

Pension's Resource-Time Trade-off: The Role of Inequalities in the Design of Retirement Schemes

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WP 2024 - Nr 20

Pension's Resource-Time Trade-off: The Role of Inequalities in the Design of Retirement Schemes*

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July 16, 2024

Abstract

Public pension schemes serve as mechanisms for inter-temporal income smoothing and within-cohort redistribution. This paper examines the influence of income and lifespan inequalities on the structure of a democratically chosen tier-pension scheme. We use a probabilistic voting model where agents vote on the size and the degree of redistribution (i.e. the Beveridgean factor) of the pension scheme and can supplement it with voluntary contributions. Our analysis reveals that when all agents can supplement the public scheme with private contributions, their voting behavior depends solely on the share of total income redistributed through the pension system, referred to as the redistributive power of the pension. Income inequality positively correlates with the equilibrium redistributive power, while lifespan inequality exhibits the opposite effect, leading to a resource-time trade-off; particularly when both inequality measures are correlated. In scenarios where low earners are hand-to-mouth and unable to make voluntary contributions, the effects on pension size (through mandatory contributions) and degree of redistribution become disentangled. Income inequality diminishes pension size while augmenting redistribution, whereas lifespan inequality increases pension size while reducing redistribution. We provide empirical evidence from OECD countries supporting these theoretical findings and calibrate the model on French data to quantify the effects.

Keywords: Tier pensions, inequality, income, lifespan, intra-generational redistribution.

*We thank Pierre Boyer, Philippe De Donder, Mathieu Lefebvre, Marie-Louise Leroux, Pierre Pestieau, Avner Seror, Alain Trannoy and participants at the CREST Firms and Markets Seminar, the 20th Journées Louis-André Gérard-Varet and the 10th meeting of the Society for the Study of Economic Inequality for valuable comments and discussions. The project leading to this publication has received funding from the French government under the "France 2030" investment plan managed by the French National Research Agency (reference :ANR-17-EURE-0020) and from Excellence Initiative of Aix-Marseille University – A*MIDEX.

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1 Introduction

Retirement systems are key institutions on the policy agenda of countries, serving as critical mechanisms for addressing old-age poverty and income stability throughout individuals' working and retirement phases. However, the design and features of public pension schemes vary significantly among countries, with their redistributive properties interacting with the economic environment and influencing their political support. This paper aims to explore the impact of income and lifespan inequalities on the structure of tiered pension schemes chosen through political processes, highlighting the central role of socioeconomic disparities in explaining the observed diversity in pension characteristics across different nations.

The World Bank advocates for tier pension schemes as the most effective approach to achieving poverty reduction and income sufficiency in old age (World Bank, 1994). This model involves multiple layers of retirement benefits, consisting in: (i) a mandatory-redistributive first tier, (ii) a mandatory-income-related second tier, and (iii) a voluntary-income-related third tier. Since the publication of this report, many countries have moved towards implementing this type of pension structure. However, the size of the pension and its degree of redistribution (by favoring the first or the second tier) varies significantly between countries, and little is known about the role of inequalities in shaping political support for different tier configurations.

Income inequality amplifies the benefits from redistribution, thereby enhancing the redistributive properties of public policies (see, *e.g.*, Meltzer and Richard, 1981). However, retirement systems introduce a temporal dimension where income differentials also influence resource allocation over time. We first highlight the importance of income-related constraints on voluntary contributions to the third tier in shaping political support for public pension schemes. We show that when all individuals have access to the third tier and can optimally choose their voluntary contributions, the public pension scheme influences their utility and, consequently, their voting behavior only through the share of income redistributed through the first tier - the redistributive power of the pension scheme. Pension schemes with varying sizes and degrees of redistribution may have the same redistributive power, leading to different levels of voluntary contribution. This equivalence could explain the negative relationship observed in OECD countries between public pension sizes and their degrees of redistribution (see Figure 1 and Appendix B for details on its computation).

The equivalence breaks down when poorer individuals cannot smooth consumption through the voluntary tier of the pension system, *i.e.*, when they live hand-to-mouth during the working period. Indeed the income-related second tier acts as a substitute for voluntary contributions (crowding-out effect), while the redistributive first tier reduces the contributions of low earners (substitution effect), limiting the size of the pension when individuals cannot borrow against their future pension (*i.e.* when contributions have to be positive). At this limit size, poorer agents do not contribute. We show that while income inequality increases the equilibrium redistributive power; it reduces the size of the pension (and increases its degree of redistribution) in these situations. In other words, the equilibrium scheme becomes more Beveridgean as income inequality increases. This finding is also consistent with data on OECD countries, as detailed in Appendix B.

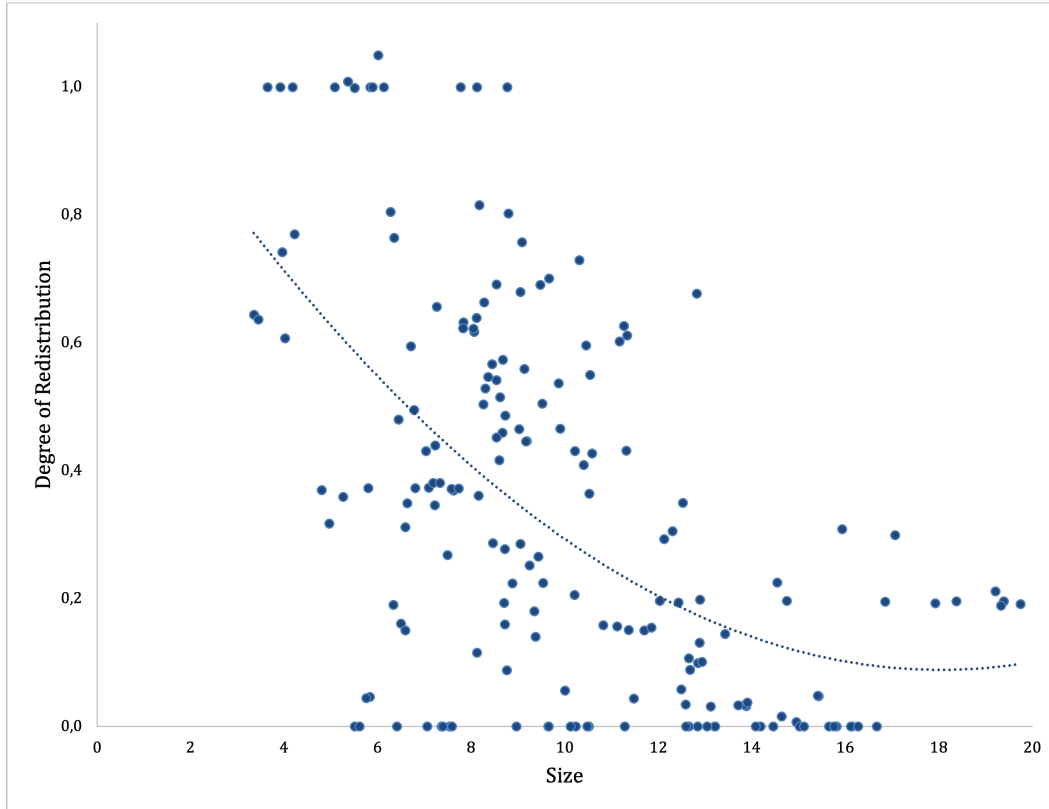


Figure 1: Pension's size vs degree of redistribution (2014 - 2022)

Source: Own elaboration with data from OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023).

Lecture note: Figure 1 plots the size of public pension – measured as the replacement rate for the average earner – against the degree of redistribution of pension – measured as the coefficient of variation in replacement rates – for OECD countries, and shows a decreasing relationship. See Appendix B.1 for details on the computation.

Through intertemporal redistribution, the first-tier component of pensions benefits individuals with longer lifespans. Differences in the length of the retirement period also matter, with richer individuals typically enjoying longer life expectancies. Consequently, income and lifespan inequality exert opposite effects through the redistributive component of the pension system. Specifically, we show that lifespan inequality, favoring richer individuals, reduces the equilibrium redistributive power of the pension. High earners benefit from their longevity premium, thereby diminishing the potential gains from redistributive pensions for low earners. We refer to this phenomenon as the resource-time trade-off. In settings where low earners cannot voluntarily contribute, an increase in lifespan inequality has the opposite effect of income inequality, reducing the degree of redistribution of pensions but increasing pension size. The effect of lifespan inequality on redistribution is consistent with cross-country data in OECD countries; and we recover the negative effect on pension size observed in data when lifespan and income inequality are correlated.¹

Additionally, we provide a calibration using French data. Our findings indicate that a one percent increase in the share of total income earned by the top quintile of the income distribution (our measure of income

¹The effect of lifespan inequality is aligned with data for both degree of redistribution and pension size when lifespan and income inequality are mildly correlated.

inequality) more than doubles the equilibrium redistributive power of pensions. This increase predominantly translates into a significant increase in the degree of redistribution in the equilibrium where low earners live hand-to-mouth, with a almost negligible negative size effect. Similarly, a one percent increase in longevity inequality leads to a one-third decrease in the equilibrium redistributive power, primarily affecting the degree of redistribution when low earners do not voluntarily contribute. Consequently, the lifespan inequality elasticity of income inequality needs to be between $1/14$ and $1/7$, approximately, for an increase in lifespan inequality to decrease all pension characteristics as the seen in the data.

Our paper represents the first comprehensive analysis of the impact of longevity inequality on democratically chosen pension schemes and sheds light on the interplay between income and longevity inequalities in shaping redistribution through public pensions. By delving into the political economy of tiered pension schemes and their relationship with socioeconomic inequalities, we contribute to the literature on social security and redistribution.

Galasso and Profeta (2002) identify five key economic factors influencing political support for pension schemes, including within-cohort redistribution and crowding-out effects on savings. We focus on these aspects by modeling a tiered pension system with intragenerational redistribution and voluntary contributions, while also accounting for dynamic efficiency concerns through lifespan inequality. Our analysis underscores the critical role of income and lifespan inequality in shaping the political support for pensions, in the context of voluntary contributions.

Moreover, we contribute to the literature on the crowding-out effect of pensions, pioneered by Kotlikoff (1989). Our analysis shows that when pensions also entail redistribution, the crowding-out effect is complemented by a substitution effect on voluntary contribution from low-income individuals (who then contribute less) and a compensation effect on contribution from high-income (who then contribute more). Additionally, our analysis provides insights into the redistributive characteristics of pensions, including the inverse relationship between pension size and degree of redistribution, as highlighted by Koethenburger et al. (2008). In our framework, when all agents privately contribute to the third tier at equilibrium, they are indifferent toward pension schemes with the same redistributive power, what results in a negative relationship between pension size and degree of redistribution.

In examining the effect of income inequality on pensions, our work extends the findings of Conde-Ruiz and Profeta (2007), who find that redistribution should increase with inequality in a median voter setting when pension size is fixed. We build upon this by allowing voters to choose both pension size and the degree of redistribution simultaneously, while also considering the potential correlation between income and longevity inequality. Additionally, our analysis is related to the literature on unequal lifespans, particularly Borck (2007), who studies political support for large unfunded pension schemes in the presence of income-dependent life expectancy. In line with current public pension schemes, we analyze a funded tiered pension scheme where agents can vote over both pension size and degree of redistribution, demonstrating how income and lifespan inequalities generate a trade-off through the redistributive first tier of the system, consistent

with data on OECD countries.

The rest of the paper is organized as follows. Section 2 outlines the modeling environment, including individual preferences and the pension scheme. Section 3 describes the democratic process and political equilibrium. We introduce lifespan inequality in Section 4 and correlate it with income inequality in Section 5. Section 6 concludes our analysis, and we provide empirical support for our findings in Appendix B.

2 Model

2.1 The Economy: Individual Preferences and Pensions

We consider an economy populated with a mass 1 of individuals who differ in their income. We assume two income groups (high and low) of respective size $n_H = \delta$ and $n_L = 1 - \delta$. Agents live for two periods: in the first, of length normalized to one, they inelastically supply labor and receive an income y_i depending on their type (with $y_H > y_L$). In the second period, of length $\psi \in (0, 1)$, agents are retired and do not receive labor income.² To smooth consumption between the two periods, they rely on social security payments and voluntary savings. We consider that agents consume all their accumulated income in the second period, that is, we assume away bequests. Following Acemoglu and Robinson (2005), we measure income inequality in the first period through the share of total income earned by the high income group, that is, $\gamma \equiv \frac{\delta y_H}{\bar{y}}$ with $\bar{y} = \delta y_H + (1 - \delta)y_L$. The first period income of both groups can then be written as $y_L = \frac{(1-\gamma)\bar{y}}{(1-\delta)}$ and $y_H = \frac{\gamma\bar{y}}{\delta}$.

We focus on additively-separable utility functions, common to both income groups:

$$U_i(c_i, d_i) = u(c_i) + \beta\psi u(d_i) \tag{1}$$

where β is the discounting factor, c_i is the first-period consumption group i individuals, and d_i their second-period consumption. We moreover assume constant inter-temporal elasticity of substitution, that is:³

$$u(c) = \begin{cases} \frac{c^{1-\mu}}{1-\mu} & \text{if } \mu \geq 1 \\ \log c & \text{if } \mu = 1 \end{cases} \tag{2}$$

with $\sigma = 1/\mu$ being the inter-temporal elasticity of substitution.

Consistent with the model of several countries, and the recommendation of the World Bank (see Introduction), we consider a tiered pension scheme composed of (i) a first mandatory fully redistributive tier aimed at reducing poverty in old age, (ii) a second mandatory and income-related tier with no redistribution, and (iii) a third private-voluntary tier (or annuity plan). We follow Casamatta et al. (2000) in using a linear pension wealth formula to model the mandatory two-tier structure, which is then composed of the

²We introduce differences in the length of the second period (*i.e.* lifespan inequality) in Section 4.

³As there is no uncertainty in our model, following de la Croix and Michel (2002), we refer to constant inter-temporal elasticity of substitution (CIES) rather than constant relative risk aversion (CRRA) utility function.

rate of social security contributions: τ (that is, the size of the public pension scheme) and the share of the public budget going to the redistributive first tier: ν . Hence, $(1 - \nu)$ corresponds to the Bismarckian factor, the income-dependent share of pension benefits. Denoting by a_i the share of income individuals in group i dedicate to the voluntary contribution to the third tier and assuming actuarial fairness on both private and public pension; consumption of group i then writes:

$$c_i(\tau, a_i) = (1 - \tau - a_i)y_i \quad (3)$$

$$d_i(\tau, \nu, a_i) = \frac{R}{\psi} [a_i y_i + \tau(\nu \bar{y} + (1 - \nu)y_i)] \quad (4)$$

with R being the common gross rate of return on mandatory and voluntary tiers of the public pension (actuarially fair return: R/ψ guarantees here balanced public budget).⁴

We assume a democratic choice for the characteristics of the public pension scheme, τ and ν , with the following timing. In the working period (i) individuals of the group i inelastically supply labor and receive income y_i ; (ii) Candidates choose their political platforms, *i.e.* public pension characteristics, non-cooperatively (described more precisely in Section 3); (iii) Agents vote for the candidate of their preference; (iv) Given the result of the electoral process and the corresponding pension scheme, individuals choose their voluntary contributions to the third tier of the system, and consume the remaining of their available income. Finally, in retirement individuals consume all their income, from their mandatory and voluntary pension benefits. We solve the model by backward induction, starting with the computation of optimal voluntary contributions, and focus on the effect of income inequality on the democratically chosen pension scheme. Lifespan inequality is introduced in Section 4.

