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Abstract: We provide the first analysis of the risk-sharing implications of altruism networks. Agents are embedded in a fixed network and care about each other. We study whether altruistic transfers help smooth consumption and how this depends on the shape of the network. We identify two benchmarks where altruism networks generate efficient insurance: for any shock when the network of perfect altruism is strongly connected and for any small shock when the network of transfers is weakly connected. We show that the extent of informal insurance depends on the average path length of the altruism network and that small shocks are partially insured by endogenous risk-sharing communities. We uncover complex structural effects. Under iid incomes, central agents tend to be better insured, the consumption correlation between two agents is positive and tends to decrease with network distance, and a new link can decrease or increase the consumption variance of indirect neighbors. Overall, we show that altruism in networks has a first-order impact on risk and generates specific patterns of consumption smoothing.

Keywords: Altruism, Networks, Risk Sharing, Informal Insurance.

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I Introduction

Informal safety nets play a major role in helping people cope with negative shocks, even in high income economies. Informal transfers generally flow through family and social networks[1] and are motivated, to a large extent, by altruism. Individuals give support to others they care about.[2] Thus, informal insurance provided by networks appears to be mediated by altruistic transfers.

We provide the first analysis of the risk-sharing implications of altruism networks. We introduce stochastic incomes into the model of altruism in networks analyzed in Bourlès, Bramoullé & Perez-Richet (2017). Agents care about each other and the altruism network describes the structure of social preferences. For each realization of incomes, agents play a Nash equilibrium of the game of transfers. Our objective is to understand how altruistic transfers affect the risk faced by the agents. Do altruism networks help smooth consumption and how does this depend on the structure of the network?

We find that altruism networks have a first-order impact on risk and generate specific patterns of consumption smoothing. In line with Becker (1974)'s intuition, altruistic transfers often mimick classical insurance schemes.[3] Altruistic agents tend to give to others when rich and receive from others when poor, which reduces the variability of consumption. These effects depend on the shape of the network, however. Our analysis unfolds in three stages.

We first identify two important benchmarks where equilibrium transfers generate efficient insurance à la Townsend (1994). They yield efficient insurance for any random incomes if and only if the network of perfect altruistic ties is strongly connected. Every agent must give another agent's utility as much weight as she gives her own utility and these strong caring relationships must indirectly connect everyone. All agents then have equal Pareto weights. Perhaps more suprisingly, altruistic transfers also generate efficient insurance for small shocks when the network of transfers is weakly connected. This happens, for instance,

[1] See, e.g., Fafchamps & Gubert (2007), Fafchamps & Lund (2003), De Weerdt & Dercon (2006).
[2] See, e.g., Foster & Rosenzweig (2001), Leider et al. (2009), Ligon & Schechter (2012).
[3] In a context of household decision-making, "The head's concern about the welfare of other members provides each, including the head, with some insurance against disasters.", Becker (1974, p.1076).

in the presence of a rich benefactor in a connected community. Pareto weights then reflect individuals' positions in the transfer network. In either case, noncooperative transfer adjustments in response to shocks operate as if agents were following the directives of a social planner.

We next look at the general case. For utilities satisfying Constant Absolute Risk Aversion (CARA) or Constant Relative Risk Aversion (CRRA), we are able to bound the expected deviation between equilibrium and efficient consumption for arbitrary shocks. We find, in particular, that bridges play a major role under altruism and that informal insurance tends to be better when the average path length in the altruism network is shorter. As discussed below, these features appear to be specific to the model of altruism in networks and may help identify the motives behind informal transfers. We then characterize what happens for small shocks, leaving the structure of giving relationships invariant, and for arbitrary utilities. We show that altruistic transfers yield efficient insurance within the weak components of the network of transfers. Moreover, the reverse property holds generically: If altruistic transfers generate efficient insurance within groups, the structure of giving relationships must be invariant and these groups must be the weak components of the network of transfers. For small shocks, the extent of informal insurance thus critically depends on the number and sizes of the weak components of the transfer network.

Third, we study how informal insurance depends on the network's structure, with the help of numerical simulations. We consider a network of informal lending and borrowing relations in a village in rural India, from the data of Banerjee et al. (2013). Under iid incomes, we find that various measures of an agent's centrality are negatively correlated with consumption variance. Thus, a more central agent in the altruism network tends to have less variable consumption. We then show, analytically, that altruistic transfers generate positive correlation in consumption streams across agents. Shocks propagate in the altruism network. We find, numerically, that these correlations tend to decrease as the distance in the altruism network decreases. Finally, we look at the impact of adding a link within the network. A new link connecting two agents generally reduces their consumption variance. By contrast, it can decrease or increase the consumption variance of indirect neighbors.

Our analysis contributes to a growing literature studying informal transfers in networks.[4] With stochastic incomes, Ambrus, Mobius & Szeidl (2014) characterize Pareto-constrained risk-sharing arrangements under network capacity constraints. In a recent paper, Ambrus, Milan & Gao (2017) adopt a similar approach, focusing on local informational constraints. By contrast, with non-stochastic incomes, Bourlès, Bramoullé & Perez-Richet (2017) characterize the Nash equilibria of a game of transfers where agents care about others' well-being. We introduce stochastic incomes to this setup and analyze the risk sharing implications of altruism networks. This allows us to connect the analysis of altruism in networks with the study of informal insurance.

We notably show that there are important differences between the anatomy of risk sharing in a model of altruism in networks and in models of network-constrained risk-sharing arrangements. Structurally important links, like bridges or long-distance connections, have a strong impact on risk sharing under altruism but not in the other models. Predictions on the effect of shock size also differ. Starting from a situation with similar incomes, small shocks generate no transfer under altruism but are perfectly insured under capacity-constrained risk sharing, as in Ambrus, Mobius & Szeidl (2014). By contrast, arbitrarily large shocks yield arbitrarily large transfers under altruism but saturate capacity constraints. These findings could help empirically distinguish between the different models and motives.[5]

Our analysis further advances the economics of altruism, pioneered by Becker (1974) and Barro (1974). With the exception of Bernheim & Bagwell (1988) and Laitner (1991), this literature has abstracted away from the complex structures of real family networks. Economic studies of altruism consider either small groups of completely connected agents (e.g. Alger & Weibull (2010), Bernheim & Stark (1988), Bruce & Waldman (1991)) or linear dynasties (e.g. Altig & Davis (1992), Galperti & Strulovici (2017), Laitner (1988)).

[4] One branch of this literature looks at network formation and stability, see e.g. Bloch, Genicot & Ray (2007), Bramoullé & Kranton (2007a, 2007b).

[5] In an unpublished PhD dissertation, Karner (2012) derives differential implications of altruism and informal insurance on transfers and tests these implications on data from Indonesia. We thank Dilip Mookherjee for bringing our attention to this interesting work.

These structures are irrealistic. As is well-known from human genealogy, strong family ties form complex networks. Bourlès, Bramoullé & Perez-Richet (2017) introduce networks into a model of altruism à la Becker, for non-stochastic incomes. We build on this previous analysis and look at whether and how altruism networks help agents smooth consumption. Despite the key role played by altruistic support in helping real-world agents cope with shocks, there has been surprisingly little work on altruism and risk, even in simple structures.[6] Our analysis represents a leap forward for the literature, filling this gap: we analyze the combined effect of risk and complex networks on transfers and consumption.

The remainder of the paper is organized as follows. We introduce the model of altruism in networks under stochastic incomes in Section 2. We analyze large shocks in Section 3 and characterize what happens with small shocks in Section 4. We investigate structural effects in Section 5 and conclude in Section 6.

II Setup

We introduce stochastic incomes into the model of altruism in networks analyzed in Bourlès, Bramoullé & Perez-Richet (2017). Society is composed of $n \geq 2$ agents who can care about each other. Incomes are stochastic. Once incomes are realized, informal transfers are obtained as Nash equilibria of a non-cooperative game of transfers. We first describe how transfers are determined conditional on realized incomes. We then introduce risk and the classical notion of efficient insurance.

A Transfers conditional on incomes

Agent i has income $y_i^0 \geq 0$ and can give $t_{ij} \geq 0$ to agent j. By convention, $t_{ii} = 0$. The collection of bilateral transfers $\mathbf{T} \in \mathbb{R}_+^{n^2}$ defines a network of transfers. Income after

[6]Foster & Rosenzweig (2001) introduce altruism in a model of risk-sharing arrangements under limited commitment between two agents. They derive predictions through simulations and test these predictions on data from rural South Asia.

transfers, or consumption, y_i is equal to

$$y_i = y_i^0 - \sum_j t_{ij} + \sum_k t_{ki} \tag{1}$$

where $\sum_j t_{ij}$ represents overall transfers made by i and $\sum_k t_{ki}$ overall transfers received by i. Private transfers redistribute income among agents and aggregate income is conserved: $\sum_i y_i = \sum_i y_i^0$.

Agent i chooses her transfers to maximize her altruistic utility:

$$v_i(\mathbf{y}) = u_i(y_i) + \sum_{j \neq i} \alpha_{ij} u_j(y_j) \tag{2}$$

under the following assumptions. Private utility $u_i : \mathbb{R} \to \mathbb{R}$ is twice differentiable and satisfies $u_i' > 0$, $u_i'' < 0$ and $\lim_{y \to \infty} u_i'(y) = 0$. Coefficient $\alpha_{ij} \in [0, 1]$ captures how much i cares about j's private well-being. By convention $\alpha_{ii} = 1$. The *altruism network* $\boldsymbol{\alpha} = (\alpha_{ij})_{i,j=1}^n$ represents the structure of social preferences.[7] In addition, we assume that

$$\forall i, j, \forall y, u_i'(y) \geq \alpha_{ij} u_j'(y) \tag{3}$$

which guarantees that an agent's transfer to a friend never makes this friend richer than her.

