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Guillaume Bataille
Benteng Zou

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Guillaume Bataille † Benteng Zou‡

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Abstract

Motivated by recent examples, this study proposes a dynamic multistage optimal control problem to explain the instability of International Fishery Agreements (IFAs). We model two heterogeneous countries that exploit shared fishery resources, and investigate the conditions that lead to a shift from cooperation to competition. We assume that countries differ in their time preferences, initially behave as if the coalition will last indefinitely, use fixed sharing rules during cooperation, and adopt Markovian strategies after withdrawal. Our findings reveal that, for any sharing rule, coalitions of heterogeneous players always break down in finite time. We use the dynamic Shapley Value to decompose the coalition's aggregate worth over time, thereby eliminating the incentive to leave the agreement. Additionally, we show that a fishing moratorium policy accelerates the recovery of near-extinct fish stocks; however, fishing should resume under a cooperative regime once sustainable levels are achieved.

Keywords: Fisheries, International Fishery Agreements, Dynamic games, Multistage Optimal Control.
JEL Codes: C71, C72, Q22.

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†Aix Marseille Univ, CNRS, AMSE, Marseille, France. Address: AMSE, Maison de l'économie et de la gestion d'Aix, 424 chemin du viaduc, 13080 Aix-en-Provence, France Email: guillaume.bataille@univ-amu.fr.

‡DEM, University of Luxembourg, Luxembourg. Address:University of Luxembourg, 6, rue Richard Coudenhove-Calergi, L-1359, Luxembourg. Email: benteng.zou@uni.lu.

1 Introduction

The 1982 establishment of Exclusive Economic Zones (EEZs) has redefined the property rights of marine fisheries by transitioning fish resources from a common pool to coastal state assets. A fish stock is internationally shared if the species' habitat is located between several EEZs or if the fish migrate between EEZs. These stocks constitute up to one-third of the global marine-capture fishery yield (Munro et al., 2004). This requires multinational management strategies to prevent resource depletion induced by noncooperative use. Fish management is generally a regional issue among nations exploiting stocks in a specific location, which led to the establishment of Regional Fishery Management Organizations (RFMOs) as the principal tool for international fishery governance. However, recent challenges, including coalition breakdowns among RFMOs, highlight the fragility of International Fishery Agreements (IFAs). *This paper propose a dynamic model of endogenous exit that explains coalition breakdowns, and suggests a time-consistent sharing mechanism to stabilize IFAs. Furthermore, we propose an optimal exploitation strategy under international cooperation based on the stock of fish resources.*

The strategic nature of managing straddling fish stocks among countries requires game-theoretic models to better understand and solve issues related to the exploitation of fish resources. Munro (1979) was the first to revisit the standard fishery model of Gordon (1954) and its dynamic version proposed by Clark and Munro (1975) by introducing strategic components. The game-theoretic literature addresses two main issues: the economic and ecological impacts of noncooperative behaviors among countries sharing a fish stock (see Mirman (1979), Levhari and Mirman (1980), and their followers) and the conditions enabling the stability of IFAs over time. In this study, we are more interested in the latter issue, which has been analyzed through both non-cooperative and cooperative game approaches.

The coalition formation approach using a partition function game is mainly noncooperative and was first developed by Bloch (2003) and applied to fisheries by Pintassilgo and Lindroos (2008). This part of the literature provides the conditions for the endogenous formation of a coalition, as shown in Long and Flaaten (2011) and Breton and Keoula (2012)[1]. On the contrary, the stability of coalitions has mainly been explored using cooperative game theory through characteristic function games (see Kaitala and Lindroos (1998) and Kronbak and Lindroos (2007), among others). The goal is to determine the *ex-ante* sharing mechanism

[1] A number of empirical applications of the partition function game has been developed such as Ekerhovd (2010) who applied p-games to the blue withing fishery in the Northeast Atlantic, or even Kulmala et al. (2013), who explored the management of Atlantic salmon stocks in the Baltic Sea.

between members that satisfies some optimality criterion[2]. We follow this strand of the literature by assuming coalition formation and focusing on the impact of sharing mechanisms on IFA stability. We argue that time-invariant sharing mechanisms, even if optimally established during coalition formation, fail to maintain stability because of the strong economic incentives for members to exit as time passes. Essentially, regardless of the initial sharing arrangement among the coalition members, a point in time arises when the future benefits of exiting the coalition surpass those of the remaining members. This rigidity implies coalition breakdowns due to dynamic time inconsistency.

Recent decades witnessed a growing focus on time inconsistency in resource management. In fisheries, Ekeland et al. (2015) investigated the dynamic time inconsistency resulting from policymakers' inability to commit, manifested by the constant discounting of future utilities between both current and not-yet-born individuals. We argue that heterogeneous discounting can induce dynamic inconsistencies in coalitions. Motivated by the growing focus on discounting in environmental economics (see Gollier (2010) and Scarborough (2011)), we examine how the time preference diversity among fishing nations affects the dynamic stability of IFAs. In the fishery context, such differences in discounting are usually interpreted as divergence between management objectives. The observed diversity in the discounting of countries within a coalition can be attributed to the political parties' preferences for resource conservation and economic activity. An additional motivation for heterogeneous time preferences is the link between different zonal attachments or fish runs among countries (Munro, 1990). More advantageous migratory patterns within the fish stock are usually associated with a lower discount rate, and therefore, a higher preference for future rewards. An example of coalition breakdown induced by divergence in fish runs is the management of Atlanto-Scandian Herring stock by the North East Atlantic Fisheries Commission (NEAFC). In 2002, following the fish run changes after stock recovery, Norway exited the coalition due to disagreements over the initial quota allocations set when stocks were low, transitioning from cooperation to competition in fish stock management. This example motivated our analysis in this study. Additional examples are available in the recent book by Grønbæk et al. (2020).

The main contribution of this study is that it explores the dynamic aspects of coalition stability, allowing for time-varying decisions regarding coalition membership. As Pintassilgo et al. (2015) point out, this has not been studied much in the fishery literature, partly because of its theoretical complexity. This is a major contribution to the literature on the stability of

[2]Some applications that empirically derives different sharing imputations includes Arnason et al. (2000) for the Norwegian spring-spawning herring fishery or Duarte et al. (2000) for the Northern Atlantic Bluefin tuna fishery.

a coalition and can be applied to current IFA issues.

An example that we have in mind is the EU/UK post-Brexit fishing-related problem, where EU fishers will be gradually excluded from UK waters until 2026, when the share of quotas will be decided every year. While the EU favoring a permanent deal, the UK expects an annual renewal of quota shares. Within a dynamic framework, our study investigates whether these renegotiations in catch share could enhance cooperation stability over adjacent EU and UK fish stocks. An additional element that influences coalition (in)stability is the health status of fish stocks before coalition formation. Considering that many internationally shared fish stocks are still overexploited (FAO 2005), we argue that more depleted stocks prior to cooperation may help stabilize and reduce the cost of sustaining such cooperation for members over time.

A time-consistent sharing rule is required to promote IFAs. We build on insights from cooperative dynamic game literature. The main theoretical contributions of Petrosyan (1995) and Zaccour (2008) offer a framework for developing imputation rules that sustain cooperation over time using dynamic individual rationality as a key concept. Our analysis draws on these theoretical results to derive efficient catch sharing over time, especially in applying the Shapley value Petrosjan and Zaccour (2003)'s application of the Shapley value (Shapley, 1953) for time-based pollution cost allocation.

We develop a theoretical framework in which two asymmetric countries characterized by distinct discount rates form a coalition at $t = 0$. Within a coalition, it is assumed that a group's time preference lies within the range of its individual time preferences (Breton and Keoula, 2014). Despite potential objections, it seems unlikely that the collective time preference exceeds the individual ranges, particularly as it may be subject to negotiations during coalition formation[3]. Once part of the coalition, countries believe that it will last indefinitely and share the aggregate lifetime utility derived from cooperative management using a time-invariant, possibly optimal, sharing rule.

To examine the conditions under which the coalition may split, we consider withdrawing an endogenous variable allowing countries to leave the coalition at any time. When a potential split arises, both countries engage in a noncooperative game, considering their respective individual time preferences. Our analysis focuses on Markovian equilibria post-split, although extending it to non-Markovian strategies, as in Zou (2016), remains feasible.

The contributions of Hannesson (1997); Ekerhovd et al. (2021); Laukkanen (2003); Bediako and Nkuiya (2022) investigate cooperation, defection, and full competition in fishery games under different scenarios; however, all of them remain silent on regime (or game) changes.

