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Abstract

This paper contributes to the literature interested in the new factors that may determine fertility behaviors. Many studies underlay that environmental concerns have a direct effect on households' fertility decisions. We present a dynamic model that explicitly examines this interplay, considering whether the number of children and environmental concerns may be complementary or substitutable. Interesting results occur when environmental concerns and the number of children are substitutable. At a stable steady state, a stronger effect of environmental concerns on household's preferences reduces the number of children, as also stressed by a recent literature. The dynamics can be described by an inversely U-shaped relationship between fertility and environmental indicators reflecting the impact of economic production, such as the carbon intensity, as we illustrate using data on US States. The dynamics also explain that regions with lower carbon intensity are those with lower fertility.

JEL Classification: J11, J13, Q56.

Keywords: Fertility, Environmental concerns, Quantity-quality trade-off, Transitional dynamics.

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1 Introduction

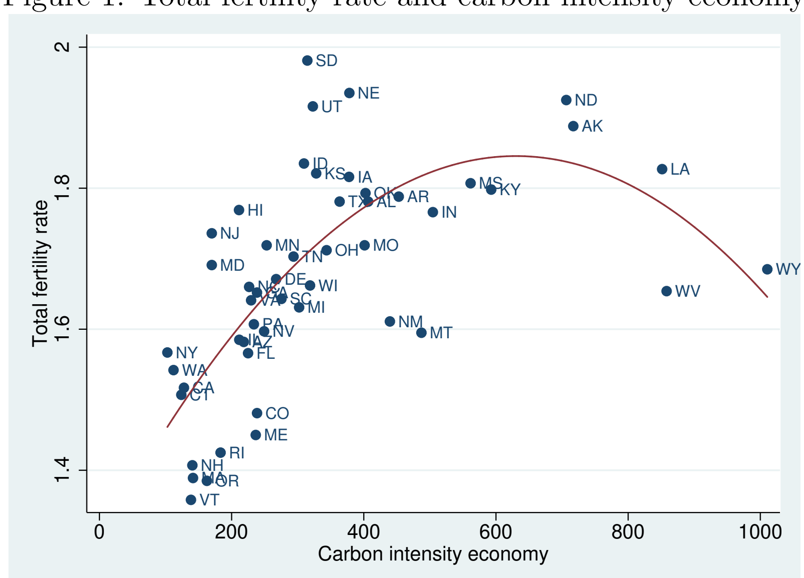
In recent years, there is growing recognition of evidence suggesting that increased socio-economic development has reshaped one of the most robust empirical findings in the economic literature: The negative association between fertility and economic development. For instance, a study by Myrskyla et al. (2009) examined data from 1975 to 2005 for countries that achieved a Human Development Index (HDI) score of at least 0.9 by 2005. Their results demonstrated that continued socio-economic development has the potential to alter the nature of the negative relationship between development and fertility, and even reverse the declining trend in fertility. More recently, Doepke et al. (2022) analyze the main reasons behind this shifting in the relationship between income and fertility. They observe that the economics of fertility has entered a new era because in high-income countries, which are in general positively correlated with countries with larger HDI, the income-fertility relationship has flattened and in some cases reversed. The authors claim that in these economies, the empirical relationship between women's labor force participation and fertility has reverted, pointing out different channels helping women to combine a career with a larger family, such as supportive family policies, cooperative parents, flexible labor markets and changing social norms.

There is no doubt that these factors underlie changes in the fertility choices of families in high-income economies. However, other non-economic factors might directly or indirectly shape the relationship between income and fertility. Among others, one important factor that deserves to be analyzed is the emergence of environmental concerns related to climate change. Schneider-Mayerson and Leong (2020) surveyed 607 US-Americans aged between 27 and 45, and found that 96.5% of respondents were at least very concerned about the negative effect of climate change on the welfare of their expected, hypothetical or existing children.

Not surprisingly, environmental concerns are increasingly present in the public debate in developed societies, prompting many individuals to adopt more sustainable practices, such as recycling behaviors, consumption of organic food, investments in green and less polluting technologies, but also to demand more sustainable public policies, and even embrace non-violent civil disobedience. Environmental concerns arise from human activities and generates consequences for both the environment and human society. Therefore, they could also potentially impact family planning decisions as well as the decisions to have children and how many. Even though individual choices are, of course, strongly influenced by socio-economic conditions, government policies and social norms, several studies suggest a potential association between increasing environmental concerns and decreasing fertility desires, particularly in younger generations.[1]

Environmental concerns are closely linked to both the production processes and technological characteristics of an economy. In particular, they are strongly influenced by one of the primary drivers of climate change: The carbon intensity of the economy, which measures the amount of carbon dioxide (CO2) emissions generated per unit of Gross Domestic Product (GDP). Figure 1 plots the relationship between total fertility rate and carbon intensity of the economy in the US in 2020.[2] The figure is particularly interesting because it highlights the existence of a non-monotonic relationship between total fertility rate and the level of carbon intensity at the US State level.