2.2 Voluntary Third-Tier Contributions and Consumption

First, we study how the optimal voluntary contribution a_i^* varies with pension characteristics (τ and ν). We show that, when these contributions are strictly positive, consumption levels in both periods only depend on the product $\tau \times \nu$, that is the redistributive power. We also analyze the role of liquidity constraints, and how they are impacted by redistribution and income inequality.

For given values of τ and ν , the optimal voluntary contribution of individual i to third-tier solves

$$\max_{a_i} U_i(c_i(\tau, a_i), d_i(\tau, \nu, a_i)) \quad (5)$$

with $c_i(\cdot)$ and $d_i(\cdot)$ defined in (3) and (4). Optimal contributions then solve the usual Euler equation: $u'(c_i) = \beta R u'(d_i)$ guaranteeing consumption-smoothing and the interaction between voluntary contribution

⁴By assuming the same rate of return on private and public contribution, we abstract away for now from the dynamic inefficiency motives in the support for public pension (see *e.g.* Galasso and Profeta, 2002). Lifespan inequality, in Section 4, will create a dynamic inefficiency motive for high earners to support the pension.

and public pension features is characterized by the following lemma.

Lemma 1. *Voluntary contributions to third-tier react to public pension schemes through:*

1. a crowding-out effect, higher for low earners: $\frac{\partial a_L^*(\tau, \nu)}{\partial \tau} < \frac{\partial a_H^*(\tau, \nu)}{\partial \tau} < 0$; complemented by
2. a substitution effect for low earners: $\frac{\partial a_L^*(\tau, \nu)}{\partial \nu} < 0$; and
3. a compensation effect for high earners: $\frac{\partial a_H^*(\tau, \nu)}{\partial \nu} > 0$

Proof. See Appendix (A.1) □

Pension systems affect contributions in two ways. First, assuming away redistribution (*i.e.* in a full Bismarckian system, with $\nu = 0$), pensions have a one-to-one crowding-out effect on contributions. Now, in a redistributive system ($\nu > 0$), contributions also react to gains and losses from pension's redistribution. High earners perceive redistribution as a tax on their future consumption, which they compensate for, by increasing their contribution rate. In contrast, low earners perceive it as a subsidy, and reduce their contribution rate, through a substitution effect. Under CIES utility, optimal reactions to pension schemes then result in consumption levels that depend on pension characteristics only through the product $\tau\nu$. Indeed:

$$c_H(\tau, \nu) = \frac{(\gamma - \tau\nu(\gamma - \delta))\bar{y}}{(1 + \eta)\delta}, \quad c_L(\tau, \nu) = \frac{(1 - \gamma + \tau\nu(\gamma - \delta))\bar{y}}{(1 + \eta)(1 - \delta)} \quad (6)$$

$$\text{and } d_i(\tau, \nu) = (\beta R)^\sigma c_i(\tau, \nu), \quad i = \{L, H\} \quad (7)$$

where $\eta = \beta^\sigma R^{\sigma-1} \psi$.

Proposition 1. *Through optimal contributions to the third tier, the characteristics of public pension impact the preferences of agents only towards the redistributive power: the share of total income redistributed through the pension scheme. High earners' consumption and utility levels decrease with the redistributive power, while those of low earners increase.*

In other words, when agents are able to smooth consumption through the third tier, public pension characteristics only matter toward the share of redistributed income. This creates a de facto negative relationship between the size of the public pension and its degree of redistribution, consistent with the cross-country evidence highlighted in Figure 1 in the Introduction.

Still, due to liquidity constraints, all agents might not be able to smooth consumption using the third tier of pension. This might happen in our setting if we assume that agents are not allowed to take advantage of generous pension schemes by borrowing in their working period against their future pensions (in line with [Cremer and Pestieau, 2011](#)); that if we assume non negative voluntary contributions ($a_i \geq 0$).

Remark 1. *If agents cannot borrow against their future public pension payments, low earners voluntarily contribute to the third tier only when mandatory contributions are low, that is, when $\tau < \frac{\eta(1-\gamma)}{(1+\eta)(1-\gamma)+\nu(\gamma-\delta)} \equiv \bar{\tau}$. If $\bar{\tau} \leq \tau$ only high earners possibly voluntarily contribute: $a_H^* \geq 0$ and $a_L^* = 0$. The upper bound on public pension size at which low earners contribute is (i) decreasing with the degree of redistribution of pensions ($\partial\bar{\tau}/\partial\nu < 0$) and (ii) decreasing with income inequality ($\partial\bar{\tau}/\partial\gamma < 0$). Moreover, the negative effect of redistribution on the upper bound worsens with income inequality ($\partial^2\bar{\tau}/\partial\nu\partial\gamma < 0$).*

Proof. See Appendix A.2 □

On top of the usual crowding-out effect, low earners' voluntary contributions are dampened in a tier system by a substitution effect coming from redistribution (see Lemma 1). The liquidity constraint of low earners then binds first, and the level of mandatory contribution for which they stop voluntarily contributing is negatively affected by public pension redistribution (either through higher degree of redistribution of pension; or higher income inequality for a given redistribution degree). Through its effect on actual redistribution, income inequality amplifies the influence of redistributive degree on voluntary contribution; thereby limiting the range of possible pension schemes (τ, ν) when individuals face liquidity constraints.

3 Democratic Process

We assume that the characteristics of the public pension scheme are determined according to a probabilistic voting model (see *e.g.* the seminal work by Lindbeck and Weibull (1987), or Banks and Duggan (2005) for probabilistic voting in a multi-dimensional policy space). We more precisely consider two purely office-motivated candidates, proposing political platforms simultaneously and non-cooperatively. In our context, these platforms correspond to public pension characteristics $(\tau$ and $\nu)$. Both candidates aim to maximize their probability of winning the election (or, equivalently, their expected vote share), taking into account the uncertainty in voters' choices. Uncertainty is modeled through a random variable (unknown to the candidates) entering the choice of each voter in addition to its economic preferences on the platforms. This variable represents unobservable factors that influence voters' idiosyncratic preferences for candidates. We focus on cases where the equilibrium is symmetric, with both candidates offering the same platform and having an equal probability of winning (see Banks and Duggan, 2005, for specific conditions on idiosyncratic voters preferences leading to such policy coincidence at equilibrium).

Each candidate $N = \{A, B\}$ selects a political platform ρ^N from a multidimensional policy space (τ^N, ν^N) with $\tau^N, \nu^N \in [0, 1]$. Anticipating their voluntary contribution to the third-tier, voters then choose their candidate based on their idiosyncratic preferences and their indirect utility, defined as:

$$V_i(\tau, \nu) = U_i(c_i(\tau, a_i^*(\tau, \nu)), d_i(\tau, \nu, a_i^*(\tau, \nu))). \quad (8)$$

As established in the literature on probabilistic voting (see for example [Acemoglu and Robinson, 2005](#); [Banks and Duggan, 2005](#)), the symmetric solution to the candidates' problem corresponds to maximizing a weighted-utilitarian welfare function, where each group's weight reflects its political power (coming from the dispersion of idiosyncratic preferences in each group).⁵ In our model, both equilibrium political platforms $\rho^N = (\tau^N, \nu^N)$ are solutions to:

$$\max_{\rho^N} W^N(\rho^N) \equiv \sum_i \phi^i n_i V_i(\rho^N), \quad \forall N = \{A, B\} \quad (9)$$

where n_i represents the size of group i and $\phi^i \in \mathbb{R}^+$ is the political power of that group. For simplicity, we normalize the political power of low earners to one and define ϕ as the political power of high earners relative to low earners.

This formulation allows for different welfare considerations depending on the value of ϕ . When $\phi = 0$ (high earners have no political power), the equilibrium aligns with the objective of a Rawlsian social planner, prioritizing the welfare of the worst-off. With $\phi = 1$, corresponding to equal political power among all groups, the equilibrium mirrors the utilitarian social planner's solution. Intermediate values of ϕ represent a weighted utilitarian welfare maximization with varying weights among different groups.

We discuss in the following the impact of political power and wealth inequality on the equilibrium platform, first considering cases where both income groups contribute voluntarily to the third tier. We then analyze in section 3.3 scenarios where low earners rely solely on public pensions, and introduce longevity inequality in section 4.

3.1 Political Equilibrium with Positive Contribution Rates

Following Proposition 1, when both type of agents optimally use voluntary contributions to smooth consumption between periods, the equilibrium political platform, *i.e.* the redistributive power of the public pension, is then determined by:

$$\max_{\tau\nu^N} W^N(\tau\nu^N) = \sum_i \phi^i n_i V_i(\tau\nu^N) \quad (10)$$

where $V_i(\tau\nu^N) = U_i(c_i(\tau, \nu), d_i(\tau, \nu))$ with $c_i(\cdot)$ and $d_i(\cdot)$ defined in (6) and (7). Given that $\tau \times \nu$ must lie between 0 and 1 and as we assume that agents cannot borrow against their future pension (*i.e.* $\tau < \bar{\tau}$), the equilibrium platform is the solution of (10) when political power of high earners is neither too high (which would result in a non-redistributive public pension) nor too low (leading to a fully redistributive pension that suffices to smooth low earners' consumption).

Lemma 2. *The equilibrium redistributive power of pension is the interior solution of program (10) when*

⁵This implies that the political equilibrium results in Pareto optimality of the chosen platform.

$\underline{\phi} < \phi < \bar{\phi}$ (defined in Appendix A.3.1). When $\phi \leq \underline{\phi}$, $\nu^* = 1$ and $\tau^* = \bar{\tau}$. Conversely, when $\bar{\phi} \leq \phi$, $\tau\nu^* = 0$.

Proof. See Appendix A.3.1 □

The interior solution is given by:

$$\tau\nu^* = \frac{\gamma(1-\delta) - (1-\gamma)\delta\phi^\sigma}{(\gamma-\delta)(\delta\phi^\sigma + 1-\delta)} \quad (11)$$

and,

Theorem 1. *When all agents optimally choose a positive contribution rate, the interior equilibrium redistributive power: (i) decreases with the political power of high earners ($\partial\tau\nu^*/\partial\phi < 0$) and (ii) increases with ex-ante income inequality ($\partial\tau\nu^*/\partial\gamma > 0$).*

Proof. See Appendix A.3.2 □

Higher political power of high earners intuitively reduces the redistributive power of the pension, as high earners tend to lose from the first-tier's redistribution. Conversely, income inequality increases the potential welfare gains from redistribution due to the concavity of utility functions. As a result, income inequality tends to increase the redistributive power of the equilibrium public pension scheme, meaning a greater share of income is redistributed through the first tier. As we detail in the Appendix (see section B.3), this negative relationship between income inequality and the redistributive power of pensions is confirmed by data for OECD countries (see Figure 5). We size more precisely this effect in the French case in the next section before discussing in section 3.3 settings where low earners cannot contribute to third tier. In such cases, inequality-implied redistribution entails a negative effect on the pension-size upper bound, through the substitution effect highlighted in Lemma 1. Income inequality then increases the degree of redistribution of pension (ν) but decreases pension size (τ).

3.2 Simulations on French Data

To quantify the impact of income inequality on the degree of redistribution of public pensions, we use data from the World Inequality Database for France in 2022. We define high earners as the top quintile of the post-tax income distribution. According to the latest OECD "Pensions at a Glance" report (OECD, 2023), we assume a yearly return rate of 1.25% for both pensions and contributions, with a defined contribution conversion ratio of 0.9 and a capitalization period of 43 years. Considering a life expectancy at retirement of approximately 25 years, we estimate that the retirement period is nearly 60% of the working period. Given these assumptions, the French pension system is projected to redistribute about 1.31% of the overall income ($\tau\nu = 1.31\%$) by 2065 (see section B.1 in the Appendix for details on the estimation of pension

characteristics). Assuming perfect foresight in the form of financial discounting, this aligns with our equilibrium redistributive power of public pensions (in equation (11)) provided $\phi^\sigma = 2.67$.⁶ Data on high earners' political power (ϕ) and inter-temporal elasticity of substitution ($\sigma = 1/\mu$) being missing, we rely on this estimation to run our calibration exercise. Table 1 summarizes the key calibration parameters and the derived values for our model:

Table 1: Parameter calibration - French Simulation

Parameter		Value	Source
Share of high-income agents	δ	0.2	High earners as top quintile of the income distribution
Share of income earned by high-income	γ	0.4027	World Inequality Database Post-Tax Income Distribution
Gross rate of return on pension	R	2.6024	2023 OECD "Pensions at a Glance" Report
Discounting factor	β	0.3843	Financial discounting ($1/R$)
Length of the retirement period	ψ	0.5842	2023 OECD "Pensions at a Glance" Report
Redistributive power of public pension	$\tau\nu$	1.3107%	Own estimations, see Appendix B.1 for details
High earner political power and IES	ϕ^σ	2.6672	Calibrated value to replicate French pension data

To understand the effects of varying income inequality, we simulate different values of income inequality (parameter γ). Table 2 illustrates the impact of changes in income inequality on the redistributive power of the pension scheme:

Table 2: The Impact of Changes in Income Inequality on Equilibrium Redistributive Power

$\Delta\gamma$	(%)	New $\tau\nu$	$\Delta\tau\nu$	(%)
0.002	0.5%	2.28%	0.0097	74%
0.004	1%	3.23%	0.0192	147%
0.006	1.5%	4.16%	0.0285	218%

Source: Own computations with data from the WID Database and the OECD's Pensions at a Glance 2023 report.

Lecture note: Table 2 shows the impact of an increase (of 0.5%, 1% and 1.5%) in income inequality on the equilibrium redistribution power of public pension (that is the share of income redistributed through the first tier) calibrated on French data.

Our calibration exercise indicates that income inequality has a substantial impact on the redistributive power of the pension scheme. For example, a 1% increase in income inequality more than double the redistributive power, as reflected in the share of income redistributed through the first tier. Figure 2 depicts this effect on the (τ, ν) map.

⁶Details on the estimation of ϕ^σ can be found in Appendix A.4.

