportion falls as listings age. Therefore, as in the directed search allocation, it is efficient to allocate more job seekers per listing to younger listing ages. The rate of filling jobs decreases over the lifetime of a listing. Lastly, the marginal productivity of job seekers is larger at age A than at age 0. When the elasticity α is well-behaved, i.e., when $\alpha'(\theta)\leq 0$, this also implies that the job-finding rate tends to decrease with listing age.

Proposition 4 (Constrained efficiency) The following properties hold:

(i) The random search allocation is never constrained efficient.

(ii) The directed search allocation is generically not constrained efficient.

The proof is given in Appendix A and is a straightforward implication of Propositions 1 to 3.

Randomizing over the different job listings without accounting for their age is not efficient. This result is not surprising. In the constrained efficient allocation, market tightness varies with the nonphantom proportion, whereas random search implies that tightness is constant over listing age. This is in contrast to the result from standard search models in which applying for the different jobs with equal probability is efficient. The reason that random search is inefficient in our setting is, of course, that older job listings are more likely to be phantoms.

The directed search allocation is also not constrained efficient. The rules allocating job seekers across listing ages in the directed search and constrained efficient allocations are, respectively

$$\begin{aligned}\pi^{\mathrm{ds}}(a)m(1,\theta^{\mathrm{ds}}(a)) &= m(1,\theta^{\mathrm{ds}}(0))\\ (s_2(a)-s_1(a))(1-\alpha(\theta^{\mathrm{eff}}(a)))\pi^{\mathrm{eff}}(a)m(1,\theta^{\mathrm{eff}}(a)) &= 1.\end{aligned}$$

Comparing them reveals two differences. The first one is unrelated to phantoms. In the directed search allocation, what matters is the job-finding rate $\pi m(1,\theta)$, which must be constant over listing age. This implies that its growth rate $\dot{\pi}/\pi+\alpha\dot{\theta}/\theta=0$. In the efficient allocation, what matters is the marginal productivity of job seekers $(1-\alpha)\pi m(1,\theta)$, i.e., the job-finding rate times the elasticity $1-\alpha$. This elasticity may vary with age, which is reflected in the term $-[\theta\alpha'/(1-\alpha)]\dot{\theta}/\theta$ on the left-hand side of equation (15).

The second difference is due to the presence of an intertemporal externality that is internalized in the social planner allocation. The planner accounts for phantom birth and phantom persistence. Allocating a job seeker to listings of a given age translates into more matches at that age, which fuels the phantom stock. The magnitude of this informational externality decreases with age. This occurs for two reasons. First, a young phantom persists for a long time and thus has the potential to affect many job seekers during its lifetime. Second, given the concentration of job seekers at young listing ages, the phantom has the potential to affect a large number of job seekers. Symmetrically, an old phantom has a short life and only impacts the distribution of job seekers where the density is low.

Formally, the shadow value of a phantom is negative and strictly increases with listing age. It is equal to 0 when the termination age A is reached. After this age all listings, including phantoms, are destroyed, which explains why phantoms have a smaller effect as A gets closer.

3.4 Efficient wage schedule

As we noted above, the distribution of job seekers over listing age in the directed search allocation is generally not constrained efficient. If the wage were conditioned on vacancy age, it could alter this distribution and thereby internalize the externality caused by phantoms. We now discuss the characteristics of such an efficient wage schedule. This allows us to provide an alternative measure of the informational externality induced by directed search. Of course, we emphasize that we do not actually see age-dependent wage schedules. In section 5.4, we discuss the case of wage bargaining.

Let r be the discount rate. In the directed search allocation, suppose that a vacancy of age a pays wage $w(a)$ when the worker obtains the corresponding job. Let $\omega(a) = w(a)/w(0)$ be the *wage schedule*. The value of having a job paying $w(a)$ is $W(a)$, whereas the value of job search is U:

$$\begin{aligned} rW(a) &= w(a) + \lambda[U - W(a)], \\ rU &= \mu(a)[W(a) - U]. \end{aligned}$$

The no-arbitrage condition across submarkets implies that

$$\pi(a)m(1, \theta(a))[W(a) - U] = m(1, \theta_0)[W(0) - U], \tag{17}$$

which is equivalent to

$$\frac{\pi(a)m(1, \theta(a))}{r + \lambda + \pi(a)m(1, \theta(a))}\omega(a) = \frac{m(1, \theta_0)}{r + \lambda + m(1, \theta_0)}. \tag{18}$$

When $r \to 0$, we obtain $[1 - ur(a)]\omega(a) = 1 - ur(0)$, where $ur(a) = \lambda/(\lambda + \pi(a)m(1, \theta(a))$ is the unemployment rate at listing age a, i.e., the unemployment rate among job seekers who target listings of age a. Given any wage schedule, the employment probability $1 - ur(a)$ adjusts so that the product of this employment probability and the wage schedule equals the employment probability when there are no phantoms, i.e., when workers search for new vacancies.

An *efficient wage schedule* $\omega^{\text{eff}} : [0, A] \to \mathbb{R}$ is such that tightness is the same in the directed search and in the constrained efficient allocations, that is, $\theta^{\text{eff}}(a) = \theta^{\text{ds}}(a)$ for all $a \in [0, A]$. The function ω^{eff} satisfies:

$$\omega^{\text{eff}}(a) = \frac{m(1, \theta^{\text{eff}}(0))}{\lambda + m(1, \theta^{\text{eff}}(0))} \frac{\lambda + \pi^{\text{eff}}(a) m(1, \theta^{\text{eff}}(a))}{\pi^{\text{eff}}(a) m(1, \theta^{\text{eff}}(a))}. \tag{19}$$

We can characterize it as follows:

$$\ln \omega^{\text{eff}}(a) = \ln[1 - ur^{\text{eff}}(0)] - \ln[1 - ur^{\text{eff}}(a)] \approx ur^{\text{eff}}(a) - ur^{\text{eff}}(0), \tag{20}$$

when ur^{eff} is sufficiently small. The log efficient wage schedule at listing age a compensates for the unemployment rate differential between listing ages a and zero.

The efficient wage schedule satisfies $\omega^{\text{eff}}(0) = 1$. As $\dot{\omega}^{\text{eff}}/\omega^{\text{eff}} = \dot{ur}^{\text{eff}}/[1 - ur^{\text{eff}}]$, we have

$$\frac{\dot{\omega}^{\text{eff}}}{\omega^{\text{eff}}} = -ur^{\text{eff}} \left[\alpha \frac{\dot{\theta}^{\text{eff}}}{\theta^{\text{eff}}} + \frac{\dot{\pi}^{\text{eff}}}{\pi^{\text{eff}}} \right].$$

The growth rate of the efficient wage schedule is the product of the unemployment rate at listings of age a and the negative of the growth rate of the job-finding rate at the same listing age. This product is positive when the job-finding rate declines with age in the constrained efficient allocation and negative otherwise.

The efficient wage schedule specifies the wage growth required to internalize the externality caused by phantoms. However, it does not specify the wage level. In our model with a fixed number of jobs, the wage level redistributes welfare between firms and workers without affecting employment probabilities. Therefore the optimal wage level is indeterminate and only wage growth matters.

4 Calibrations

The purpose of this section is to illustrate the theoretical model, quantify the dynamic externality associated with phantom vacancies, and characterize the job-seekers' behavior by listing age. We calibrate our model to simulate the working of a typical job board, Craigslist, and reproduce aggregate US labor market data. We assume the contact technology is Cobb-Douglas, i.e., $m(u, v + p) = m_0 u^{1-\alpha}(v + p)^{\alpha}$, where $m_0 > 0$ and $\alpha \in (0, 1)$. We first characterize the policy function linking tightness to the nonphantom proportion in the constrained efficient allocation. We then choose the various parameters and discuss the features of the three allocations. Lastly we consider alternative values for the elasticity α and the termination age A.

4.1 Policy function

The policy function expresses the control variable, θ, as a function of the state variables, v and p, parameterized by the initial condition θ_0. In both the directed search and the constrained efficient allocations, θ depends on v and p only through the nonphantom proportion, π. Conditional on θ_0, we can solve explicitly for θ as a function of π in the Cobb-Douglas case.

Proposition 5 (Cobb-Douglas case) Let $\boldsymbol{\theta}^{\mathrm{ds}}$ and $\boldsymbol{\theta}^{\mathrm{eff}}$ be two functions such that:

$$\begin{aligned} \boldsymbol{\theta}^{\mathrm{ds}}(\pi, \theta_0) &= \theta_0 \pi^{-1/\alpha}, && (21) \\ \boldsymbol{\theta}^{\mathrm{eff}}(\pi, \theta_0) &= \frac{\theta_0}{1 - \theta_0(1-\pi)B(1-\alpha)/\alpha}. && (22) \end{aligned}$$

The following properties hold:

(i) $\theta^{\mathrm{ds}}(a) = \boldsymbol{\theta}^{\mathrm{ds}}(\pi^{\mathrm{ds}}(a), \theta^{\mathrm{ds}}(0))$ and $\theta^{\mathrm{eff}}(a) = \boldsymbol{\theta}^{\mathrm{eff}}(\pi^{\mathrm{eff}}(a), \theta^{\mathrm{eff}}(0))$ for all $a \in [0, A]$;

(ii) $\theta^{\mathrm{eff}}(0) > \theta^{\mathrm{ds}}(0)$.

The proof is given in Appendix A.

Knowledge of the policy functions facilitates the comparison between the constrained efficient and directed search allocations. The functions $\boldsymbol{\theta}^{\mathrm{ds}}$ and $\boldsymbol{\theta}^{\mathrm{eff}}$ are both decreasing in the nonphantom proportion, π. In the directed search allocation, this property results from the fact that job-seekers spread over submarkets so that the job-finding rate stays constant with listing age. A similar property holds in the efficient allocation: the reason that tightness increases with listing age (see Proposition 3, part (iii)) is because the nonphantom proportion decreases with it. We have $\theta^{\mathrm{ds}}(\pi, \theta_0) > \theta^{\mathrm{eff}}(\pi, \theta_0)$ whenever $\theta^{\mathrm{eff}}(\pi, \theta_0) > 0$. Conditional on the nonphantom proportion and initial tightness, the planner allocates more job-seekers per listing than workers spontaneously do under directed search. Phantom formation is reduced and the nonphantom proportion decreases at a smaller pace.

Part (ii) shows that $\theta^{\mathrm{eff}}(0) > \theta^{\mathrm{ds}}(0)$; if not, tightness would be larger at all listing ages in the directed search allocation, which is impossible. Let the *job queue* be the ratio of job seekers to listings, i.e., the inverse tightness $1/\theta(a)$. Part (ii) predicts that the job queue is longer at low listing ages in the directed search allocation; that is, search is excessively concentrated on newly created vacancies. This also implies that $\mu^{\mathrm{eff}}(0) = m(1, \theta^{\mathrm{eff}}(0)) > \mu^{\mathrm{ds}}(0) = m(1, \theta^{\mathrm{ds}}(0))$. Workers who search for new jobs have a higher job-finding rate in the efficient allocation.

4.2 Parameters

We use BLS data for the period 2000-2008. Over this period, the monthly probability of finding a job was about $\mu_m = 0.4$. Thus $1 - \exp(-\mu) = 0.4$ and $\mu = -\ln(1-0.4) \approx 0.5108$. The monthly job loss probability was $\lambda_m = 0.03$. This gives $\lambda = -\ln(1-0.03) \approx 0.0305$. The corresponding stationary unemployment rate is $\mathsf{u} = \lambda/(\lambda+\mu) \approx 5.63\%$. We display the parameter values with a four-digit precision because, as we shall see, the comparison of unemployment rates in the different allocations involves this precision.