Figure 1: Total fertility rate and carbon intensity economy



We can especially see that low carbon intensity economies are associated with low total fertility rate. One idea behind this empirical fact is

[1]See Section 2 for a literature review on this topic.

[2]Please, see Appendix B for data source description. Note that the inverted U-shaped relationship between carbon intensity and total fertility rate is the same for years just before Covid.

that in high-income US States, citizens tend to be more environmental concerned about the carbon intensity of the economy, therefore reducing total fertility rate. However, some high-income States, such as Wyoming, Alaska or North-Dakota (all included in the 20 richest US States in term of GDP per capita in the first quarter of 2020) still exhibit high fertility rates, together with high-carbon intensity of the economy. Therefore, the relationship high-income/low fertility rate alone seems not able to explain these different patterns. While various socio-economic factors play a role in explaining these divergent trends, it is plausible that expectations concerning the future consequences of climate change may influence fertility behaviors.

In this paper, we look for a theoretical mechanism that can explain the direct impact that environmental concerns on climate change can have in shaping fertility decisions in high-income economies. Indeed, a standard economic model based on the quantity-quality trade-off between income and fertility isn't appropriate to explain the non-monotonic relationship between total fertility rate and the level of carbon intensity observed in Figure 1. Therefore, we enrich such type of model by introducing non-separable preferences between the number of children and environmental concerns. Then, the quantity-quality trade-off will directly depend on an index of environmental quality, which allows us to explain the different facts we mention earlier, such as the negative effect of environmental concerns on fertility or the emergence of a non-monotonic relationship between fertility and environmental indicators reflecting the environmental impact of production.

We consider an overlapping generations model with paternalistic altruism where two sectors produce both green and polluting goods, with the latter also serving as the investment good. Adults utility depends on the bequest given to each of their children, on the number of children they will have, and on environmental concerns. A novel aspect of our approach is the inclusion of fertility and environmental concerns, measured by an environmental quality index, in a non-separable (CES) manner within the utility function. This integration allows for an interplay between the decision to have children and environmental quality. The index of environmental quality we employ is the ratio of the production of green goods to polluting ones. It is, of course, inversely related to carbon intensity, which is the environmental index we discussed in the empirical evidence above. Furthermore, it increases with capital over labor because the production of the polluting good is also used for the capital investment and it is in accordance with US data.

In our model, when environmental quality has no impact on household utility, the quantity-quality trade-off does not play a significant role in generating a negative relationship between fertility and capital over labor. However, if the environmental index and the number of children are substitutable, we can exhibit an inversely U-shaped relationship between fertility and capital over labor. As the latter ratio increases, the environmental index rises as well, pushing down the marginal utility of having children, which requires a lower marginal utility of bequest, i.e. a higher quality of children. Environmental concerns can generate a negative relationship between fertility and capital over labor or the environmental index at some levels of economic development, because it reinforces the effect of child quality. The analysis of steady states allows us to show that at a stable one, an increase of the degree of environmental concerns induces a lower number of children, as found in the inter-disciplinary literature we will refer in Section 2.1. Indeed, when the effect of environmental concerns increases, the same level of utility can be reached with a lower number of children. Exploiting the inversely U-shaped relationship between fertility and capital over labor, which occurs when fertility and environmental concerns are substitutes, the analysis of dynamics allows us to explain that regions with lower carbon intensity are those with lower fertility. We will illustrate this using recent data on US States (see Section 2.2). These dynamics are also in accordance with the non-linear relationship between fertility and carbon intensity presented in Figure 1.