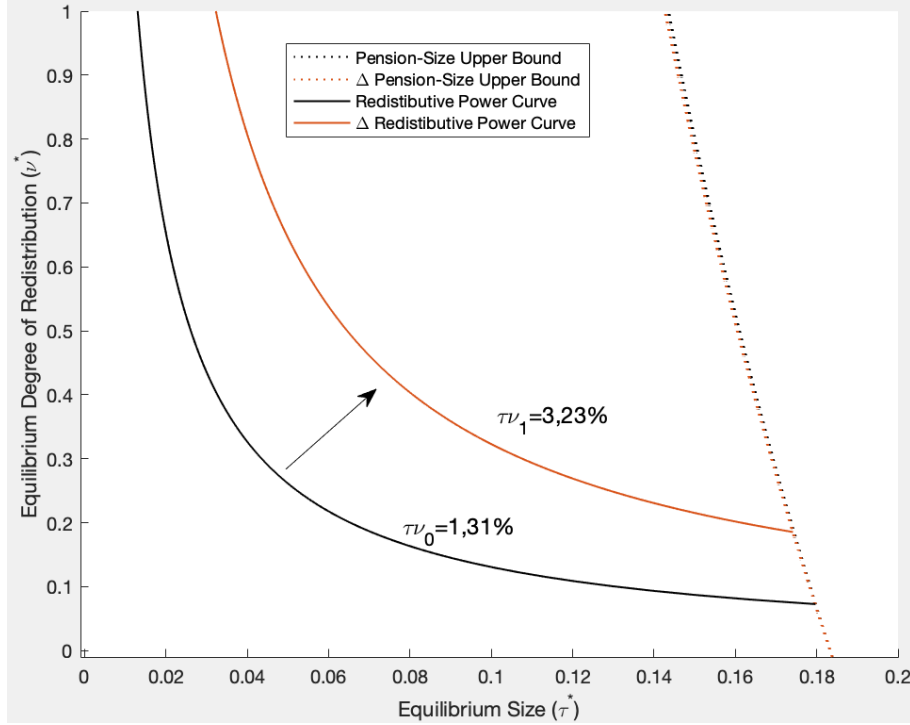


Figure 2: Effect of income inequality on the equilibrium redistributive power - French Simulation

Source: Own Simulations with data from WID Database, and OECD's Pensions at a Glance 2023 report.

Lecture note: Figure 2 plots the effect of a 1% increase in income inequality (orange curves) from the calibrated characteristics on current French public pension system (black curves). It shows an increase in the redistributive power of pension, from 1.31% to 3.23%, consistently with Theorem 1.

This figure shows in black the set of pension size (τ) and degree of redistribution (ν) compatible with our estimated equilibrium redistributive power for France; and how it would evolve after a 1% increase in income inequality. This analysis highlights the positive effect on redistributive power ($\tau\nu$) but does not provide details on changes in pension size and degree of redistribution. We study further these effects in the next section by focusing on the equilibrium in which low earners do not contribute to the third tier of the pension scheme. This will notably entail studying the pension size upper-bound (defined in Remark 1), which is plotted with dashed lines in Figure 2.

3.3 Political Equilibrium with Hand-to-Mouth Agents

In this section, we focus on cases where low earners are constrained not to contribute to the third tier of pensions, that is a setting in which $a_L = 0$. This case seems empirically relevant as, in developed countries, lower income deciles typically have zero savings rates (ING, 2019; Ahnert et al., 2020). The proportion of people who do not contribute to voluntary pension plans, *i.e.*, long-term savings, is even larger (see the OECD report "Pension Market in Focus 2022" OECD, 2019).⁷ In this context, we focus on the equilibrium

⁷There are various reasons why people might not contribute to the third tier of pensions, such as myopia, low income, or reliance on other sources of retirement income. Here, we focus on the equilibria where it occurs, without delving into the specific reasons.

where low earners consume all their post-tax income (that is, are "hand-to-mouth") during the first period.⁸

Given that consumption levels depend solely on the redistributive power of public pensions, this setting corresponds to equilibrium selection in terms of τ and ν . Put another way, there always exists a pension scheme solving problem (10) compatible with low earners not contributing to the third-tier ($a_L^* = 0$). This occurs when pension size equals the pension-size upper bound, as defined in Remark 1 ($\tau^* = \bar{\tau}$). From a political economy perspective, this means that both candidates offer pension parameters consistent with low earners not contributing to the third tier. This equilibrium is represented in Figure 2 at the intersection of redistributive power and the pension-size upper-bound curves.

Corollary 1. *When low earners are hand-to-mouth in the first period ($a_L^* = 0$), the equilibrium is characterized by the intersection of the redistributive power defined by equation (11) and the pension-size upper bound defined in Remark 1. At this equilibrium: (i) the degree of redistribution increases with ex-ante income inequality ($\partial\nu^*/\partial\gamma > 0$), and (ii) pension size decreases with ex-ante income inequality ($\partial\tau^*/\partial\gamma < 0$).*

Proof. See Appendix A.5 □

As noted earlier, income inequality amplifies the potential gains of redistribution, increasing the equilibrium redistributive power of pensions while reducing the pension-size upper bound through the substitution effect for low earners. These effects lead to a smaller equilibrium pension size and a higher degree of redistribution. The resulting negative (for pension size) and positive (for degree of redistribution) relationships with inequality are in line with evidences from OECD countries studied in Appendix B.3 (see, in particular, Figures 6, and 7 therein).

These results rely on how redistribution affects the well-being of both groups, and particularly on the relative loss from redistribution for high earners. Now, as detailed in the introduction, public pension schemes not only entail within-cohort redistribution but also inter-temporal redistribution. In this context, differences in longevity also drive the effect of pension redistribution on well-being and thus impact the characteristics of democratically chosen pension schemes. In the following section, we examine the effect of lifespan inequality on equilibrium pension size and degree of redistribution, highlighting a resource-time trade-off in how redistribution influences income group preferences.

4 Lifespan Inequality

While average lifespan has increased over the past century (Cutler et al., 2006), this increase has not been uniform across income levels. Permanyer and Scholl (2019) notably point out that lifespan inequality has risen among individuals over the age of 65, potentially affecting the sustainability of health and pension systems. This section examines the impact of lifespan inequality on political support for redistributive

⁸Ampudia et al. (2018) and Brevoort (2022) found that around 24% of European and 35% of US households are liquidity constrained.

pensions. We assume that individuals from different income groups experience varying lengths of retirement periods, with wealthier individuals generally living longer: $\psi_L < \psi_H$.⁹

4.1 Public Budget Balance

Public pension budgets are impacted by lifespan inequality, as it alters the period during which pension benefits are received across income groups. When high earners live longer, they receive a longevity premium, drawing benefits for a longer period than the average individual. Public pensions need to adjust for this to maintain sustainability, which can be achieved by ensuring actuarial fairness across the pension tiers. This comes to adjusting the real return on each tier to reflect the average lifespan of recipients. The mandatory-first tier, common to all agents, then earns a return $R/\bar{\psi}$, where $\bar{\psi} \equiv \delta\psi_H + (1 - \delta)\psi_L$ is the average lifespan in the population; whereas the income-specific second tier earns a return based on the specific lifespan of each group R/ψ_i . Note that to ensure a balanced budget, the (actuarially fair) return for high earners in the second tier is lower than the return on the first tier ($R/\psi_H < R/\bar{\psi} < R/\psi_L$). Through differences in returns, high earners now benefits from the first-tier. As shown in equation (12), this adjustment allows public pensions to achieve sustainability despite unequal lifespans:

$$\underbrace{R\tau\bar{y}}_{\text{Fiscal Revenue}} = \underbrace{\frac{R}{\bar{\psi}}\tau\nu\bar{y}(\delta\psi_H + (1 - \delta)\psi_L)}_{\text{First Tier Payments}} + \underbrace{\frac{R}{\psi_H}\tau(1 - \nu)\delta\psi_H y_H}_{\text{High Earners Second Tier}} + \underbrace{\frac{R}{\psi_L}\tau(1 - \nu)(1 - \delta)\psi_L y_L}_{\text{Low Earners' Second Tier}} \quad (12)$$

As highlighted in the previous sections, low earners are direct beneficiaries of a tier pension scheme without lifespan inequality. However, the return of the redistributive first tier favors those with longer lifespans. When longer lifespans are positively correlated with higher income, this creates a dynamic inefficiency motive for high earners to support pensions. Ultimately, whether an individual benefits from the redistributive tier depends on their position in the income and lifespan distributions, leading to what we call the "resource-time trade-off".

Since winners and losers from the redistributive part of pensions now depend on both income and lifespan inequalities, we measure lifespan inequality similarly to income inequality, using $\theta \equiv \delta\psi_H/\bar{\psi}$. This measure, with $\theta \in (\delta, 1)$, indicates the share of total retirement time held by high earners. Based on studies by the United Nations (Hertog, 2013) and our comparison between Ginis of disposable income and lifespan (see Table 6 in Appendix B.2), we focus in the following on situations where income inequality exceeds lifespan inequality, $\gamma > \theta$.

⁹For more evidence on differences in longevity across income groups, see Kanbur and Mukherjee (2007) or Lefebvre et al. (2019). These longevity disparities may be due to factors such as working conditions or greater health investments by high earners.

4.2 Optimal Voluntary Contributions with Lifespan Inequality

We now examine how lifespan inequality impacts the characteristics of public pensions chosen democratically, assuming a balanced budget. In this case, the consumption levels are given by:

$$c_i(\tau, a_i) = (1 - \tau - a_i)y_i \quad (13)$$

$$d_i(\tau, \nu, a_i) = R \left[\frac{\tau \nu \bar{y}}{\bar{\psi}} + \frac{\tau(1 - \nu)y_i}{\psi_i} + \frac{a_i y_i}{\psi_i} \right]. \quad (14)$$

The impact of lifespan inequality on optimal voluntary contributions to the third-tier can be summarized as follows:

Lemma 3. *If income inequality is larger than lifespan inequality ($\gamma > \theta$), the effects presented in Lemma 1 persists. Additionally, lifespan inequality (i) worsens the crowding-out effect of pension size on high earners' contribution rates ($\partial^2 a_H^* / \partial \tau \partial \theta < 0$), and (ii) reduces the compensation effect of pension degree of redistribution on high earners' contribution rates ($\partial^2 a_H^* / \partial \nu \partial \theta < 0$).*

Proof. See Appendix A.6 □

Although high earners' contributions still exhibit a crowding-out effect from pension size ($\partial a_H^* / \partial \tau < 0$) and a compensation effect from pension's degree of redistribution ($\partial a_H^* / \partial \nu > 0$), lifespan inequality intensifies the former and reduces the latter. Because high earners have a longer expected lifespan, they suffer less from the first tier's redistribution, which increases the crowding-out effect by making the mandatory tiers a better substitute for their contributions. Similarly, lifespan inequality reduces the perceived tax component of the pension for high earners, thereby diminishing the compensation effect.

In the following section, we explore how these mechanisms impact the voting behavior of high earners and, consequently, how lifespan inequality influences public pension characteristics.

4.3 Political Equilibrium with Lifespan Inequality

To examine the impact of lifespan inequality on public pension characteristics, we follow the same reasoning as above. Consider first cases where all agents contribute voluntarily to the third tier. With the above optimal contribution, consumption and utility levels still depend on public pension characteristics only through its redistributive power. Specifically, the utility functions for low and high earners are:

$$V_L(\tau, \nu) = \frac{(1 - \delta + \tilde{\eta}(1 - \theta))}{(1 - \delta)(1 - \mu)} \left(\frac{1 - \gamma + \tau \nu (\gamma - \theta)}{1 - \delta + \tilde{\eta}(1 - \theta)} \bar{y} \right)^{1 - \mu} \quad (15)$$

$$V_H(\tau, \nu) = \frac{(\delta + \tilde{\eta}\theta)}{\delta(1 - \mu)} \left(\frac{\gamma - \tau \nu (\gamma - \theta)}{\delta + \tilde{\eta}\theta} \bar{y} \right)^{1 - \mu} \quad (16)$$

As long as income inequality is larger than lifespan inequality ($\gamma > \theta$), the redistributive power of pensions ($\tau \nu$) increases the utility of low earners at the expense of high earners. However, lifespan inequality decreases

low earners' gains from redistribution, reducing the equilibrium redistributive power. This result again aligns with data for OECD countries, as analyzed in Appendix B.4 (see in particular Figure 8).

Theorem 2. *In the presence of unequal lifespans (with $\gamma > \theta$), the equilibrium redistribution power of pensions, when interior, increases with income inequality and decreases with lifespan inequality ($\partial\tilde{\tau}\nu^*/\partial\theta < 0 < \partial\tilde{\tau}\nu^*/\partial\gamma$).*

Proof. See Appendix A.8 □

To disentangle the effects of inequality on pension size and degree of redistribution, we focus on equilibria where low earners are hand-to-mouth and do not contribute to the first tier. This occurs when the pension size reaches its upper bound, denoted $\bar{\tau}'$ when lifespans differ.

Remark 2. *In the presence of unequal lifespans, the pension-size upper bound decreases with both income inequality and lifespan inequality ($\partial\bar{\tau}'/\partial\gamma, \partial\bar{\tau}'/\partial\theta < 0$). Additionally, while income inequality exacerbates the effect of the degree of redistribution on the upper bound, lifespan inequality mitigates it ($\partial^2\bar{\tau}'/\partial\nu\partial\gamma < 0 < \partial^2\bar{\tau}'/\partial\nu\partial\theta$).*

Proof. See Appendix A.7 □

Lifespan inequality tightens the pension-size upper bound, discouraging low earners from allocating resources for the future. This occurs for two main reasons: first, an increase in lifespan inequality shortens the relative retirement period for low earners, reducing their motivation to contribute; and second, actuarial neutrality means that shorter retirement periods lead to higher returns from the second tier relative to the third, further crowding out contributions.

As stated in Corollary 1, the equilibrium at which low earners do not contribute is given by the intersection of the redistributive power curve and the pension-size upper bound. Given Theorem 2, in the presence of lifespan inequality, income inequality increases the redistributive power of pensions while tightening the pension-size upper bound. These effects go in the same direction, income inequality still reduce the equilibrium pension size and increases the degree of redistribution.

On the contrary, the effects of lifespan inequality on the redistributive power curve and the pension-size upper bound generate conflicting pressures on equilibrium pension characteristics. The reduction of the redistributive power creates a downward pressure on redistribution and an upward pressure on pension size. However, a tighter pension-size upper bound decreases pension size while increasing redistribution. We prove that, ultimately, the redistributive power effect dominates, leading to overall opposite effects between income and longevity inequality on pension characteristics.

Corollary 2. *Income and lifespan inequality have opposite effects on pension characteristics in equilibrium where low earners are hand-to-mouth. Lifespan inequality decreases the equilibrium degree of redistribution*

and increases pension size; income inequality has the opposite effect, increasing the equilibrium degree of redistribution and reducing pension size. ($\partial\nu^*/\partial\theta < 0 < \partial\nu^*/\partial\gamma$; $\partial\tau^*/\partial\gamma < 0 < \partial\tau^*/\partial\theta$).