[3]The aggregation of time preference when agents are heterogeneous is studied in Gollier and Zeckhauser (2005).

Our regime-switch model is essentially a two-stage optimal control problem, where the second stage is a standard non-cooperative differential game (Dockner et al., 2000). Recent studies, including Boucekkine et al. (2013) and Moser et al. (2014), developed multistage optimal control models, but few integrated dynamic games. Boucekkine et al. (2024) addressed this by examining a linear-quadratic framework for managing public bads, such as pollution. We adapt their approach with two key modifications: (i) we use a capital asset accumulation model to represent fish growth within a fisheries context, and (ii) we introduce a time-varying sharing mechanism to mitigate the coalition breakdown induced by player heterogeneity.

The principal findings of this study are as follows. Sustaining the coalition over time with a static (optimal) sharing rule established at the beginning of the coalition is impossible. This outcome is directly linked to the disparity in discount rates among the participating countries. Specifically, a more patient country is incentivized to remain longer for increased cooperative benefits but will leave once sufficient stock recovery is achieved. Conversely, the less-patient country leaves the coalition earlier as its proportion of aggregate lifetime utility from cooperation increases. Finally, it was shown that a dynamic welfare allocation mechanism using the time-dependent Shapley value as a tool can foster cooperative stability over time. Additionally, we demonstrated that a moratorium is an optimal cooperative strategy when the fish stock is nearing extinction. However, the moratorium should end once the fish stock reaches a minimum sustainable level.

The remainder of this paper is organized as follows. In Section 2, we introduce the model, and in Section 3, we compute both the cooperative and non-cooperative regimes as a function of the splitting time. The optimal splitting time is derived in Section 4. In Section 5, we propose a time-consistent payoff imputation rule to maintain cooperation. Furthermore, when the fish stock is severely depleted as part of cooperative management, we introduce the possibility of a harvest moratorium in Section 6 and provide the optimal moratorium duration, if any. Section 7 offers a discussion with alternative or relaxed assumptions compared to the previous sections, and Section 8 concludes the paper.

2 The Model

We analyze a shared resource, specifically, a fish stock harvested by two heterogeneous players or countries, differentiated by their time preference rates, ρ_l, for the more patient and ρ_h for the less patient. Initially, at $t = 0$, these countries created a Regional Fishery Management Organization (RFMO) designed to manage fish stocks. Within the coalition, players aim to maximize their joint payoff by exploiting the fish population at a common (and agreed-upon) discount rate, ρ. They share the payoff according to a fixed sharing rule, where γ is the share of the joint payoff received by player l, which could be the result of prior negations when the coalition has been formed. When players decide on the coalition's catch quota allocation, they are assumed to believe that the coalition will persist. However, if at some future date $0 \leq T \leq \infty$, one of them realizes that they would be better off quitting the coalition, then the coalition breaks down and the resulting situation is a standard Markovian competition under the individual's rates of time preferences. Motivated by the recent coalition breakdown in the fishery context, such a withdrawal is expected to occur due to the asymmetry in time preferences (or management objectives), in addition to the fixed sharing from which players could disagree as time passes.

To obtain an explicit formula, we consider a parametric example in which the dynamics of the fish population subject to consumption $c_k(t)$ $\forall k = l, h$ are as follows:

$$\dot{x}(t) = Ax(t)^{\theta} - \delta x(t) - c_l(t) - c_h(t), \qquad x(0) > 0, \tag{1}$$

where $A > 0$ and $\delta > 0$ denote the intrinsic growth and natural mortality rates of the fish, respectively. Parameter $0 < \theta < 1$ scales the production function relative to population size, making the growth function $Ax(t)^{\theta} - \delta x(t)$ concave with an inverted U-shaped curve. Importantly, the growth rate becomes infinite as the stock level approaches zero, excluding the Allee effect. Nevertheless, for given $x(0) > 0$ this framework ensures positive steady-state stocks. Without fishing, that is, $c_h(t) = c_l(t) = 0$, Equation (1) is a Bernoulli equation and the explicit solution can be obtained by letting $X(t) = x(t)^{1-\theta}$:

$$X_n(t) = (X(0) - \overline{X}_n)e^{-(1-\theta)\delta t} + \overline{X}_n, \tag{2}$$

where the initial condition satisfies $X(0) = x(0)^{1-\theta} > 0$ and the natural steady state without consumption is $\overline{X}_n = \frac{A}{\delta} > 0$, which is also referred to as the maximum sustainable yield (see Clark (2010)).

We now introduce the economic activities. The two asymmetric players enjoy consuming fish according to the following utility function, which has constant marginal utility elasticity:

$$\mathcal{U}_k\Big(c_k(\cdot)\Big)_{k=l,h} = \int_0^{\infty} \frac{c_k(t)^{1-\nu}}{1-\nu} \exp^{-\rho t} dt, \tag{3}$$

where ν denotes the constant elasticity of marginal utility. The following parametric restriction is imposed.

Assumption 1 *Suppose that* $\nu = \theta \quad \forall \;\; \theta \in (0, 1)$.

This assumption links the shape of the species growth function with the utility function.

Despite the lack of empirical justification, it facilitates analytical solutions, and thus, clear economic intuitions. Given that the results are not driven by this assumption, relaxing it is unlikely to alter the outcomes, but will imply loose tractability. Hence, θ and ν are embodied in a single parameter for the subsequent analyses.

Suppose that when the players are embodied in a coalition, the common discount factor of coalition ρ satisfies $\rho_l < \rho < \rho_h$. Cooperation includes the joint choice of the optimal catch and sharing strategy, but the discounting rate ρ could also be a result of negotiation, which could be the reason for the coalition breakdown. Therefore, it is unlikely that this rate was outside the range of individual time preferences. The objective of the players within the coalition is to jointly choose their consumption paths for the fish stock to maximize the sum of their intertemporal utility flows. Because they believe that the coalition is everlasting, the joint optimal control is

$$\mathcal{W}(x(0)) = \max_{c_l(\cdot), c_h(\cdot)} \int_0^\infty \left[\frac{c_l(t)^{1-\theta}}{1-\theta} + \frac{c_h(t)^{1-\theta}}{1-\theta} \right] e^{-\rho t} dt,$$

subject to the dynamics constraint in Equation (1).

Suppose either player l or player h decides to quit the coalition at time $0 \leq T \leq \infty$. The regime changes from cooperative management to full competition. Following Van Long (2010) and Gaudet and Lohoues (2008), we search for stationary linear Markovian strategies in the following form:

$$\forall k = l, h \quad c_k(t) = \omega_k x(t). \tag{4}$$

Therefore, the problem of a specific player $k = l, h$ can be written as

$$\begin{aligned} \mathcal{V}_k(x(T)) = \max_{c_k(\cdot)} & \int_T^\infty \frac{c_k(t)^{1-\theta}}{1-\theta} \exp^{-\rho_k t} dt \\ \text{s.t.} \quad & \dot{x}(t) = Ax(t)^\theta - \delta x(t) - c_k(t) - \omega_{-k} x(t), \quad t \geq T, \\ & x(T) > 0, \end{aligned} \tag{5}$$

where $c_k(x)$ is the catch of player k and $\omega_{-k} x(t)$ represents the Markovian strategy of the other player. Our model employs a forward-looking approach with anticipations. Players make decisions at $t = 0$, and in the non-cooperative regime, they discount payoff over the entire time period.