In a Nash equilibrium, each agent chooses her transfers to maximize her altruistic utility conditional on transfers made by others. Transfer network $\mathbf{T} \in \mathbb{R}_+^{n^2}$ is a Nash equilibrium if and only if the following conditions are satisfied:

$$\forall i, j, u_i'(y_i) \geq \alpha_{ij} u_j'(y_j) \text{ and } t_{ij} > 0 \Rightarrow u_i'(y_i) = \alpha_{ij} u_j'(y_j) \tag{4}$$

In particular under CARA utilities $u_i(y) = -e^{-Ay}$, equilibrium conditions become: $\forall i, j, y_i \leq y_j - \ln(\alpha_{ij})/A$ and $t_{ij} > 0 \Rightarrow y_i = y_j - \ln(\alpha_{ij})/A$.

Our analysis builds on equilibrium properties established in our previous paper.[8] In par-

[7] These preferences could be exogenously given, or could be generated by primitive preferences where agents care about others' private and social utilities, see Bourlès, Bramoullé & Perez-Richet (2017, p.678).

[8] Our assumptions differ slightly from the assumptions made in Bourlès, Bramoullé & Perez-Richet

ticular, an equilibrium always exists, equilibrium consumption is unique, and the network of equilibrium transfers is generically unique and has a forest structure. Formally, $\mathbf{T}$ has a forest structure when it contains no non-directed cycle, i.e., sets of agents $i_1, i_2, ..., i_l = i_1$ such that $\forall s < l$, $t_{i_s i_{s+1}} > 0$ or $t_{i_{s+1} i_s} > 0$.

Proposition 1 *(Bourlès, Bramoullé & Perez-Richet 2017) A Nash equilibrium exists. Equilibrium consumption* $\mathbf{y}$ *is unique and continuous in* $\mathbf{y}^0$ *and* $\boldsymbol{\alpha}$*. Generically in* $\boldsymbol{\alpha}$*, the network of equilibrium transfers is unique and is a forest.*

B Stochastic incomes

We now consider stochastic incomes. Following each income realization, agents make equilibrium transfers to each other. Proposition 1 ensures that there is a well-defined mapping from incomes to consumption. Let $\tilde{\mathbf{y}}^0$ denote the stochastic income profile and $\tilde{\mathbf{y}}$ the resulting stochastic consumption profile.[9]

To illustrate how altruistic transfers affect risk, consider the following simple example. Two agents care about each other with $\alpha_{12} = \alpha_{21} = \alpha$. They have common CARA utilities $u(y) = -e^{-y}$. Let $c = -\ln(\alpha)$. Agents' incomes are iid with binary distribution: $y_i^0 = \mu - \sigma$ with probability $\frac{1}{2}$ and $y_i^0 = \mu + \sigma$ with probability $\frac{1}{2}$, with $\sigma > c/2$. When one agent has a positive shock and the other a negative one, the lucky agent makes a positive transfer to the unlucky one. Altruistic transfers lead to the following stochastic consumption: $(y_1, y_2) = (\mu - c/2, \mu + c/2)$ with probability $\frac{1}{4}$, $(\mu + c/2, \mu - c/2)$ with probability $\frac{1}{4}$, $(\mu - \sigma, \mu - \sigma)$ with probability $\frac{1}{4}$, $(\mu + \sigma, \mu + \sigma)$ with probability $\frac{1}{4}$.

In this example, consumption $\tilde{\mathbf{y}}$ is less risky than income $\tilde{\mathbf{y}}^0$ for Second-Order Stochastic Dominance. The reason is that altruism entails giving money when rich and receiving money when poor. Altruistic transfers in this case mimick a classical insurance scheme. While informal insurance provided by altruistic transfers is generally imperfect, $\tilde{\mathbf{y}}$ becomes less and less risky as α increases and idiosyncratic risks are fully eliminated when $\alpha = 1$. In

(2017), to cover situations where altruism may be perfect and $\alpha_{ij} = 1$. We describe in Appendix how our previous results generalize to this extended setup.

[9]Throughout the paper, we denote random variables with tilde and specific realizations of these random variables without tilde.

the rest of the paper, we study how these effects and intuitions extend to complex networks and risks.

Our analysis relies on the classical notion of efficient insurance, see e.g. Gollier (2001).

Definition 1 *Informal transfers generate efficient insurance if there exist Pareto weights* $\boldsymbol{\lambda} \geq \mathbf{0}$, $\boldsymbol{\lambda} \neq \mathbf{0}$ *such that consumption* $\tilde{\mathbf{y}}$ *solves*

$$\max_{\substack{\mathbf{y} \\ \sum_i y_i = \sum_i y_i^0}} \sum_i \lambda_i E u_i(y_i)$$

Efficient insurance is a central notion, describing the ex-ante Pareto frontier with respect to private utilities. It provides the conceptual foundation of a large empirical literature, following Townsend (1994), which attempts to assess the extent of actual insurance in real contexts. Note that $\sum_i \mu_i E v_i = \sum_i (\sum_j \alpha_{ji} \mu_j) E u_i$. Therefore, a Pareto optimum with respect to expected altruistic utilities always generates efficient insurance. The converse may not be true, however, and efficient insurance situations may not constitute altruistic Pareto optima.[10]

Let us next recall some well-known properties of efficient insurance. When $\boldsymbol{\lambda} > \mathbf{0}$, efficient insurance is such that $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$ for every income realization $\mathbf{y}^0$. The ratio of two agents' marginal utilities is constant across states of the world. Define $\bar{y}^0 = \frac{1}{n}\sum_i y_i^0$. When agents have common utilities and equal Pareto weights $\lambda_i = \lambda_j = \lambda$, this leads to equal income sharing $y_i = \bar{y}^0$. When agents have CARA utilities and $\sum_k \ln(\lambda_k) = 0$, this yields $y_i = \bar{y}^0 + \frac{1}{A}\ln(\lambda_i)$. An agent's consumption is then equal to the average income plus a state-independent transfer. In general, an agent's consumption is a function of average income depending on Pareto weights and utilities.

[10] This concerns the extreme parts of the private Pareto frontier. If i is altruistic towards others, the dictatorial private Pareto optimum where $\lambda_j = 0$ if $j \neq i$ is not an altruistic Pareto optimum. In general if $\det(\boldsymbol{\alpha}^T) \neq 0$, a private Pareto optimum with weights $\boldsymbol{\lambda}$ is an altruistic Pareto optimum iff $(\boldsymbol{\alpha}^T)^{-1}\boldsymbol{\lambda} \gneq \mathbf{0}$. In the literature on welfare evaluation, some researchers argue that social preferences should not be taken into account when evaluating welfare, see e.g. Section 5.4 in Blanchet & Fleurbaey (2006).

III Large shocks

A Perfect altruism

We first identify a natural benchmark where altruistic transfers generate efficient insurance for any random incomes. Say that agent i is perfectly altruistic towards agent j if $\alpha_{ij} = 1$. The network of perfect altruism is the subnetwork of $\boldsymbol{\alpha}$ which contains perfect altruistic ties. The network of perfect altruism is *strongly connected* if any two agents are connected through a path of perfect altruistic ties. Formally, for any $i \neq j$ there exist a set of l agents $i_1 = i$, i_2,..., $i_l = j$ such that $\forall s < l, \alpha_{i_s i_{s+1}} = 1$. Detailed proofs are provided in the Appendix.

Proposition 2 *Informal transfers generate efficient insurance for every stochastic income if and only if the network of perfect altruism is strongly connected. In this case, agents have equal Pareto weights.*

To prove sufficiency, we show how to combine equilibrium conditions to obtain the first-order conditions of the planner's program. To prove necessity, we assume that the network of perfect altruism is not strongly connected. We build instances of income distribution for which altruistic transfers do not generate efficient insurance.

Proposition 2 complements earlier results on equal income sharing, see Bloch, Genicot & Ray (2008) and Proposition 1 in Bramoullé & Kranton (2007a).[11] Consider, for example, common utilities and suppose that any altruistic link is perfect $\alpha_{ij} \in \{0, 1\}$. Agent i's best response is to equalize consumption with her poorer friends. Proposition 1 shows that when all agents seek to equalize consumption with their poorer friends and when the altruism network is strongly connected, private transfers necessarily lead to overall equal income sharing, i.e., $y_i = \bar{y}^0$.

This result is straightforward when the network of perfect altruism is complete, as all agents then seek to maximize utilitarian welfare. Proposition 2 shows, however, that

[11] Bloch, Genicot & Ray (2008) show that equal sharing in components is the only allocation consistent with the social norm of bilateral equal sharing. Bramoullé & Kranton (2007a) show that if linked pairs meet at random and share income equally, consumption converges to equal sharing in components. By contrast, Proposition 2 identifies conditions under which equal sharing in components emerges as the unique Nash equilibrium of a game of transfers.

perfect altruism also generates efficient insurance in sparse networks such as the star and the line or when two communities are connected by a unique bridge. In these cases, agents' interests are misaligned. Agents potentially care about distinct subsets of people. Still, under connectedness, the interdependence in individual decisions embedded in equilibrium behavior leads noncooperative agents to act as if they were following a planner's program.

B Imperfect altruism

We next look at imperfect altruism. In general, how far can informal insurance induced by altruistic transfers move away from efficient insurance with equal Pareto weights? And how does this depend on the structure of the altruism network?

To answer these questions, we consider common utilities and introduce a measure of distance from equal income sharing, $DISP$, as in Mobius, Ambrus & Szeidl (2014). Formally given income realization $\mathbf{y}^0$,

$$DISP(\mathbf{y}) = \frac{1}{n}\sum_i |y_i - \bar{y}^0|$$

where $DISP(\mathbf{y}) \geq 0$ and $DISP(\mathbf{y}) = 0 \Leftrightarrow \forall i, y_i = \bar{y}^0$. We can then compute the expected value over all income realizations $EDISP(\tilde{\mathbf{y}}) = E\frac{1}{n}\sum_i |y_i - \bar{y}^0|$ such that $EDISP(\tilde{\mathbf{y}}) = 0 \Leftrightarrow \forall \mathbf{y}^0, \forall i, y_i = \bar{y}^0$.