Proof. See Appendix A.9 □

These opposing effects constitute what we call the "resource-time trade-off" of pensions. Income inequality drives smaller pension systems with larger redistributive first tiers, while lifespan inequality leads to larger pensions with closer links between benefits and mandatory contributions. These results mostly align with OECD data (see Figures 8, and 9 in Appendix B.4). However, lifespan inequality and pension size show an inverse correlation with the predicted one (see Figure 10), which might be due to the observed correlation between income and lifespan inequality. We analyze this correlation further in Section 5.

4.4 Lifespan Inequality in French Pension Simulations

Before considering correlated lifespan and income inequalities, we size the effects found in the previous section with data on different lifespans from France. We keep the same parameters as in Section 3.2 and compute lifespan inequality using data from the French National Institute of Statistics and Economic Studies (INSEE). We estimate the share of overall life expectancy captured by high earners by comparing the life expectancy at age 65 of white-collar workers to that of the entire population. We find that high earners capture nearly 23% of overall life expectancy when retired, indicating that high earners have a retirement period length about 17.5% longer than low earners. Table 3 summarizes the parameter values with lifespan inequality and the estimated ϕ^σ required to replicate French pension characteristics with lifespan inequality:

Table 3: Parameter calibration with lifespan inequality - French Simulation

Parameter		Value	Source
Share of high-income agents	δ	0.2	High earners considered as top quintile of the income distribution
Share of income earned by high earners	γ	0.4027	World Inequality Database Post-Tax Income Distribution
Gross rate of return on pension	R	2.6024	2023 OECD "Pensions at a Glance" Report
Discounting factor	β	0.3843	Financial discounting ($1/R$)
Length of the retirement period	ψ	0.5842	2023 OECD "Pensions at a Glance" Report
Share of life expectancy enjoyed by high earners	θ	0.2271	INSEE, Échantillon Démographique Permanent et état civil 2009-2013. Using white-collar (cadres) as a proxy for high earners.
Redistributive power of public pension	$\tau\nu$	1.3107%	Own estimations, see Appendix B.1 for details
High earner political power and IES	ϕ^σ	2.5902	Calibrated value to replicate French pension data

Introducing lifespan inequality reduces the estimated ϕ^σ that replicates French pension data due to the resource-time trade-off. Since lifespan inequality decreases the equilibrium redistributive power (as established in Theorem 1), this adjustment requires less political power of high earners to match French pension data. Next, we simulate increases in income and lifespan inequality by raising γ and θ by one percent (indicating that high earners' share of income and lifespan during the working period increases by

one percent). The simulation predicts that a one percent increase in γ raises the share of income redistributed through the pension system by 2.2 percentage points ($\Delta\tau\nu = 0.022$, consistent with the results in Section 3.2). However, a one percent increase of θ reduces the share of income redistributed through the pension system by 0.3 percentage points ($\Delta\tau\nu = -0.003$). This suggests that the effect of income inequality is more significant than the effect of lifespan inequality. The graphical representation is shown in Figure 3.

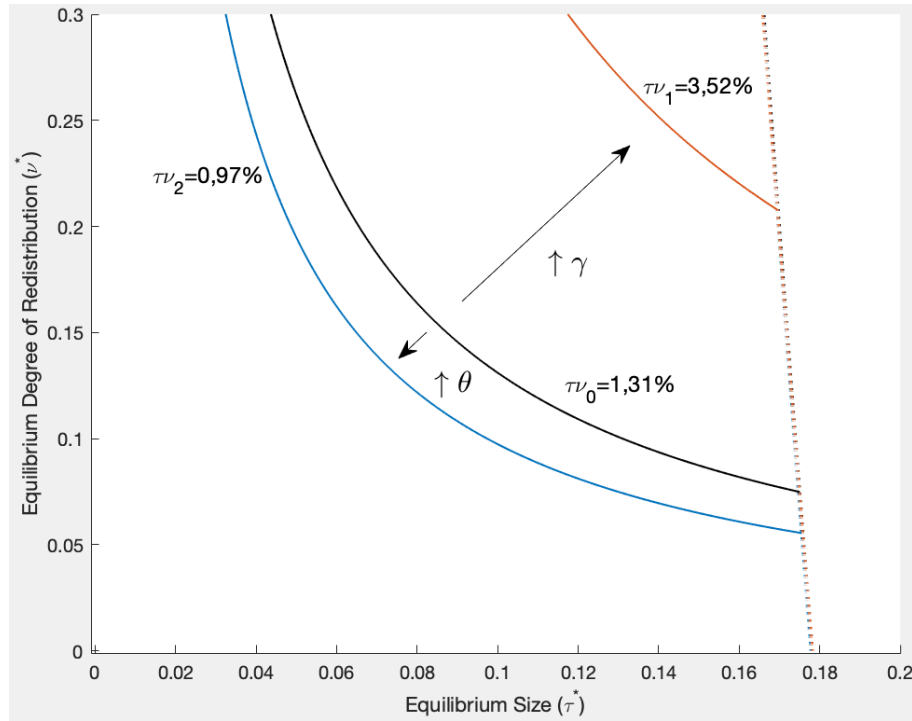


Figure 3: Impact of inequalities on the equilibrium redistributive power with lifespan inequality

Source: Own Simulations with data from WID Database, OECD’s Pensions at a Glance 2023 report, and INSEE.

Lecture note: Figure 3 plots the effect of a 1% increase in income inequality (orange curves) and in lifespan inequality (blue curves) from the calibrated characteristics on current French public pension system (black curves). It shows an increase in the redistributive power of pension (measured as the product of equilibrium degree of redistribution and pension size) following an increase in income inequality, and a decrease in the redistributive power of pension following an increase in lifespan inequality consistently with Theorem 2.

Regarding the scenario where low earners are hand-to-mouth, Figure 4 indicates that most of the effects come from changes in the degree of redistribution of public pensions, with relatively minor changes in pension size. This observation could explain the negative correlation between lifespan inequality and pension size observed in data, especially if income and lifespan inequalities are not independent.

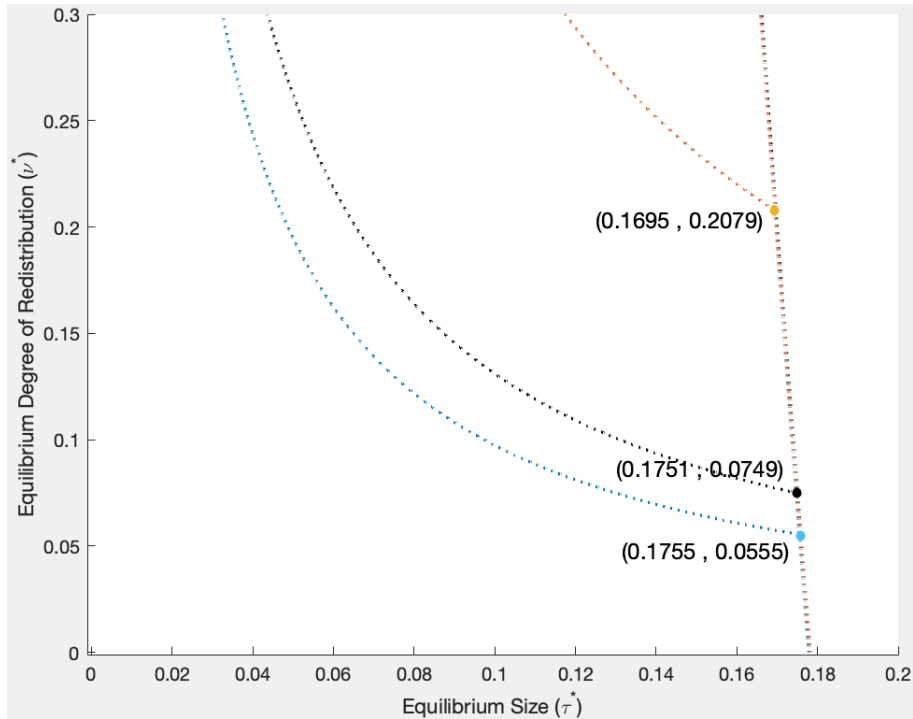


Figure 4: Impact of inequalities on the equilibrium parameters with H-2-M and lifespan inequality

Source: Own Simulations with data from WID Database, OECD’s Pensions at a Glance 2023 report, and INSEE.

Lecture note: Figure 4 plots the effect of a 1% increase in income inequality (orange curves) and in lifespan inequality (blue curves) from the calibrated characteristics on current French public pension system (black curves), when low earners are hand-to-mouths and cannot voluntarily contribute to the third tier. It shows (i) an increase in the degree of redistribution and a decrease in pension size following an increase in income inequality; and (ii) a decrease in the degree of redistribution and an increase in pension size following an increase in lifespan inequality consistently with Corollary 2.

5 When Income Inequality and Lifespan Inequality are Correlated

Until now, we have considered income and lifespan inequality as independent phenomena. However, there is a positive correlation between income and lifespan. For example, [Chetty et al. \(2016\)](#) highlights the direct link between income and life expectancy in the U.S., while [Lefebvre et al. \(2013, 2018, 2019\)](#) demonstrate the effect of income-dependent mortality rates on poverty measures. Regarding inequality more specifically, [Hertog \(2013\)](#) provides evidence of a positive association across countries between the Gini index of income and indices of lifespan inequality (such as Gini and Atkinson indices). This study finds that, controlling for life expectancy at birth, income inequality explains between 73% and 89% of the variation in lifespan inequality. Our analysis confirms this positive correlation for OECD countries (see Figure 11 in Appendix B.5).

In this section, we explore the impact of lifespan inequality on pension characteristics when it is also correlated with income inequality. Our aim is to understand whether such a correlation can explain the negative relationship observed in data between lifespan inequality and pension size (Figure 10), contrary to our initial predictions; while maintaining a negative relationship between lifespan inequality and redistribution

(redistributive power and degree of redistribution) consistent with our initial prediction and data (Figures 8 and 9). When lifespan inequality and income inequality are correlated, we can express γ as a function of θ , and the overall impact of lifespan inequality on the equilibrium pension size writes:

$$\frac{d\tau^*}{d\theta} = \frac{\partial\tau^*}{\partial\theta} + \frac{\partial\tau^*}{\partial\gamma} \frac{\partial\gamma}{\partial\theta} \quad (17)$$

Since income and lifespan inequalities have opposite effects on pension characteristics, a positive correlation creates an ambiguous effect on political support for tiered pension schemes. Reorganizing Equation (17), and defining the lifespan inequality elasticity of income inequality as $\varepsilon = \frac{\partial\gamma}{\partial\theta} \frac{\theta}{\gamma}$, we more precisely find that:

Proposition 2. *When the correlation between lifespan inequality and income inequality is large, that is when $\varepsilon > \underline{\varepsilon}$, lifespan inequality decreases pension size. Moreover, there exist a threshold of the elasticity ($\underline{\varepsilon} < \varepsilon < \bar{\varepsilon}$) for which lifespan inequality decreases pension size but still decreases the degree of redistribution and redistributive power (consistent with Figures 8, 9, and 10). The lower bound on elasticity increases with the political power of high earners, the discount rate and the rate of return when inter-temporal elasticity of substitution is high enough ($\partial\underline{\varepsilon}/\partial\phi, \partial\underline{\varepsilon}/\partial\beta > 0$ and $\partial\underline{\varepsilon}/\partial R > 0$ if $\sigma > 1$).*

Proof. See Appendix A.10 □

In other words, the strength of the correlation needed to change the relationship between lifespan inequality and the degree of redistribution is higher than that needed to change the relationship between lifespan inequality and pension size. The observation of Figures 8, 9, and 10 is thus consistent with our model for intermediate levels of the elasticity of lifespan-income inequality.

Larger political power of high earners increases the overall effect of lifespan inequality on pension characteristics. Lifespan inequality increases pension size while decreasing the share of income being redistributed through the first tier. However, when lifespan inequality is correlated with income inequality, both effects are dampened. Additionally, with larger political power among high earners, any increase in lifespan inequality leads to a larger decrease in redistribution, because high earners have less incentive to support a redistributive pension system.

A higher discount factor suggests that agents place greater value on future benefits, indicating a more forward-looking perspective. When low earners are constrained this means that they will push for larger pensions given the lifespan-income inequality correlation. Likewise, a larger inter-temporal elasticity of substitution means that agents are more willing to shift consumption between periods. Therefore, a higher rate of return or a larger discount factor can exacerbate the impact of lifespan inequality on pension characteristics.

5.1 French Pension Simulation with Income-Lifespan Elasticity

To size the effects discussed earlier, we simulate the impact of income-lifespan inequality elasticity using French data. Based on the parameters from Table 3, we find that $\underline{\varepsilon} = 0.074$ and that the level of elasticity at which the relationship between lifespan inequality and the degree of redistribution change sign: $\tilde{\varepsilon} = 0.146$. Table 4 more generally summarizes the impact of a one percent increase in the share of lifespan captured by high earners at different levels of the lifespan-income elasticity.

Table 4: Impact of a 1% increase in lifespan inequality on the equilibrium pension characteristics for different values of ε

ε	0	$\underline{\varepsilon}$	0.1	$\tilde{\varepsilon}$	0.3
$\Delta\tau^*$	0.23%	-	-0.11%	-0.23%	-0.74%
$\Delta\nu^*$	-25.9%	-12.68%	-8.14%	-	27.5%
$\Delta\tau\nu^*$	-25.95%	-12.98%	-8.4%	-0.05%	26.72%

Source: Own Simulations with data from the WID Database, the OECD’s Pensions at a Glance 2023 report, and INSEE.

Lecture note: Table 4 shows the impact of a 1% increase in lifespan inequality on the equilibrium characteristics of public pension calibrated on French data, for various levels of the lifespan inequality elasticity of income inequality.

As income and lifespan inequalities have opposite effects on the share of income redistributed through the system, a larger correlation (measured by ε) reduces the impact of an increase in lifespan inequality. If the elasticity is equal to the lower bound ($\varepsilon = \underline{\varepsilon}$), then lifespan inequality has no effect on the equilibrium pension size, and it reduces the redistributive power. If the elasticity is equal to the upperbound ($\varepsilon = \tilde{\varepsilon}$), then lifespan has no effect on the degree of redistribution and it reduces pension size. The third column in table 4 ($\varepsilon = 0.1$) corresponds to a case where the elasticity (that is the correlation between income inequality and lifespan inequality) can explain the observed relationships between lifespan inequality, pension size and redistribution in OECD countries (see Figures 8, 9, and 10 in Appendix B.4).