We set the termination age to one month, i.e., $A = 1$. This corresponds to Craigslist where ads are posted for one month. Craigslist is interesting because it corresponds well to our basic model. There is clear evidence that employers do not withdraw ads on this website. In Appendix B, we study a cross-section of more than 500,000 ads that were posted on the US part of Craigslist's website as of June 10, 2015. The distribution of listings by age is roughly uniform. This means that employers do not delete obsolete ads. We consider alternative values for A in Section 4.4.

To set the total number of jobs $1 - \mathsf{u} + \mathsf{v} = K$, we need an estimate of the number of vacancies. In the JOLTS dataset, firms report how many vacancies they have. The vacancy-to-unemployed ratio, $x \equiv \mathsf{v}/\mathsf{u}$, is approximately equal to 1/2. However, many of these vacancies are filled by employed job seekers and our model does not consider on the job search. On the other hand, Davis et al. (2013) suggest that the JOLTS data underestimate the number of vacancies because of time aggregation issues. Therefore we take a different approach and set x from the mean duration of a vacancy. In our model, this duration is $d = \int_0^A \pi(a)[1-\pi(A)]^{-1}da$. As $\pi(a) = v(a)/v(0)$ and $v(0) = \lambda(1-\mathsf{u})/[1-\pi(A)]$, we have $d = \mathsf{v}/[\lambda(1-\mathsf{u})] = x\mathsf{u}/[\lambda(1-\mathsf{u})]$. Davis et al. (2013) argue that d lies between 14 and 25 days. We set $x = 0.33$ so that d is about 19.5 days, the mean of this interval.

As for α, we reproduce the elasticity of the aggregate job-finding rate with respect to the vacancy-to-unemployed ratio used by Shimer (2005). This elasticity is about 0.4. The aggregate elasticity does not coincide with α. Instead, it depends on the nonphantom proportion at each age and on the allocation of job seekers across submarkets. Intuitively, this elasticity should be larger than α: having more vacancies increases the nonphantom proportion, thereby increasing the number of matches (see Chéron and Decreuse (2017) for the random search case). This strategy leads to $\alpha = 0.15$. We consider alternative values in Section 4.4.

We set the scale parameter m_0 so that the unemployment rate in the directed search allocation is equal to $\mathsf{u} = 5.63\%$. Finally, we use the resource constraint to set $K = 0.9623$. The interpretation is that if there were no frictions in the labor market, i.e., no meeting frictions and no phantoms, then 3.77% of the workforce would nonetheless remain unemployed.

Table 1 gives the set of parameters in the baseline calibration.

m_0	α	A	λ	K	μ	u
1.0671	0.15	1	0.0305	0.9623	0.5108	0.0563

Table 1: Parameter values in the baseline calibration

4.3 Baseline calibration

We solve for the directed search allocation in the baseline calibration and compare it with the random search and efficient allocations. These latter allocations are found using the same parameter values, but changing the rule allocating job seekers across the different listing ages.

By construction the unemployment rate is $\mathsf{u}^{\mathrm{ds}} = 5.63\%$ in the directed search allocation, whereas it is $\mathsf{u}^{\mathrm{eff}} = 5.39\%$ in the efficient allocation and $\mathsf{u}^{\mathrm{rs}} = 5.64\%$ in the random search allocation.

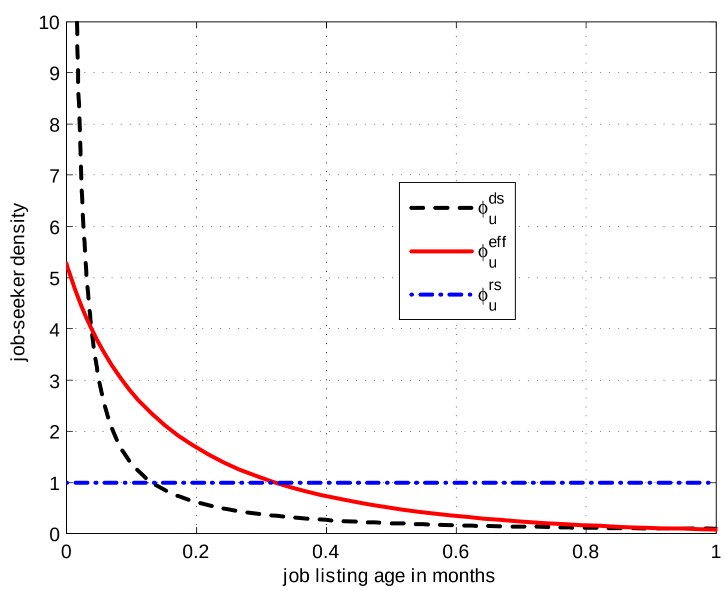

Figure 1: Distribution of job-seekers by listing age

Figure 1 displays the density function of job seekers by listing age in the three allocations: random search, directed search and efficient search. This figure shows dramatic differences in

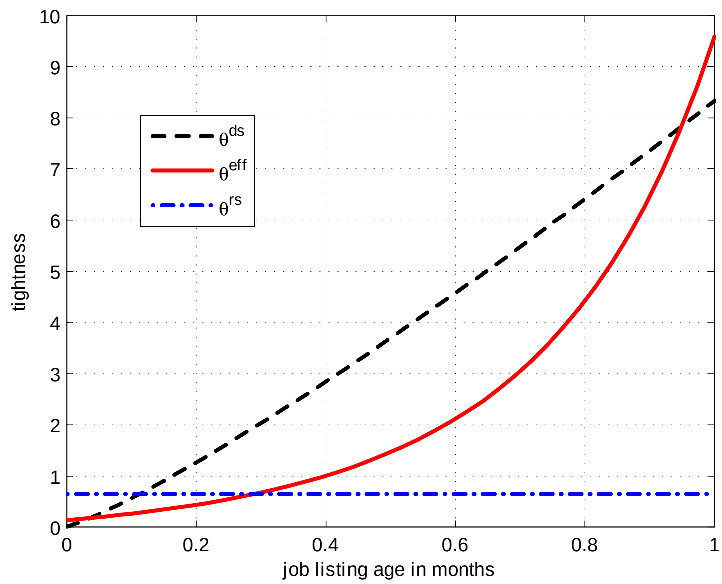

Figure 2: Tightness by listing age (in months)

search behavior. In the random search allocation, the density is uniform (see Proposition 1). In the directed search allocation, the density has a spike for newly created vacancies and then strongly decreases. The constrained efficient allocation reveals an intermediate behavior: the spike is much less pronounced for new vacancies (see Proposition 5).

Figure 2 shows the resulting tightness by listing age in the three allocations. Tightness is constant under random search, whereas it strictly increases with listing age in the other cases. Because job seekers are concerned about phantoms, they direct their search towards younger listings, and thus the ratio of advertised jobs to searchers increases with age. One of the key predictions of our model with directed search is that workers primarily search for recent listings. In our calibration, the bulk of job applications happen in the very first days of a listing's existence. Table 2 shows that more than 50% of the job seekers focus on listings aged less than 24 hours in the directed search allocation. This probability mass falls to 10% for listings aged between 24 and 48 hours. Only 15% of the job seekers send applications to listings older than one week. The corresponding figures in the model with random search are, respectively, 3.5%, 3.5% again and 75%.

Another way to quantify this bias towards young listings is to compute job queues at various listing ages. The mean job queue between two dates, say a_1 and a_2, is $(a_2 - a_1)^{-1} \int_{a_1}^{a_2} \theta(a)^{-1} da$. In the directed search allocation, Table 2 shows that the mean job queue is 21 job seekers per listing for listings aged less than 24 hours. It falls to 4 for listings aged between 24 and 48 hours.

	Density of job seekers	Mean job queue
Day 1	0.5834	20.8422
Day 2	0.1044	3.7329
Day 3	0.0569	2.0321
Day 4	0.0382	1.3697
Week 1	0.8527	4.3530
Week 2	0.078	0.3981
Week 3	0.0415	0.2120
Week 4	0.0278	0.1417

Table 2: Mass of job seekers and job queues at various intervals of listing age.

The fall in job queues with listing age is larger than the one reported by Faberman and Menzio (2017). However, they use data from the 1980s. More recently, internet facilitates the diffusion of information about new vacancies. Therefore the distribution of job applications has likely shifted to the left, i.e., job seekers apply for younger vacancies. Moreover, Faberman and Menzio (2017) focus on vacancy age, whereas we have a theory of listing age. Firms with an old vacancy may post a new listing and receive many applications as a result.

To understand why the directed search allocation is not efficient, it is useful to look at the job-finding rate (shown in Figure 3) and the nonphantom proportion (shown in Figure 4). The job-finding rate is by definition constant under directed search. It strictly decreases over listing age in the random search allocation. It is also such that $\mu(0) > \mu(A)$ in the constrained efficient allocation.[6] In both the random search and efficient allocations, reallocating individual search effort towards younger job listings would increase the individual odds of finding a job. This individual gain would also lead to a social gain in the random search allocation. However, it would decrease welfare in the constrained efficient allocation. Figure 4 shows why. When job seekers direct their search towards more recent listings, the nonphantom proportion decreases very rapidly when a is low. This strong decline in the nonphantom proportion persists and affects all listing ages. The constrained efficient allocation accounts for this effect. The pace of phantom creation is smoother over listing age, and the nonphantom proportion declines less rapidly.

Directed search leads job seekers to over-apply to young job listings. Reallocating some job seekers from younger listings to older ones leads to efficiency gains. As explained in Section 3, the reason is that directed search imposes a negative informational externality that persists across

[6] There is a modest increase by the end of the one-month period. It is induced by the rise of the shadow value of a vacancy, s_1, as the renewal age, A, gets closer.

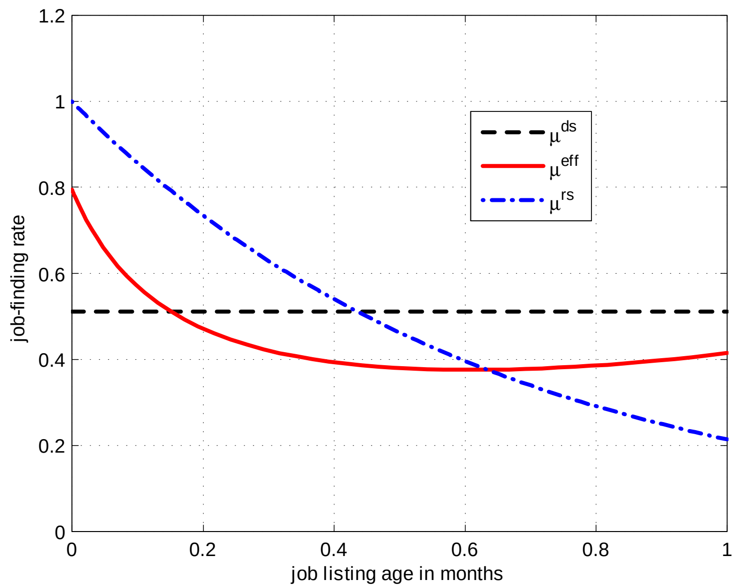

Figure 3: Job-finding rate by listing age (in months)