Next, we extend the notion of network distance to altruism networks. Following Bourlès, Bramoullé & Perez-Richet (2017), introduce $c_{ij} = -\ln(\alpha_{ij})$ if $\alpha_{ij} > 0$ as the virtual cost of the altruistic link. Stronger links have lower costs. Define the cost of a path as the sum of the costs of the links in the path. If i and j are connected through a path of altruistic links in $\boldsymbol{\alpha}$, define $\hat{c}_{ij}$ as the lowest virtual cost among all paths connecting i to j. For instance when all links have the same strength $\alpha_{ij} \in \{0, \alpha\}$, then $\hat{c}_{ij} = -\ln(\alpha)d_{ij}$ where d_{ij} is the usual network distance between i and j, that is, the length of a shortest path connecting them. When links have different strengths, $\hat{c}_{ij}$ is a measure of altruism distance between i and j accounting for the strength of altruistic ties in indirect paths connecting the two agents. In particular, $\hat{c}_{ij} = 0$ if and only if there is a path of perfect altruistic links connecting i to j.

In our next result, we show that under CARA utilities and for any income realization, distance to equal income sharing is bounded from above by a simple function of distances in the altruism network.

Proposition 3 *Assume that agents have common CARA utilities. If the altruism network is strongly connected, then for any income realization:*

$$DISP(\mathbf{y}) \leq \frac{1}{An^2} \sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})$$

For every altruism network which is not strongly connected, $EDISP(\tilde{\mathbf{y}})$ can take arbitrarily large values.

We prove this result by combining, in different ways, inequalities appearing in equilibrium conditions (4). In the Appendix, we show how to obtain similar bounds for other measures of distance to equal income sharing and for other utility functions. For CRRA utility functions, we show that the ratio $DISP(\mathbf{y})/\bar{y}^0$ is bounded from above by a simple function of the altruism network. Note that for CARA utilities, the first part of Proposition 2 follows directly from Proposition 3. When the network of perfect altruism is strongly connected, $\forall i, j, \hat{c}_{ij} = 0$ and hence $\forall \mathbf{y}^0, DISP(\mathbf{y}) = 0$.

Proposition 3 identifies specific structural features governing the extent of informal insurance provided by altruistic transfers. It shows, in particular, that bridges play a critical role. Suppose that the altruism network is formed of two separate, strongly connected communities. Community-level shocks are not insured, and expected distance from equal sharing can be arbitrarily large. Next, add a single link between the two communities. Ex-post distance from equal sharing is now bounded from above and this bound is independent of the size of the shocks. A large negative shock in one community generates large transfers flowing through the bridge. Both bridge agents play the role of transfer intermediaries and help ensure that informal support from the rich community reaches the poor community.

More generally, Proposition 3 says that the quality of informal insurance depends on the average altruism distance in the network. For instance if links are undirected and have the same strength $\alpha_{ij} = \alpha_{ji} \in \{0, \alpha\}$, the upper bound becomes $\frac{1}{A}\frac{n(n-1)}{n^2}\bar{d}$ where $\bar{d}$ is the average path length in the network, $\bar{d} = \frac{2}{n(n-1)}\sum_{i<j} d_{ij}$. In general, $\frac{1}{n}\sum_j \hat{c}_{ij}$ measures the average altruism distance from i to other agents, while $\frac{1}{n}\sum_j \hat{c}_{ji}$ measures the average altruism distance from other agents to i. Then, $\frac{1}{n}\sum_i \max(\frac{1}{n}\sum_j \hat{c}_{ij}, \frac{1}{n}\sum_j \hat{c}_{ji})$ is a measure of average altruism distance in the network. This notably implies that informal insurance induced by altruism is subject to small-world effects, see Watts & Strogatz (1998). Starting from a spatial network with long average path length, adding a few long-range connections leads to a strong drop in average path length and hence to a potentially strong increase in the quality of informal insurance.

These structural effects may help distinguish altruism from network-constrained risk sharing. In the social collateral model (Ambrus, Mobius & Szeidl (2014)), adding a bridge to separate communities does not have much impact. A large negative shock on one community saturates the bridge's capacity constraint, and the distance to equal income sharing can be arbitrarily large. Similarly, a few long-range connections have little impact.[12] Rather, the extent of informal insurance in that model depends on the expansiveness of the network, i.e., how the number of connections a group has with the rest of society varies as group size increases. Average path length and expansiveness capture different aspects of a network's geometry, indicating the profoundly different effects the network structure can have on informal insurance.

Proposition 3 applies to any income distribution. The bound's tightness may vary, however, and tighter bounds can be obtained through other arguments or by making specific assumptions on income shocks. In the Appendix, we show that when $\boldsymbol{\alpha}$ is strongly connected, $DISP(\mathbf{y}) \leq \frac{1}{A}\frac{1}{2}\max_{i,j}\hat{c}_{ij}$. For undirected binary networks, $\max_{i,j}\hat{c}_{ij} = cd_{\max}$ where $d_{\max}$ is the network's diameter, i.e., the length of the longest shortest path. This improves on Proposition 3 in some cases.

Alternatively, consider a strongly connected altruism network and income distributions where a single agent i is subject to large shocks. If the shock is positive, the agent's transfers irrigate the whole community. Money flows, directly or indirectly, from i to any other agent.

[12] Bridges and long-distance connections also have little impact on overall informal insurance in a model of risk sharing under local information constraints, see Ambrus, Gao & Milan (2017).

Conversely if the shock is negative, money flows, directly or indirectly, from any other agent to i. Consider a binary and undirected network and introduce $\bar{d}_i = \frac{1}{n}\sum_j d_{ij}$, a measure of the average distance between i and other agents in society. In this case we can show that under CARA, $DISP(\mathbf{y}) = \frac{c}{A}\frac{1}{n}\sum_j |\bar{d}_i - d_{ij}|$, see the Appendix. When a single agent is subject to large shocks, the quality of informal insurance depends on the dispersion in network distances to this agent.[13]

IV Small shocks

In this section, we characterize what happens with small shocks. More precisely, we consider shocks that do not affect transfer relationships - who gives to whom. Formally, given equilibrium transfers $\mathbf{T}$, introduce the directed binary graph of transfers $\mathbf{G}$ such that $g_{ij} = 1$ if $t_{ij} > 0$ and $g_{ij} = 0$ if $t_{ij} = 0$. In Bourlès, Bramoullé & Perez-Richet (2017), we showed that generically in $\boldsymbol{\alpha}$ and in $\mathbf{y}^0$ there exists $\eta > 0$ such that if $||\hat{\mathbf{y}}^0 - \mathbf{y}^0|| \leq \eta$ then the unique equilibrium $\hat{\mathbf{T}}$ for incomes $\hat{\mathbf{y}}^0$ has the same graph of transfers as the equilibrium $\mathbf{T}$ for incomes $\mathbf{y}^0$, and this graph is a forest. Thus, income variations which are relatively small in magnitude generically leave $\boldsymbol{G}$ unchanged.[14] They affect, of course, the amounts transferred and we next characterize the insurance properties of these transfer adjustments.

To present our main result, we need to introduce some additional notions and notations. A *weak component* of $\mathbf{G}$ is a component of the undirected binary graph where i and j are connected if $g_{ij} = 1$ or $g_{ji} = 1$. When i and j belong to the same weak component of forest graph $\mathbf{G}$, define

$$\bar{c}_{ij} = \sum_{s:g_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:g_{i_{s+1} i_s}=1} c_{i_{s+1} i_s}$$

for the unique path $i_1 = i$, i_2,..., $i_l = j$ such that $\forall s, g_{i_s i_{s+1}} = 1$ or $g_{i_{s+1} i_s} = 1$. Note that $\bar{c}_{ij}$ is generally distinct from $\hat{c}_{ij}$. While the altruism distance $\hat{c}_{ij}$ is greater than or equal to zero and only depends on the altruism network $\boldsymbol{\alpha}$, the parameter $\bar{c}_{ij}$ can take negative

[13]We see, again, the differences in insurance patterns between altruism and social collateral. Under social collateral, large shocks on one agent saturate all transfer capacities, leading to arbitrarily large departures from equal income sharing.

[14]Note that some large income variations also leave $\mathbf{G}$ invariant. For instance with 2 agents and CARA utilities, i gives to j in equilibrium iff $y_i^0 \geq y_j^0 + \frac{c_{ij}}{A}$.

values and also depends on who gives to whom.[15] The interior of a set is the largest open set included in it.

Theorem 1 *(1) Let* $\tilde{\mathbf{y}}^0$ *be an income distribution and* $\mathbf{G}$ *a forest graph such that, for any supported income realization, there exists a Nash equilibrium of the transfer game with transfer graph* $\mathbf{G}$*. Then altruistic transfers generate efficient insurance within weak components of* $\mathbf{G}$*. If agent* i *belongs to weak component* C *of size* n_C*, his Pareto weight* λ_i *is such that* $\ln(\lambda_i) = \frac{1}{n_C}\sum_{j\in C}\bar{c}_{ij}$ *under normalization* $\sum_{j\in C}\ln(\lambda_j) = 0$*.*
(2) Consider an income distribution whose support's interior is non-empty. Generically in $\boldsymbol{\alpha}$*, if society is partitioned in communities and altruistic transfers generate efficient insurance within communities, then the graph of transfers is constant across income realizations in the support's interior and these communities are equal to the weak components of the transfer graph.*

To prove the first part of Theorem 1, we compare equilibrium conditions with the first-order conditions of the planner's program. When i makes transfers to j in equilibrium, the ratio of their marginal utilities is equal to the altruistic coefficient: $u_i'(y_i)/u_j'(y_j) = \alpha_{ij}$. Under efficient insurance, we would have $u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i$. We thus look for Pareto weights such that $\lambda_j/\lambda_i = \alpha_{ij}$. This equality can of course generally not be satisfied for all pairs of agents. We show in the Appendix how to exploit the forest structure of equilibrium transfers to find appropriate Pareto weights. Our proof is constructive and based on the explicit formulas provided in the Theorem. Note that the Pareto weights only depend on $\boldsymbol{\alpha}$ and $\mathbf{G}$ and hence do not depend on the specific income realization. Since money flows within but not between weak components, this leads to efficient insurance within weak components.