6 Conclusion

We present a simple framework to explain the heterogeneity in pension characteristics across countries, yielding results consistent with correlations observed in OECD countries. When individuals with perfect foresight can contribute to the third tier of pensions, they do so by considering only the share of income redistributed through the first tier, that we define as the redistributive power of the pension. Candidates for political office anticipate this behavior and choose a tiered pension scheme that maximizes their probability of winning office. Ex-ante income inequality increases the potential gains of the first tier for low earners, thus increasing the redistributive power of pensions chosen by the candidates.

In settings where low earners are hand-to-mouth and can not supplement public pensions with private

contributions, we find that higher income inequality slightly reduces pension size while increasing the share of income redistributed through the first tier. This adjustment prompts high earners to increase their voluntary contributions, responding to the rise in redistribution through a compensation effect. Our simulations for the French case suggest that a one percent increase in the share of income captured by high earners leads to a 1.9 percentile point increase in the share of income redistributed through the pension system.

When we introduce lifespan inequality and ensure public pension budget balance, the longevity premium for high earners allows them to benefit from the redistributive first tier of pensions. However, when income is more concentrated than lifespan, the net benefit from pension redistribution is still captured by low earners. An increase in lifespan inequality has the opposite effect on the political support for tiered pensions—it reduces the share of the first tier while slightly increasing overall pension size. Our French simulations predict that a one percent increase in the share of lifespan captured by high earners reduces the share of income redistributed through the system by 0.34 percentile points.

When income inequality correlates with lifespan inequality, the effects on pension characteristics are dampened. This interdependence reduces the individual effect of both inequalities on pension characteristics. Our model helps explain why more unequal countries tend to have smaller pension systems with higher redistribution through their first tier. The opposing effect of lifespan inequality represents the resource-time trade-off of pensions. Future increases in lifespan inequality – either through a higher pass-through of income inequality or through independent changes – should further reduce the redistributive first tier and increase the income-related second tier.

Most of the mechanisms presented in the paper should be robust to the specification of more than two income groups and to differences in returns on public and private pensions. The first generalization would entail more technicalities regarding the probabilistic voting equilibrium and a more complex relationship between income inequality and lifespan inequality. Differences in returns on public and private pensions would create an arbitrage between the second and third tiers but would not alter the forces at play in the political equilibrium. Generalizations to overlapping generations (OLG) or the inclusion of other intra-period redistribution mechanisms would require further investigation. In particular, including voting in an OLG setting generally entails assuming commitment to policy variables (so that the youth anticipates the chosen policy would persist when they are old), which does not appear satisfactory in our setting. Moreover, allowing redistribution through non-pension-related policies would modify individual preferences and is also likely to impact longevity in our model. These investigations are left for future research.

References

Aburto, J. M., F. Villavicencio, U. Basellini, S. Kjærgaard, and J. W. Vaupel (2020). Dynamics of life expectancy and life span equality. *Proceedings of the National Academy of Sciences* 117(10), 5250–5259.

- Acemoglu, D. and J. A. Robinson (2005). *Economic Origins of Dictatorship and Democracy*. Cambridge University Press. [5](#), [9](#)
- Ahnert, H., I. K. Kavonius, J. Honkkila, and P. Sola (2020). Understanding household wealth: linking macro and micro data to produce distributional financial accounts. Statistics Paper Series 37, European Central Bank. [12](#)
- Ampudia, M., D. Georgarakos, J. Slacalek, O. Tristani, P. Vermeulen, and G. L. Violante (2018). Monetary policy and household inequality. Working Paper Series 2170, European Central Bank. [13](#)
- Banks, J. S. and J. Duggan (2005). *Probabilistic Voting in the Spatial Model of Elections: The Theory of Office-motivated Candidates*, pp. 15–56. Springer Berlin Heidelberg. [8](#), [9](#)
- Borck, R. (2007). On the choice of public pensions when income and life expectancy are correlated. *Journal of Public Economic Theory* 9(4), 711–725. [4](#)
- Brevoort, K. P. (2022). Do low mortgage balances limit refinancing opportunities? *Available at SSRN 4163151*. [13](#)
- Casamatta, G., H. Cremer, and P. Pestieau (2000). The political economy of social security. *The Scandinavian Journal of Economics* 102(3), 503–522. [5](#)
- Chetty, R., M. Stepner, S. Abraham, S. Lin, B. Scuderi, N. Turner, A. Bergeron, and D. Cutler (2016). The Association Between Income and Life Expectancy in the United States, 2001–2014. *JAMA* 315(16), 1750–1766. [19](#)
- Conde-Ruiz, J. I. and P. Profeta (2007). The Redistributive Design of Social Security Systems. *The Economic Journal* 117(520), 686–712. [4](#)
- Cremer, H. and P. Pestieau (2011). Myopia, redistribution and pensions. *European Economic Review* 55(2), 165–175. [7](#)
- Cutler, D., A. Deaton, and A. Lleras-Muney (2006). The determinants of mortality. *Journal of Economic Perspectives* 20(3), 97–120. [13](#)
- de la Croix, D. and P. Michel (2002). *A Theory of Economic Growth*. Cambridge University Press. [5](#)
- Disney, R., T. Boeri, and T. Jappelli (2004). Are contributions to public pension programmes a tax on employment? *Economic Policy* 19(39), 269–311. [33](#)
- Galasso, V. and P. Profeta (2002). The political economy of social security: a survey. *European Journal of Political Economy* 18(1), 1–29. [4](#), [6](#)

- Hertog, S. (2013). The association between two measures of inequality in human development: Income and life expectancy. Technical Report 2013/7, United Nations Department of Economic and Social Affairs - Population Division. [14](#), [19](#)
- ING (2019). Saving woes stretch retirement outlook. Technical Report 2019/2, ING International Survey. [12](#)
- Kanbur, R. and D. Mukherjee (2007). Premature mortality and poverty measurement. *Bulletin of Economic Research* 59(4), 339–359. [14](#)
- Koethenbueger, M., P. Poutvaara, and P. Profeta (2008). Why are more redistributive social security systems smaller? a median voter approach. *Oxford Economic Papers* 60(2), 275–292. [4](#)
- Kotlikoff, L. J. (1989). *What Determines Savings?*, Volume 1 of *MIT Press Books*. The MIT Press. [4](#)
- Lefebvre, M., P. Pestieau, and G. Ponthiere (2013). Measuring poverty without the Mortality Paradox. *Social Choice and Welfare* 40(1), 285–316. [19](#)
- Lefebvre, M., P. Pestieau, and G. Ponthiere (2018). Fgt old-age poverty measures and the mortality paradox: Theory and evidence. *Review of Income and Wealth* 64(2), 428–458. [19](#)
- Lefebvre, M., P. Pestieau, and G. Ponthiere (2019). Premature mortality and poverty measurement in an old economy. *Journal of Population Economics* 32(2), 621–664. [14](#), [19](#)
- Lindbeck, A. and J. W. Weibull (1987). Balanced-budget redistribution as the outcome of political competition. *Public Choice* 52(3), 273–297. [8](#)
- Meltzer, A. H. and S. F. Richard (1981). A rational theory of the size of government. *Journal of Political Economy* 89(5), 914–927. [2](#)
- OECD (2019). *Pension Markets in Focus 2022*. OECD Publishing Paris. [12](#)
- OECD (2023). *Pensions at a glance 2023: OECD and G20 indicators*. OECD Publishing Paris. [10](#)
- Permanyer, I. and N. Scholl (2019). Global trends in lifespan inequality: 1950-2015. *PloS one* 14(5), e0215742. [13](#)
- World Bank (1994). *Averting the old age crisis: Policies to protect the old and promote growth*. The World Bank. [2](#)

A Theoretical Proofs

A.1 Contribution Rates and Consumption

With a CIES utility function, the first order conditions of problem (5) can be summarized as:

$$(\beta R)^\sigma c_i = d_i \quad (18)$$

leading to voluntary contributions:

$$a_H^*(\tau, \nu) = \frac{\eta}{1 + \eta} - \tau + \frac{\tau\nu(\gamma - \delta)}{(1 + \eta)\gamma} \quad \text{and} \quad a_L^*(\tau, \nu) = \frac{\eta}{1 + \eta} - \tau - \frac{\tau\nu(\gamma - \delta)}{(1 + \eta)(1 - \gamma)} \quad (19)$$

where $\eta = \beta^\sigma R^{\sigma-1} \psi$; and corresponding consumption levels:

$$c_H(\tau, \nu) = \frac{(\gamma - \tau\nu(\gamma - \delta))\bar{y}}{(1 + \eta)\delta} \quad d_H(\tau, \nu) = (\beta R)^\sigma c_H(\tau, \nu) \quad (20)$$

$$c_L(\tau, \nu) = \frac{(1 - \gamma + \tau\nu(\gamma - \delta))\bar{y}}{(1 + \eta)(1 - \delta)} \quad d_L(\tau, \nu) = (\beta R)^\sigma c_L(\tau, \nu) \quad (21)$$

Consumption of low earners increases with the redistributive power of pensions ($\tau\nu$), high earners' decreases.

By construction, $\gamma > \delta$, then the effect of pension characteristics on contribution rates:

$$\frac{\partial a_L^*}{\partial \tau} = -1 - \frac{\nu(\gamma - \delta)}{(1 + \eta)(1 - \gamma)} < 0 \quad \text{and} \quad \frac{\partial a_H^*}{\partial \tau} = -1 + \frac{\nu(\gamma - \delta)}{(1 + \eta)\gamma} < 0 \quad (22)$$

with $\frac{\partial a_L^*}{\partial \tau} < \frac{\partial a_H^*}{\partial \tau} < 0$ if $\nu \neq 0$. Moreover:

$$\frac{\partial a_L^*}{\partial \nu} = -\frac{\tau(\gamma - \delta)}{(1 + \eta)(1 - \gamma)} < 0 \quad \text{and} \quad \frac{\partial a_H^*}{\partial \nu} = \frac{\tau(\gamma - \delta)}{(1 + \eta)\gamma} > 0. \quad (23)$$

Regarding the effect of income inequality:

$$\frac{\partial^2 a_L^*}{\partial \tau \partial \gamma} = -\frac{(1 - \delta)\nu}{(1 + \eta)(1 - \gamma)^2} < 0 \quad \text{and} \quad \frac{\partial^2 a_H^*}{\partial \tau \partial \gamma} = \frac{\delta\nu}{(1 + \eta)\gamma^2} > 0 \quad (24)$$

$$\frac{\partial^2 a_L^*}{\partial \nu \partial \gamma} = -\frac{(1 - \delta)\tau}{(1 + \eta)(1 - \gamma)^2} > 0 \quad \text{and} \quad \frac{\partial^2 a_H^*}{\partial \nu \partial \gamma} = \frac{\delta\tau}{(1 + \eta)\gamma^2} > 0 \quad (25)$$

A.2 Pension-Size Upper Bound

The condition that guarantees $a_L^* \geq 0$ is:

$$a_L^* = \frac{\eta}{1+\eta} - \tau - \frac{\tau\nu(\gamma-\delta)}{(1+\eta)(1-\gamma)} \geq 0 \Leftrightarrow \tau \leq \frac{\eta(1-\gamma)}{(1+\eta)(1-\gamma) + \nu(\gamma-\delta)} \equiv \bar{\tau}_L \quad (26)$$

$$a_H^* = \frac{\eta}{1+\eta} - \tau + \frac{\tau\nu(\gamma-\delta)}{(1+\eta)\gamma} \geq 0 \Leftrightarrow \tau \leq \frac{\eta\gamma}{(1+\eta)\gamma - \nu(\gamma-\delta)} \equiv \bar{\tau}_H \quad (27)$$

As $\bar{\tau}_L < \bar{\tau}_H$, the upper bound of pension size guaranteeing positive contributions from both group is $\bar{\tau} = \bar{\tau}_L$. Then:

$$\frac{\partial \bar{\tau}}{\partial \nu} = -\frac{\eta(1-\gamma)(\gamma-\delta)}{[(1+\eta)(1-\gamma) + \nu(\gamma-\delta)]^2} < 0 \quad (28)$$

$$\frac{\partial \bar{\tau}}{\partial \gamma} = -\frac{\eta(1-\delta)\nu}{[(1+\eta)(1-\gamma) + \nu(\gamma-\delta)]^2} < 0 \quad (29)$$

$$\frac{\partial^2 \bar{\tau}}{\partial \nu \partial \gamma} = -\frac{\eta(1-\delta)\{(1-\gamma)^2(1+\eta)^2 - \nu^2(\gamma-\delta)^2\}}{[(1+\eta)(1-\gamma) + \nu(\gamma-\delta)]^4} < 0 \quad (30)$$

A.3 Political Equilibrium with Positive Contributions

When agents choose positive contributions ex-post utilities write:

$$V_L(\tau, \nu) = \frac{(1+\eta)}{(1-\mu)} \left(\frac{(1-\gamma + \tau\nu(\gamma-\delta))\bar{y}}{(1+\eta)(1-\delta)} \right)^{1-\mu} \quad (31)$$

$$V_H(\tau, \nu) = \frac{(1+\eta)}{(1-\mu)} \left(\frac{(\gamma - \tau\nu(\gamma-\delta))\bar{y}}{(1+\eta)\delta} \right)^{1-\mu} \quad (32)$$

Now, as the inter-temporal elasticity of substitution is $\sigma = 1/\mu$, the first order conditions that solve candidates' problem in (10) is given by

$$\delta\phi \frac{\partial V_H}{\partial \tau\nu} + (1-\delta) \frac{\partial V_L}{\partial \tau\nu} = 0 \Leftrightarrow c_H = \phi^\sigma c_L \quad (33)$$

and using the optimal consumption levels in (20) and (21), the equilibrium redistributive power writes:

$$\tau\nu^* = \frac{\gamma(1-\delta) - (1-\gamma)\delta\phi^\sigma}{(\gamma-\delta)(\delta\phi^\sigma + 1-\delta)} \quad (34)$$

A.3.1 High Earners' Political Power Boundaries

The maximum achievable level of redistributive power $\tau\nu$ is reached at the frontier of contributions, $\bar{\tau}$ with complete redistribution $\nu = 1$. The maximum redistributive power writes:

$$\tau\nu_{max} \equiv \frac{\eta(1-\gamma)}{\eta(1-\gamma) + 1 - \delta} \text{ with } \tau \equiv \frac{\eta(1-\gamma)}{\eta(1-\gamma) + 1 - \delta} \text{ and } \nu_{max} \equiv 1$$

The conditions for $0 < \tau\nu^* < \tau\nu_{max}$ are then:

$$\tau\nu^* > 0 \Leftrightarrow \phi < \bar{\phi} \equiv \left(\frac{\gamma}{1-\gamma} \frac{1-\delta}{\delta} \right)^{\frac{1}{\sigma}} \quad (35)$$

$$\tau\nu^* < \tau\nu_{max} \Leftrightarrow \phi > \underline{\phi} \equiv \left(\frac{\gamma(1-\delta) + \eta\delta(1-\gamma)}{(1+\eta)\delta(1-\gamma)} \right)^{\frac{1}{\sigma}} \quad (36)$$

with $1 < \underline{\phi} < \bar{\phi}$.