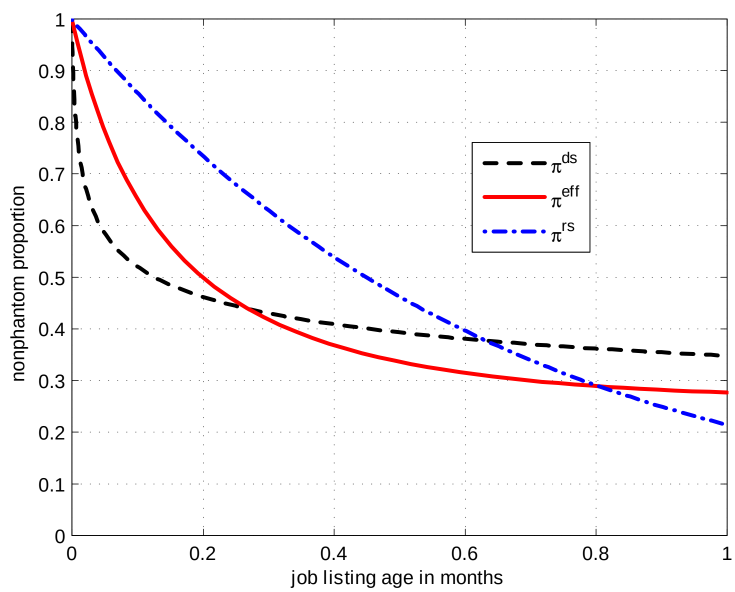

Figure 4: Nonphantom proportion by listing age (in months)

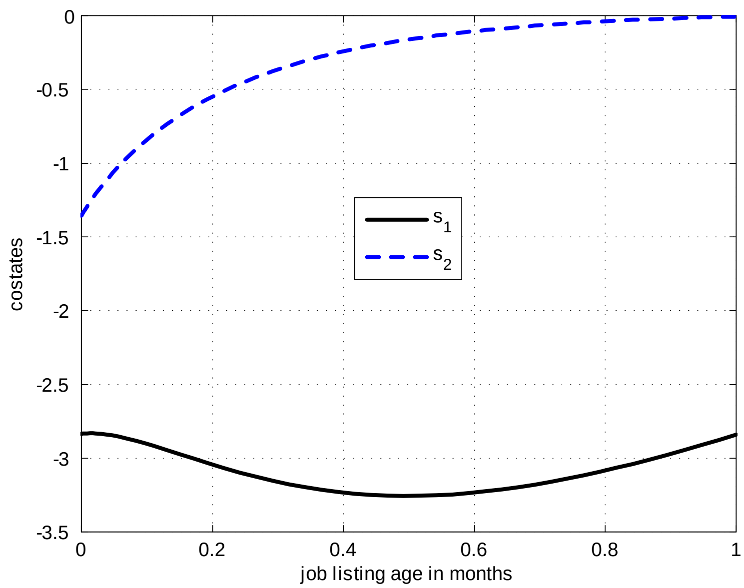

Figure 5: Shadow values by listing age (in months)

different listing ages. Applying to job listings of a given age, say a, sometimes means finding jobs of that age. This creates phantoms, and these phantoms then haunt the market. Because of the nature of directed search, phantoms only hurt agents who respond to job listings of age a or more. The magnitude of the externality thus decreases with listing age. When workers apply to newly created vacancies, they potentially affect all the other agents through phantom creation. When they direct their applications to old listings, they affect almost no one. This can be seen in Figure 5 which shows the patterns of the shadow values of vacancies and phantoms in the constrained efficient allocation. As indicated in Proposition 3, $s_1(a) < s_2(a) \leq 0$, $s_1(0) = s_1(A)$ and the shadow value of phantoms strictly increases with the listing age.

Finally, Figure 6 shows the efficient wage schedule in the baseline calibration. It also plots its approximation by the unemployment rate differential $ur(a) - ur(0)$. The wage increases by 4% over two to three weeks and experiences a small decline thereafter. This pattern compensates for the change in the job-finding rate.

Quantitatively, the efficiency gains achieved by the constrained efficient allocation are rather modest. Going from the worst (random search) to the best search method (efficient search) reduces the unemployment rate by 0.25 percentage points, i.e., a 5% decrease. The unemployment rate is only slightly affected by the way agents search for jobs. This is because any search strategy that produces more matches also increases the steady-state stock of phantoms. The nonphantom proportion, therefore, decreases as search becomes more efficient. In our baseline

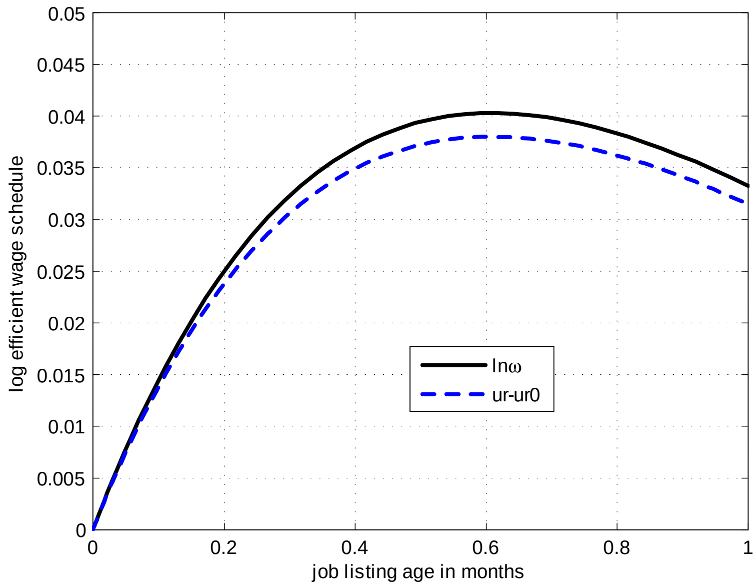

Figure 6: Efficient wage schedule in the baseline calibration

calibration, we have $\pi^{\text{rs}} = 51.00\%$, $\pi^{\text{ds}} = 42.10\%$, $\pi^{\text{eff}} = 40.53\%$.

This, however, does not mean that phantom vacancies do not have a large quantitative impact. In our calibration, removing all phantoms from the market leads to large employment gains. The corresponding allocation obtains as $A \to 0$. The unemployment rate is then $\mathsf{u}^{\text{np}} = 4.06\%$.

We can compute the contribution of phantoms to unemployment. If there were no frictions, employment would be determined by the short side of the market, and the unemployment rate would be $\min(1 - K, 0)$. In our calibration, $K = 0.9623$, so the nonfrictional unemployment rate is equal to 3.77%. Under directed search the unemployment rate is $\mathsf{u}^{\text{ds}} = 5.63\%$. It follows that matching frictions account for $(5.63 - 3.77)/\ 5.63 \approx 33\%$ of the unemployment rate. Without phantoms, the unemployment rate would decrease to 4.06%. Hence, phantoms account for $(5.63 - 4.06)/\ (5.63 - 3.77) \approx 85\%$ of overall frictions and $(5.63 - 4.06)/5.63 \approx 28\%$ of unemployment.

4.4 Alternative parameterizations

We examine how the magnitude of the inefficiency varies when we let the parameters α and A take different values. For each value of α and A, we recalibrate the directed search allocation by adjusting m_0 so that the predicted unemployment rate u^{ds} matches the mean US unemployment rate over the period 2000-2008. The two other allocations are then found by changing the rule

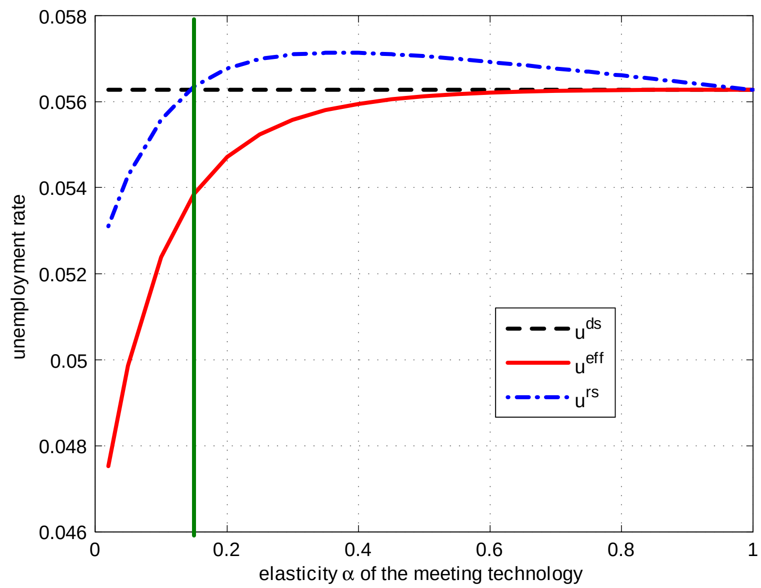

Figure 7: Unemployment rates in the three allocations as functions of α. (The green line corresponds to $\alpha = 0.15$, the baseline value.)

$\theta(a)$ that assigns job seekers to listing ages.

Figure 7 depicts the unemployment rate as a function of α. The vertical line indicates our choice of $\alpha = 0.15$ in the baseline calibration. The unemployment rate differential between the directed search and the efficient allocation decreases with α. The three allocations coincide when $\alpha = 1$. In this case, the matching technology is $M(a) = m_0 v(a)$ for all $a \in [0, A]$ and phantoms do not affect it.

Directed search does better than random search when α is sufficiently large. However, random search outperforms directed search for small values of α. In such cases, the matching process is more efficient when workers do not pay attention to listing ages and simply randomize over the different listings. This feature illustrates the dynamic externality induced by directed search. As α approaches 0, most of the congestion externalities are due to phantoms and these intertemporal frictions are neglected by the job seekers in the directed search allocation. Therefore they only search for newly created vacancies. This creates many phantoms in the beginning of a listing existence, thereby justifying the search focus on new listings. The social planner accounts for the dynamic externality and does not concentrate job seekers on new listings to the same extent. The random search allocation becomes relatively closer to the constrained efficient allocation as a result.

To understand what the various values of α mean, it is useful to relate them to the corresponding long-run elasticity of the job-finding rate with respect to $x = \mathsf{v}/\mathsf{u}$, which is what is commonly reported in the literature. For a given rule assigning job seekers to the different submarkets, the average job-finding rate is $\mu = \int_0^A \phi_u(a)\pi(a)m(1,\theta(a))da$. At each listing age, an increase in $x = \mathsf{v}/\mathsf{u}$ may modify the ratio $\theta(a)$ of listings to job seekers, the nonphantom proportion $\pi(a)$ and the density of job seekers $\phi_u(a)$.

Under random search, the density ϕ_u is uniform and θ does not vary with listing age. Therefore changes in x cause θ to change uniformly across listing ages and change μ through changes in $m(1,\theta)$ and in the mean nonphantom proportion $\bar{\pi} = \frac{1-\exp(-Am(1,\theta)/\theta)}{Am(1,\theta)/\theta}$. The long-run elasticity of the job-finding rate with respect to x is $d\ln\mu/d\ln x = \{\alpha + (1-\alpha)\left[1-\bar{\pi}^{-1}\exp(-Am(1,\theta)/\theta)\right]\} \times d\ln\theta/d\ln x$. The term between braces is larger than α: increasing θ reduces the nonphantom proportion at each listing age, which improves the chances of finding a job. Meanwhile $d\ln\theta/d\ln x > 1$ because $\theta = \mathsf{x} + \mathsf{p}/\mathsf{u}$ and u decreases with x. It follows that $d\ln\mu/d\ln x > \alpha$.

Under directed search, the average job-finding rate is $\mu = m(1,\theta(0))$. Therefore the three effects mentioned previously are embodied into changes in $\theta(0)$. The long-run elasticity of the job-finding rate with respect to x is $d\ln m(1,\theta(0))/d\ln x = \alpha \times (d\ln\theta(0)/d\ln x)$. This elasticity is larger than α. In the absence of phantoms, initial tightness would respond one-for-one to x. With phantoms, the general increase in job availability translates into larger nonphantom proportions at all ages. In turn, the job seekers reallocate their search effort to older job listings, implying that tightness increases more at low listing ages than at older ones. Thus the elasticity $(d\ln\theta(0)/d\ln x) > 1$ and the long-run elasticity of the job-finding rate with respect to x is $d\ln m(1,\theta(0))/d\ln x > \alpha$.