In the second part of Theorem 1, we show that small shocks are, generically, the only situations where altruistic transfers generate constrained efficient insurance. We provide a sketch of the proof here. The main idea is to exploit the first part of the Theorem: locally around some income profile, altruistic transfers generate constrained Pareto efficiency with

[15] In fact, $\hat{c}_{ij} = \bar{c}_{ij}$ iff i is connected to j in $\mathbf{G}$ via a path of giving relationships: $g_{ii_2} > 0$, $g_{i_2i_3} > 0$,..., $g_{i_{l-1}j} > 0$.

known features (communities and Pareto weights). These features must then be consistent with the original assumed pattern of constrained efficiency, and we show that this can only happen when the graph of transfers is invariant. An important step in the proof is to show that generically in $\boldsymbol{\alpha}$, the Pareto weight mapping $\mathbf{G} \to \boldsymbol{\lambda}(\mathbf{G})$ is injective. Overall, this result provides a generic characterization of situations of constrained efficient insurance.

The first part of Theorem 1 extends Theorem 3 in Bourlès, Bramoullé & Perez-Richet (2017). It characterizes the income-sharing functions uncovered in that result and shows that the transfer graph's weak components actually form endogenous risk-sharing communities.

Theorem 1 shows that, following small shocks, adjustments in altruistic transfers satisfy a property of constrained efficiency. Within a weak component of $\mathbf{G}$, agents act as if they were following a planner's program. The quality of informal insurance provided by altruistic transfers then depends on the connectivity of the transfer graph. Informal insurance is efficient if $\mathbf{G}$ is weakly connected. This happens, for instance, when one agent is much richer or much poorer than all other agents. By contrast, agents fully support their income risks when $\mathbf{G}$ is empty. This happens when $\forall i, j, \alpha_{ij} < 1$ and $\tilde{\mathbf{y}} = \tilde{y}^0 \mathbf{1} + \tilde{\boldsymbol{\varepsilon}}$ for $\tilde{\boldsymbol{\varepsilon}}$ small enough. When differences in incomes among agents are small in all realizations, agents make no altruistic tranfers in equilibrium. By contrast, such small shocks would be efficiently insured in the social collateral model.

More generally, the extent of informal insurance depends on the number and sizes of $\mathbf{G}$'s weak components. Under common CARA utilities, the equilibrium consumption of agent i in component C is equal to $y_i = \bar{y}_C^0 + \frac{1}{A}\ln(\lambda_i)$. Under iid income shocks, this implies that $Var(y_i) = \frac{1}{n_C} Var(y_i^0)$ and an increase in components' sizes leads to a decrease in consumption variance for all agents.[16]

The Pareto weights capture how the private preferences of an agent are represented in the equivalent planner's program. They reflect agents' positions in the graph of transfers

[16] Effects are more complex when shocks are not iid. When shocks are independent but not identical, $Var(y_i) = \frac{1}{n_C^2}\sum_{j\in C} Var(y_j^0)$. Consumption variance may be greater than income variance for an agent with relatively low income variance. Note, however, that $\sum_{i\in C} Var(y_i) = \frac{1}{n_C}\sum_{i\in C} Var(y_i^0) < \sum_{i\in C} Var(y_i^0)$. Increases in variance for some agents would be more than compensated by decreases in variance for others.

and depend on the graph's full structure. For instance, a giving line where $t_{i_1 i_2} > 0$, $t_{i_2 i_3} > 0$, ..., $t_{i_{n-1} i_n} > 0$ yields $\lambda_1 > \lambda_2 > ... > \lambda_n$. More generally an agent's preferences tend to be well-represented in the equivalent planner's program when this agent has a relatively "higher" position in the network of transfers. This happens when he tends to give to others towards whom he is not too altruistic, inducing higher c's.

A further implication is that local changes may have far-reaching consequences. Suppose, for instance, that $g_{ij} = 1$ and consider a small increase in α_{ij} that does not change the pattern of giving relationships. Let C be the weak component of i and j and define C_i as the weak component of i in the graph obtained from $\mathbf{G}$ by removing the link ij, and similarly for C_j. Note that $C = C_i \cup C_j$ and $C_i \cap C_j = \varnothing$. Informally, C_i represents agents indirectly connected to the giver while C_j represents agents indirectly connected to the receiver.

Proposition 4 *Suppose that $g_{ij} = 1$ and consider a small increase in α_{ij} leaving $\mathbf{G}$ unaffected. Then, λ_k decreases if $k \in C_i$ and increases if $k \in C_j$.*

Therefore the normalized Pareto weights of the giver and of agents indirectly connected to her decrease, while the normalized Pareto weights of the receiver and of agents indirectly connected to her increase. This implies that the consumption of agents in C_i decreases while the consumption of agents in C_j increases, and hence Proposition 4 extends the first part of Theorem 4 in Bourlès, Bramoullé & Perez-Richet (2017).

V Network structure and informal insurance

In this Section, we study the impact of the network structure on consumption smoothing. How is the position of an agent in the altruism network related to her consumption variance? How do altruistic transfers affect the correlation structure of consumption streams across individuals? How does a new link between two agents affect their consumption variance? How does it affect the consumption variance of other agents in the network? We uncover some complex effects, which we analyze through a combination of analytical results and numerical simulations.

As a preliminary remark, note that altruistic transfers generally affect all moments of the consumption distribution. Expected consumption may thus differ from expected income. While these redistributive aspects are potentially interesting, we wish to focus here on the risk-sharing implications of altruistic transfers. To do so, we identify a natural benchmark where expected consumption is invariant. Altruistic ties are undirected when $\forall i, j, \alpha_{ij} = \alpha_{ji}$. Say that the distribution of stochastic income $\tilde{\mathbf{y}}^0$ is symmetric if individuals have the same expected income and if the whole profile is distributed symmetrically around its expectation. Formally, $\tilde{\mathbf{y}}^0 = \mu\mathbf{1} + \tilde{\boldsymbol{\varepsilon}}$ with $E(\tilde{\boldsymbol{\varepsilon}}) = \mathbf{0}$ and $f(\boldsymbol{\varepsilon}) = f(-\boldsymbol{\varepsilon})$ where f is the pdf of $\tilde{\boldsymbol{\varepsilon}}$. This covers iid symmetric distributions as well as distributions with income correlation.

Proposition 5 *Suppose that agents have common CARA utilities, that altruistic ties are undirected, and that income distribution is symmetric. Then* $\forall i, Ey_i = Ey_i^0$.

To prove this result, we prove that if equilibrium transfers $\mathbf{T}$ are associated with shock $\boldsymbol{\varepsilon}$, then reverse transfers $\mathbf{T}^t$ are equilibrium transfers for shock $-\boldsymbol{\varepsilon}$.[17] Symmetry assumptions then guarantee the absence of redistribution in expectations.

We present results of numerical simulations based on the following parameter values. We consider a real network of informal lending and borrowing relationships, connecting 111 households in a village in rural India drawn from the data analyzed in Banerjee et al. (2013). The network is depicted in Figure 1. Altruistic links have strength α and agents have CARA utilities $u_i(y) = -e^{-Ay}$ with $-\ln(\alpha)/A = 3$. Incomes are iid binary: $y_i^0 = 0$ with probability 0.5 and 20 with probability 0.5. We consider $10,000$ realizations of incomes and, for each realization, we compute equilibrium transfers and consumption. The analysis was replicated with lognormal incomes with the same mean and variance, and all the results reported below were found to be robust.

[17] We thank Adam Szeidl for having first made the connection between this property and the result of no redistribution in expectation.

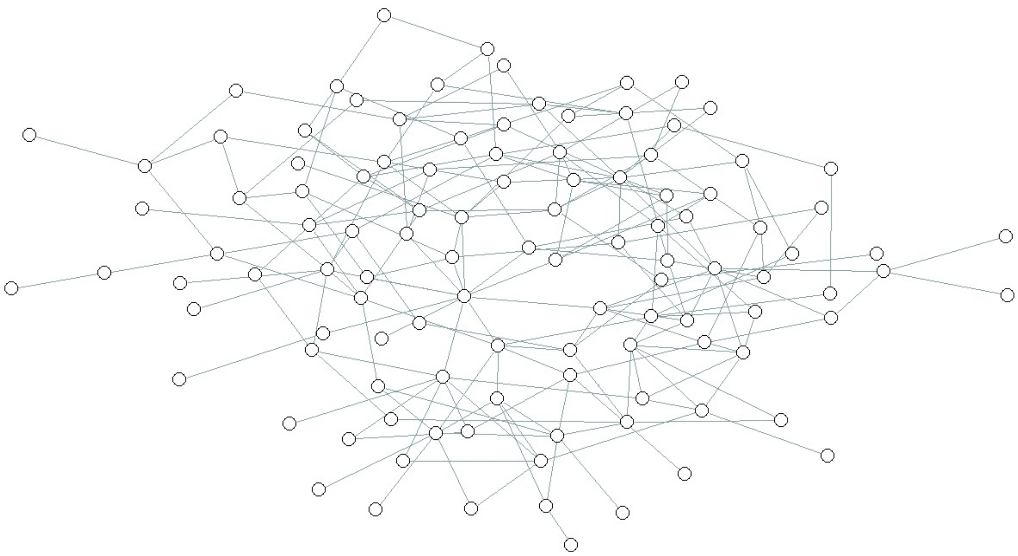

Figure 1. A Network of Informal Risk Sharing

We start by looking at the relation between the network structure and the consumption variance - covariance matrix. Are more central agents better insured? We compute correlation coefficients between consumption variance and different measures of centrality (degree, betweenness centrality, eigenvector centrality), see Table 1. Correlation is clearly negative and both quantitatively and statistically significant.