The theoretical literature exploring the link between environmental concerns and fertility decisions within the field of economics is quite limited and focuses on different questions than ours. Among others, in Casey et al. (2019), climate change has an impact on the cost and benefit of having children. The effect through the quantity-quality trade-off depends on the latitude of the country. For some countries, climate change promotes unskilled labor in the agricultural sector and therefore fertility rather than education. De la Croix and Grosseries (2012) and more recently Gerlagh et al. (2022) study the effect of carbon price when fertility is endogenous. An increase of the carbon price reduces the quality of children and therefore pushes up the number of children. This creates a pervasive effect on environmental quality because of a higher population size. In the same vein, Chou (2002) and Jöst and Quaas (2009) analyze optimal policies. As in Harford (1997, 1998), they show that a Pigouvian tax is not sufficient to reach the optimal allocation. Other papers study the long term relationship between pollution, income and fertility. In Constant et al. (2014), household's utility for having children and disutility for pollution allow to generate a trap above which the economy converges to a long run equilibrium explaining the polluting industrialization. Varvarigos and Zakaria (2017) exhibit a N-shaped relationship between pollution and income par capita in a model where the effect of environmental quality goes through the longevity, while Lehmijoki and Palokangas (2009) analyze how trade liberalization can generate an environmental Kuznets curve in a two-sector small open economy where decisions of having children are not directly affected by environmental quality. Marsiglio (2017) studies an endogenous growth model with fertility and environmental quality in which emerges a non-monotonic relationship between fertility and growth at the balanced growth path, but without transitional dynamics. Finally, Bosi and Desmarchelier (2013) address a different issue. Considering a monetary overlapping generations model with pollution and endogenous fertility, they show the existence of endogenous cycles when there is a strong income effect. Hence, none of these contributions study the dynamics of fertility according to the substitutability or complementarity between having children and households' environmental concerns. The contribution of this paper is therefore to fill this gap.

The paper is organized as follows. In the next section, we discuss the recent interdisciplinary literature on the role of environmental concerns on fertility decisions and empirical facts between total fertility rate and carbon intensity. In Section 3, we present the model. In Section 4, we define an equilibrium and discuss the dynamic behavior of fertility. In Section 5, we study the existence and multiplicity of steady states. In Section 6, we analyze the effect of environmental concern on long run fertility rates. Section 7 is devoted to the analysis of dynamics, with a focus on the link between fertility and our index of carbon intensity. Section 8 concludes, whereas technical details are relegated to an Appendix.

2 Recent evidence and stylized facts

First, drawing on a large inter-disciplinary literature, we highlight that fertility decisions are affected by environmental concerns and awareness. In general, a higher environmental consciousness is associated to a lower number of children. Second, using data from different US states, we demonstrate a strong correlation between total fertility rates and carbon emission. This correlation appears to be at least as satisfactory as the negative one between fertility and income per capita, as illustrated using maps of the United States.

2.1 Environmental concern on fertility decisions

As mentioned earlier, fertility decisions are impacted by various socio-economic factors. Recent research has shed light on an additional aspect that has gained attention, which is the potential link between growing environmental concerns and fertility choices. Several surveys and empirical studies conducted in developed countries have studied this subject, emphasizing how environmental concerns can influence individuals' inclinations towards family size and their decisions on the ecological consequences of having children.

Using a sample of 139 Canadian undergraduates, Arnochy et al. (2012) have investigated the potential relationship between different types of environmental concerns and fertility intentions. They found that respondents concerned about natural environment and the consequences of pollution on mental and physical health are less inclined to have children or desire to have a smaller number of children throughout their lives. Davis et al. (2019) corroborate these results, analyzing the interaction between environmental concerns, pro-environmental behavior, and reproductive attitudes among 200 Canadian university students. Helm et al. (2021) performed 24 semi-structured interviews in New Zealand and the USA, and found similar results. Focusing on a sample of 607 US-Americans aged less than 45 years-old, Schneider-Mayerson and Leong (2020) conclude that climate change concerns are likely to impact the fertility decisions of environmentally aware young people. Schneider-Mayerson (2022) confirms this result and further identifies several dimensions of the connection between reproductive choices and environmental politics in the age of climate change.

Using birth data rather than survey data on attitudes towards children, Lockwood et al. (2022) found that people who are strong environmentalists are less likely to have children in the future.[3] Rackin et al. (2023) focus on the environmental attitudes and fertility desires among 34104 US adolescents from 2005-2019. They found that environmental issues could be linked to a reduced desire to have children, particularly among young individuals. All these empirical studies clearly indicate that, in order to understand recent fertility behaviors, in particular in young generations, the new era of fertility studies must account for environmental concerns.

[3]The authors use the Understanding Society UKHLS (the annual United Kingdom Household Longitudinal Survey), a random sample of size approximately 10000 of the UK population. Their study concentrates on a sample of 6000 UK people with and without children and on childless people in 2012, approximately 2300 people.