Figure 8 plots the long-run elasticity of the job-finding rate with respect to x against α. It also shows the corresponding scale parameter m_0. We compute the long-run elasticity as follows: Let $\ln x_\alpha$ and $\ln\mu_\alpha$ denote, respectively, the log vacancy-to-unemployed ratio and the log job-finding rate of the directed search allocation conditional on α. By construction, $\ln x_\alpha = \ln x = -\ln 3$ and $\ln\mu_\alpha = \ln(0.5108)$. We then decrease the total number of jobs K by one percentage point and recompute the directed search allocation. Let $\ln x_-$ and $\ln\mu_-$ denote, respectively, the associated log tightness and log job-finding rate. We similarly consider a one-percentage-point increase in the number of jobs and compute $\ln x_+$ and $\ln\mu_+$. The long-run elasticity of the job-finding rate is approximated by

$$\frac{d\ln\mu}{d\ln x} \approx \frac{1}{2}\left(\frac{\ln\mu_+ - \ln\mu_\alpha}{\ln x_+ - \ln x_\alpha} + \frac{\ln\mu_- - \ln\mu_\alpha}{\ln x_- - \ln x_\alpha}\right). \tag{23}$$

As explained above, the long-run elasticity is larger than α. It is close to 0.4, the value used by Shimer (2005), when α lies between 0.1 and 0.2. The vertical line indicates our choice of $\alpha = 0.15$.

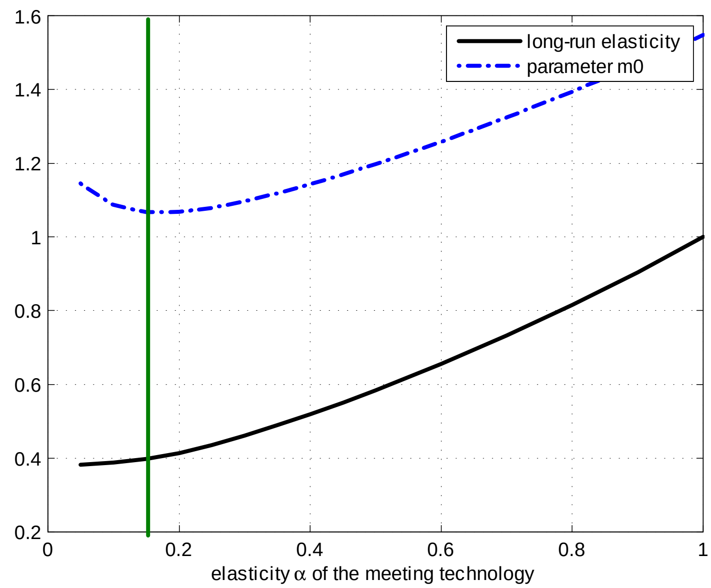

Figure 8: Long-run elasticity of the matching technology as a function of the elasticity of the contact technology. (The green line corresponds to $\alpha = 0.15$, the baseline value.)

Figure 9 depicts the unemployment rate as a function of the termination age A. This parameter sets the magnitude of externalities induced by information obsolescence. The three allocations coincide when $A = 0$. Then the unemployment rate differential between the directed search and the efficient one gradually increases with A.

Random search does slightly better than directed search when A is lower than one month, but does worse for higher values. When A is large, the uniform spreading of workers costs too much because the nonphantom proportion is very small at old listing ages. Therefore the random search allocation leads to higher unemployment than the directed search allocation. When A is smaller, the uniform density is not too costly because the nonphantom proportion remains high until the termination age.

Random search does better than directed search when α and A are small. This is explained by the large magnitude of the dynamic externality when α is low and the small inefficiency of random search when A is short. This issue is investigated in more detail in the Web Appendix.

5 Extensions

In this section, we present four extensions to our model. First, we allow for the possibility that some phantoms may be withdrawn before the termination age, A. Second, we allow for vacancy

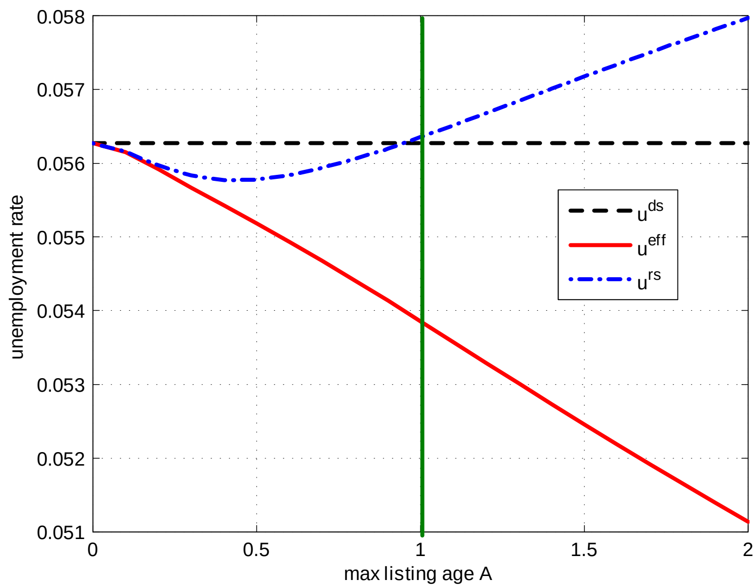

Figure 9: Unemployment rate in the three allocations as a function of the max listing age. The green line corresponds to $A = 1$, the value used in the baseline calibration.

renewal, i.e., that employers may relist their vacancies as new ones before the termination age is reached. Third, we consider the possibility that some job listings may be "lemons," i.e., jobs that no workers are willing to take. Finally, we consider wage bargaining and compare it to the efficient wage schedule.

5.1 Phantom death rate

In this extension, we introduce a new parameter, δ, the Poisson rate at which phantoms die. When this parameter is zero, we are back to the basic model and obsolete information only disappears when job listings reach the termination age A. When this parameter tends to infinity, phantoms die immediately. The law of motion for phantoms is now $\dot{p} = M - \delta p$ and the corresponding motion of the nonphantom proportion is $\dot{\pi}/\pi = -(m(1,\theta)/\theta) + \delta(1 - \pi)$.

We start from the baseline calibration displayed in Section 4, Table 1, and progressively increase the phantom death rate. Figure 10 shows the resulting unemployment rate. It decreases from $\mathsf{u}^{\mathrm{ds}} = 5.63\%$ when $\delta = 0$ (the baseline case) to $\mathsf{u}^{\mathrm{ds}} = 4.06\%$ when δ tends to infinity (the case with no phantoms). For example, suppose employers withdraw their obsolete listings in one week on average rather than waiting for the termination age of A. This corresponds $\delta = 4.0$. The unemployment rate would then decrease to 4.85%.

Why do phantoms survive even though removing them would result in employment gains?

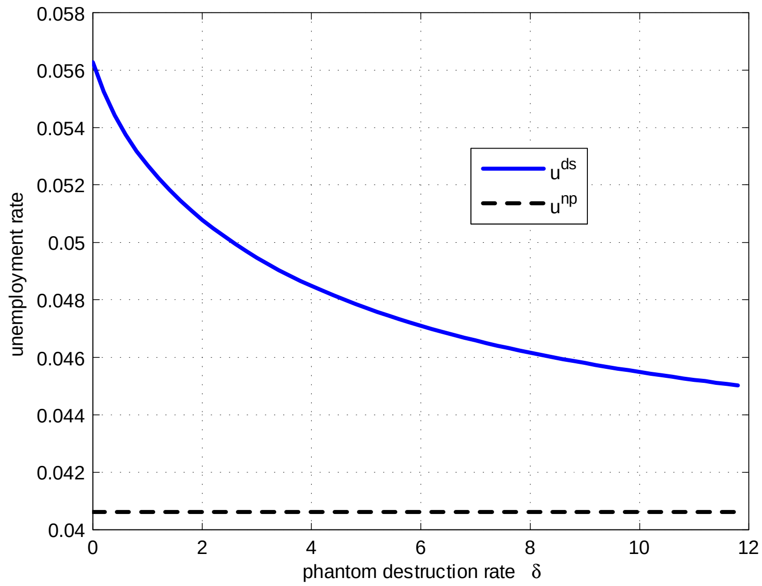

Figure 10: Unemployment rate and phantom destruction

On job platforms, the cost of removing phantoms is generally small, but firms do not perceive any private benefit in return. On Craigslist, for example, the direct cost involves only editing the job listing and deleting it. However, there are additional private costs to removing the listing related to the uncertainty surrounding the actual hiring of the recruitee. On net, there is no reason why a successful recruiter would make the effort to remove the listing.

One might also ask, if phantoms cause significant inefficiencies, why don't job search platforms do something to minimize the problem? One can find many complaints on the web about obsolete information on job boards (see the supplementary appendix of Chéron and Decreuse, 2017), but such customer dissatisfaction has not been directly addressed yet. One reason may be that it is the firms that pay the job boards, but, even so, firms have an interest in the job board attracting many applicants. Job search engines compete over pricing schemes and services offered to employers and job seekers. They provide different packages and some aspects of these packages may affect the nonphantom proportion on the website. The case of indeed.com is interesting. Indeed.com uses pay-per-click pricing. Employers only pay when someone clicks to view their jobs. Employers then have an incentive to remove obsolete ads to avoid paying for additional clicks.

The employment gains indicated by Figure 10 do not take into account that some listings specify an application deadline. Application deadlines would mitigate the phantom problem from the perspective of job seekers. However, this is at the cost of creating another problem of obsolete information, but this time for the employers. By committing to wait until the application deadline, employers risk losing some job seekers who may find another job in the meantime. In the terminology of our paper, these applicants would represent phantom job seekers.

5.2 Vacancy renewal

Some websites offer employers the possibility to relist vacancies as new before the age limit is reached. If the previous ad disappears, the vacancy is characterized by a single ad. If it does not, then there may be several ads for the same vacancy at one time, each corresponding to a different age. The first case allows us to examine the pure effect of renewing vacancies. In the second case, the pure renewal effect is mixed with the effect of differing search intensity. We focus on the first case so that we can look at the pure renewal effect.

We assume that vacancy renewal occurs between ages 0 and A randomly at rate γ. This modifies the dynamics of vacancies as follows:

$$\dot{v} = -M - \gamma v, \tag{24}$$

$$v(0) = \lambda(1 - \mathsf{u}) + \gamma \mathsf{v} + v(A). \tag{25}$$

At each age $a \leq A$, the measure of vacancies decreases by the flow of matches formed with vacancies of that age and by the flow of vacancies that are relisted. The inflow of new vacancies is then equal to newly destroyed employment relationships $\lambda(1-\mathsf{u})$ plus the numbers of randomly renewed vacancies, $\gamma\mathsf{v}$, and deterministically renewed vacancies, $v(A)$. The rest of the model is unchanged.

We now proceed as before. We first derive the directed search allocation, and then construct the other allocations. Under directed search, the allocation of job seekers across the different listing ages equalizes the job-finding rates. Thus $\mu(a) = \pi(a)m(1, \theta(a)) = m(1, \theta(0))$. The nonphantom proportion is now affected by vacancy renewal:

$$\frac{\dot{\pi}}{\pi} = -m(1, \theta)/\theta - \gamma(1 - \pi), \tag{26}$$

with $\pi(0) = 1$. The additional term $-\gamma(1 - \pi)$ captures the fact that vacancy renewal speeds up the decline of the nonphantom proportion with listing age. The random search allocation results when $\theta(a) = \theta(0)$ for all $a \in [0, A]$. The constrained efficient allocation is computed from Theorem 1, proof of Proposition 3 in the Appendix. In this case, unlike in the basic model, we cannot analytically solve for the policy function described in Proposition 5.