Table 1. Correlation between Centralities and Consumption Variance

	Variance
Degree	-0.7657***
Between	-0.5148***
Eigen	-0.6800***

***denotes statistical significance at the 1% level

Simulation Result 1: *More central agents tend to have lower consumption variance.*

On this dimension, the model of altruism in networks generates predictions similar to those of the model of social collateral. It differs from the model of local information constraints, which generates positive correlation between consumption variance and centrality, see Ambrus, Gao & Milan (2017).

We next look at correlations in consumption streams across individuals. We show that, starting from independent incomes, altruistic transfers necessarily induce weakly positive covariance in consumption across agents. This holds for any pair of agents, any altruism network and any utility functions.

Proposition 6 *Suppose that incomes are independent across agents.* $\forall i, j, cov(\tilde{y}_i, \tilde{y}_j) \geq 0$.

We obtain this result by relying on the global comparative statics of consumption with respect to incomes, see Theorem 3 in Bourlès, Bramoullé & Perez-Richet (2017). This result says that y_i is weakly increasing in y_j^0 for any i, j. A positive shock on any agent's income thus induces weakly positive changes in the consumption of every agent in society, and conversely for negative shocks. To prove the result, we then combine this property with two classical properties of the covariance operator.

Altruistic transfers thus tend to generate positive correlation across individuals' consumption streams. We next explore through simulations how these correlations depend on the network distance between agents. Figure 2 depicts the correlogram of consumption correlation between y_i and y_j as a function of network distance between i and j. We consider all pairs at given distance d and compute the average correlation coefficient (plain line) as well as the 5th and 95th percentiles of the correlation distribution (dashed lines). We see that consumption correlation is generally positive, consistentl with Proposition 6. Furthermore,

Simulation Result 2: *Consumption correlation tends to decrease with network distance.*

Consumption correlation can reach very high levels for direct neighbors and then tends to decrease at a decreasing rate as network distance increases.

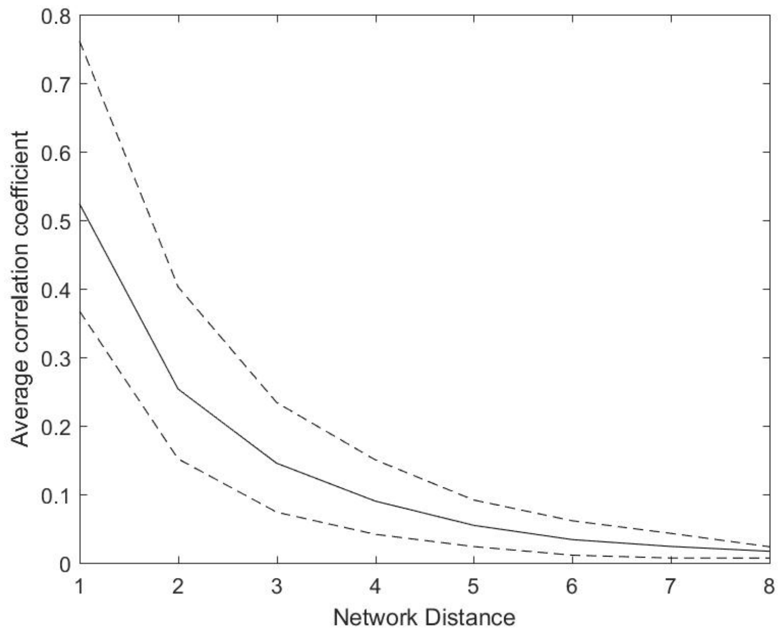

Figure 2. Consumption Correlation as a Function of Network Distance

Finally, we study the impact of adding one altruistic link on agents' consumption variances. We ran extensive numerical simulations for a variety of income distributions and network structures. With iid incomes and under the assumptions underlying Proposition 5, the consumption variance of the two agents becoming connected generally drops.[18] This is consistent with Simulation Result 1: acquiring more links, or a better position, in the network allows agents to reduce consumption variability in this framework. By contrast, the new link may increase or decrease the consumption variance of other agents in the network. Two opposite forces are at play here. On the one hand, the new link provides a source of additional indirect support, which can help further smooth consumption. On the other hand, the new neighbor is also a competitor for the support of the existing neighbor, which can reduce the consumption smoothing.

For instance, with 3 agents, iid binary incomes and CARA utilities, we can show the

[18] We provide a simple example in the Appendix showing that if incomes are correlated, obtaining a new connection may lead to an increase in consumption variance.

following result (proof in Appendix). Start from a situation where agent 1 is connected to agent 2 but not to agent 3. Add the connection between 2 and 3 to form a line, and $Var(y_1)$ drops. Next, close the triangle by adding the connection between 1 and 3, and $Var(y_2)$ increases. Connecting the two peripheral agents of a 3-agent line leads to an increase in consumption variance for the center.

We next look at the impact of adding a link to a complex, real-world network, as shown in Figure 3. We depict the new link in bold and focus on the region of the network close to the new link. No change in variance is detected outside this region. Nodes for which we detect a change in consumption variance are depicted in grey, with a symbol describing the direction of the change.[19] We observe both increases and decreases in consumption variance for indirect neighbors.[20] To sum up,

Simulation Result 3: *Connecting two agents generally leads to a decrease in their consumption variance and can lead to a decrease or an increase in the consumption variance of other agents.*

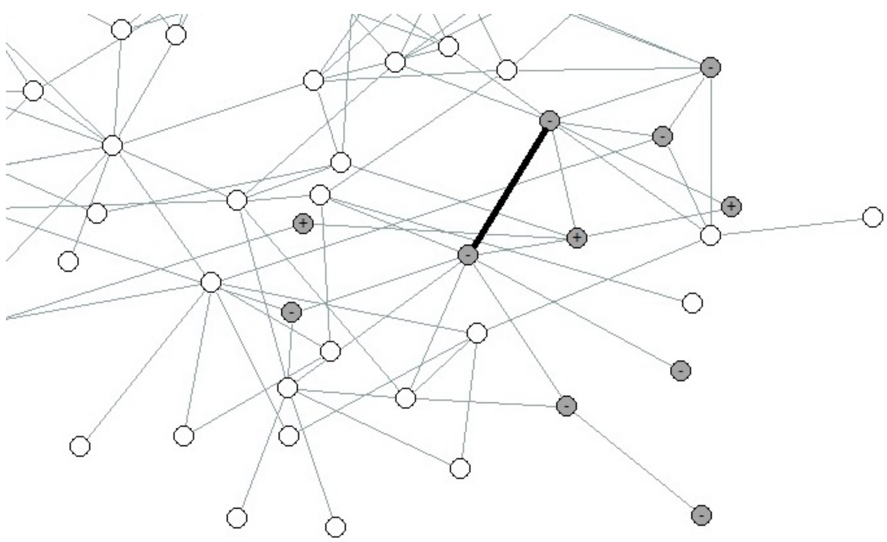

Figure 3. Impact of a New Link on Consumption Variances

[19]Because of numerical variability, we set a relatively high detection threshold t and only report variance changes $\Delta Var(y_i)$ such that $|\Delta Var(y_i)| \geq t$. Thus, Figure 3 likely does not report false positives (detected changes are likely true changes) and may report false negatives (some true changes may not be detected).

[20]In unreported simulations, we also detected simultaneous decreases and increases in consumption variance due to adding a new link in simple networks, such as when connecting the two peripheral agents of a 5-agent line.

VI Conclusion

We analyze the risk-sharing implications of altruism in networks. We find that altruistic transfers have a first-order impact on risk. When the network of perfect altruistic ties is strongly connected, altruistic transfers generate efficient insurance with equal Pareto weights for any shock. More generally, the distance to equal income sharing tends to decrease with the average path length of the network, revealing a disproportionate impact of bridges and long-distance connections. We then show that for shocks leaving the structure of giving relationships unchanged, altruistic transfers generate efficient insurance within the weak components of the transfer network. Conversely, we show that generically these are the only situations where altruistic transfers generate constrained efficient insurance. Finally, we uncover and investigate complex structural effects.

We establish a connection between the analysis of altruism networks and the literature on informal insurance. There are many interesting lines of research to be pursued in future investigations. For instance, how do altruism networks affect agents' incentives to take risks, see Alger & Weibull (2010)? How do altruism networks interact with classical risk-sharing motives, see Foster & Rosenzweig (2001)? How can network data be exploited empirically to identify motives behind informal transfers? More generally, how can network models of informal transfers be applied to data?

APPENDIX

Extension of previous results to perfect altruism. Bourlès, Bramoullé & Perez-Richet (2017) assume that $\alpha_{ij} < 1$ and $u_i'(y) > \alpha_{ij} u_j'(y)$. We relax these assumptions slightly here by assuming that $\alpha_{ij} \leq 1$ and $u_i'(y) \geq \alpha_{ij} u_j'(y)$, allowing for perfect altruism. Perfect altruism gives rise to unbounded Nash equilibria, caused by cycles in transfers. For instance if two agents are perfectly altruistic towards each other $\alpha_{12} = \alpha_{21} = 1$ and have the same utility functions and incomes, Nash equilibria are transfer profiles of the form $(t_{12} = t, t_{21} = t)$, leaving incomes unaffected. Theorems 1-4 in Bourlès, Bramoullé & Perez-Richet (2017) then still hold under the new assumptions with two caveats. (1) Equilibrium transfers are now not necessarily acyclic. An acyclic Nash equilibrium still exists, however. To see why, suppose that there is a cycle in transfers: $t_{i_1 i_2} > 0$,..., $t_{i_l i_1} > 0$. This implies that $u_{i_1}'(y_{i_1})/u_{i_2}'(y_{i_2}) = \alpha_{i_1 i_2}$,... , $u_{i_l}'(y_{i_l})/u_{i_1}'(y_{i_1}) = \alpha_{i_l i_1}$. Multiplying all equalities yields $1 = \alpha_{i_1 i_2} ... \alpha_{i_l i_1}$ and hence $\alpha_{i_l i_1} = ... = \alpha_{i_l i_1} = 1$. Cycles in transfers can only happen in cycles of perfect altruistic ties. Then, let $t = \min(t_{i_1 i_2}, ..., t_{i_l i_1})$. Removing t from all transfers in the cycle yields another Nash equilibrium, and repeating this operation leads to an acyclic Nash equilibrium. (2) The genericity condition in $\boldsymbol{\alpha}$ must be supplemented by the condition that $\boldsymbol{\alpha}$ does not contain directed cycles of perfect altruistic ties. This then guarantees that Nash equilibria are acyclic.