2.2 Relationship between fertility, environmental concerns and carbon intensity

As discussed in the Introduction, environmental concerns are shaped by the economic production and the technological characteristics of the economy. Figure 2 analyses the relationship between carbon intensity at the State level in the US, defined as the amount of carbon dioxide (CO2) emissions produced per unit of GDP and two measures of environmental concerns for environmental policy in 2014: The percentage of Americans thinking that stricter environmental laws and regulations are worth the cost *versus* considering they cost too many jobs and hurt the economy.[4]

Figure 2: Environmental concern and carbon intensity in the US

Figure 2 clearly shows a strong correlation between the carbon intensity of the economy and environmental concerns about environmental policy at the State level in the US. More precisely, the negative (resp. positive) correlation between carbon intensity and the percentage of individuals thinking that stricter environmental laws and regulations are worth the cost (resp.

[4]See Appendix B for data source description.

stricter environmental laws and regulations cost too many jobs and hurt the economy) is equal to -0.8239 (resp. 0.8004), with significance level at 1%.

As it is well recognized in the economic literature, economic factors play a major role in shaping fertility decisions. Figure 3 maps the quintiles of the total fertility rate and income per capita in 2020 in the US. As expected, the maps show a standard negative relationship between income and fertility. However, as discussed in the Introduction, environmental concerns about climate change and its determinants, such as the level of carbon intensity of the economy, can also significantly shape fertility decisions.

Figure 3: Total fertility rate and income per capita in the US

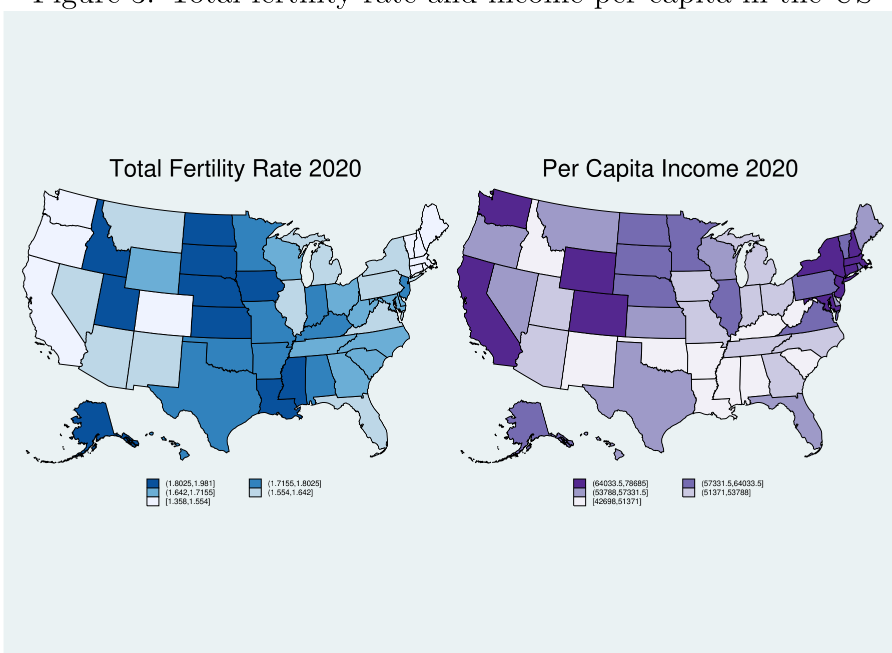
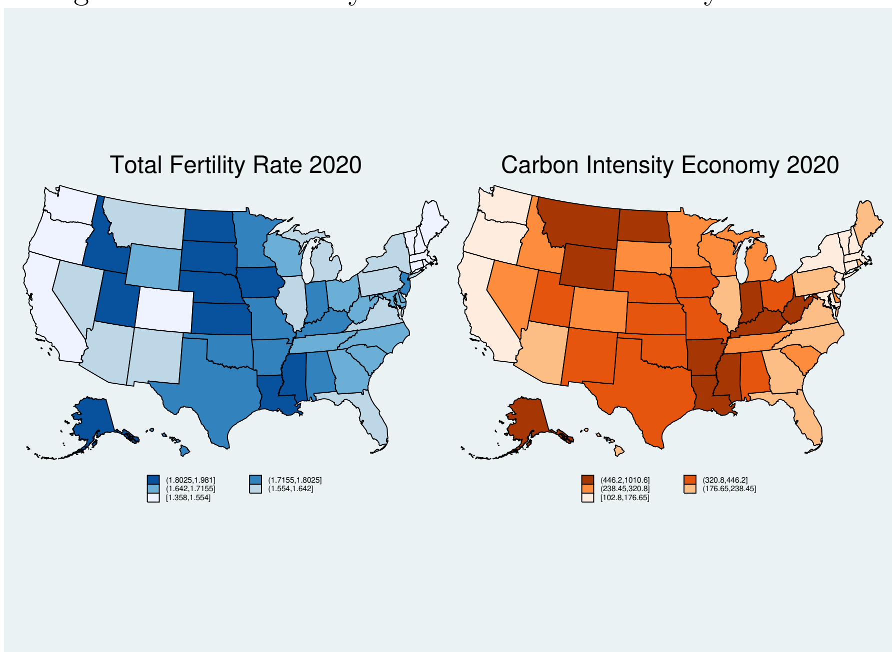