In this context, the choice of splitting time T depends on the sharing strategy chosen by the coalition members, γ, the optimal payoff at a given time $\mathcal{W}(T)$ and the payoffs flow after breaking, $\mathcal{V}_k(x(T))$. In particular, the optimal switching times for players l and h are

$$\max_T \Big[\gamma \mathcal{W}(T) + \mathcal{V}_l(x(T))\Big], \quad \text{and} \quad \max_T \Big[(1-\gamma)\mathcal{W}(T) + \mathcal{V}_h(x(T))\Big]. \tag{6}$$

Arguably, $\mathcal{W}(T)$ should be increasing in T, meaning that the longer the coalition, the higher the joint payoff and, therefore, the higher the payoff after splitting; otherwise, splitting occurs immediately. Conversely, we expect that as the cooperative becomes longer, the non-cooperative equilibrium payoff, $\mathcal{V}_k(x(T))$, after withdrawal decreases. The first-order conditions for (6) are

$$\gamma \frac{d\mathcal{W}(T)}{dT} + \frac{d\mathcal{V}_l(x(T))}{dT} = 0, \quad \text{and} \quad (1-\gamma)\frac{d\mathcal{W}(T)}{dT} + \frac{d\mathcal{V}_h(x(T))}{dT} = 0. \tag{7}$$

Therefore, the optimal splitting time is the lowest time T solving one of the two first conditions above, provided that the second-order sufficient conditions

$$\gamma \frac{d^2\mathcal{W}(T)}{dT^2} + \frac{d^2\mathcal{V}_l(x(T))}{dT^2} < 0, \quad \text{or} \quad (1-\gamma)\frac{d^2\mathcal{W}(T)}{dT^2} + \frac{d^2\mathcal{V}_h(x(T))}{dT^2} < 0 \tag{8}$$

are satisfied. This suggests that T is indeed a global optimum characterized more precisely by diminishing marginal cooperative gains and decreasing marginal non-cooperative losses associated with an extended cooperative stage. We can now characterize the cooperative and noncooperative outcomes before computing the optimal splitting time.

3 Cooperative and non-cooperative outcomes

First, we proceed with the characterization of the equilibrium under the cooperative regime, followed by an analysis of the non-cooperative Markovian strategies post-splitting, and finally conclude with a comparison of the outcomes under both regimes.

3.1 Cooperative stage

The optimal control problem in (4) is standard and can be solved using dynamic programming with the typical Hamilton-Jacobi-Bellman (HJB thereafter) equation. Denoting the optimal value function as $\mathcal{W}$, the HJB is

$$\rho \mathcal{W}(x) = \max_{c_l, c_h \geq 0} \left\{ \frac{c_l^{1-\theta}}{1-\theta} + \frac{c_h^{1-\theta}}{1-\theta} + \mathcal{W}(x)' \Big(A x^\theta - \delta x - c_l - c_h \Big) \right\}, \tag{9}$$

at state x.

The first-order necessary conditions on the right side are

$$c_l^{-\theta} = \mathcal{W}(x)' \quad \text{and} \quad c_h^{-\theta} = \mathcal{W}(x)'. \tag{10}$$

Considering the Bernoulli transformation $x(t)^{1-\theta} = X(t)$, we assume that the optimal value function is linear in the transformed state variable

$$\mathcal{W}(x) = \alpha \frac{x^{1-\theta}}{1-\theta} + \beta = \alpha \frac{X}{1-\theta} + \beta, \tag{11}$$

where α and β are coefficients identified using (9), (10), and (11). The following proposition summarizes the cooperative management regime under an everlasting coalition.

Proposition 1 *There exists a unique set of coefficients β, α that satisfy Equation (9), where the optimal choices of players $k = l, h$ are*

$$c_l = c_h = \alpha^{-1/\theta} x = \omega_c x, \tag{12}$$

where

$$\beta = \frac{A}{\rho}\alpha, \quad \alpha^{-1/\theta} = \omega_c = \frac{1-\theta}{2\theta}\left(\frac{\rho}{1-\theta} + \delta\right) (> 0). \tag{13}$$

Given that $x(t)^{1-\theta} = X(t)$, the trajectory of the fish stock is

$$X_c(t) = e^{-at}\left(X(0) - \bar{X}_c\right) + \bar{X}_c, \tag{14}$$

where $a = (1-\theta)(\delta + 2\omega_c) > 0$, and the long-run steady state is $\bar{X}_c = \frac{A}{\delta + 2\omega_c}$.

Proof. The proof is straightforward and is based on the identification of (9). ■

As expected, the long-run steady state stock $\bar{X}_c$ is lower than the maximum sustainable yield $\bar{X}_n$. The convergence speed to the steady state, that is, a, is significantly influenced by the group's time preference ρ through fishing intensity ω_c but not by the intrinsic growth rate of the fish, A. Thus, fish recovery is slower because of ongoing exploitation.

3.2 Non-cooperative stage after the split T

Thus far, we characterized cooperative equilibrium outcomes for the coalition over the entire timeframe. Suppose a player decides to exit the coalition at date T and adopts purely egoistic behavior. When only two players are involved, one player's departure leads to a scenario of pure competition. Our focus shifts to Markovian strategies after withdrawal, considering that players differ in their time preferences. Among the potentially infinite Markovian Nash equilibria (Dockner and Van Long, 1993), we aim to identify a pair of linear Markovian strategies $\{c_l(t), c_h(t)\} = \{\omega_l x(t), \omega_h x(t)\}$ that are the best responses to each other. For each player $k = h, l$, the individual value function is denoted by $\mathcal{V}_k(x)$, and the maximization problem in (5) is formulated recursively, as follows:

$$\rho_k \mathcal{V}_k(x) = \max_{c_k \geq 0} \left\{ \frac{c_k^{1-\theta}}{1-\theta} + \mathcal{V}_k(x)' \Big(A x^\theta - \delta x - c_k - \omega_{-k} x \Big) \right\}, \quad \forall k = l, h \text{ and } -k \neq k. \tag{15}$$

The first-order conditions on the right-hand side of (15) yield

$$c_k^{-\theta} = \mathcal{V}(x)' \quad \forall k = l, h. \tag{16}$$

Supposing that the individual value function is linear in the transformed state variable for all players,

$$\mathcal{V}_k(x) = \alpha_k \frac{x^{1-\theta}}{1-\theta} + \beta_k = \alpha_k \frac{X}{1-\theta} + \beta_k \quad \forall k = l, h, \tag{17}$$

where α_k, β_k are the coefficients that must be identified using (15) and (16), respectively. The following proposition provides the results for the non-cooperative stage with linear Markovian strategies after potential withdrawal.

Proposition 2 *Let player $k = h, l$ exit the coalition at time T and adopt linear Markovian strategies after withdrawal. There exists a unique set of coefficients α_k and β_k satisfying Equation (15). The strategies are*

$$c_k = \alpha_k^{-1/\theta} x = \omega_k x, \tag{18}$$

where

$$\beta_k = \frac{A}{\rho_k}\alpha_k, \quad \alpha_k^{-1/\theta} = \omega_k = \left(\frac{\rho_k}{1-\theta} + \frac{\delta + \rho_{-k}}{\theta} \right) \frac{(1-\theta)\theta}{\theta^2 - (1-\theta)^2}, \quad -k \neq k \tag{19}$$

which are positive if $1 > \theta > \frac{1}{2}$. Given the fish stock value after the first cooperative stage, $X_c(T)$, the fish stock trajectory under Markovian strategies in the second stage is

$$X_{nc}(t) = e^{-b(t-T)}(X_c(T) - \bar{X}_{nc}) + \bar{X}_{nc}, \quad \forall t \geq T, \tag{20}$$

where $b = (1-\theta)(\delta + \omega_h + \omega_l) > 0$ and $\bar{X}_{nc} = \frac{A}{\delta + \omega_h + \omega_l}$ is the long-run steady state.

Proof. The proof is straightforward and is based on the identification of (15). ■

Not surprisingly, condition (19) states that the patient player exploits the stock at a lower rate than the impatient player does; that is, $\rho_h > \rho_l \Longleftrightarrow \omega_h > \omega_l$. In other words, more patient players can more effectively postpone their current consumption to future consumption. Similarly, comparing the aggregate catch rates in the cooperative and non-cooperative regimes yields the expected results. Specifically, comparing $2\omega_c$ in (13) with $\omega_l + \omega_h$ in (19) shows that

$$2\omega_c = \frac{\rho}{\theta} + \frac{\delta(1-\theta)}{\theta} < \frac{\rho_l + \rho_h}{\theta^2 - (1-\theta)^2} + \frac{2\delta(1-\theta)}{\theta^2 - (1-\theta)^2} = \omega_l + \omega_h \tag{21}$$

Thus, for any fish stock x, the cooperative catch rate $2\omega_c$ is always lower than the competitive catch rate $\omega_l + \omega_h$. Consequently, in the long-run steady state, the fish stock under cooperation is larger than that under competition; that is, $\bar{X}_c > \bar{X}_{nc}$.

4 The optimal splitting time

To determine the optimal coalition splitting time, we first compute the individual payoffs in both stages for a given splitting time, T. Then, using (7) and a given sharing rule γ, we identify the conditions under which player $k \in h, l$ will withdraw in finite time.