Proof of Proposition 2. We will make use of the following properties established in Bourlès, Bramoullé & Perez-Richet (2017). Define $\hat{\alpha}_{ij} = e^{-\hat{c}_{ij}}$ if $\hat{c}_{ij} < \infty$ and $\hat{\alpha}_{ij} = 0$ otherwise. Then, $\forall i, j, u_i'(y_i) \geq \hat{\alpha}_{ij} u_j'(y_j)$ and $u_i'(y_i) = \hat{\alpha}_{ij} u_j'(y_j)$ if there is a directed path connecting i to j in $\mathbf{T}$. Next, suppose that i is much richer than everyone else. Then money indirectly flows from i to every other agent j such that $\hat{\alpha}_{ij} > 0$ and $\forall i, j : \hat{\alpha}_{ij} > 0, u_i'(y_i) = \hat{\alpha}_{ij} u_j'(y_j)$.

Observe that the network of perfect altruistic ties is strongly connected if and only if $\forall i, j, \hat{\alpha}_{ij} = 1$. If this holds, then $\forall i, j, u_i'(y_i) \geq u_j'(y_j)$ and hence $u_i'(y_i) = u_j'(y_j)$. These are the first-order conditions of the problem of maximizing utilitarian welfare. Next, suppose that there exist i and j such that $\hat{\alpha}_{ij} < 1$. Define $\mathbf{y}^0$ such that $y_i^0 = Y$ and $\forall k \neq i, y_k^0 = 0$. If $\hat{\alpha}_{ij} = 0$, money cannot flow from i to j. As Y increases, consumption y_i tends to ∞ while $y_j = 0$. If Y is large enough, $u_i'(y_i) < u_j'(y_j)$. If $\hat{\alpha}_{ij} > 0$, then $u_i'(y_i) = \hat{\alpha}_{ij} u_j'(y_j) < u_j'(y_j)$ if Y is large enough. Similarly, define $\tilde{\mathbf{y}}^0$ such that $\tilde{y}_j^0 = Y$ and $\forall k \neq j, \tilde{y}_k^0 = 0$. Since $\hat{\alpha}_{ji} \leq 1$, $u_j'(\tilde{y}_j) \leq u_i'(\tilde{y}_i)$ if Y large enough. Under efficient insurance, we would then have $\lambda_j < \lambda_i$ and $\lambda_j \geq \lambda_i$, a contradiction. Therefore, altruistic transfers do not generate efficient insurance. QED.

Proof of Proposition 3. Recall; $\forall i, j, u_i'(y_i) \geq \hat{\alpha}_{ij} u_j'(y_j)$. This is equivalent to: $(u_j')^{-1}(\frac{1}{\hat{\alpha}_{ij}} u_i'(y_i)) \leq y_j$. Summing over j leads to:

$$\sum_j (u_j')^{-1}(\frac{1}{\hat{\alpha}_{ij}} u_i'(y_i)) \leq n\bar{y}^0$$

We also have $\forall i, j, u'_j(y_j) \geq \hat{\alpha}_{ji} u'_i(y_i)$ and hence $y_j \leq (u'_j)^{-1}(\frac{1}{\hat{\alpha}_{ji}} u'_i(y_i))$, leading to

$$n\bar{y}^0 \leq \sum_j (u'_j)^{-1}(\frac{1}{\hat{\alpha}_{ij}} u'_i(y_i))$$

Under common CARA utilities, this yields

$$\frac{1}{An} \sum_j \hat{c}_{ji} \leq y_i - \bar{y}^0 \leq \frac{1}{An} \sum_j \hat{c}_{ij}$$

and hence $|y_i - \bar{y}^0| \leq \frac{1}{An} \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})$. Finally, $DISP(\mathbf{y}) \leq \frac{1}{An^2} \sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})$.

Next, we illustrate how to compute similar bounds for other measures of distance and other utility functions. Introduce $SDISP(\tilde{\mathbf{y}}) = [E\frac{1}{n} \sum_i (y_i - \bar{y}^0)^2]^{1/2}$ as in Ambrus, Mobius & Szeidl (2014). We obtain:

$$SDISP(\tilde{\mathbf{y}}) \leq \frac{1}{A} \frac{1}{n^{3/2}} [\sum_i \max(\sum_j \hat{c}_{ij}, \sum_j \hat{c}_{ji})^2]^{1/2}$$

When the network is binary and undirected, the bound becomes $\frac{-\ln(\alpha)}{A} \frac{1}{n^{3/2}} [\sum_{i,j} d_{ij}^2]^{1/2}$. Then, $\frac{1}{n(n-1)} \sum_{i,j} d_{ij}^2 = \bar{d}^2 + V(\mathbf{d})$ where $V(\mathbf{d})$ is the variance of path lengths. Thus, $SDISP$ tends to be lower when average path length and path length variance are lower.

Alternatively, consider common CRRA utilities: $u(y) = y^{1-\gamma}/(1-\gamma)$ if $\gamma \neq 1$ and $u(y) = \ln(y)$ if $\gamma = 1$. This yields

$$(\frac{1}{n} \sum_j \hat{\alpha}_{ji}^{1/\gamma} - 1)\bar{y}^0 \leq y_i - \bar{y}^0 \leq (\frac{1}{\frac{1}{n}\sum_j \hat{\alpha}_{ij}^{1/\gamma}} - 1)\bar{y}^0$$

and hence

$$DISP(\mathbf{y}) \leq \frac{1}{n} \sum_i \max(1 - \frac{1}{n} \sum_j \hat{\alpha}_{ji}^{1/\gamma}, \frac{1}{\frac{1}{n}\sum_j \hat{\alpha}_{ij}^{1/\gamma}} - 1)\bar{y}^0$$

which simplifies to $DISP(\mathbf{y}) \leq \frac{1}{n} \sum_i (\frac{1}{\frac{1}{n}\sum_j \alpha^{d_{ij}/\gamma}} - 1)\bar{y}^0$ for undirected, binary networks.

Finally, consider common CARA utilities and suppose that the network of altruism is not strongly connected. Then, there exists a set S such that $S \neq \varnothing$, $N - S \neq \varnothing$, there exists a path between any two agents in S in $\boldsymbol{\alpha}$, and no agent in S cares about an agent not in S. Consider the income distribution such that $y_i^0 = Y > 0$ if $i \in S$ and $y_i^0 = 0$ if $i \notin S$. Then, there is no transfer in equilibrium and $\mathbf{y} = \mathbf{y}^0$. This yields $DISP(\tilde{\mathbf{y}}) = \frac{n - n_S}{n} Y$. QED.

Proof of alternative bound on p.11. Denote by $\hat{c}_{\max} = \max_{i,j} \hat{c}_{ij}$. Since $\forall i, j, u'_i(y_i) \geq \hat{\alpha}_{ij} u'_j(y_j)$, $\forall i, j, y_i \leq y_j + \hat{c}_{ij}/A \leq y_j + \hat{c}_{\max}/A$. This implies that $y_{\max} - y_{\min} \leq \hat{c}_{\max}/A$ where $y_{\max} = \max_i y_i$ and $y_{\min} = \min_i y_i$. Consider the problem of maximizing $DISP(\mathbf{y})$ under the constraint that $y_{\max} - y_{\min} = \Delta$ where Δ is some arbitrarily fixed value. The solution to this problem is to set $y_i = y_{\max}$ for $n/2$ agents if n is even and for $(n+1)/2$ agents if n is odd and $y_i = y_{\min}$ for $n/2$ agents if n is even and for $(n-1)/2$ agents if n is odd. This yields $DISP(\mathbf{y}) = \frac{1}{2}\Delta$ if n is even and $= (\frac{1}{2} - \frac{1}{2n^2})\Delta$ if n is odd. This implies that, in general, $DISP(\mathbf{y}) \leq \frac{1}{2}(y_{\max} - y_{\min}) \leq \frac{1}{2}\hat{c}_{\max}/A$. QED.

Proof of computations on p.11. Suppose agent i is subject to large shocks. If the shock is positive, $\forall j, u_i'(y_i) = \hat{\alpha}_{ij}u_j'(y_j)$. This yields $y_i = y_j + \frac{c}{A}d_{ij}$. Taking the average over j yields $y_i = \bar{y}^0 + \frac{c}{A}\bar{d}_i$ and $y_j = \bar{y}^0 + \frac{c}{A}(\bar{d}_i - d_{ij})$. If the shock is negative, $\forall j, u_j'(y_j) = \hat{\alpha}_{ji}u_i'(y_i)$ and hence $y_j = y_i + \frac{c}{A}d_{ij}$ and $y_i = \bar{y}^0 - \frac{c}{A}\bar{d}_i$ and $y_j = \bar{y}^0 + \frac{c}{A}(d_{ij} - \bar{d}_i)$. This leads to $\frac{1}{n}\sum_i |y_i - \bar{y}^0| = \frac{c}{nA}\sum_i |\bar{d}_i - d_{ij}|$. QED.

Proof of Theorem 1

Lemma A1 *Fix a transfer graph* $\mathbf{G}$. F*or any* i, j, k, *we have:* $\bar{c}_{ji} = -\bar{c}_{ij}$, $\bar{c}_{ij} + \bar{c}_{jk} = \bar{c}_{ik}$ *and* $\ln(\lambda_i) - \ln(\lambda_j) = \bar{c}_{ij}$. *Further,* $\sum_i \ln(\lambda_i) = 0$.