A.3.2 Equilibrium Redistributive Power - Properties

$$\frac{\partial \tau\nu^*}{\partial \gamma} = \frac{(1-\delta)\delta(\phi^\sigma - 1)}{(\gamma - \delta)^2(\delta\phi^\sigma + (1-\delta))} > 0 \text{ when } \tau\nu^* < 1 \text{ (i.e. } \phi^\sigma > 1) \quad (37)$$

$$\frac{\partial \tau\nu^*}{\partial \phi} = -\frac{(1-\delta)\delta\sigma\phi^{\sigma-1}}{(\gamma - \delta)(\delta\phi^\sigma + (1-\delta))} < 0 \quad (38)$$

$$\frac{\partial \tau\nu^*}{\partial \sigma} = -\frac{(1-\delta)\delta\phi^\sigma \ln \phi}{(\gamma - \delta)(\delta\phi^\sigma + (1-\delta))} < 0 \quad (39)$$

A.4 Political Equilibrium - French Simulation

Given the real return used in the OECD pension model, and the funded conversion rate, the gross return of the tier pension system and the corresponding discount factor given financial discounting write:

$$R = (0.90)(1.025)^{43} = 2.6024 \text{ and } \beta = 1/R = 0.3843$$

In order to estimate ϕ^σ , we reorganize Equation (34):

$$\phi^\sigma = \left(\frac{1-\delta}{\delta} \right) \left(\frac{\gamma - (\gamma - \delta)\tau\nu}{1 - \gamma + (\gamma - \delta)\tau\nu} \right) \quad (40)$$

Replacing for the values of the parameters and the French pension characteristics we are able to find ϕ^σ .

A.5 Political Equilibrium with Hand-to-Mouth Agents

When low earners cannot contribute ($a_L = 0$), their consumption profiles depend on the size and the redistribution degree independently, which generates a trade-off between consumption in both periods. Low-

earner's consumption profiles then write:

$$c_L = \frac{(1-\tau)(1-\gamma)\bar{y}}{1-\delta} \text{ and } d_L = \frac{R\tau(1-\gamma+\nu(\gamma-\delta))\bar{y}}{(1-\delta)\psi} \quad (41)$$

and while ex-post utility of high earners (V_H) remains unchanged:

$$V_L(\tau, \nu) = \frac{1}{1-\mu} \left(\frac{(1-\tau)(1-\gamma)\bar{y}}{1-\delta} \right)^{1-\mu} + \frac{\beta\psi}{1-\mu} \left(\frac{R\tau(1-\gamma+\nu(\gamma-\delta))\bar{y}}{(1-\delta)\psi} \right)^{1-\mu} \quad (42)$$

The problem in (10) then turns into a two-variable maximization:

$$\max_{\tau^N, \nu^N} W^N = \sum_i \phi^i n_i V_i(\tau^N, \nu^N) \quad (43)$$

whose first orders conditions write:

$$\sum_i n_i \phi^i \frac{\partial V_i(\rho^N)}{\partial \tau^*} = 0 \text{ and } \sum_i n_i \phi^i \frac{\partial V_i(\rho^N)}{\partial \nu^*} = 0 \quad (44)$$

The maximum is reached when low earners perfectly smoothing consumption, that is when $(\beta R)^\sigma c_L = d_L$ with $a_L = 0$. Then:

$$\tau^* = \frac{(1-\gamma)(1+\eta)\delta\phi^\sigma + (1-\delta)(\eta-\gamma(1+\eta))}{(1-\gamma)(1+\eta)(\delta\phi^\sigma + (1-\delta))} \quad (45)$$

$$\nu^* = \frac{(1-\gamma)(1+\eta)\{\gamma(1-\delta) - (1-\gamma)\delta\phi^\sigma\}}{(\gamma-\delta)\{(1-\gamma)\delta(1+\eta)\phi^\sigma + (1-\delta)(\eta-\gamma(1+\eta))\}} \quad (46)$$

Which correspond to the intersection of the equilibrium redistributive power curve with pension-size upper bound curve. The properties of such equilibrium characteristics are:

$$\frac{\partial \tau^*}{\partial \gamma} = -\frac{(1-\delta)(1+\gamma(1+\eta)-\eta)}{(1-\gamma)^2(1+\eta)(\delta\phi^\sigma + 1-\delta)} < 0 \quad (47)$$

$$\frac{\partial \nu^*}{\partial \gamma} > 0 \quad (48)$$

This last effect is straightforward from the effect of income inequality on both the redistributive power curve and the pension-size upper bound

A.6 Contribution Rates and Consumption with Lifespan Inequality

The first order conditions of the problem described in (5), with CIES utility functions, again gives:

$$(\beta R)^\sigma c_i = d_i \quad (49)$$

That is, using (13) and (14):

$$a_H^* = \frac{\tilde{\eta}\theta}{\delta + \tilde{\eta}\theta} - \tau + \frac{\tau\nu(\gamma - \theta)\delta}{(\delta + \tilde{\eta}\theta)\gamma} \quad (50)$$

$$a_L^* = \frac{\tilde{\eta}(1 - \theta)}{1 - \delta + \tilde{\eta}(1 - \theta)} - \tau - \frac{\tau\nu(\gamma - \theta)(1 - \delta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma)} \quad (51)$$

$$c_H = \frac{(\gamma - \tau\nu(\gamma - \theta))\bar{y}}{\delta + \tilde{\eta}\theta} \text{ and } c_L = \frac{(1 - \gamma + \tau\nu(\gamma - \theta))\bar{y}}{1 - \delta + \tilde{\eta}(1 - \theta)} \quad (52)$$

where $\tilde{\eta} = \beta^\sigma R^{\sigma-1} \bar{\psi}$

A.6.1 Effect of pension characteristics on contribution rates and consumption with Lifespan Inequality

Assuming $\gamma > \theta$:

$$\frac{\partial a_L^*}{\partial \tau} = -1 - \frac{\nu(\gamma - \theta)(1 - \delta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma)} < 0 \text{ and } \frac{\partial a_H^*}{\partial \tau} = -1 + \frac{\nu(\gamma - \theta)\delta}{(\delta + \tilde{\eta}\theta)\gamma} < 0 \quad (53)$$

with $\frac{\partial a_L^*}{\partial \tau} < \frac{\partial a_H^*}{\partial \tau} < 0$ if $\nu \neq 0$. Moreover

$$\frac{\partial a_L^*}{\partial \nu} = -\frac{\tau(\gamma - \theta)(1 - \delta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma)} < 0 \text{ and } \frac{\partial a_H^*}{\partial \nu} = \frac{\tau(\gamma - \theta)\delta}{(\delta + \tilde{\eta}\theta)\gamma} > 0 \quad (54)$$

$$\frac{\partial^2 a_L^*}{\partial \tau \partial \gamma} = -\frac{(1 - \delta)\nu(1 - \theta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma)^2} < 0 \text{ and } \frac{\partial^2 a_H^*}{\partial \tau \partial \gamma} = \frac{\delta\nu\theta}{(\delta + \tilde{\eta}\theta)\gamma^2} > 0 \quad (55)$$

$$\frac{\partial^2 a_L^*}{\partial \tau \partial \theta} = \frac{(1 - \delta)\nu(1 - \delta + \tilde{\eta}(1 - \gamma))}{(1 - \delta + \tilde{\eta}(1 - \theta))^2(1 - \gamma)} > 0 \text{ and } \frac{\partial^2 a_H^*}{\partial \tau \partial \theta} = -\frac{\delta\nu(\delta + \tilde{\eta}\gamma)}{(\delta + \tilde{\eta}\theta)^2\gamma} < 0 \quad (56)$$

$$\frac{\partial^2 a_L^*}{\partial \nu \partial \gamma} = -\frac{(1 - \delta)\tau(1 - \theta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma)^2} < 0 \text{ and } \frac{\partial^2 a_H^*}{\partial \nu \partial \gamma} = \frac{\delta\tau\theta}{(\delta + \tilde{\eta}\theta)\gamma^2} > 0 \quad (57)$$

$$\frac{\partial^2 a_L^*}{\partial \nu \partial \theta} = \frac{(1 - \delta)\tau(1 - \delta + \tilde{\eta}(1 - \gamma))}{(1 - \delta + \tilde{\eta}(1 - \theta))^2(1 - \gamma)} > 0 \text{ and } \frac{\partial^2 a_H^*}{\partial \nu \partial \theta} = -\frac{\delta\tau(\delta + \tilde{\eta}\gamma)}{(\delta + \tilde{\eta}\theta)^2\gamma} < 0 \quad (58)$$

A.7 Pension-Size Upper Bound with Lifespan Inequality

The condition that guarantees $a_L^* \geq 0$ is:

$$a_L^* = \frac{\tilde{\eta}(1 - \theta)}{1 - \delta + \tilde{\eta}(1 - \theta)} - \tau - \frac{\tau\nu(\gamma - \theta)(1 - \delta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma)} \geq 0 \quad (59)$$

$$\Leftrightarrow \tau \leq \frac{\tilde{\eta}(1 - \gamma)(1 - \theta)}{(1 - \delta + \tilde{\eta}(1 - \theta))(1 - \gamma) + (1 - \delta)\nu(\gamma - \theta)} \equiv \bar{\tau}' \quad (60)$$

With:

$$\frac{\partial \bar{\tau}'}{\partial \nu} = -\frac{\tilde{\eta}(1-\gamma)(1-\delta)(\gamma-\theta)}{[(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)+(1-\delta)\nu(\gamma-\theta)]^2} < 0 \quad (61)$$

$$\frac{\partial \bar{\tau}'}{\partial \gamma} = -\frac{\tilde{\eta}(1-\delta)(1-\theta)^2\nu}{[(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)+(1-\delta)\nu(\gamma-\theta)]^2} < 0 \quad (62)$$

$$\frac{\partial \bar{\tau}'}{\partial \theta} = -\frac{\tilde{\eta}(1-\delta)(1-\gamma)^2(1-\nu)}{[(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)+(1-\delta)\nu(\gamma-\theta)]^2} < 0 \quad (63)$$

$$\frac{\partial^2 \bar{\tau}'}{\partial \gamma \partial \nu} = -\frac{\tilde{\eta}(1-\delta)(1-\theta)^2\{(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)-(1-\delta)\nu(\gamma-\theta)\}}{[(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)+(1-\delta)\nu(\gamma-\theta)]^3} < 0 \quad (64)$$

$$\frac{\partial^2 \bar{\tau}'}{\partial \theta \partial \nu} = \frac{\tilde{\eta}(1-\delta)(1-\gamma)^2\{(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)+(1-\delta)(\gamma-\theta)(2-\nu)\}}{[(1-\delta+\tilde{\eta}(1-\theta))(1-\gamma)+(1-\delta)\nu(\gamma-\theta)]^3} > 0 \quad (65)$$

A.8 Equilibrium Redistributive Power Curve with Lifespan Inequality

When agents are able to use contributions to smooth consumption, ex-post utilities write:

$$V_L(\tau, \nu) = \frac{(1-\delta+\tilde{\eta}(1-\theta))}{(1-\delta)(1-\mu)} \left(\frac{(1-\gamma+\tau\nu(\gamma-\theta))\bar{y}}{1-\delta+\tilde{\eta}(1-\theta)} \right)^{1-\mu} \quad (66)$$

$$V_H(\tau, \nu) = \frac{(\delta+\tilde{\eta}\theta)}{\delta(1-\mu)} \left(\frac{(\gamma-\tau\nu(\gamma-\theta))\bar{y}}{\delta+\tilde{\eta}\theta} \right)^{1-\mu} \quad (67)$$

The first order condition (FOC) of candidates' problem in (10) still writes:

$$\delta\phi \frac{\partial V_H}{\partial \tau\nu} + (1-\delta) \frac{\partial V_L}{\partial \tau\nu} = 0 \Rightarrow c_H = \phi^\sigma c_L \quad (68)$$

And using consumption levels in (52), the equilibrium redistributive power solves:

$$\tau\nu^* = \frac{\gamma(\tilde{\eta}(1-\theta)+1-\delta)-(1-\gamma)\phi^\sigma(\tilde{\eta}\theta+\delta)}{(\gamma-\theta)\{\phi^\sigma(\tilde{\eta}\theta+\delta)+\tilde{\eta}(1-\theta)+1-\delta\}} \quad (69)$$

A.8.1 High Earners' Political Power Boundaries with Lifespan Inequality

The maximum level of redistributive power is achieved when $\tau = \bar{\tau}'$ and $\nu = 1$:

$$\tau\nu_{max} \equiv \frac{\tilde{\eta}(1-\gamma)}{\tilde{\eta}(1-\gamma)+1-\delta}$$

and the conditions for an interior solution $0 < \tau\nu^* < \tau\nu_{max}$ are:

$$\phi < \bar{\phi}' \equiv \left(\frac{\gamma}{1-\gamma} \frac{\tilde{\eta}(1-\theta)+1-\delta}{\tilde{\eta}\theta+\delta} \right)^{\frac{1}{\sigma}}, \text{ and} \quad (70)$$

$$\phi > \underline{\phi}' \equiv \left(\frac{(\tilde{\eta}(1-\theta)+1-\delta)\{\gamma(1-\delta)+\tilde{\eta}\delta(1-\gamma)\}}{(\tilde{\eta}\theta+\delta)(1+\eta)(1-\delta)(1-\gamma)} \right)^{\frac{1}{\sigma}} \quad (71)$$

A.8.2 Impact of Income and Lifespan Inequalities on the Equilibrium Redistributive Power of Pensions

- Effect of Income Inequality on the Redistributive Power:

$$\frac{\partial \tau \nu^*}{\partial \gamma} = \frac{\phi^\sigma (\tilde{\eta} \theta + \delta)(1 - \theta) - \theta (\tilde{\eta}(1 - \theta) + 1 - \delta)}{(\gamma - \theta)^2 \{ \phi^\sigma (\tilde{\eta} \theta + \delta) + \tilde{\eta}(1 - \theta) + 1 - \delta \}} > 0 \quad (72)$$

when

$$\phi \geq \underline{\phi}' > \left(\frac{\theta}{1 - \theta} \frac{(\tilde{\eta}(1 - \theta) + 1 - \delta)}{(\tilde{\eta} \theta + \delta)} \right)^{\frac{1}{\sigma}} > 0 \text{ since } \gamma > \theta \quad (73)$$

- Effect of Longevity Inequality on Redistributive Power:

Longevity inequality decreases the redistributive power. For the proof we use Corollary 1: the equilibrium when low earners can not contribute ex ante is the intersection between the equilibrium redistributive power and the pension-size upper bound curves. When low earners can not contribute ex ante, the equilibrium pension size writes:

$$\tau^* = \frac{(1 - \gamma) \phi^\sigma (\tilde{\eta} \theta + \delta) + (1 - \gamma) \tilde{\eta}(1 - \theta) - \gamma(1 - \delta)}{(1 - \gamma) \{ \phi^\sigma (\tilde{\eta} \theta + \delta) + \tilde{\eta}(1 - \theta) + (1 - \delta) \}} \quad (74)$$

and is increasing with lifespan inequality:

$$\frac{\partial \tau^*}{\partial \theta} = \frac{(1 - \delta) \tilde{\eta} (\phi^\sigma - 1)}{(1 - \gamma) \{ (\tilde{\eta} \theta + \delta) \phi^\sigma + \tilde{\eta}(1 - \theta) + 1 - \delta \}^2} > 0 \quad (75)$$

Now, by equation (63), the pension size upper bound curve shift left (in the (τ, ν) map) when lifespan inequality increase. As both curves are decreasing, this can be consistent with (75) only if lifespan inequality also shifts left the redistributive power curve, i.e. only if $\frac{\partial \tau \nu^*}{\partial \theta} < 0$.