We continue to assume that $A = 1$. The model calibration then involves one new parameter, γ, the rate of vacancy renewal. To set this parameter, we use information from Craigslist. In

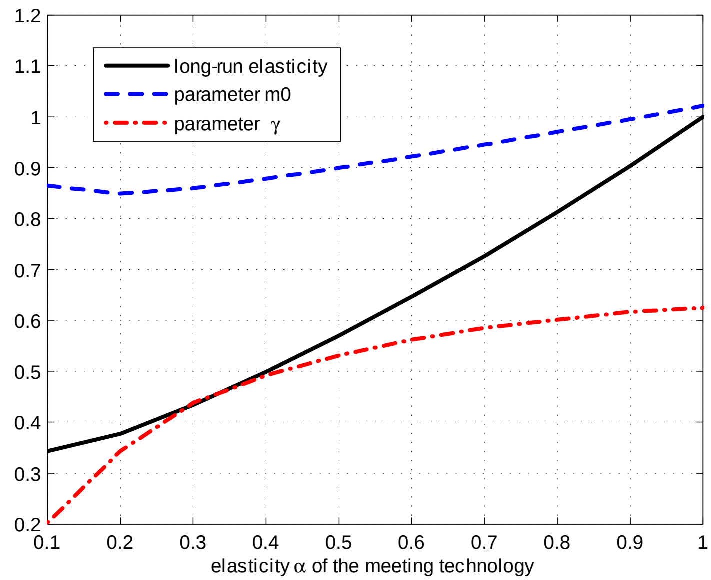

Figure 11: Parameters γ and m_0 as functions of the elasticity α of the meeting technology

Appendix B, we use a cross-section of ads from June 10, 2015 suggesting that the proportion of newly created vacancies in a new cohort of vacancies is about 50%. The corresponding theoretical moment is $\lambda(1-\mathsf{u})/[\lambda(1-\mathsf{u})+\gamma\mathsf{v}+v(1)]$, i.e., the ratio of newly destroyed employment relationships to overall new job listings. We let α vary from 0 to 1, and fix γ and m_0 so that $\lambda(1-\mathsf{u})/[\lambda(1-\mathsf{u})+\gamma\mathsf{v}+v(1)] = 50\%$ and $\mathsf{u} = 5.63\%$.

Figure 11 plots the parameters m_0 and γ as functions of α. This figure also displays the corresponding long-run elasticity of the mean job-finding rate with respect to the vacancy-to-unemployed ratio v/u. This elasticity is slightly lower than in the baseline case for given α. It is about 0.4 when $\alpha = 0.2$, substantially more than $\alpha = 0.15$ in the baseline calibration. This implies $m_0 = 0.8488$ and $\gamma = 0.3438$.

The three allocations corresponding to this parameterization closely resemble the baseline case. (See the Web Appendix for a complete description.) Therefore we only comment here on the distributions of job seekers by listing age. The spike at newly created listings is less pronounced than in the basic model: 51% of the job seekers focus on job listings aged less than 48 hours, against 69% in the baseline calibration. This obtains even though vacancy renewal concentrates the distribution of listings at low ages. The reason is that the calibrated value of α is larger in the renewal case than in the baseline.

Vacancy renewal does not alter the main conclusions of Section 4. In particular, phantoms have a substantial impact on overall unemployment. The unemployment rate declines to 4.26%

in the absence of phantoms, i.e., when $A \to 0$. Thus obsolete information accounts for $(5.63 - 4.26)/(5.63 - 3.77) \approx 75\%$ of overall frictions and $(5.63 - 4.26)/5.63 \approx 25\%$ of unemployment.

As we saw in Section 4, modest employment gains can be expected from more efficient job search. We have $\mathsf{u}^{\text{eff}} = 5.51\%$, $\mathsf{u}^{\text{ds}} = 5.63\%$ and $\mathsf{u}^{\text{rs}} = 5.68\%$.

5.3 Lemons vs phantoms

In our model, the only reason that workers pay attention to listing age is because they are concerned about phantoms. We now turn to another possible reason for searchers to be concerned with listing age, namely, the existence of jobs that are lemons. Like phantoms, lemons do not lead to profitable matches and so create congestion for job seekers. Older job listings are more likely to be lemons. Workers therefore have an incentive to search for young listings.

Our main result is that lemons are unlikely to affect job seekers as much as phantoms do. Phantoms are a by-product of match formation and so increase with listing age. In contrast, to have a large effect, the proportion of lemons must be very large among cohorts of new vacancies to generate the kind of job search behavior caused by phantoms.

To focus on the role of lemons, we shut down phantom creation. Let l_0 be the number of lemons in each new cohort of vacancies. The number of matches at age a is $M(a) = u(a)\pi(a)m(1, \theta(a))$, where now $\pi(a) = v(a)/(v(a) + l_0)$, the nonlemon proportion, and tightness is $\theta(a) = (v(a) + l_0)/u(a)$. We assume that the inflow of new lemons is proportional to the inflow of newly created vacancies, i.e., $l_0 = \beta v(0) \geq 0$. Therefore each new vacancy comes at the cost of having β lemons. These lemons disappear at age A like other listings. The initial nonlemon proportion is $\pi(0) = v(0)/(v(0) + l_0) = 1/(1 + \beta)$.

Under directed search, workers spread themselves across listing ages so that the job-finding rate is the same at all ages. Thus $\pi(a)m(1, \theta(a)) = \pi(0)m(1, \theta(0))$ for all $a \in [0, A]$. Under random search, job queues are the same at all listing ages. That is, $\theta(a) = \theta(0)$ for all $a \in [0, A]$. Under constrained efficient search, job seekers are allocated to mitigate the externality caused by lemons. This allocation is computed from Theorem 1 in the proof of Proposition 3 in Appendix A.

We consider a parameterization similar in spirit to the one used in Section 4, Table 1. However, we modify it to account for lemons. The key new parameter is the lemon proportion among new ads. We set β so that $\pi_0 = 90\%$, which we believe is a realistic number. The other important parameter is α, the elasticity of the contact technology. Here again we choose α so that the long-run elasticity of the matching function is 0.4. We continue to use $x = \mathsf{v}/\mathsf{u} = 0.33$ and adjust the scale parameter m_0 so that the predicted unemployment rate in the directed search allocation equals 5.63%.

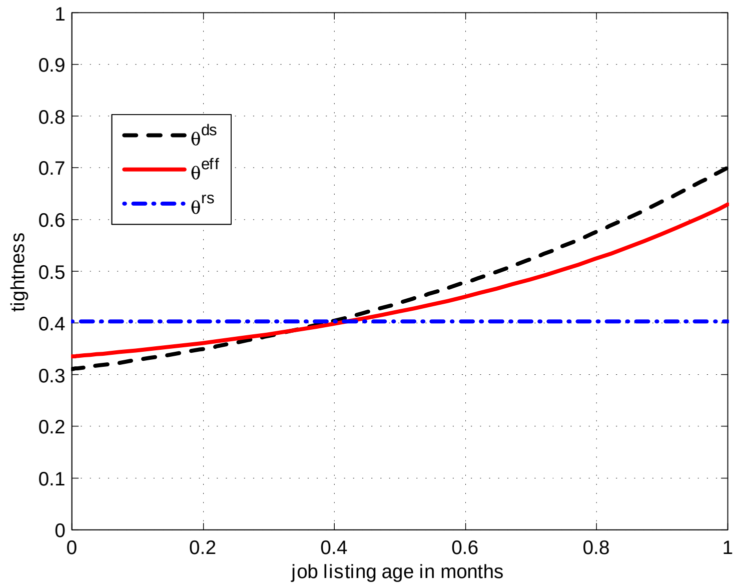

Figure 12: Tightness by listing age with lemons

This parameterization strategy leads to $\alpha = 0.35$ and $m_0 = 0.8541$. On the one hand, the scale parameter m_0 of the contact technology is lower than in the baseline model with phantoms (0.8541 against 1.0671). On the other hand, the elasticity of the contact technology is larger (0.35 against 0.2). Changes in the vacancy to unemployed ratio $x = \mathsf{v}/\mathsf{u}$ still affect the nonlemon proportion and the distribution of job seekers across listing ages. However, such effects are small and a higher elasticity α is required to match a long-run elasticity equal to 0.4.

In line with these comments, the striking feature of the directed search allocation is its high nonlemon proportion. It starts at 90% and falls to 70% in one month with an average of 82%. Job seekers adjust their behavior to account for this overall decline in the nonlemon proportion. However, this adjustment is necessarily small. Figure 12 shows the equilibrium tightness by listing age in the three allocations. Tightness in the directed search allocation increases from 0.3 to 0.7 in one month. Interpreting the inverse of tightness as the typical job queue, i.e., the number of job seekers per job listing, Figure 12 implies that the average job queue is roughly the same during the first and second days of the listing. In the same spirit, the density of job seekers is roughly constant during the first 48 hours of a listing.

Figure 12 also shows the (small) difference between the constrained efficient allocation and the directed search allocation. This difference between the two allocations is due to another kind of externality. The constrained efficient allocation takes into account that the creation of a new job goes along with β lemons. Therefore the social marginal value of vacancies is smaller at the end of the age span (where they are about to be replaced by other vacancies plus lemons) than at its beginning (where this replacement has already occurred). The magnitude of the externality is small. The unemployment rate is 5.63% in all three allocations.

Matching does not generate the kind of intertemporal frictions that we observed with phantoms. Thus the extent of additional frictions is limited by the lemon proportion among newly created vacancies. The directed search allocation fails to reproduce the patterns of tightness and job seeker density by listing age shown with phantoms unless the initial proportion is pushed to very large values – well over 30%. In the Web Appendix, we describe the three allocations completely.

5.4 Wage bargaining

Wage contracts that condition the wage on the listing age may internalize the externality induced by phantom vacancies. However, we do not observe such contracts being offered and it would be unrealistic to consider them. However, decentralized bargaining between firms and workers could lead to age-dependent wages. The purpose of this section is to consider this possibility.

We focus on Nash bargaining, by far the most popular way to endogenize wages in the search and matching literature. The case for Nash bargaining is intuitive: the bargained wage generally depends on each agent's outside option and the value of keeping the position vacant changes with the listing age. We rule out wage renegotiation and assume that the wage is fixed once an agreement is reached between the firm and the worker.

Let $w(a)$ be the bargained wage for a vacancy filled at age a and U and $W(a)$ denote, respectively, the values of being unemployed and employed. Similarly, $V(a)$ and $J(a)$ denote, respectively, the values of a vacancy and a filled job. These values are defined as follows:

$$
\begin{aligned}
rU &= \max_{a\geq 0}\{\pi(a)m(1,\theta(a))[W(a)-U]\}, &(27)\\
rW(a) &= w(a)+\lambda[U-W(a)], &(28)\\
rV(a) &= \eta(a)[J(a)-V(a)]+V'(a), &(29)\\
rJ(a) &= 1-w(a)+\lambda[V(0)-J(a)], &(30)
\end{aligned}
$$

In equation (29), the flow value of a vacancy equals the job-filling rate $\eta(a)$ times the change in value $J(a)-V(a)$ plus the change in value due to aging $V'(a)$. As the listing gets renewed at age A, the value of a vacancy also satisfies $V(0)=V(A)$.