Proof: (1) The path leading from j to i reverses all directions from the path leading from i to j, leading to the first property. (2) Suppose that j lies on the path connecting i to k. By definition, $\bar{c}_{ik} = \bar{c}_{ij} + \bar{c}_{jk}$. If k lies on the path connecting i to j, we then have $\bar{c}_{ij} = \bar{c}_{ik} + \bar{c}_{kj} = \bar{c}_{ik} - \bar{c}_{jk}$. Next, suppose that l is the last agent lying both on the path from i to k and on the path from i to j. Then, $\bar{c}_{ik} = \bar{c}_{il} + \bar{c}_{lk}$ and $\bar{c}_{ij} = \bar{c}_{il} + \bar{c}_{lj}$. Moreover, the path from k to j is formed of the path from k to l and of the path from l to j. Therefore, $\bar{c}_{kj} = \bar{c}_{kl} + \bar{c}_{lj}$. This yields: $\bar{c}_{ik} + \bar{c}_{kj} = \bar{c}_{il} + \bar{c}_{lk} + \bar{c}_{kl} + \bar{c}_{lj} = \bar{c}_{il} + \bar{c}_{lj} = \bar{c}_{ij}$. (3) Applying these two properties, we obtain:

$\ln(\lambda_i) - \ln(\lambda_j) = \frac{1}{n_C}\sum_{k\in C}(\bar{c}_{ik} - \bar{c}_{jk}) = \frac{1}{n_C}\sum_{k\in C}(\bar{c}_{ik} + \bar{c}_{kj}) = \frac{1}{n_C}\sum_{k\in C}\bar{c}_{ij} = \bar{c}_{ij}$

(4). Finally, note that $\sum_i \ln(\lambda_i) = \frac{1}{n_C}\sum_{i,j}\bar{c}_{ij} = \frac{1}{n_C}\sum_{i<j}(\bar{c}_{ij} + \bar{c}_{ji}) = 0$. QED.

Lemma A2 *Consider an income realization* $\mathbf{y}^0$, *equilibrium transfers* $\mathbf{T}$ *with transfer graph* $\mathbf{G}$. *Let* C *be a weak component of* $\mathbf{G}$. *Then, equilibrium consumption profile* $\mathbf{y}_C$ *on* C *solves the planner's program:* $\max_{\tilde{\mathbf{y}}_C}\sum_i \lambda_i u_i(\tilde{y}_i)$ *under the constraint* $\sum_{i\in C}\tilde{y}_i = \sum_{i\in C} y_i^0$ *and with* λ_i *such that* $\ln(\lambda_i) = \frac{1}{n_C}\sum_{j\in C}\bar{c}_{ij}$.

Proof: Consider i and j in C, connected through the path $i_1 = i$, i_2,..., $i_l = j$. If $g_{i_s i_{s+1}} = 1$, then equilibrium conditions imply that $\ln(u_{i_s}'(y_{i_s})) - \ln(u_{i_{s+1}}'(y_{i_{s+1}})) = -c_{i_s i_{s+1}}$. If $g_{i_{s+1}i_s} = 1$, then $\ln(u_{i_s}'(y_{i_s})) - \ln(u_{i_{s+1}}'(y_{i_{s+1}})) = c_{i_{s+1}i_s}$. Summing over all agents in the path yields

$$\ln(u_i'(y_i)) - \ln(u_j'(y_j)) = -\bar{c}_{ij} = \ln(\lambda_j) - \ln(\lambda_i)$$

by Lemma A1. These correspond to the first-order conditions of the planner's program. In addition, no money flows from C to $N - C$ or from $N - C$ to C. Therefore, $\sum_{i\in C} y_i^0 = \sum_{i\in C} y_i$ and aggregate income is preserved within C. QED.

Suppose that for any income realization, there is a Nash equilibrium with transfer graph $\mathbf{G}$. Then, the λ_i's do not depend on the income realization and the first part of Theorem 1 follows directly from Lemma A2.

For the second part, consider an altruism network $\boldsymbol{\alpha}$ satisfying the following property. Consider an undirected cycle, that is, a binary graph $\mathbf{U}$ connecting l agents i_1,..., $i_l = i_1$

such that either $u_{i_s i_{s+1}} = 1$ or $u_{i_{s+1} i_s} = 1$ and $u_{ij} = 0$ if i and j are not two consecutive agents in the set . Then, $\sum_{s:u_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:u_{i_{s+1} i_s}=1} c_{i_{s+1} i_s} \neq 0$. In Bourlès, Bramoullé & Perez-Richet (2017), we showed that such networks are generic and that they always have a unique Nash equlibrium. Given a binary directed forest $\mathbf{G}$, define $Y_0(\mathbf{G}) = \{\mathbf{y}^0 \in Y_0 :$ the transfer graph of the Nash equilibrium is $\mathbf{G}\}$ the set of income realizations leading to $\mathbf{G}$ and $\mathring{Y}_0(\mathbf{G})$ its interior. Observe that the non-empty sets $Y_0(\mathbf{G})$ define a finite partition of Y_0. Define $\boldsymbol{\lambda}(\mathbf{G})$ the profile of Pareto weights as defined in the first part of Theorem 1. This mapping satisfies the following useful property.

Lemma A3 *Consider two binary directed trees* $\mathbf{G}$ *and* $\mathbf{H}$. *Then,* $\boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}(\mathbf{H}) \Rightarrow \mathbf{G} = \mathbf{H}$.

Proof: Let $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}(\mathbf{H})$ and suppose that $\mathbf{G} \neq \mathbf{H}$. There exists i, j such that $g_{ij} = 1$ and $h_{ij} = 0$. Since $g_{ij} = 1$, $\ln(\lambda_i) - \ln(\lambda_j) = c_{ij}$. Since $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\mathbf{H})$, $\ln(\lambda_i) - \ln(\lambda_j) = \bar{c}_{ij} = \sum_{s:h_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:h_{i_{s+1} i_s}=1} c_{i_{s+1} i_s}$ for an undirected path connecting i to j. The set $i, i_2, ..., i_l = j, i$ then defines an undirected cycle satisfying $\sum_{s:u_{i_s i_{s+1}}=1} c_{i_s i_{s+1}} - \sum_{s:u_{i_{s+1} i_s}=1} c_{i_{s+1} i_s} = 0$, which is impossible given our assumptions on $\boldsymbol{\alpha}$. QED.

Suppose first that there is only one community in the partition. In other words, altruistic transfers generate efficient insurance for Pareto weights $\boldsymbol{\mu}$. This implies that there exist functions f_i such that $\forall \mathbf{y}^0 \in Y_0, y_i = f_i(\sum_j y_j^0)$. Let $\mathbf{G}$ be any graph such that $\mathring{Y}_0(\mathbf{G}) \neq \varnothing$. Such a graph exists by the assumption that $\mathring{Y}_0 \neq \varnothing$. Suppose that $\mathbf{G}$ is disconnected. Then by the first part of Theorem 1, there exist functions g_i such that $\forall \mathbf{y}^0 \in \mathring{Y}_0(\mathbf{G})$, $y_i = g_i(\sum_{j \in C} y_j^0) = f_i(\sum_{j \in C} y_j^0 + \sum_{j \in N-C} y_j^0)$. This implies that $\forall \mathbf{y}^0 \in \mathring{Y}_0(\mathbf{G}), \sum_{j \in N-C} y_j^0 = L$ which contradicts the fact that $\mathring{Y}_0(\mathbf{G})$ is a non-empty open set.

Therefore, $\mathbf{G}$ is connected and hence is a tree. By the first part of Theorem 1, there is efficient risk sharing on $\mathring{Y}_0(\mathbf{G})$ for Pareto weights $\boldsymbol{\lambda}(\mathbf{G})$. $\forall i, j, u_i'(y_i)/u_j'(y_j) = \lambda_j/\lambda_i = \mu_j/\mu_i$. This implies that there exists $t > 0$ such that $\boldsymbol{\mu} = t\boldsymbol{\lambda}(\mathbf{G})$. Next consider another graph $\mathbf{H}$ for which $\mathring{Y}_0(\mathbf{H}) \neq \varnothing$. By the same reasoning, there exists $t' > 0$ such that $\boldsymbol{\mu} = t'\boldsymbol{\lambda}(\mathbf{H})$. Since $\boldsymbol{\lambda}(\mathbf{G})$ and $\boldsymbol{\lambda}(\mathbf{H})$ satisfy the same normalization $\sum_j \ln(\lambda_j) = 0$, then $\boldsymbol{\lambda}(\mathbf{G}) = \boldsymbol{\lambda}(\mathbf{H})$. By Lemma A3, $\mathbf{G} = \mathbf{H}$. Therefore, $\mathring{Y}_0 = \mathring{Y}_0(\mathbf{G})$.

Finally, suppose that the partition is composed of several communities. Apply, first, the previous reasoning to each community C considered separately. There exists a tree graph $\mathbf{G}_C$ connecting agents in C and such that $\boldsymbol{\mu}_C = t_C\boldsymbol{\lambda}(\mathbf{G}_C)$ for $t_C > 0$ and $\boldsymbol{\mu}_C$ Pareto weights within C and $\mathbf{G}_C$ describes the pattern of giving relationships within C. Next, let us show that for any income realization in the support's interior, an agent in one community cannot give to an agent in another. Constrained efficiency implies income conservation within communities: $\forall C, \sum_{i \in C} y_i = \sum_{i \in C} y_i^0$. Suppose that for some $\mathbf{y}^0 \in Y_0$, there are some intercommunity transfers. The graph connecting communities is also a forest. Therefore, there exists a community connected to other communities through a single link. Formally, there exists $C \neq C'$ such that $i \in C$, $j \in C'$ and $t_{ij} > 0$ or $t_{ji} > 0$ and where there is no other giving link connecting C and $N - C$. If $t_{ij} > 0$, this implies $\sum_{i \in C} y_i = \sum_{i \in C} y_i^0 - t_{ij}$. If $t_{ji} > 0$, this implies $\sum_{i \in C} y_i = \sum_{i \in C} y_i^0 + t_{ij}$. In either case, $\sum_{i \in C} y_i \neq \sum_{i \in C} y_i^0$, which contradicts the original assumption. QED.