4.1 The cooperative payoff

Suppose that γ, the share of the cooperative payoff allocated to player l, is constant over time. The aggregate payoff flow from time zero to time T is then

$$\mathcal{W}(T) = \int_0^T \left[2\frac{\omega_c^{1-\theta} X_c(t)}{1-\theta}\right] e^{-\rho t} dt, \tag{22}$$

where ω_c and $X_c(t)$ are given by (13) and (14), respectively. Welfare (22) can be rewritten as

$$\mathcal{W}(T) = 2\frac{\omega_c^{1-\theta}}{1-\theta}\left[\frac{(X(0) - \bar{X}_c)}{a+\rho}\left(1 - e^{-(a+\rho)T}\right) + \frac{\bar{X}_c}{\rho}\left(1 - e^{-\rho T}\right)\right], \tag{23}$$

where $a = (1-\theta)(\delta + 2\omega_c) > 0$. Therefore, player l receives $\gamma\mathcal{W}(T)$ and player h receives $(1-\gamma)\mathcal{W}(T)$. To assess how the splitting time affects each individual's cooperative payoff, we compute the partial derivative of the total payoff with respect to T in (23), given that the sharing rule is time independent.

$$\frac{\partial \mathcal{W}(T)}{\partial T} = 2\frac{\omega_c^{1-\theta}}{1-\theta}\left[(X(0) - \bar{X}_c)e^{-(a+\rho)T} + \bar{X}_c e^{-\rho T}\right] = 2\frac{\omega_c^{1-\theta}}{1-\theta} e^{-\rho T}\left[(X(0) - \bar{X}_c)e^{-aT} + \bar{X}_c\right] > 0 \tag{24}$$

using the trajectory of the fish stock (14). However, the second-order conditions with respect to the splitting time hold:

$$\frac{\partial^2 \mathcal{W}(T)}{\partial T^2} = -2\frac{\omega_c^{1-\theta}}{1-\theta}e^{-\rho T}\left[(\rho+a)(X(0)-\bar{X}_c)e^{-aT}+\rho\bar{X}_c\right] < 0. \tag{25}$$

As expected, the longer the cooperative stage, the larger the individual payoffs, although the marginal returns decrease over time.

4.2 Non-cooperative payoff

To determine the optimal splitting time, we also need to compute the payoff flows for each player after splitting. Using the second-stage results from Proposition 2, the payoff for player $k = l, h$ is

$$\mathcal{V}_k(X_c(T)) = \int_T^\infty \frac{\omega_k^{1-\theta}}{1-\theta}X_{nc}(t)e^{-\rho_k t}dt, \tag{26}$$

where $X_c(T)$ is the stock value at the end of the cooperative period. Computing (26) using (20) yields

$$\mathcal{V}_k(X_c(T)) = \frac{\omega_k^{1-\theta}}{1-\theta}\left[\frac{(X_c(T)-\bar{X}_{nc})}{b+\rho_k}e^{-\rho_k T}+\frac{\bar{X}_{nc}}{\rho_k}e^{-\rho_k T}\right], \tag{27}$$

where $b = (1-\theta)(\delta+\omega_l+\omega_h) > 0$. Taking the derivative with respect to T yields:

$$\frac{\partial \mathcal{V}_k(x(T))}{\partial T} = \frac{\omega_k^{1-\theta}e^{-\rho_k T}}{1-\theta}\left[-\left(\frac{\rho_k(X_c(T)-\overline{X}_{nc})}{b+\rho_k}+\overline{X}_{nc}\right)+\frac{1}{b+\rho_k}\frac{dX_c}{dT}\right] \tag{28}$$

where $\frac{dX_c(T)}{dT} = -a(X(0)-\overline{X}_c)e^{-aT}(>0)$ denotes the instantaneous stock gain at T. This term is positive if the initial stock is below the potential long-run steady state, $X(0)-\overline{X}_c < 0$, which is generally true given the aim of increasing the long-run fish stock under cooperation. The first part of (28), $\left(\frac{\rho_k(X_c(T)-\overline{X}_{nc})}{b+\rho_k}+\overline{X}_{nc}\right)$, measures the instantaneous loss in the value function at T because this delays the end of cooperation. Therefore, the sign of (28) depends on the dominant factor. Combining the two parts, rewriting, and using the fact that $a\overline{X}_c = b\overline{X}_{nc} = A(1-\theta)$, it follows that

$$\frac{\partial \mathcal{V}_k(x(T))}{\partial T} = \frac{\omega_k^{1-\theta}\ e^{-\rho_k T}\ (a+\rho_k)}{(1-\theta)(b+\rho_k)}\left[(\overline{X}_c-X(0))e^{-aT}-\overline{X}_c\right] < 0, \qquad \forall T > 0. \tag{29}$$

Additionally, we can readily verify that $\frac{\partial^2 \mathcal{V}_k(x(T))}{\partial T^2} < 0$, which, combined with (29), indicates that a longer cooperation period decreases the individual payoff post-withdrawal, with losses becoming increasingly significant over time.

4.3 Optimal splitting time

The optimal splitting time must satisfy the first-order conditions in Equation (7). By substituting Equations (29) and (24) into Equation (7), the optimal switching time T_k for player $k = l, h$ is

$$e^{(\rho_l - \rho)T_l} = \frac{1}{2\gamma} \left(\frac{\omega_l}{\omega_c}\right)^{1-\theta} \left(\frac{a+\rho_l}{b+\rho_l}\right), \quad \text{and} \quad e^{(\rho_h - \rho)T_h} = \frac{1}{2(1-\gamma)} \left(\frac{\omega_h}{\omega_c}\right)^{1-\theta} \left(\frac{a+\rho_h}{b+\rho_h}\right). \tag{30}$$

We directly obtain the following results.

Proposition 3 *Given that $\rho_l < \rho < \rho_h$ and parameters $A, \theta, \rho_l, \rho_h$, and δ,*

(a) ***For Player** l**:*** *If $\gamma \geq \underline{\gamma} = \frac{1}{2}\left(\frac{\omega_l}{\omega_c}\right)^{1-\theta}\left(\frac{a+\rho_l}{b+\rho_l}\right) (\in (0,1))$, then Player l exits the coalition at*

$$T_l = \frac{1}{\rho_l - \rho} \ln\left(\frac{1}{2\gamma}\left(\frac{\omega_l}{\omega_c}\right)^{1-\theta}\left(\frac{a+\rho_l}{b+\rho_l}\right)\right) \geq 0, \tag{31}$$

which increases with player l's share γ for $\gamma \geq \underline{\gamma}$ due to $\rho_l - \rho < 0$. The latest exit time is

$$T_l^{max} = \frac{1}{\rho_l - \rho} \ln\left(\frac{1}{2}\left(\frac{\omega_l}{\omega_c}\right)^{1-\theta}\left(\frac{a+\rho_l}{b+\rho_l}\right)\right) > 0.$$

(b) ***For Player** h**:*** *If $\gamma \geq \overline{\gamma} = 1 - \frac{1}{2}\left(\frac{\omega_h}{\omega_c}\right)^{1-\theta}\left(\frac{a+\rho_h}{b+\rho_h}\right) (\in (0,1))$; that is, the share of player h checks $1-\gamma < 1-\overline{\gamma}$, then Player h exits the coalition at*

$$T_h = \frac{1}{\rho_h - \rho} \ln\left(\frac{1}{2(1-\gamma)}\left(\frac{\omega_h}{\omega_c}\right)^{1-\theta}\left(\frac{a+\rho_h}{b+\rho_h}\right)\right) \geq 0, \tag{32}$$

which decreases with share $1-\gamma$ for $\gamma \geq \overline{\gamma}$ given $\rho_h - \rho > 0$.

Proof. The proof is straightforward, from (30). ■

The equality condition $a\overline{X}_c = b\overline{X}_{nc} = A(1-\theta)$ is crucial for proving the above proposition. This result reveals key insights into the decision-making process when a player decides to stop the coalition. Recalling the definitions of a and b in Equations (14) and (20), respectively, they can be considered effective discounting factors. Consequently, the equality $a\overline{X}_c = b\overline{X}_{nc} = A(1-\theta)$ implies that the instantaneous long-run steady state is identical under both cooperation and competition conditions. Therefore, maintaining the coalition offers no further benefit when competition can yield better results despite the fact that the long-run fish stock is higher under cooperation ($\overline{X}_c > \overline{X}_{nc}$).