Nash bargaining involves the division of the match surplus $S(a)\equiv W(a)-U(a)+J(a)-V(a)$ between the two parties. That is, $W(a)-U=\rho S(a)$ and $J(a)-V(a)=(1-\rho)S(a)$, where $\rho\in(0,1]$ is workers' bargaining power. In equilibrium, all submarkets are open and $\pi(a)m(1,\theta(a))S(a)=m(1,\theta(0))S(0)$ for all $a\in[0,A]$. Let $s(a)=S(a)/S(0)$, then we have

$\pi(a)m(1,\theta(a))s(a)/m(1,\theta(0)) = 1$. At each age, job seekers spread over submarkets so that the product of the job-finding rate and the match surplus stays constant.

There is a close connection between the rules allocating job seekers across listing ages in the constrained efficient allocation and the directed search allocation with Nash bargaining. In the constrained efficient allocation, Proposition 3, part (i), gives the rule allocating the job seekers to the different submarkets, and, above, we have a similar rule in the directed search allocation with Nash bargaining (denoted $^{\text{nb}}$ hereafter). We reproduce these rules here for convenience:

$$\begin{aligned} \pi^{\text{nb}}(a)m(1,\theta^{\text{nb}}(a))s(a)/m(1,\theta^{\text{nb}}(0)) &= 1, & (31)\\ (1-\alpha(\theta^{\text{eff}}(a)))\pi^{\text{eff}}(a)m(1,\theta^{\text{eff}}(a))(s_2(a)-s_1(a)) &= 1. & (32) \end{aligned}$$

Generically, the two rules highlighted by equations (31) and (32) cannot hold simultaneously. On the contrary, suppose they do. This requires that $s(a) = [s_2(a) - s_1(a)](1 - \alpha(\theta^{\text{eff}}(a)))m(1,\theta^{\text{eff}}(0))$ for all $a \in [0, A]$. In particular, $s(0) = 1$ and $s(A) = m(1,\theta^{\text{eff}}(0)/[\pi^{\text{eff}}(A)m(1,\theta^{\text{eff}}(A)]$. A key result given in Proposition 3, part (iii), is that the job-finding rate at listing age 0, $m(1,\theta^{\text{eff}}(0))$, generally differs from the job-finding rate at age A, $\pi^{\text{eff}}(A)m(1,\theta^{\text{eff}}(A)$. The former is actually larger than the latter when $\alpha'(.) \leq 0$.

What about the directed search allocation with Nash bargaining? The equality $s(0) = 1$ is a direct implication of the definition of the relative match surplus and, therefore, obviously holds. As for the second equality, we use equations (27)-(30) to show $(r+\lambda)S(a) = 1 + \lambda V(0) - (r+\lambda)V(a) - rU$. Therefore $V(0) = V(A)$ implies that $S(0) = S(A)$. In turn, $s(0) = 1 = s(A)$. Hence the relative match surplus is the same at the beginning and at the end of possible listing ages. Proposition 3 shows that the shadow value of vacancies s_1 has the same property, i.e., $s_1(0) = s_1(A)$. However, what matters for constrained efficiency is the differential $s_2 - s_1$. The private match surplus does not account for the shadow value of phantoms. Therefore it does not correctly price the informational externality induced by match formation – it does not price it at all. This explains why Nash bargaining cannot restore efficiency.

Another way to understand this result is to examine the wage schedule. Combining equations (27) and (28) gives

$$\omega^{\text{nb}}(a) = \frac{r+\lambda+\pi^{\text{nb}}(a)m(1,\theta^{\text{nb}}(a))}{r+\lambda+m(1,\theta^{\text{nb}}(0))}s(a),$$

which implies that $\omega^{\text{nb}}(0) = 1 = \omega^{\text{nb}}(1)$. Nash bargaining cannot predict $\omega^{\text{nb}}(A) > \omega^{\text{nb}}(0)$ as required to decentralize the efficient allocation.

6 Conclusion

Our paper has focused on two important features of the labor market. First, much of the information available to job seekers is out of date; many job listings are for phantom vacancies, and the probability that a listing corresponds to an unfilled vacancy decreases with listing age. Second, job seekers are aware of these facts, and they adapt their search behavior accordingly. In particular, workers pay attention to listing age when deciding how to direct their search.

We present three main results. First, we characterize the directed search allocation of job seekers across listing ages by a simple no-arbitrage condition. The allocation of job seekers across submarkets must be such that the job-finding rate is the same at all listing ages. Second, we characterize the constrained efficient allocation, that is, the allocation of job seekers across listing ages that a social planner – who, like job seekers, is unable to distinguish between real vacancies and phantoms – would choose. We describe a dynamic congestion externality associated with phantoms; specifically, the social cost caused by a young phantom exceeds the corresponding cost of an older one. Third, by calibrating our model to US labor market data over 2000-2008, we provide evidence that phantoms are quantitatively important. Our baseline calibration indicates that phantoms account for approximately 28% of overall unemployment and 85% of the unemployment caused by matching frictions. Despite the fact that phantoms contribute significantly to unemployment, the social planner allocation offers scant improvement relative to the directed search allocation. The reason is simply that efficient job search leads to more filled jobs, and these additional matches add to the stock of phantoms.

In addition to our main results, we consider several extensions. Specifically, we allow for the possibility that some phantoms "die" at an exogenous Poisson rate as well as for the possibility that some employers repost their vacancies, i.e., reset the listing age of their vacancies to zero, again at an exogenous Poisson rate. These extensions do not change our qualitative results. We also consider an alternative congestion externality, namely, that some jobs are "lemons," that is, jobs that no one accepts. We show that an unrealistically large mass of lemons would be required to reproduce the patterns we see in the data. Finally, we consider the possibility, albeit not one observed in practice, of wages that vary with listing age. We derive the efficient wage schedule, that is, the schedule $\omega(a)$ that implements the constrained efficient allocation under directed search. We show that Nash bargaining gives wages that vary with listing age but that irrespective of the value taken by the share parameter, the constrained efficient allocation cannot be achieved using this wage schedule.

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APPENDIX

A Proofs

Proof of Proposition 1:

(i). We have $\mathsf{v} = v(0)\int_0^A \pi(a)da$. Thus $\mathsf{v} = \lambda(1-\mathsf{u})\theta/m(1,\theta)$, where u is given by (7). Consider the function $\psi : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ such that

$$\psi(\theta, A) = 1 - \mathsf{u} + \mathsf{v} - K.$$

The steady-state tightness, if any, is such that $\psi(\theta^{\mathrm{rs}}, A) = 0$. The properties of the function m imply that $\lim_{\theta\to 0}\psi(\theta, A) = -K < 0$ and $\lim_{\theta\to\infty}\psi(\theta, A) = \infty$. By continuity, there is $\theta^{\mathrm{rs}} > 0$ such that $\psi(\theta^{\mathrm{rs}}, A) = 0$. Moreover, $\psi_\theta(\theta, A) > 0$ because u strictly decreases with θ, whereas v strictly increases with it. It follows that θ^{rs} is unique.

(ii) is straightforward.

(iii). Consider the function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f(x) = [1 - \exp(-\eta x)]/(\eta x)$, with $\eta > 0$. We have $f(0) = 1$ and $\lim_{x\to\infty} f(x) = 0$. Taking the derivative of f with respect to x, we obtain $f'(x) = g(x)/(\eta x^2)$, with $g(x) = e^{-\eta x}(1+\eta x) - 1$. Since $g(0) = 0$ and $g'(x) = -\eta^2 x e^{-\eta x} < 0$, we have $f'(x) < 0$. It follows that $\psi_A(\theta, A) < 0$ and $d\theta^{\mathrm{rs}}/dA > 0$ by the implicit function theorem. Manipulating the condition $\psi(\theta^{\mathrm{rs}}, A) = 0$, we can show that the job-finding rate of the random search allocation is $\mu = \lambda K/(1 - K + \lambda\theta^{\mathrm{rs}}/m(1, \theta^{\mathrm{rs}}))$. The right-hand side of this equality strictly decreases with θ^{rs}. Therefore $d\mu/dA < 0$. It follows from (7) that u strictly increases with A.

Proof of Proposition 2:

(i). This property follows from the fact that workers equalize the job finding rate across submarkets and repeats equation (8).

(ii). The ODE (12) for θ^{ds} follows from differentiating (8) with respect to age a. That $\dot{\pi}/\pi = -m(1,\theta)/\theta$ follows from equation (4) with $\partial\pi(a,t)/\partial t = 0$.

We now prove existence of a θ_0 that is defined implicitly by the resource constraint, $1 - \mathsf{u} + \mathsf{v} = K$. As indicated in the text,

$$\mathsf{u} = \frac{\lambda}{\lambda + m(1, \theta_0)}.$$

The inflow of new vacancies is $v(0) = \lambda(1 - \mathsf{u}) + v(A)$, whereas $v(a) = v(0)\pi(a)$. Thus $v(0) = \lambda(1-\mathsf{u})/(1 - \pi(A))$ and

$$\mathsf{v} = \frac{\lambda(1-\mathsf{u})}{(1 - \pi(A))}\int_0^A \pi(a, \theta_0)da$$

with $\pi(A) = \exp\left[-\int_0^A m(1, \theta(a, \theta_0))/\theta(a, \theta_0)da\right]$.

At given θ_0, equation (12) determines a unique function $\theta(a, \theta_0)$. This function strictly increases with a and θ_0. Finding a θ_0 that satisfies the resource constraint is then equivalent to finding θ_0 such that $\psi(\theta_0) = 0$, where $\psi : \mathbb{R}_+ \to \mathbb{R}$ is defined as

$$\psi(\theta_0) = 1 - \mathsf{u} + \mathsf{v} - K$$

First, $\lim_{\theta_0 \to 0}(1-\mathsf{u}) = 0$ and $\lim_{\theta_0 \to 0} \mathsf{v} = \lim_{\theta_0 \to 0}[\frac{\lambda(1-\mathsf{u})}{(1-\pi(A))} \int_0^A \pi(a, \theta_0) da] = 0$. Thus, $\lim_{\theta_0 \to 0} \psi(\theta_0) = -K$. Second, $\lim_{\theta_0 \to \infty}(1 - \mathsf{u}) = 1$ and $\lim_{\theta_0 \to \infty} \mathsf{v} = \infty$ so that $\lim_{\theta_0 \to \infty} \psi(\theta_0) = \infty$. Thus there is $\theta_0 > 0$ such that $\psi(\theta_0) = 0$.

(iii). The job-finding rate is constant as a result of the directed search assumption. As for the nonphantom proportion π, we have $\dot{\pi} = -\pi m(1, \theta)/\theta < 0$. The ODE implies that $\dot{\theta}$ has the sign of $-\dot{\pi}$; therefore $\dot{\theta} > 0$. Lastly, we have $\eta(a) = m(1, \theta(a))/\theta(a)$, which strictly decreases with a because $m(1, \theta)/\theta$ strictly decreases with θ.

Proof of Proposition 3:

Preamble.—We consider a generalized problem where vacancies last at most $A > 0$. Then they are renewed. Renewal also occurs randomly at constant rate γ, and phantoms depreciate at rate $\delta > 0$. Each cohort of new listings includes $l_0 = \beta v(0)$ jobs that are lemons. The basic model of Sections 3 and 4 obtains when $\delta = \gamma = l_0 = 0$. The model with phantom destruction of Section 5.1 involves $\delta > 0$ and $\gamma = l_0 = 0$. The model with vacancy renewal of Section 5.2 requires $\gamma > 0$ and $\delta = l_0 = 0$. The model with lemons of Section 5.3 requires δ to tend to infinity.

Planners' problem in raw form.—The planner's problem is the following

$$\max_{\theta(.)} - \int_0^A v(a) da \tag{$*$}$$

subject to

$$\begin{aligned}
\dot{v} &= -\eta(\theta)v - \gamma v, \\
\dot{p} &= \eta(\theta)v - \delta p, \\
v(0) &= \lambda \left(K - \int_0^A v(a) da \right) + \gamma \int_0^A v(a) da + v(A), \\
1 - \int_0^A [(v(a) + p(a) + l_0)/\theta(a)] da + \int_0^A v(a) da &= K, \\
l_0 - \beta v(0) &= 0, \\
p(0) &= 0.
\end{aligned}$$

There are two state variables v and p, and one control variable θ.