Proof of Proposition 4 If α_{ij} increases, c_{ij} decreases. Then, $\bar{c}_{kl}$ decreases if the link ij lies on the path connecting k to l. By contrast, $\bar{c}_{kl}$ increases if the link ji lies on the path connecting k to l. Agents in C_i are connected through agents in C_j through the link ij, and this link does not appear on the path connecting agents in C_i. This implies that $\sum_{l\in C}\bar{c}_{kl}$ decreases if $k\in C_i$. Similarly, $\sum_{l\in C}\bar{c}_{kl}$ increases if $k\in C_j$. QED.

Proof of Proposition 5. Given transfers $\mathbf{T}$, observe that $\mathbf{T}^t$ represent reverse transfers with identical amounts flowing in opposite directions. We first establish that reverse transfers form an equilibrium for the opposite shock. Denote by $\mathbf{y}^0(\boldsymbol{\varepsilon})=\mu\mathbf{1}+\boldsymbol{\varepsilon}$ and by $\mathbf{y}(\boldsymbol{\varepsilon})$ the associated equilibrium consumption.

Lemma A4 *Let* $\mathbf{T}$ *be a Nash equilibrium for incomes* $\mathbf{y}^0(\boldsymbol{\varepsilon})$ *leading to consumption* $\mathbf{y}(\boldsymbol{\varepsilon})$*. Then,* $\mathbf{T}^t$ *is a Nash equilibrium for incomes* $\mathbf{y}^0(-\boldsymbol{\varepsilon})$ *and* $\mathbf{y}(\boldsymbol{\varepsilon})-\mathbf{y}^0(\boldsymbol{\varepsilon})=\mathbf{y}^0(-\boldsymbol{\varepsilon})-\mathbf{y}(-\boldsymbol{\varepsilon})$*.*

Proof: Note that $\mathbf{y}=\mu\mathbf{1}+\boldsymbol{\varepsilon}-\mathbf{T1}+\mathbf{T}^t\mathbf{1}$. Denote by $\mathbf{y}'$ the consumption associated with transfers $\mathbf{T}^t$ when incomes are $\mu\mathbf{1}-\boldsymbol{\varepsilon}$. Then, $\mathbf{y}'=\mu\mathbf{1}-\boldsymbol{\varepsilon}-\mathbf{T}^t\mathbf{1}+\mathbf{T1}$. Comparing yields: $\mathbf{y}-\mu\mathbf{1}-\boldsymbol{\varepsilon}=\mu\mathbf{1}-\boldsymbol{\varepsilon}-\mathbf{y}'$ and hence $\mathbf{y}(\boldsymbol{\varepsilon})-\mathbf{y}^0(\boldsymbol{\varepsilon})=\mathbf{y}^0(-\boldsymbol{\varepsilon})-\mathbf{y}'$. Equilibrium conditions on $\mathbf{T}$ are: (1) $\forall i,j,y_i-y_j\leq c_{ij}/A$, and (2) $t_{ij}>0\Rightarrow y_i-y_j=c_{ij}/A$. Next, let us check that $\mathbf{T}^t$ satisfy the equilibrium conditions for incomes $\mathbf{y}^0(-\boldsymbol{\varepsilon})$. We have: $y_i'=2\mu-y_i$. This implies that $y_i'-y_j'=y_j-y_i$. Therefore, $\forall i,j,y_i'-y_j'=y_j-y_i\leq c_{ji}/A=c_{ij}/A$ since the ties are undirected. In addition, $(\mathbf{T}^t)_{ij}=t_{ji}$ and $t_{ji}>0\Rightarrow y_j-y_i=c_{ji}/A\Rightarrow y_i'-y_j'=c_{ij}/A$. QED.

We have: $E(y_i-y_i^0)=\int_{\boldsymbol{\varepsilon}}[y_i(\boldsymbol{\varepsilon})-y_i^0(\boldsymbol{\varepsilon})]f(\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}$. In the integral, the term associated with no shock is equal to 0, $y_i(\mathbf{0})=y_i^0(\mathbf{0})$. The term associated with shock $\boldsymbol{\varepsilon}$ is equal to $[y_i(\boldsymbol{\varepsilon})-y_i^0(\boldsymbol{\varepsilon})]f(\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}$. The term associated with shock $-\boldsymbol{\varepsilon}$ is equal to $[y_i(-\boldsymbol{\varepsilon})-y_i^0(-\boldsymbol{\varepsilon})]f(-\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}=[y_i^0(\boldsymbol{\varepsilon})-y_i(\boldsymbol{\varepsilon})]f(\boldsymbol{\varepsilon})d\boldsymbol{\varepsilon}$ by Lemma A4 and by shock symmetry. The sum of these terms is then equal to 0 and the integral aggregates such sums. QED.

A new connection can increase consumption variance under income correlation. Consider agents 1, 2 and 3 with incomes $(12,0,0)$ with probability 1/2 and $(0,12,12)$ with probability 1/2. Note that this satisfies the symmetry assumption of Proposition 5. Agents have common CARA utilities with $-\ln(\alpha)/A=2$. In the original network, 1 and 2 are connected and 3 is isolated. Consumption is $(7,5,0)$ with proba 1/2 and $(5,7,12)$ with proba 1/2. Next, connect 2 and 3. Consumption becomes $(6,4,2)$ with proba 1/2 and $(6,8,10)$ with proba 1/2. Agent 2 faces a more risky consumption profile. Here, the income streams of agent 2 and 3 are perfectly positively correlated. Agent 2's consumption becomes lower when poor and higher when rich, due to this new connection.

Variance computations on p.20. With 3 agents, there are 8 states of the world. Consider, first, the network where 1 and 2 are connected and 3 is isolated, see the example in Section 2.1. Since $c<2\sigma$, the variance of y_1 and y_2 drops from σ^2 to $\frac{1}{2}\sigma^2+\frac{1}{4}c^2$. Next, connect agents 2 and 3 to form a line. We assume that altruism is high enough to induce transfer paths of length 2 in situations where a single peripheral agent has a positive or a negative shock. This is satisfied iff $c<\frac{2}{3}\sigma$. Computing transfers and consumption for each state of the world, we find, $Var(y_1) = Var(y_3) = \frac{1}{3}\sigma^2 + \frac{1}{18}\sigma c + \frac{19}{36}c^2$ and $Var(y_2) = \frac{1}{3}\sigma^2 - \frac{1}{9}\sigma c + \frac{1}{9}c^2$. All variances drop. Finally, connect agents 1 and 3 to form the triangle. Consumption variance for any agent is now equal to $\frac{1}{3}\sigma^2 + \frac{1}{6}c^2$. $Var(y_2)$ increases while $Var(y_1) = Var(y_3)$ decreases. QED.

Proof of Proposition 6. Our proof makes use of the following classical properties of the covariance operator, see e.g. Gollier (2001). First, if f and g are non-decreasing functions and $\tilde{X}$ is some random variable, then, $cov(f(\tilde{X}), g(\tilde{X})) \geq 0$. Second, the law of total covariance states that if $\tilde{X}, \tilde{Y}, \tilde{Z}$ are three random variables, then $cov(\tilde{X}, \tilde{Y}) = E(cov(\tilde{X}, \tilde{Y}|Z)) + cov(E(\tilde{X}|Z), E(\tilde{Y}, Z))$.

Given set of agents S, denote by $\mathbf{y}^0_{-S}$ the vector of incomes of agents not in S. Apply the law of total covariance to variables $\tilde{y}_i, \tilde{y}_j$ and $\tilde{\mathbf{y}}^0_{-1}$. This yields $cov(\tilde{y}_i, \tilde{y}_j) = E(cov(\tilde{y}_i, \tilde{y}_j|\mathbf{y}^0_{-1})) + cov(E(\tilde{y}_i|\mathbf{y}^0_{-1}), E(\tilde{y}_j|\mathbf{y}^0_{-1}))$. Note that conditional on $\mathbf{y}^0_{-1}$, y_i and y_j are deterministic, non-decreasing functions of y^0_1 by Theorem 3 in Bourlès, Bramoullé & Perez-Richet (2017). By the property of the covariance of monotone functions, this implies that $\forall \mathbf{y}^0_{-1}$, $cov(\tilde{y}_i, \tilde{y}_j|\mathbf{y}^0_{-1}) \geq 0$ and hence $E(cov(\tilde{y}_i, \tilde{y}_j|\mathbf{y}^0_{-1})) \geq 0$. Next, let f_1 denote the pdf of $\tilde{y}^0_1$. By independence,

$$E(\tilde{y}_i|\mathbf{y}^0_{-1}) = \int y_i(y^0_1, \mathbf{y}^0_{-1}) f_1(y^0_1) dy^0_1$$

Since $y_i(y^0_1, \mathbf{y}^0_{-1})$ is non-decreasing in y^0_2, this implies that $E(\tilde{y}_i|\mathbf{y}^0_{-1})$ is also non-decreasing in y^0_2. We can therefore repeat the argument: $cov(E(\tilde{y}_i|\mathbf{y}^0_{-1}), E(\tilde{y}_j|\mathbf{y}^0_{-1})) = E(cov(\tilde{y}_i, \tilde{y}_j|\mathbf{y}^0_{-1,2})) + cov(E(\tilde{y}_i|\mathbf{y}^0_{-1,2}), E(\tilde{y}_j|\mathbf{y}^0_{-1,2}))$ where $E(cov(\tilde{y}_i, \tilde{y}_j|\mathbf{y}^0_{-1,2})) \geq 0$ by monotonicity. Dimensionality is reduced at each step, and all terms are non-negative. QED.

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