Thus, the effect of inequalities on the redistributive power curve always dominate the effect on the pension-size upper-bound. This can be seen graphically in Figures 2 and 3, in which the impact of both inequalities on the pension-size upper bound are negligible compared to the effect on the redistributive power curve.

A.9 Political Equilibrium with Hand-to-Mouth Agents and Lifespan Inequality

From Corollary 1, the equilibrium with no low earners' contributions lies at the intersection of the redistributive power curve with the pension-size upper bound. The equilibrium pension size is given by (74) and its properties are studied on the previous subsection. Regarding the equilibrium degree of redistribution, it writes in this case:

$$\nu^* = \frac{(1-\gamma)\{\gamma(\tilde{\eta}(1-\theta) + 1 - \delta) - (1-\gamma)\phi^\sigma(\tilde{\eta}\theta + \delta)\}}{(\gamma - \theta)\{(1-\gamma)\phi^\sigma(\tilde{\eta}\theta + \delta) + (1-\gamma)\tilde{\eta}(1-\theta) - \gamma(1-\delta)\}} \quad (76)$$

It then increases with income inequality and decreases with lifespan inequality: $\frac{\partial \nu^*}{\partial \theta} < 0 < \frac{\partial \nu^*}{\partial \gamma}$ (consistently with the above observation that the impact of inequalities on the redistributive power curve dominates their impact on the pension-size upper bound).

A.10 When Income Inequality impacts Lifespan Inequality

As $\frac{\partial \tau^*}{\partial \theta} \geq 0$ and $\frac{\partial \tau^*}{\partial \gamma} \leq 0$, by equation (17): $\frac{d\tau^*}{d\theta} \leq 0$ if and only if:

$$\frac{\partial \gamma}{\partial \theta} \geq -\frac{\partial \tau^*/\partial \theta}{\partial \tau^*/\partial \gamma} = \frac{(1-\gamma)\tilde{\eta}(\phi^\sigma - 1)}{\phi^\sigma(\tilde{\eta}\theta + \delta) + \tilde{\eta}(1-\theta) + 1 - \delta} \quad (77)$$

that is after defining $\underline{\varepsilon} \equiv -\frac{\partial \tau^*/\partial \theta}{\partial \tau^*/\partial \gamma} \frac{\gamma}{\theta}$: $\varepsilon \geq \underline{\varepsilon}$. If the elasticity of lifespan-income inequality is large enough, lifespan inequality decreases pension size.

Similarly, as $\frac{\partial \nu^*}{\partial \theta} \leq 0$ and $\frac{\partial \nu^*}{\partial \gamma} \geq 0$: $\frac{d\nu^*}{d\theta} \geq 0$ if and only if $\varepsilon \geq \tilde{\varepsilon}$ with $\tilde{\varepsilon} \equiv -\frac{\partial \nu^*/\partial \theta}{\partial \nu^*/\partial \gamma} \frac{\gamma}{\theta}$.

And, as $\frac{\partial \tau \nu^*}{\partial \theta} \leq 0$ and $\frac{\partial \tau \nu^*}{\partial \gamma} \geq 0$: $\frac{d\tau \nu^*}{d\theta} > 0$ if and only if $\varepsilon \geq \bar{\varepsilon}$ with $\bar{\varepsilon} \equiv -\frac{\partial \tau \nu^*/\partial \theta}{\partial \tau \nu^*/\partial \gamma} \frac{\gamma}{\theta}$.

Now, using the pension size upper bound, we can rank these boundaries: $\underline{\varepsilon} < \tilde{\varepsilon} < \bar{\varepsilon}$. Indeed, by definition:

$$\tau^*(\theta) = \bar{\tau}(\nu^*, \theta) = \frac{\tilde{\eta}(1-\gamma)(1-\theta)}{(1-\delta + \tilde{\eta}(1-\theta))(1-\gamma) + (1-\delta)\nu^*(\gamma-\theta)} \quad (78)$$

and as proven in Section A.7: $\partial \bar{\tau}/\partial \nu > 0$, $\partial \bar{\tau}/\partial \theta < 0$, $\partial \bar{\tau}/\partial \gamma < 0$. Now, at $\varepsilon = \tilde{\varepsilon}$, an increase of lifespan inequality from θ_1 to $\theta_2 > \theta_1$ doesn't impact ν^* ($\nu^*(\theta_1) = \nu^*(\theta_2)$). Therefore, as $\partial \bar{\tau}/\partial \theta < 0$, at $\varepsilon = \tilde{\varepsilon}$:

$$\tau^*(\theta_1) = \bar{\tau}(\nu^*(\theta_1), \theta_1) = \bar{\tau}(\nu^*(\theta_2), \theta_1) > \bar{\tau}(\nu^*(\theta_2), \theta_2) = \tau^*(\theta_2) \quad (79)$$

Since $\frac{\partial \tau^*}{\partial \theta} \geq 0$ and $\frac{\partial \tau^*}{\partial \gamma} \leq 0$ this means that $\underline{\varepsilon} < \tilde{\varepsilon}$ (meaning that the level of elasticity at which the effect of lifespan inequality on pension size change size is lower than the level at which the effect on degree of redistribution change sign). Similarly, by (79), at $\varepsilon = \tilde{\varepsilon}$:

$$\tau \nu^*(\theta_1) = \tau^*(\theta_1)\nu^*(\theta_1) = \tau^*(\theta_1)\nu^*(\theta_2) > \tau^*(\theta_2)\nu^*(\theta_2) = \tau \nu^*(\theta_2) \quad (80)$$

and, as $\frac{\partial \tau \nu^*}{\partial \theta} \leq 0$ and $\frac{\partial \tau \nu^*}{\partial \gamma} \geq 0$: $\tilde{\varepsilon} < \bar{\varepsilon}$. This means that an equilibrium where lifespan inequality decreases size, degree of redistribution, and redistributive power of the pension system is possible. This happens exactly when $\underline{\varepsilon} < \varepsilon < \tilde{\varepsilon}$, and it is in line with Figures 8, 9, and 10. We now focus on the properties of the lower bound as it defines this scenario.

$$\frac{\partial \underline{\varepsilon}}{\partial \phi} = -\frac{\sigma \phi^{\sigma-1} (1-\gamma) \tilde{\eta} \{2(\tilde{\eta} \theta + \delta) + 1 - \delta\}}{\{(1-\gamma) \tilde{\eta} (\phi^\sigma - 1)\}^2} < 0 \quad (81)$$

$$\frac{\partial \underline{\varepsilon}}{\partial \beta} = -\frac{\sigma (\phi^\sigma \delta + 1 - \delta)}{\beta (1-\gamma) \tilde{\eta} (\phi^\sigma - 1)} < 0 \quad (82)$$

$$\frac{\partial \underline{\varepsilon}}{\partial R} = -\frac{(\sigma - 1) (\phi^\sigma \delta + 1 - \delta)}{R (1-\gamma) \tilde{\eta} (\phi^\sigma - 1)} < 0 \text{ iff } \sigma > 1 \quad (83)$$

B Pension Empirics

B.1 Estimation of Pension Characteristics

In this section, we analyze how our theoretical predictions align with correlations observed in OECD countries. We focus on the main mandatory pension scheme in each country, *i.e.*, the system that covers the largest share of pensioners, and examine how pension size and degree of redistribution correlate with income and lifespan inequalities. We estimate these pension characteristics using replacement rates at different income levels from the OECD Pensions at a Glance Reports (2015, 2017, 2019, 2021, and 2023). Replacement rates (RR in the following) refer to the share of labor income that individuals receive as pensions, based on full-career workers.¹⁰ We use these replacement rates to estimate the degree of redistribution and size of pensions, following Disney et al. (2004); and consider replacement rates for men, when gender-specific differences exist. We measure the redistribution degree as the part of pension benefits that is not income-related, calculated as the actual coefficient of variation of replacement rates (normalized by the hypothetical coefficient of variation of a fully redistributive pension, so that $\nu = 1$ in that case):

$$\nu = \frac{\sqrt{\text{var}(RR)}}{RR} \bigg/ \frac{\sqrt{\text{var}(RR^f)}}{RR^f} \quad (84)$$

and consider three level of replacement rates: half-average income, average income and one and a half average income: $RR = [RR_{0.5\bar{y}}; RR_{\bar{y}}; RR_{1.5\bar{y}}]$. The vector of replacement rates under a fully redistributive pension RR^f then becomes, with the replacement rate of average income as reference: $RR^f = [2RR_{\bar{y}}; RR_{\bar{y}}; (2/3)RR_{\bar{y}}]$. Pension size is computed from the replacement rate for average-income individuals, as from equation (4) for $y_i = \bar{y}$: $RR_{\bar{y}} = R\tau/\psi$.¹¹

$$\tau = \psi \cdot \frac{RR_{\bar{y}}}{R} = \frac{|d|}{|c|} \frac{RR_{\bar{y}}}{\xi(1+r)^{|c|}} \quad (85)$$

¹⁰Replacement rates are calculated assuming identical economic characteristics (e.g., yearly returns, inflation rate, and wage growth) among countries, representing intrinsic differences in pension schemes.

¹¹To account for differences in contribution and retirement period length, we correct replacement rate by period length rather than by support ratios as Disney et al. (2004).

with $|c|$ and $|d|$ respectively the length of the working and the retired period, ξ the conversion factor of pension (i.e. dead-weight loss from pension funds) and r the real return rate. We calibrate $\xi = 0.9$ and $r = 1.25\%$ based on the OECD pension model.

Table 5: Average Pension Size and Degree of Redistribution (2014-2022)

Country	Size	Degree of Redistribution	Country	Size	Degree of Redistribution
Australia	0,064	0,890	Japan	0,072	0,401
Austria	0,159	0,001	Korea	0,075	0,595
Belgium	0,089	0,401	Latvia	0,074	0,087
Canada	0,081	0,506	Lithuania	0,036	0,629
Chile	0,068	0,235	Luxembourg	0,192	0,197
Colombia	0,165	0,304	Mexico	0,076	0,361
Costa Rica	0,146	0,041	Netherlands	0,134	0,073
Czechia	0,087	0,678	New Zealand	0,085	0,912
Denmark	0,110	0,563	Norway	0,088	0,423
Estonia	0,062	0,483	Poland	0,058	0,018
Finland	0,103	0,000	Portugal	0,135	0,034
France	0,126	0,109	Slovakia	0,093	0,191
Germany	0,078	0,041	Slovenia	0,097	0,299
Greece	0,141	0,243	Spain	0,160	0,039
Hungary	0,100	0,011	Sweden	0,114	0,163
Iceland	0,116	0,257	Switzerland	0,091	0,545
Ireland	0,057	0,999	Turkey	0,143	0,000
Israel	0,101	0,664	United Kingdom	0,060	0,820
Italy	0,129	0,000	United States	0,071	0,372

Source: OECD Pension at a Glance 2015, 2017, 2019, 2021, 2023

B.2 Inequality Measures

We measure income inequality using the OECD Gini index for disposable (adjusted) income and lifespan inequality using the Gini index for men from Our World in Data.¹² Table 6 presents the average of these inequality measures for the period between 2012 and 2021, indicating that income dispersion is larger than lifespan dispersion.

¹²The measure uses data from the Human Mortality Database and the methodology of [Aburto et al. \(2020\)](#).

Table 6: Average Income and Lifespan Inequality (2012-2021)

Country	Gini of Income	Gini of Lifespan	Country	Gini of Income	Gini of Lifespan
Australia	0,33	0,09	Japan	0,33	0,09
Austria	0,28	0,09	Korea	0,36	0,09
Belgium	0,26	0,09	Latvia	0,35	0,12
Canada	0,31	0,10	Lithuania	0,36	0,13
Chile	0,46	0,11	Luxembourg	0,31	0,09
Colombia	N/A	0,13	Mexico	0,44	0,15
Costa Rica	0,48	0,11	Netherlands	0,30	0,09
Czechia	0,25	0,10	New Zealand	0,33	0,10
Denmark	0,26	0,09	Norway	0,26	0,08
Estonia	0,32	0,12	Poland	0,28	0,11
Finland	0,26	0,10	Portugal	0,33	0,10
France	0,30	0,10	Slovakia	0,24	0,11
Germany	0,29	0,10	Slovenia	0,25	0,09
Greece	0,33	0,10	Spain	0,34	0,09
Hungary	0,28	0,11	Sweden	0,28	0,08
Iceland	0,25	0,08	Switzerland	0,30	0,08
Ireland	0,30	0,09	Turkey	0,40	0,10
Israel	0,35	0,09	United Kingdom	0,36	0,10
Italy	0,33	0,09	United States	0,39	0,12

Source: Gini of after-tax income OECD, and Gini of lifespan of men from Our World in Data

Our theoretical model assumes away informality by presuming that all working-period income is taxable. However, informality is prevalent in some countries in our sample. To address this, we exclude countries with the highest levels of average Gini of disposable income, namely Chile, Colombia, Costa Rica, Mexico, and Turkey, as informality should be reflected in these measures through income misreporting.

B.3 Income inequality and Pension Characteristics

Next, we analyze the effect of income inequality on pension characteristics. Figure 5 shows the correlation between income inequality and the redistributive power of public pensions. Consistent with Theorem 1, this figure reveals a positive relationship.