We now prove the following result.

Theorem 1 In the efficient allocation, the following properties hold:

$$(s_2 - s_1)(1 - \alpha(\theta))\pi m(1, \theta) = 1, \tag{33}$$

where the functions $s_1 : [0, A] \to \mathbb{R}$ and $s_2 : [0, A] \to \mathbb{R}$ are such that

$$\dot{s}_1 = Z + \gamma s_1 - \frac{1 - (1 - \alpha)\pi}{(1 - \alpha)\pi\theta}, \tag{34}$$

$$\dot{s}_2 = \delta s_2 + 1/\theta, \tag{35}$$

where $Z \in \mathbb{R}$ and $s_1(a) < s_2(a) \leq s_2(A) = 0$ for all $a \in [0, A]$.

Planner's problem in standard form.—To solve $(*)$, we transform the two integral constraints into differential equations with associated boundary conditions. Let $x_1(a) = \int_0^a v(s)ds$ and $x_2(a) = \int_0^a \{[v(s) + p(s) + l_0]/\theta(s) - v(s)\}ds$ for all $a \in [0, A]$. The planner's problem is now the following:

$$\max_{\theta(.)} - \int_0^A v(a)da \tag{$*$}$$

subject to

$$\begin{aligned}
\dot{v} &= -\eta(\theta)v - \gamma v, \\
\dot{p} &= \eta(\theta)v - \delta p, \\
\dot{x}_1 &= v, \\
\dot{x}_2 &= (v + p + l_0)/\theta - v, \\
\lambda K - v(0) + v(A) + (\gamma - \lambda)x_1(A) &= 0, \\
K - 1 + x_2(A) &= 0, \\
l_0 - \beta v(0) &= 0, \\
p(0) &= x_1(0) = x_2(0) = 0.
\end{aligned}$$

Solving.—Let $H : \mathbb{R}^4 \times \mathbb{R}^4 \times \mathbb{R} \to \mathbb{R}$ be the Hamiltonian such that:

$$H(y, \sigma, \theta) = -v - \sigma_1[\eta(\theta) + \gamma]v + \sigma_2[\eta(\theta)v - \delta p] + \sigma_3 v + \sigma_4[(v + p + l_0)/\theta - v],$$

where σ_1 and σ_2 are the costates associated with the state variables v and p, and σ_3 and σ_4 are the costates associated with the auxiliary variables x_1 and x_2, $y = (v, p, x_1, x_2)$ and $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$. We define $\phi_1(y(0), y(A)) = \lambda K - v(0) + v(A) + (\gamma - \lambda)x_1(A)$, $\phi_2(y(0), y(A)) = K - 1 + x_2(A)$, $\phi_3(y(0), y(A)) = l_0 - \beta v(0)$, $\phi_4(y(0), y(A)) = p(0)$, $\phi_5(y(0), y(A)) = x_1(0)$, $\phi_6(y(0), y(A)) = x_2(0)$.

We now introduce the main result that we need, a version of Theorem 11.1 in Hestenes (1966), which accounts for equality constraints on the state variables at the beginning and at the end of possible ages.

Theorem 2 (*Maximum principle*) Suppose $\theta^*(.)$ is optimal for the optimization problem $(*)$ and let y^* be the corresponding trajectory of the state variables. Then there exists $\sigma^* : [0, A] \to \mathbb{R}^4$ and $\rho^* \in \mathbb{R}^6$ such that for all $a \in [0, A]$:

A. Maximization principle

$$H(y^*(a), \sigma^*(a), \theta^*(a)) = \max_{\theta \geq 0} H(y^*(a), \sigma^*(a), \theta) \tag{36}$$

B. Adjoint equations

$$\dot{\sigma}_i^* = -\frac{\partial H(y^*(a), \sigma^*(a), \theta^*(a))}{\partial y_i(a)}, i = 1, 2, 3, 4 \tag{37}$$

C. Transversality conditions

$$\begin{aligned} \sigma_i^*(0) &= \sum_{j=1}^{6} \rho_j^* \frac{\partial \phi_j}{\partial y_i(0)}, i = 1, 3, 4, & (38) \\ \sigma_i^*(A) &= -\sum_{j=1}^{6} \rho_j^* \frac{\partial \phi_j}{\partial y_i(A)}, i = 1, 3, 4, & (39) \\ \sigma_2^*(A) p^*(A) &= 0. & (40) \end{aligned}$$

The transversality conditions feature the Lagrange multiplier ρ^*, which is associated with the constraints $\phi_i(.) = 0$, $i = 1, ..., 6$. In practice, we neglect the trivial constraints $\phi_4(.) = \phi_5(.) = \phi_6(.) = 0$. There are no constraints involving the initial or final number of phantoms. The associated transversality condition is $\sigma_2(A)p(A) = 0$.

Applying Theorem 2.—Hereafter we neglect the star * notation. We first focus on the adjoint equations and transversality conditions. This gives

$$\begin{aligned} \frac{\partial H}{\partial v} &= -1 + (\sigma_2 - \sigma_1)\eta(\theta) - \sigma_1 \gamma + \sigma_3 + \sigma_4(\theta^{-1} - 1) = -\dot{\sigma}_1, & (41) \\ \frac{\partial H}{\partial p} &= -\delta \sigma_2 + \sigma_4 / \theta = -\dot{\sigma}_2, & (42) \\ \frac{\partial H}{\partial x_1} &= \frac{\partial H}{\partial x_2} = 0 = -\dot{\sigma}_3 = -\dot{\sigma}_4, & (43) \\ \sigma_1(0) &= -\rho_1 - \beta \rho_3, & (44) \\ \sigma_1(A) &= -\rho_1 & (45) \\ \sigma_2(A) p(A) &= 0, & (46) \\ \sigma_3(A) &= \rho_1(\lambda - \gamma), & (47) \\ \sigma_4(A) &= -\rho_2. & (48) \end{aligned}$$

It follows that $\sigma_3(a) = \rho_1(\lambda - \gamma)$ and $\sigma_4(a) = -\rho_2$ for all $a \in [0, A]$.

We now turn to the maximization principle given in A above. Suppose that the first-order condition is necessary. Then we have

$$\frac{\partial H}{\partial \theta} = (\sigma_2 - \sigma_1)\eta'(\theta)v - \sigma_4 \frac{v + p + l_0}{\theta^2} = 0. \tag{49}$$

Re-arranging terms, we obtain

$$-(\sigma_2 - \sigma_1)(1 - \alpha(\theta))\pi m(1, \theta) = \sigma_4 \tag{50}$$

Taking the second derivative of the Hamiltonian gives

$$\begin{aligned}
\frac{\partial^2 H}{\partial \theta^2} &= (\sigma_2 - \sigma_1)\eta''(\theta)v + 2\sigma_4 \frac{v + p + l_0}{\theta^3} \\
&= \frac{v + p + l_0}{\theta^3}\left[(\sigma_2 - \sigma_1)\frac{\theta\eta''(\theta)}{\eta'(\theta)}\frac{\theta\eta'(\theta)}{\eta(\theta)}\theta\eta(\theta)\pi + 2\sigma_4\right] \\
&= \frac{v + p + l_0}{\theta^3}\left[-(\sigma_2 - \sigma_1)\frac{\theta\eta''(\theta)}{\eta'(\theta)}(1 - \alpha(\theta))\pi m(1, \theta) + 2\sigma_4\right].
\end{aligned}$$

Once evaluated in the proposed maximum, we have

$$\frac{\partial^2 H}{\partial \theta^2} = \sigma_4 \frac{v + p + l_0}{\theta^3}\left[\frac{\theta\eta''(\theta)}{\eta'(\theta)} + 2\right]. \tag{51}$$

But $\theta\eta''(\theta)/\eta'(\theta) = m_{\theta\theta}(1, \theta)/\eta'(\theta) - 2 > -2$. Thus the candidate solution satisfies the second-order condition provided that $\sigma_4 = -\rho_2 < 0$, which implies $\rho_2 > 0$. By virtue of (50), this also implies $\sigma_2 - \sigma_1 > 0$ for all $a \in [0, A]$.

Putting things together.—From (41) and (42), we obtain

$$\dot{s}_1 = Z + \gamma s_1 - \frac{1 - (1 - \alpha)\pi}{(1 - \alpha)\pi\theta}, \tag{52}$$

$$\dot{s}_2 = \delta s_2 + 1/\theta, \tag{53}$$

where $s_1 \equiv \sigma_1/\rho_2$ and $s_2 \equiv \sigma_2/\rho_2$ are the normalized costates.

Equation (53) combined with the boundary condition $s_2(A) = 0$ implies that $s_2(a) < 0$ for all $a < A$. In turn, equation (50) implies that $s_1(a) < s_2(a) \leq 0$ for all $a \in [0, A]$.

Proof of Proposition 3. (i) is an immediate implication of Theorem 1 with $\gamma = \delta = l_0 = 0$. We have $s_1(0) = s_1(A) = -\rho_1/\rho_2 < 0$. Equation (52) implies that $Z > 0$. If not, $\dot{s}_1 < 0$, leading to $s_1(A) < s_1(0)$.

Using the different boundary constraints together with equation (50), we obtain $s_1(0) = s_1(A) = -[(1 - \alpha(\theta(A)))\pi(A)m(1, \theta(A))]^{-1}$ and $s_2(0) = [(1 - \alpha(\theta(0)))m(1, \theta(0))]^{-1} - [(1 - \alpha(\theta(A)))\pi(A)m(1, \theta(A))]^{-1}$.

To find B, we use the fact that the Hamiltonian stays constant over age. Then we derive B from the equality $H(y(0), \sigma(0), \theta(0)) = H(y(A), \sigma(A), \theta(A))$.

(ii) Differentiating equation (50) with respect to age, it comes

$$(\dot{s}_2 - \dot{s}_1)(1-\alpha(\theta))\pi m(1,\theta) + \left[\frac{-\dot{\alpha}}{1-\alpha} + \frac{\dot{\pi}}{\pi} + \alpha\frac{\dot{\theta}}{\theta}\right] = 0. \tag{54}$$

Using Proposition 3, part (i), we have $\dot{s}_2 - \dot{s}_1 = -B + [(1-\alpha)\pi\theta]^{-1}$. Therefore

$$\frac{-\dot{\alpha}}{1-\alpha} + \frac{\dot{\pi}}{\pi} + \alpha\frac{\dot{\theta}}{\theta} = B(1-\alpha(\theta))\pi m(1,\theta) - m(1,\theta)/\theta. \tag{55}$$

Noting that $\dot{\alpha} = \alpha'(\theta)\dot{\theta}$ and $\dot{\pi}/\pi = -m(1,\theta)/\theta$, we finally have

$$\frac{-\dot{\alpha}}{1-\alpha} + \alpha\frac{\dot{\theta}}{\theta} = B(1-\alpha(\theta))\pi m(1,\theta). \tag{56}$$

(iii) We know $\dot{\pi} < 0$. As $\alpha' = (1-\alpha)m'/m + \theta m''/m' \times m'/m$, we have $-\theta\alpha'/(1-\alpha) = -\alpha - \theta m''/m' \times \alpha/(1-\alpha)$. Therefore $\alpha - \theta\alpha'/(1-\alpha) > 0$ and $\dot{\theta} > 0$. In turn, this implies $\dot{\eta} < 0$. Since $s_2(0) = [(1-\alpha(\theta(0)))m(1,\theta(0))]^{-1} - [(1-\alpha(\theta(A)))\pi(A)m(1,\theta(A))]^{-1} < 0$, we have $(1-\alpha(\theta(0)))m(1,\theta(0)) > (1-\alpha(\theta(A)))\pi(A)m(1,\theta(A))$. This implies that $\mu(0) = m(1,\theta(0)) > \mu(A) = \pi(A)m(1,\theta(A))$ when $\alpha'(.) \leq 0$.