The comparative statics of the splitting time T_k with respect to the sharing rule γ provide valuable insights. Player l, the more patient player, exits later as the share obtained from cooperation increases; that is, $\frac{\partial T_l}{\partial \gamma} > 0$, and always exits in finite time regardless of the share received from the cooperative stage. By contrast, player h, the more impatient player, exits sooner as the share obtained from cooperation increases; that is, $\frac{\partial T_h}{\partial(1-\gamma)} < 0$. The intuition is that the more player h receives from the cooperative stage, the sooner they will exit the coalition and enter competition.

The above proposition provides insights into the players' individual optimal exit times. However, to determine who is the first to dissolve the coalition, we must compare T_l and T_h in (31) and (32). Specifically, we define γ^e as the share of player l that leads to simultaneous existence; that is, $T_l = T_h$:

$$\gamma^e = \frac{e^{\frac{1}{\rho_l - \rho}} {\omega_l}^{1-\theta} \left(\frac{a+\rho_l}{b+\rho_l}\right)}{e^{\frac{1}{\rho_l - \rho}} {\omega_l}^{1-\theta} \left(\frac{a+\rho_l}{b+\rho_l}\right) + e^{\frac{1}{\rho_h - \rho}} {\omega_h}^{1-\theta} \left(\frac{a+\rho_h}{b+\rho_h}\right)} \in (0,1). \tag{33}$$

Thus, we should consider four cases, depending on the range of values for γ. This leads to the following conclusions:

Proposition 4 *For given parameters $A, \theta, \rho_l, \rho_h$, and δ,*

(1) If the share of player l satisfies $\underline{\gamma} > \gamma > \overline{\gamma}$, player l quits the coalition first at the time defined in (31).

(2) If the share of player l satisfies $\underline{\gamma} < \gamma < \overline{\gamma}$, then player h quits the coalition first at the time defined in (32).

(3) If the share of player l satisfies $\gamma \leq \min\{\overline{\gamma}; \underline{\gamma}\}$, then the coalition breaks immediately, $T = 0$.

(4) If the share of player l satisfies $\gamma > \max\{\overline{\gamma}; \underline{\gamma}\}$, then the coalition breaks in finite time either by player l if $\gamma < \gamma^e$ or by player h if $\gamma > \gamma^e$.

In contrast to the cases above where the joint time preference is located between the low and high time preferences ($\rho_l < \rho < \rho_h$), if the two players are identical, that is, $\rho_l = \rho_h = \rho$, then equality (30) provides no solution. [4] We thus conclude the following.

Corollary 1 *For any given parameters, A, θ, and δ, the coalition, if it exists, will last forever; that is, $T = +\infty$, if and only if the players are identical; that is, $\rho_l = \rho_h = \rho$.*

5 Time-consistent sharing to prevent breakdowns

The analysis above highlights some disappointing results for any fishery coalition. As long as the players are not identical, the coalition will eventually break in a finite time. Nonetheless, some cooperation does persist for a long time. One powerful tool to ensure successful cooperation is side payments, that is, sharing extra fishing resources and benefits fairly. The time-variant sharing rule between the UK and the EU mentioned in the introduction is an example. Other examples include the Pacific Salmon Treaty between the United States and Canada, and the North Pacific Fur Seal Treaty among Canada, Japan, Russia, and the US. More detailed descriptions of these cases are provided in Sections 4.4.2 and 4.4.3 of Grønbæk et al. (2020).

Recently, Bergantiños et al. (2023) applied Shapley values to study a sharing strategy for tuna fisheries among different vessels belonging to the same firm. They noted that although the Shapley value is standard in the game theory literature, its application to fisheries is rather limited. Both Grønbæk et al. (2020) and Bergantiños et al. (2023) apply a time-invariant Shapley value. However, as showed above, a fair Shapley value at one point in time may become unfair as time passes. In this section, we present a time-dependent Shapley value tailored to the current fishery cooperation situation.

5.1 Time-variant Shapley values

Let $\Phi_l(X)$ and $\Phi_h(X)$ be the Shapley values for players l and h, respectively. Following Petrosjan and Zaccour (2003), these Shapley values are

$$\Phi_k(X(t)) = \frac{\mathcal{V}(X(t)) + \mathcal{V}_k(X(t)) - \mathcal{V}_{-k}(X(t))}{2} \quad \text{for } k = l, h \text{ and } -k \neq k. \tag{34}$$

[4] The reason is straightforward. It is easy to check that $\omega_l = \omega_h = \frac{\rho+(1-\theta)\delta}{2\theta-1} > \frac{\rho+(1-\theta)\delta}{2\theta} = \omega_c$ and $\frac{a+\rho}{b+\rho} = \frac{2\theta-1}{2\theta}$. Thus, the first-order condition for player l in Equation (30) becomes $1 = \frac{1}{2\gamma}\left(\frac{2\theta-1}{2\theta}\right)^{\theta}$, which holds, if and only if $2\gamma = \left(\frac{2\theta-1}{2\theta}\right)^{\theta} > 1$. This is impossible given that the two players are identical and there is no particular reason one player shares more of the fish stock while the other players share less. Thus, there is no solution to (30).

The allocation of welfare to player $k(=l,h)$ at time $t \in [T,\infty)$ is

$$\Gamma_k(t) = \rho\Phi_k(X(t)) - \frac{d\Phi_k(X(t))}{dt}. \tag{35}$$

In other words, at time t, player k is allocated welfare corresponding to the interest payment (i.e., the social discount rate (= interest rate) times their welfare-to-gain under cooperation given by her Shapley value) minus the variation over time of this welfare-to-gain (Petrosjan and Zaccour, 2003).

From Proposition 1 of Petrosjan and Zaccour (2003, p. 388), the allocation $(\Gamma_l(t), \Gamma_h(t))$ is a time-consistent imputation distribution process; that is, it decomposes over time the total welfare of player k as given by the Shapley value component for the whole game. Furthermore, the relationship between the dynamic Shapley value and the Nash Bargaining solution can be established as follows:

Proposition 5 *In this asymmetric two-player setting with a transferable payoff, the Shapley Values $\Phi_k(X(t))$, $k = l, andh$, coincide with the Nash bargaining solution that maximizes the Nash Product.*

Proof. The proof is straightforward. We define $\Phi_k^{\mathcal{NB}}(X)$ as the Nash Bargaining solution for player $k = h, l$ that maximizes the Nash Product:

$$\max \mathcal{NB} = \left(\Phi_h^{\mathcal{NB}}(X) - \mathcal{V}_h(X)\right)\left(\Phi_l^{\mathcal{NB}}(X) - \mathcal{V}_l(X)\right) \tag{36}$$

$$\text{s.t } \Phi_h^{\mathcal{NB}}(X) + \Phi_l^{\mathcal{NB}}(X) = \mathcal{V}(X) \tag{37}$$

The solution is $\Phi_k^{\mathcal{NB}}(X) = \frac{\mathcal{V}(X)+\mathcal{V}_k(X)-\mathcal{V}_{-k}(X)}{2}$ for $k = l, h$ and $-k \neq k$.

■

5.2 Comparing the Shapley values

Because of the fairness property of the Shapley values, and given that the players have different rates of time preferences, the value of the coalition is likely not shared equally. Comparing Equation (34), we obtain

$$\Phi_k(X(t)) > \Phi_{-k}(X(t)) \quad \Leftrightarrow \quad \mathcal{V}_k(X(t)) > \mathcal{V}_{-k}(X(t)), \quad \text{for } k = l, h \text{ and } -k \neq k. \tag{38}$$

In other words, the asymmetry in the Shapley values is mainly due to the asymmetry in payoffs along the non-cooperative trajectory. Using Equation (17), we obtain

$$\begin{aligned}\mathcal{V}_h(X(t)) - \mathcal{V}_l(X(t)) &= \tfrac{X(t)}{1-\theta}\Bigg(\alpha_h - \alpha_l\Bigg) + A\Bigg(\tfrac{\alpha_h}{\rho_h} - \tfrac{\alpha_l}{\rho_l}\Bigg),\\ &= \alpha_h\Bigg(\tfrac{X(t)}{1-\theta} + \tfrac{A}{\rho_h}\Bigg) - \alpha_l\Bigg(\tfrac{X(t)}{1-\theta} + \tfrac{A}{\rho_l}\Bigg).\end{aligned} \tag{39}$$

Thus, Player l's non-cooperative payoff exceeds that of player h if and only if

$$\frac{\alpha_h}{\alpha_l} < \frac{\frac{X(t)}{1-\theta} + \frac{A}{\rho_l}}{\frac{X(t)}{1-\theta} + \frac{A}{\rho_h}}, \tag{40}$$

which always holds because $\alpha_h < \alpha_l$ and $\rho_h > \rho_l$. Therefore, $\mathcal{V}_l(X(t)) > \mathcal{V}_h(X(t))$. We thus conclude:

Corollary 2 *The Shapley values, defined in (34), check*

$$\Phi_l(X(t)) > \Phi_h(X(t)).$$

The Shapley value allocates a larger share of the cooperative payoff to the patient player, reflecting her relatively higher marginal contribution to the aggregate payoff. This aligns with Proposition 3, which explains that a larger share of player l (and a lower share of player h) implies a longer coalition, even though it breaks in finite time.