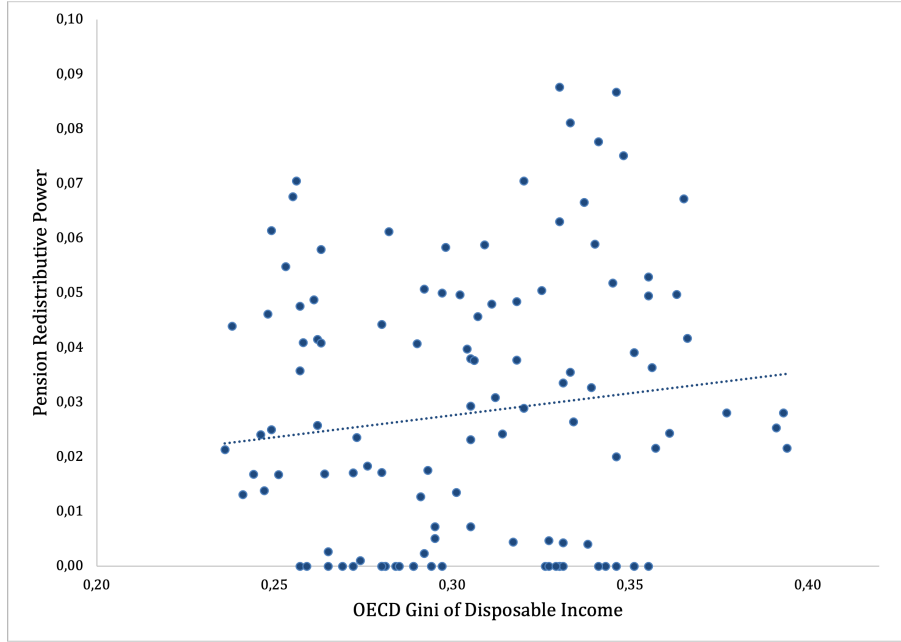


Figure 5: Gini of Disposable Income vs Pension’s Redistributive Power (2015 - 2023)
 Source: Own elaboration with data from Our World in Data, and OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023). Excluding Chile, Colombia, Costa Rica, Mexico and Turkey

Our theoretical results are also confirmed when analyzing separately the effect on income inequality on the degree of redistribution in Figure 6 and the size of pensions in Figure 7.

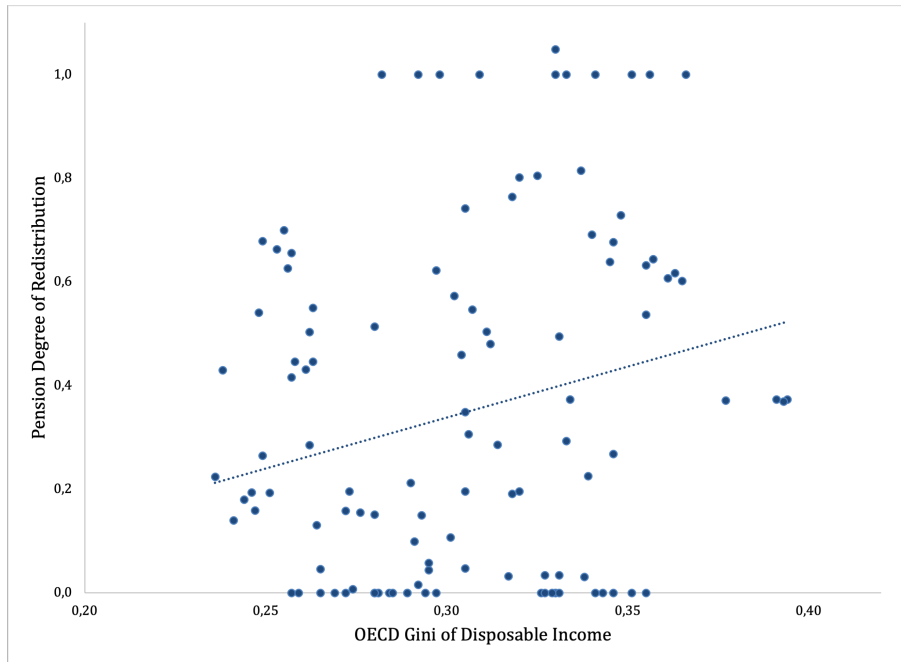


Figure 6: Gini of disposable income vs pension’s degree of redistribution (2015 - 2023)
 Source: Own elaboration with data from Our World in Data, and OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023). Excluding Chile, Colombia, Costa Rica, Mexico and Turkey

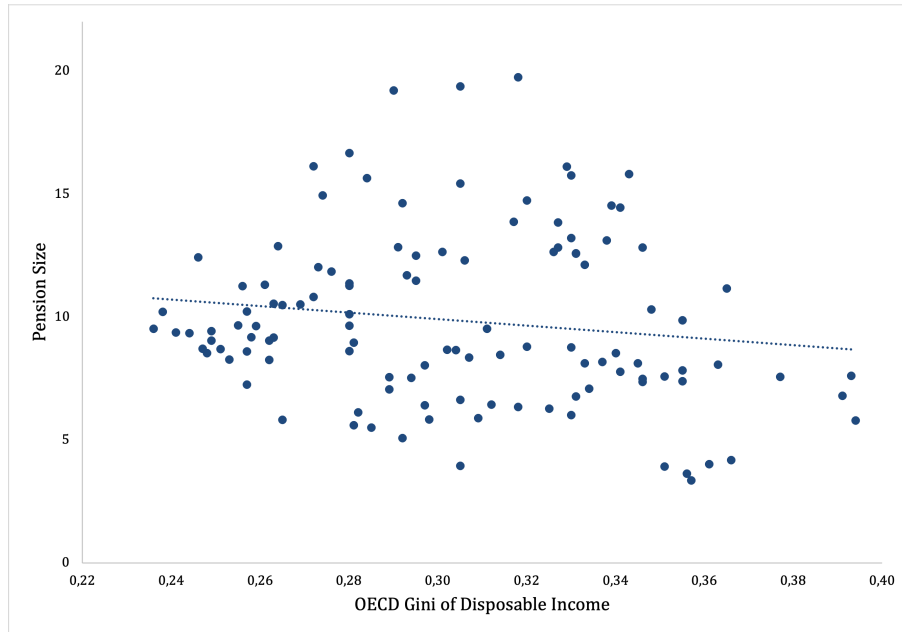


Figure 7: Gini of disposable income vs pension's size (2015 - 2023)
 Source: Own elaboration with data from Our World in Data, and OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023). Excluding Chile, Colombia, Costa Rica, Mexico and Turkey

Consistent with Corollary 1, higher income inequality is associated with more redistributive but smaller public pensions.

B.4 Lifespan inequality and Pension Characteristics

Regarding lifespan inequality, Figure 8 shows a negative correlation between lifespan inequality and the redistributive power of pensions, aligning with our Theorem 2. However, Figures 9 and 10 indicate a negative correlation between lifespan inequality and both pension degree of redistribution and pension size. This last observation doesn't align with Corollary 2, but it can be rationalized within our model's framework if lifespan inequality and income inequality are correlated (as discussed in Section 5). This positive relationship between lifespan inequality and income inequality in the data is confirmed in Figure 11.

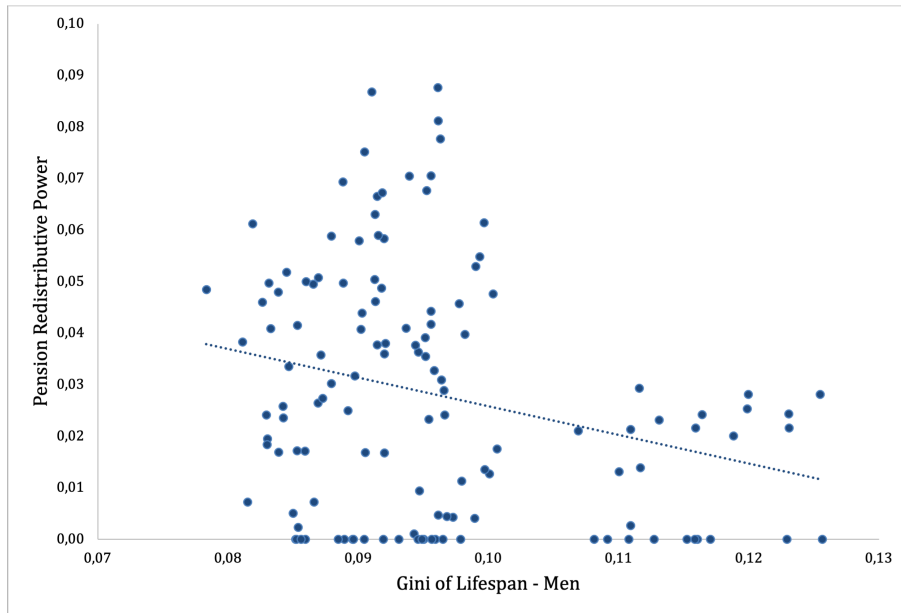


Figure 8: Gini of men lifespan vs pension’s redistributive power (2015 - 2023)

Source: Own elaboration with data from Our World in Data, and OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023). Excluding Chile, Colombia, Costa Rica, Mexico and Turkey

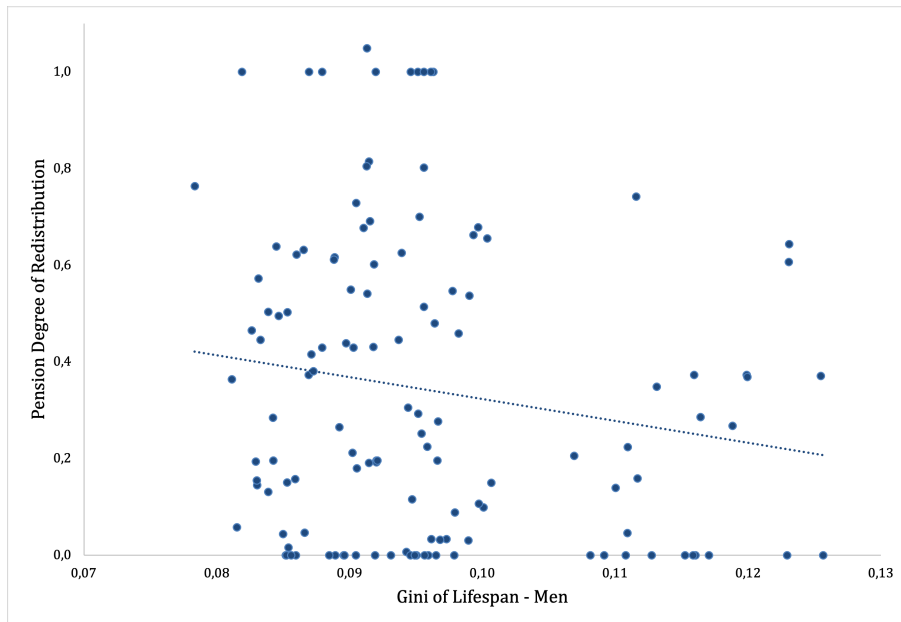


Figure 9: Gini of men lifespan vs pension’s degree of redistribution (2015 - 2023)

Source: Own elaboration with data from Our World in Data, and OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023). Excluding Chile, Colombia, Costa Rica, Mexico and Turkey

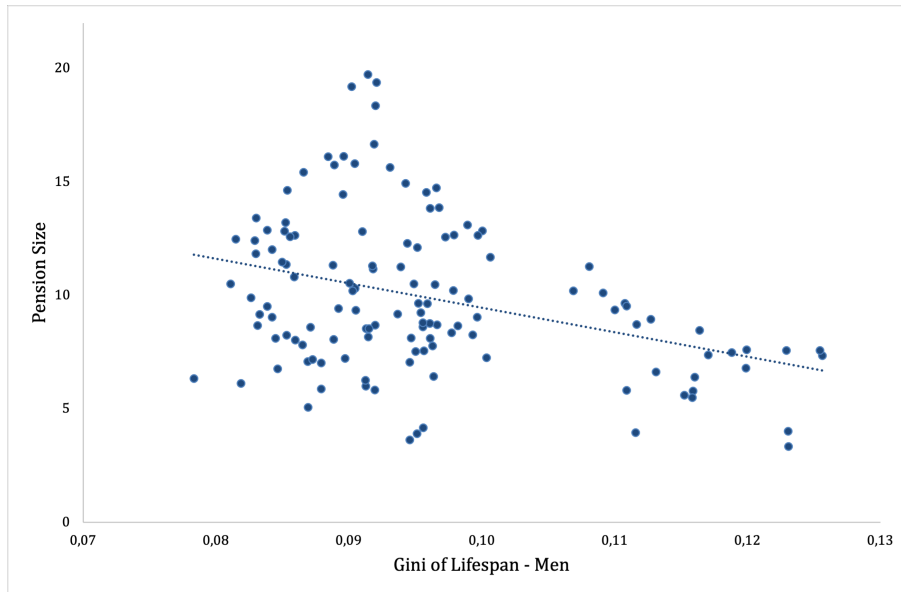


Figure 10: Gini of men lifespan vs pension's size (2015 - 2023)
 Source: Own elaboration with data from Our World in Data, and OECD Pensions at a glance reports (2015, 2017, 2019, 2021, 2023). Excluding Chile, Colombia, Costa Rica, Mexico and Turkey.

B.5 Income and Lifespan Inequality

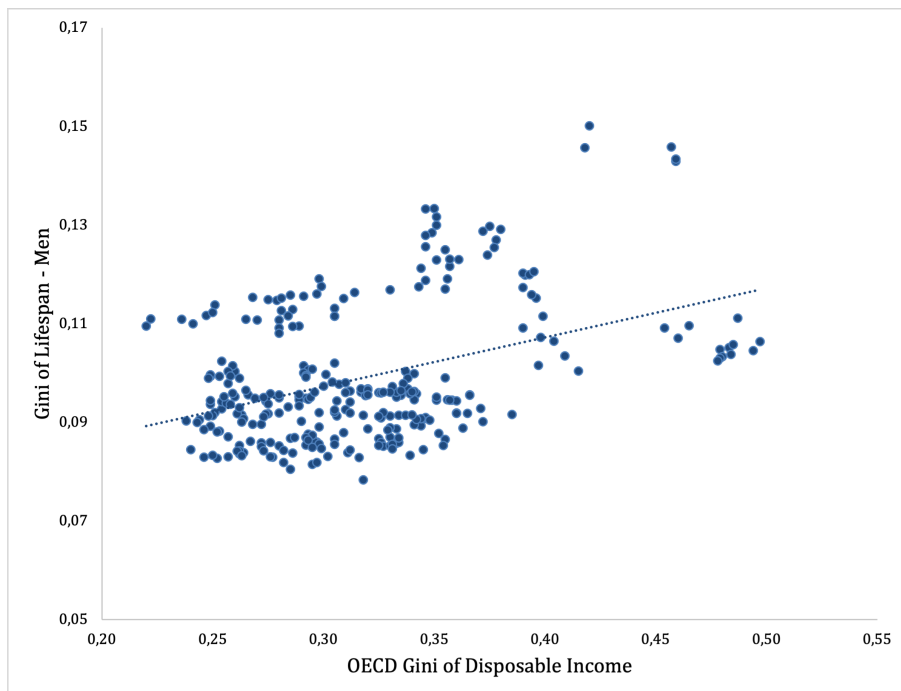


Figure 11: Gini of Disposable Income vs Gini of Men Lifespan (2015 - 2023)
 Source: Own elaboration with data from Our World in Data