Proof of Proposition 4:

(i). Efficient tightness strictly increases with listing age (Proposition 3, part (iii)), whereas tightness is constant in the random search allocation (Proposition 1, part (i)).

(ii). The job-finding rate is such that $\mu(0) > \mu(A)$ in the constrained efficient allocation (Proposition 3, part (iii)), whereas it is constant in the directed search allocation (Proposition 2, part (i)).

Proof of Proposition 5:

In the directed search allocation, $\pi^{\mathrm{ds}}(a)m(1,\theta^{\mathrm{ds}}(a)) = m(1,\theta^{\mathrm{ds}}(0))$. In the Cobb-Douglas case, this is

$$\pi^{\mathrm{ds}}(a)m_0[\theta^{\mathrm{ds}}(a)]^{\alpha} = m_0[\theta^{\mathrm{ds}}(0)]^{\alpha}$$

so that $\theta^{\mathrm{ds}}(a) = \boldsymbol{\theta}^{\mathrm{ds}}(\pi(a),\theta^{\mathrm{ds}}(0))$.

In the efficient allocation in the Cobb-Douglas case, Proposition 3, part (ii), implies that $\alpha\dot{\theta}/\theta = B(1-\alpha)\pi m(1,\theta)$. Suppose that $\theta^{\mathrm{eff}}(a) = y(\pi(a))$ for all a. Using the fact that $\dot{\theta} = y'(\pi)\dot{\pi}$ and $\dot{\pi} = -\pi m(1,y)/y$, we obtain $y'(\pi) = -B(1-\alpha)y(\pi)^2/\alpha$. For an initial condition $y(1) = \theta_0 > 0$, the solution of this differential equation is $y(\pi) = \boldsymbol{\theta}^{\mathrm{eff}}(\pi,\theta_0)$. Therefore $\theta^{\mathrm{eff}}(a) = \boldsymbol{\theta}^{\mathrm{eff}}(\pi^{\mathrm{eff}}(a),\theta^{\mathrm{eff}}(0))$.

(ii). We first establish that $[1-\pi^{\mathrm{eff}}(A)]v^{\mathrm{eff}}(0) \geq [1-\pi^{\mathrm{ds}}(A)]v^{\mathrm{ds}}(0)$ and then proceed by contradiction. The flow of new vacancies is $v(0) = \lambda(1-\mathsf{u}) + v(A)$. Moreover, $v(A) = \pi(A)v(0)$.

Therefore minimizing u is equivalent to maximizing $v(0)-v(A) = [1-\pi(A)]v(0)$. Hence $\mathsf{u}^{\text{eff}} \leq \mathsf{u}^{\text{ds}}$ is equivalent to $[1-\pi^{\text{eff}}(A)]v^{\text{eff}}(0) \geq [1-\pi^{\text{ds}}(A)]v^{\text{ds}}(0)$. Now suppose that $\theta^{\text{ds}}(0) \geq \theta^{\text{eff}}(0)$. Since $\boldsymbol{\theta}^{\text{ds}}(\pi, \theta_0) > \boldsymbol{\theta}^{\text{eff}}(\pi, \theta_0)$, $\boldsymbol{\theta}^{\text{ds}}_{\theta_0}(\pi, \theta_0) > 0$ and $\boldsymbol{\theta}^{\text{eff}}_{\theta_0}(\pi, \theta_0) > 0$, this implies $\theta^{\text{ds}}(a) > \theta^{\text{eff}}(a)$ for all $a \in [0, A]$. In turn, $\pi^{\text{ds}}(A) = \exp[-\int_0^A m(1, \theta^{\text{ds}}(b))/\theta^{\text{ds}}(b)db] < \pi^{\text{eff}}(A)$ and, therefore, $v^{\text{eff}}(0) > v^{\text{ds}}(0)$. The definition of tightness implies that $1/\theta^{\text{ds}}(a) = u^{\text{ds}}(a)/v^{\text{ds}}(0) < 1/\theta^{\text{eff}}(a) = u^{\text{eff}}(a)/v^{\text{eff}}(0) < u^{\text{eff}}(a)/v^{\text{ds}}(0)$. Summing over a gives $\mathsf{u}^{\text{ds}} < \mathsf{u}^{\text{eff}}$, a contradiction.

B Setting parameters with Craigslist

We consider the 524,948 job listings advertised on Craigslist's US website shown on June 10, 2015. Considering all ads allows us to construct the distribution of listings by age and to discuss the magnitude of vacancy renewal.

Craigslist is a diverse classified advertisement website with sections devoted to jobs, housing, personals, services, etc. Advertisers choose a city, and their listing is valid for this area only. Job ads are cheap: \$25 each in all cities except for the San Francisco Bay Area where they cost \$75. Ads are valid for one month and are then destroyed. Job seekers visiting Craigslist's website first choose a city and a job type. Then they can refine their search and see ads by inverse order of creation, i.e., the most recent ads coming first.

Each ad is characterized by an ID number, a job type (32 categories), a city (23 cities), and a date of creation. Since the termination age of a listing is one month, all job listings in our sample were created between May 10 and June 10, 2015. The creation date is missing for a small subset of observations, all located in Atlanta. Therefore we drop these observations. The remaining sample contains 523,296 observations.

Employers can post the same listing several times. This is important for large employers needing to fill several positions, or recruiting for jobs with a high turnover. In its FAQ section, Craigslist promotes this behavior by forbidding listings for multiple positions. To the question "I have three positions to fill—can I put all three in one job ad?", they answer "One job description per job post please. As a rule of thumb, job titles should be singular, not plural." In practice many employers who plan several similar hirings post several listings, each having plural job titles.

In addition, Craigslist tries to limit multiple listings for the same job. In the FAQ section again, to the question "How often can I post?", they answer "You may post to one category and in one city, no more than once every 48 hours. If you try to post something similar to an active post of yours on the site, you may get a blocked message. Removing the similar active post should help, unless it is less than 48 hours old." This policy is hardly enforced: in our sample,

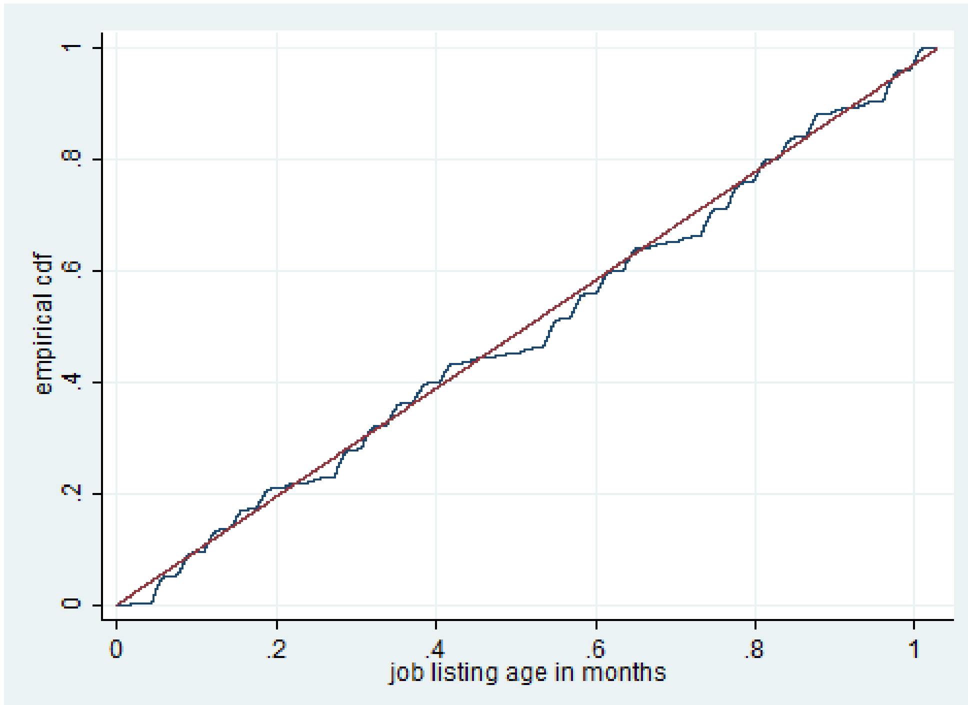

Figure 13: Empirical distribution of job listings by listing age (in months). Data source: Craigslist website, June 10, 2015

about 10% of the ads are duplicates of an ad that was posted less than 48 hours earlier.

Employers are invited to delete their obsolete listings. However, Chéron and Decreuse (2017) argue that employers are very unlikely to withdraw any ad. Indeed, they consider a particular date and find that the distribution of ads was uniform by week. The number of listings in their first week of existence was roughly equal to the number of ads in their second, third or fourth week of existence. This result is not compatible with employers withdrawing obsolete ads.

Figure 13 shows the empirical distribution of listings by age. It plots the cumulative distribution function of this distribution (blue line) and compares it to the distribution of the uniform law (red line). The maximum age is slightly above one month, because getting all these observations from the website is not instantaneous. The comparison reveals that the empirical distribution is very close to the uniform distribution. This indicates that employers do not withdraw ads before the termination age.

Data were collected city by city. In the Web Appendix, we examine the distribution by city, letting time start with the first observation in each city. Though there is evidence of a decreasing density in the very first days of a listing in some cities, roughly a quarter of the ads are aged less than 7.5 days in each city, the number predicted by the uniform law. Therefore we attribute fluctuations in the number of ads at lower quantiles of the distribution to fluctuations in the daily number of new vacancies.[7]

In Section 4, we calibrate the basic model assuming that listings cannot be renewed before the termination age. This is compatible with the Craigslist data if we assume that similar ads advertise distinct jobs. Alternatively, calibrating the model with vacancy renewal in Section 5.2 involves setting a value for the rate of vacancy renewal γ.

With our data there are two ways to identify similar ads. Some of the ads are associated with a "repost id", the id number of a previous ad used to repost the listing. Employers who repost the same ad for the same job are likely to use the initial id. Therefore this provides a simple way to quantify the magnitude of vacancy renewal. We compute the proportion of ads sharing the same repost id (which may correspond to one of these ads) in the same city and having the same job title. This proportion is about 30%.

However, some employers do not use the repost function when they renew their listings. They may do that to advertise for a different job as suggested by Craigslist. But they may also do that for alternative reasons, like avoiding being detected by Craigslist search engines when they post several similar ads in less than 48 hours. Another way to quantify similar ads consists of considering all ads with the same job title and with the same location. The proportion of ads uniquely advertising for their job decreases to 50%.

Therefore, the phenomenon of multiple ads for the same job concerns at most 50% of the ads. Many such ads actually are for similar but different jobs, or are from employers with recurrent hiring needs.

To calibrate the model of section 5.2, we limit vacancy renewal to cases where employers use the repost function when they want to advertise for the same job. To set the renewal rate γ, we compute the proportion that are not associated with a previous ad in a new cohort of listings. We obtain about 50%. In the directed search allocation, this proportion is $\lambda(1-\mathsf{u})/(\lambda(1-\mathsf{u})+\gamma\mathsf{v}+v(1))$. Therefore we fix γ and m_0 so that the theoretical proportion equals 50% and the unemployment rate is 5.63%.

[7] Going further requires following ads over time. However, the results shown here suggest there is not much to find.