5.3 Splitting under Shapley

With the above-defined Shapley value and sharing rule, in the rest of this subsection, we demonstrate that the coalition can provide higher welfare for both players, and there is no incentive for any player to quit the coalition.

To complete the proof, we argue in contradiction. Suppose that even under the dynamic Shapley sharing rule, Player k exits the coalition at $T^* < +\infty$, where $X^* = X(T^*)$; then

$$\mathcal{V}_k(X) > \Phi_k(X), \ \text{ for } \ X^* < X < \overline{X}_{nc}. \tag{41}$$

In other words, after the breakdown of the coalition, player k can do better.

By definition (34), it follows that regardless of which players would like to break the coalition,

$$\mathcal{V}_l(X) + \mathcal{V}_h(X) > \mathcal{V}(X) = \Phi_l(X) + \Phi_h(X), \ \text{ for } \ X^* < X < \overline{X}_{nc}. \tag{42}$$

Obviously, if the coalition continuous at X^*, the reversed inequality in (42) holds. Thus, at the transition point $X = X^*$, the two sides of the inequalities above are equal; that is,.

$$\mathcal{V}_l(X^*) + \mathcal{V}_h(X^*) = \mathcal{V}(X^*). \tag{43}$$

However, under cooperation, the optimal choice can be selected such that it is at least the same the optimal choice as under competition. Therefore, aggregate welfare under cooperation cannot be lower than the sum of welfare under competition; that is,.

$$\mathcal{V}_l(X) + \mathcal{V}_h(X) < \mathcal{V}(X), \ \ \forall X \in [X_0, \overline{X}_{nc}]. \tag{44}$$

The break from the previous fixed sharing rule $(\gamma, 1-\gamma)$ is mainly due to unfairness, which may be fair at the beginning: $\gamma = \Gamma(0)$; however, with time, this is no longer the case. The Shapley value provides a dynamic, time-consistent and fair sharing rule. Thus, there is no incentive to break a coalition. Consequently, Equation (43) has no solution. Therefore, the previous assumption that player k breaks the coalition at X^* does not hold true. Hence, the players have no time to enter the competitions. In other words, the cooperating players obtain their shares according to their Shapley values, which is always better than the competitive case.

6 Harvest moratorium

We conducted the analysis above based on the implicit assumption that cooperation is beneficial from the beginning. Nonetheless, cooperation among players may be triggered by severe fish stock depletion, in which the initial stock $X(0)$ is not only far below the optimal long-run cooperative steady-state level, $\overline{X}_c$, but also below the sustainability threshold, $\underline{X}$, defined by the International Council for the Exploration of the Sea (ICES). In this situation, a harvest moratorium may be the optimal target for cooperation. Assuming that cooperative management is implemented after significant fish stock depletion, recovery may be slow despite cooperative catch agreements. Although the growth rate of the fish stock is as high as $x(t) \to 0$, recovery to healthy levels may take several years. A notable example is the South Tasman Rise trawl fishery, in which Australia and New Zealand cooperatively managed orange roughy stocks. In 1998, a memorandum was established, leading to minimal catches, and a fishing ban from 2007 onwards, which remained in place. Motivated by these examples, this section examines whether a harvest moratorium policy benefits the management of severely depleted stocks, and determines the optimal duration.

Instead of continuing to fish a depleted population, we examined whether stopping fishing for a period could lead to faster fish recovery and be welfare-beneficial for the coalition. Two scenarios are considered: (A) resuming fishing after reaching the sustainability threshold, $\underline{X}$, and (B) resuming fishing after reaching the potential long-run steady state, $\overline{X}_c$. Case (B) corresponds to the bang-bang choices in linear fishery problems (see Grønbæk et al. (2020)). This allows us to complement the literature by supporting or challenging such bang-bang policies in a nonlinear context.

Our aim is not to compare welfare with and without a moratorium, as this would involve comparing constrained and unconstrained optimization. Instead, we focus on the welfare effects of different moratorium lengths based on the initial stock. The moratorium is modeled as the most rapid approach path (Noussair et al., 2015), where players agree to stop fishing until the stock reaches either the minimum sustainable level $\underline{X}$ or the long-run optimal steady state $\overline{X}_c$. Subsequently, they resume fishing, quotas are distributed, and payoffs are shared according to the dynamic Shapley values in (35). The strategy that yields the highest social welfare is optimal.

6.1 Fishing after reaching $\underline{X}$

Without fishing, fish stock dynamics follow Equation (1). This forms a Bernoulli equation with initial conditions $x(0) = X(0)^{1-\theta}$, resulting in fish stock trajectory $X_n(t)$ from (2), where $\overline{X}_n$ is the long-run steady state without fishing. The sustainable threshold $\underline{X}$ is reached in finite time t_m. Players start to optimally exploit the stock from this date and the fish population recovery time satisfies

$$X_n(t_m) = \underline{X} \iff t_m = -\frac{1}{(1-\theta)\delta} \ln\left(\frac{\overline{X}_n - \underline{X}}{\overline{X}_n - X(0)}\right)(> 0). \tag{45}$$

It is straightforward that if $\underline{X} = X(0)$, then $t_m = 0$, indicating no moratorium. Furthermore, a lower initial stock size $X(0)$ increases the difference between $\underline{X}$ and $X(0)$, thereby extending the moratorium duration. Higher mortality rates also prolong recovery time.

The aggregate welfare, assuming that cooperation will last forever, is

$$\mathcal{W}(t_m) = \int_{t_m}^{\infty} \left[2\frac{\omega_c^{1-\theta} X_c(t)}{1-\theta}\right] e^{-\rho t} dt. \tag{46}$$

Alternatively, by using the optimal stock trajectory in Equation (14) for any $t > t_m$,

$$\mathcal{W}(t_m) = 2\frac{\omega_c^{1-\theta}}{1-\theta} \int_{t_m}^{\infty} \left[e^{-a(t-t_m)-\rho t}\left(\underline{X} - \overline{X}_c\right) + e^{-\rho t}\overline{X}_c\right] dt, \tag{47}$$

where $a = (1-\theta)(\delta + 2\omega_c) > 0$ and $\overline{X}_c = \frac{A}{\delta + 2\omega_c}$ is the long-term optimal steady state. Integration leads to the following joint welfare function:

$$\mathcal{W}(t_m) = 2\frac{\omega_c^{1-\theta}}{1-\theta}e^{-\rho t_m}\left[\frac{(\underline{X} - \overline{X}_c)}{a+\rho} + \frac{\overline{X}_c}{\rho}\right] = 2\frac{\omega_c^{1-\theta}}{1-\theta}e^{-\rho t_m}\left[\rho\underline{X} + (1-\theta)A\right]. \tag{48}$$

The above results determine the minimum duration for which fishing must be prohibited to reach the minimum sustainability level $\underline{X}$ and the corresponding welfare for this policy. However, it remains unclear whether extending the waiting period results in higher or lower social welfare from fishing activities. In the following subsections, we explore an alternative approach for determining the optimal duration of the moratorium policy.

6.2 Fishing after reaching $\overline{X}_c$

To determine whether extended waiting times enhance social welfare, we examine Case (B), where the moratorium lasts until the stock reaches the long-run optimal steady state $\overline{X}_c$ rather than the minimum stock level $\underline{X}$. Seminal studies in fishery economics, such as those by Gordon (1954) and Clark and Munro (1975), assume a linear objective function in the controls. In such cases, an optimal policy consists of a combination of bang-bang and singular controls (Caputo, 2005). If the initial stock differs from the optimal steady state, the optimal policy follows a Most-Rapid Approach Path, where the stock reaches the optimal steady state as quickly as possible using a bang-bang strategy followed by singular control (Hartl and Feichtinger, 1987). We challenge this by exploring whether a nonlinear choice framework still supports such a strategy, or if an interior solution is more welfare-enhancing.

Given the fish dynamics without harvesting in Equation (2), the moratorium duration that allows the fish stock to recover from its initial state $X(0)$ to its long-run optimal value $\overline{X}_c$, denoted by t_c, is

$$X_n(t_c) = \overline{X}_c \Longleftrightarrow t_c = -\frac{1}{(1-\theta)\delta}\ln\left(\frac{\overline{X}_n - \overline{X}_c}{\overline{X}_n - X(0)}\right)(>0). \tag{49}$$

The moratorium duration t_c decreases with the distance between the initial and long-run optimal states, and becomes zero if they are equal. The joint welfare in this case is

$$\mathcal{W}(t_c) = \int_{t_c}^{\infty}\left[2\frac{\omega_c^{1-\theta}X_c(t)}{1-\theta}\right]e^{-\rho t}dt. \tag{50}$$

Given the fish stock dynamics in Equation (14), the welfare over time is the infinite discounted sum of utility derived from fishing at the steady state:

$$\mathcal{W}(t_c) = 2\frac{\omega_c^{1-\theta}}{1-\theta}\int_{t_c}^{\infty}\left[e^{-\rho t}\overline{X}_c\right]dt. \tag{51}$$

By integration, we obtain

$$\mathcal{W}(t_c) = 2\frac{\omega_c^{1-\theta}}{1-\theta}e^{-\rho t_c}\left[\frac{\overline{X}_c}{\rho}\right] = 2\frac{\omega_c^{1-\theta}}{1-\theta}e^{-\rho t_c}\left[\rho\overline{X}_c + (1-\theta)A\right]. \tag{52}$$

Now, we can move on and characterize the optimal moratorium duration.

6.3 Comparison

The goal is to compare the welfare impacts of the two different types of moratorium policies, as defined in Cases (A) and (B). Arguably, the results are not straightforward in the sense that a longer moratorium policy could allow the exploitation of a larger fish stock, which favors the payoff towards the optimal steady state; however, this should be balanced with the negative effect of a longer waiting time (either t_m or t_c). An interesting question is, assuming that the moratorium allowed stocks to reach the minimum level, $\underline{X}$, is there a welfare improvement in waiting until the stock reaches its optimal value, $\overline{X}_c$, such that future generations can benefit from larger initial stocks?

A comparison between (52) and (48) yields

$$\mathcal{W}(t_c) < \mathcal{W}(t_m) \quad \iff \quad e^{-\rho t_c}\Bigg[\rho\overline{X}_c + (1-\theta)A\Bigg] < e^{-\rho t_m}\Bigg[\rho\underline{X} + (1-\theta)A\Bigg]. \tag{53}$$

On the one hand, players can exploit a larger fish stock $\overline{X}_c > \underline{X}$, but on the other side the discounting weight negatively this value due to $e^{-\rho t_c} < e^{-\rho t_m}$, consequence of the no fishing policy. Exploration of condition (53) leads to the following results:

Proposition 6 *For the given parameters and for any sustainable level $\underline{X} \in [X(0), \overline{X}_c]$,*

(1) the social welfare checks $\mathcal{W}(t_m) > \mathcal{W}(t_c)$, where $\mathcal{W}(t_m)$, $\mathcal{W}(t_c)$, are given by (48), (52);

(2) the length of moratorium policy, t_m, given in Equation (45) is the optimal one.

Proof. From the definitions of t_c and t_m in (49) and (45), respectively, inequality (53) can be rewritten as

$$\left(\frac{\overline{X}_n - \underline{X}}{\overline{X}_n - \overline{X}_c}\right)^{\frac{\rho}{(1-\theta)\delta}} > \frac{\rho\overline{X}_c + (1-\theta)A}{\rho\underline{X} + (1-\theta)A}. \tag{54}$$

Taking the logarithm on both sides, it follows that inequality (54) is equivalent to

$$\Gamma(\underline{X}) \equiv \frac{\rho}{(1-\theta)\delta}\ \ln\left(\frac{\overline{X}_n - \underline{X}}{\overline{X}_n - \overline{X}_c}\right) - \ln\left(\frac{\rho\overline{X}_c + (1-\theta)A}{\rho\underline{X} + (1-\theta)A}\right) > 0 \qquad \forall \underline{X} \in [X(0), \overline{X}_c]. \tag{55}$$

It is straightforward that $\Gamma(\overline{X}_c) = 0$ and

$$\Gamma'(\underline{X}) = -\frac{\rho((1-\theta)\delta + \rho)\underline{X}}{[\rho\underline{X} + (1-\theta)A]\,(1-\theta)\delta\,(\overline{X}_n - \underline{X})} < 0.$$

Thus, the function $\Gamma(\underline{X})$ strictly decreases, with the minimum value $\Gamma(\overline{X}_c) = 0$. That is, $\Gamma(\underline{X}) > 0$ for any $X(0) \leq \underline{X} < \overline{X}_c$.

This completes the proof of the first part. The second part is straightforward from the first part. ■

The intuition behind this proposition is clear: when it is necessary to stop fishing to prevent the extinction of fish, that is, for sustainability reasons, it is optimal, from social welfare perspective, to start fishing immediately once the stock reaches a sustainable level. Any delay in fishing from t_m decreases social welfare.

Corollary 3 *If the initial stock $X(0)$ is above the minimum level $\underline{X}$, then a moratorium policy only yields a social welfare loss.*

This finding differs from the literature when the choice is linear, where the optimal strategy is either no fishing when the fish stock is too low, or maximum and optimal fishing when the fish stock reaches the optimal level, expect may be some singular case (Caputo, 2005). Here, with nonlinear choices, the optimal strategy is an interior choice, and the maximum fishing level can only be asymptotically reached.

7 Discussion

In this section, we discuss the main results and focus on the potential extensions and implications. The discussion is organized around the following two points: (i) is the impact of player heterogeneity on the splitting time, (ii) is the cost of the coalition, and the Shapley values.

7.1 Impacts of heterogeneity among players

An interesting point from Proposition 3 is whether the degree of heterogeneity among players, specifically the difference in individual discount rates, can increase or reduce the coalition duration. This depends on the parameter values, particularly the share of cooperative payoff $(\gamma, 1-\gamma)$. We propose numerical examples using explicit formulas for splitting time, showing how T_l and T_h evolve as $\rho_h - \rho_l$ increases. Although ρ_h (ρ_l) does not directly impact T_l (T_h), it affects them through the exploitation rates in (19). We fix all the parameters and provide figures for the splitting time of both players as a function of ρ_h for three different fixed-sharing rules: $\gamma = 1/4$, $1/2$, or $3/4$. Table 1 lists the parameter values for these three examples.

Parameter	$\theta(> 1/2)$	δ	A	ρ_l	ρ
Value	$2/3$	0.3	0.5	0.4	0.5

Table 1: Parameter values

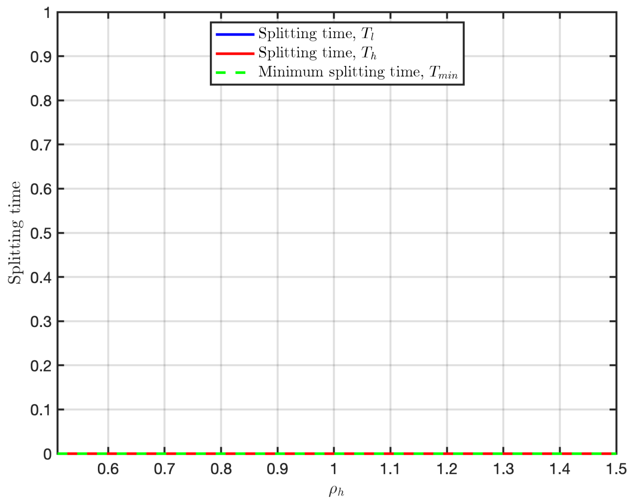
Before conducting this analysis, we recall the theoretical results of Proposition 3. The optimal splitting time of the low discount rate players increases with the share of the cooperative payoffs (γ), whereas the optimal splitting time of the high discount rate player decreases as his share gets from the cooperation $(1 - \gamma)$. At this stage, the intuition is that the shared greeting for players l should be sufficiently large for the coalition to last for some time. The question is how this would depend on the heterogeneity of discount factors. The following figures show the results for the three sharing rules: $\gamma = 1/4$, $1/2$, or $3/4$.

In the following examples, the blue line represents the splitting time of the more-patient player, the red line represents the splitting time of the less-patient player, and the green dotted line indicates the minimum of these two times, marking the moment the coalition dissolves.

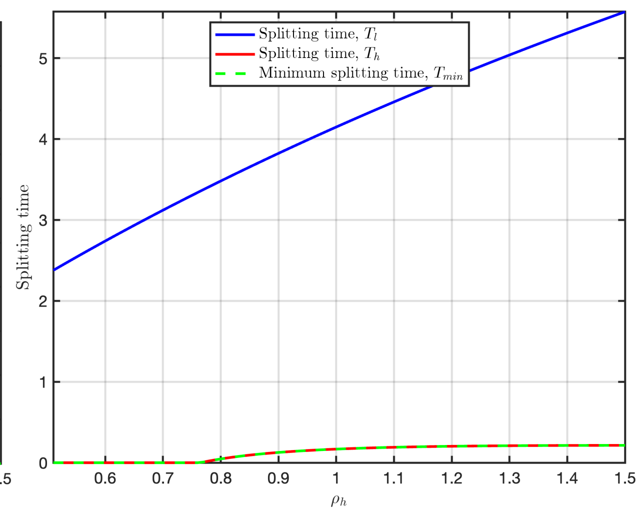
When player l receives only one-quarter of the cooperative payoff (and thus player h receives three-quarters), both players leave the coalition simultaneously. The impatient player receives a significant immediate share of the payoff, providing an incentive to exit promptly because future consumption is less valued. The patient player also prefers to leave immediately because the received payoff is insufficient and future consumption from competition is more beneficial. This scenario exemplifies a situation in which fixed sharing is suboptimal for coalition formation, resulting in the immediate dissolution of the coalition.

In the standard setting of cooperative games with symmetric players, most solution concepts advocate equal sharing of cooperative payoffs during coalition formation. That is, fixed sharing with symmetrical players is time consistent (see Corollary 1). However, as illustrated in Figure (b), this consistency does not hold for asymmetry. In this case, the duration of the coalition appears to increase with the degree of heterogeneity between the players. Predictably, under fixed equal sharing, the impatient player always exits first. To incentivize the impatient player to remain in the coalition, it is necessary to allocate a lower share of the cooperative payoffs to them. The purpose of the final example is as follows:

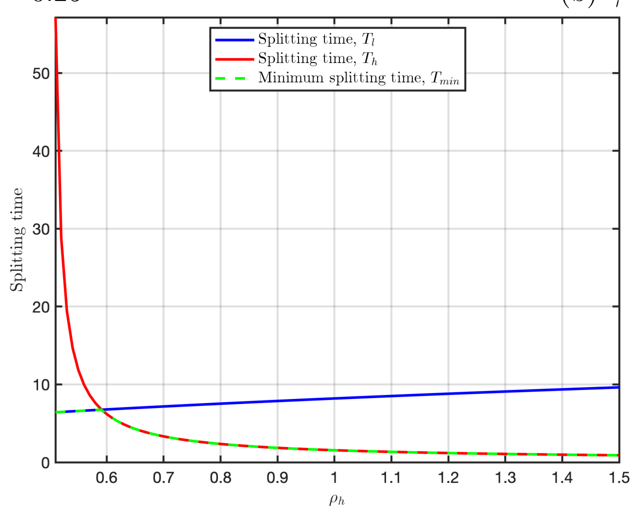
The final figure illustrates a scenario in which the patient player receives three-quarters of the cooperative payoff. The time at which a coalition breaks can be divided into two



(a) $\gamma = 0.25$



(b) $\gamma = 0.5$



(c) $\gamma = 0.75$

Figure 1: Optimal splitting time and discounting heterogeneity

phases. Initially, because the discount rates of both players are relatively close, the splitting time increases with heterogeneity, with player l withdrawing first. Subsequently, when the difference in discount rates becomes substantial, the impatient player exits first because of strong incentives for immediate consumption and the splitting time decreases as ρ_h increases.

7.2 Coalition with cost and dynamic Shapley values

Beyond the profit-driven incentives to break cooperation, continuing to cooperate may incur additional costs. These costs include, but are not limited to, the need for extensive coordination among countries. One notable example is the Northeast Atlantic Fisheries Commission (NEAFC), which involves multiple countries including the EU, Norway, Russia, and the United Kingdom. This cooperation has faced significant challenges due to climate change, which has caused fish species such as mackerel to migrate to new areas, complicating quota agreements and leading to overfishing. Further details are provided by Kapstein et al. (2023). Another example is the Marine Stewardship Council (MSC) certification process, which requires international cooperation to ensure sustainable fishing practices. This involves extensive monitoring and compliance with regulations, which can be expensive for the participating countries (Council, 2021).

These cooperative efforts are essential for sustainably managing fish stocks, and can lead to significant advancements and solutions for shared fishery resources. However, they are expensive owing to the need for scientific research, monitoring, and enforcement of regulations. Consequently, the theoretical model of a dynamic time-consistent Shapley Value may not hold when additional costs are considered.

To achieve this, we introduce the following costs into the joint value function:

$$\overline{\mathcal{V}}(X) = \mathcal{V}(X) - \Lambda(X) = \Phi_l(X) + \Phi_h(X) - \Lambda(X),$$

where $\Lambda(X) = \Lambda(X; X_0, \rho_l, \rho_h)$ is a cost function for maintaining coalition work, which may depend on the time preferences, negotiation skills, and sacrifices of both players. The cost also depends on the stock; when the initial stock is low, i.e., X_0, it is easier for the coalition to remain to protect the fish stock. However, when stock is high, there may no longer be a need to make sacrifices, at least from the perspectives of some players.

Therefore, some $\widehat{X}$ may exist depending on the different situations and costs such that

$$\mathcal{V}_l(X) + \mathcal{V}_h(X) > \overline{\mathcal{V}}(X), \quad \text{for some} \quad X \in [\widehat{X}, \overline{X}_{nc}]. \tag{56}$$

Thus, continuing the coalition is too costly for it to continue.

A new Shapley value that considers the benefits and costs is needed. Nonetheless, defining a realistic cooperative cost function and its corresponding Shapley value is beyond the scope of this study.

8 Conclusion

This study enhances the theoretical understanding of the dynamic stability of coalitions in the context of fisheries, motivated by recent issues in IFAs, such as coalition breakdowns. We propose a regime-switch model with two heterogeneous players who can revise their memberships over time. Our findings demonstrate that commonly used fixed sharing rules cannot sustain fishery coalitions. Drawing on the cooperative dynamic game literature, we propose a time-consistent sharing mechanism among coalition members that decomposes the payoff-to-go at any time, thereby creating incentives for players to remain in the coalition indefinitely. Additionally, we investigate the role of the fishing moratorium policy as part of a cooperative agreement. Our results indicate that this policy can be welfare-relevant for highly depleted stocks; however, fishing should resume once the stock recovers to sustainable levels.

This study had several limitations that should be addressed in future research. First, it does not consider scenarios involving more than two players in which both grand and partial coalitions are possible. This issue, highlighted in studies such as Breton and Keoula (2012) and Hannesson (2011), could be a valuable extension of our model. Another significant limitation was the definition of the dynamic Shapley Value in a continuous-time dynamic setting. Although theoretically convenient, continuous compensation for sharing the aggregate worth of the coalitions may be impractical for coalition members. It may be more feasible to view this problem as a renegotiation issue occurring at specific points in time, similar to current EU/UK post-Brexit fishery-related agreements. An intermediate approach could involve deciding on a new (optimal) fixed-sharing arrangement among coalition members at each coalition breakdown. This would ensure the coalition's continuation over time, albeit with a different distribution of the coalition's worth compared with our dynamic Shapley Value.

Additionally, we do not address the role of punishment strategies. Although international environmental agreements were initially not self-enforced because of strong incentives to free-ride, introducing a two-part punishment scheme for deviations from the cooperative solution, as suggested by Mason et al. (2017), could provide stability. This scheme allows one player to deviate from the cooperative strategy at any time, but they would be punished by reducing their catch in subsequent periods, whereas the other player could increase their catch. Extending our model to include this approach could yield stable outcomes for fishery coalitions.

Finally, we use the technical assumption that links θ and ν. This assumption, which is often used implicitly in similar settings, requires the use of the numerical simulations. However, we believe that the qualitative properties of our results are unaffected by this assumption. Further studies could employ the finite-difference method to solve the nonlinear HJB equations and explore this aspect in more detail.

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