Figure 4 maps the quintiles of the total fertility rate and carbon emissions produced per unit of GDP in 2020 in the US. The maps clearly show a positive correlation between the total fertility rate and the intensity of carbon emissions in the economy. In particular, the less the carbon intensity of the economy, the lower the total fertility rate. This pattern is particularly evident in the States on the West coast, but also in some States on the East coast and the South-East of the US.

Figure 4: Total fertility rate and carbon intensity in the US



Of course, the relationship between fertility rate and carbon intensity in the economy is influenced by various factors, including economic aspects, government policies, and technological advancements. However, we can note by examining both Figures 3 and 4 that in certain US states, the total fertility rate appears to be positively correlated with carbon intensity, while it seems to be negatively correlated with income per capita. Even though the latter two variables are negatively correlated, this observation highlights that certain indirect factors related to the environmental characteristics of the economy might have an essential role in explaining fertility behaviors across US States. Thus, the relationship between income, fertility rate, and the factors shaping the environmental quality of a society deserves to be analyzed in detail.

To confirm the importance of the carbon intensity characteristic of the economy for our analysis, we look at the pairwise correlation between total fertility rate (TFR), carbon intensity and income in US States. Table 1 displays the correlation coefficients between the three variables. Coefficients are strongly significative at 1% level. Moreover, the magnitude of the Pearson correlation coefficient between total fertility rate and carbon intensity economy is positive and strong (0.518), while it is negative and more moderate for both total fertility rate and per capita income (-0.413), and carbon intensity economy and per capita income (-0.424).

Table 1: Cross-correlation table

Variables	TFR	carbon intensity	per capita income
TFR	1.000		
carbon intensity	0.518^{***}	1.000	
per capita income	-0.413^{***}	-0.424^{***}	1.000

When examining the residuals of a simple linear regression between total fertility rate and carbon intensity of the economy (resp. per capita income), we observe that for a total variation of 1.152 in the total fertility rate, 0.310 (resp. 0.196) is explained by the regression, while the residuals are given by 0.843 (resp. 0.956). Moreover, the R-squared is equal to 0.269 (resp. 0.170). These statistics clearly indicate the crucial role the carbon intensity of an economy might have for our research question. Considering a model with both regressors, i.e. carbon intensity economy and per capita income, to explain total fertility rate would perform better. Indeed, looking at the residuals, we find that 0.362 of the variation of 1.152 of the total fertility rate is explained by the regression, while 0.790 by residuals. Considering the environmental characteristics of an economy, such as its carbon intensity, might help to explain the recent fertility pattern we observe in the data across US States through the channel of environmental concerns for climate change.

Based on these evidence and facts, the main aim of this paper is to provide a new framework to examine the connexion between environmental concerns and fertility decisions. We thus provide a theoretical mechanism explaining the relationship between carbon intensity of an economy and fertility. The model we develop is based on the idea that fertility and an index of environmental quality, which is negatively correlated to carbon intensity, can be substitutes or complements. In particular, in the first case, a higher environmental quality (a lower carbon intensity) may imply less children, as illustrated in Figure 4.

3 The model

Time is discrete ($t = 0, 1, ...$) and there are two types of agents: Firms and altruistic consumers. We start by presenting the production sector.

3.1 Production

There are two sectors. The green one $i = g$ produces a consumption good, the polluting one $i = p$ produces a consumption and capital good. The two technologies are similar except that they differ by the global productivity of factors.

$$Y_{pt} = A_p K_{pt}^a L_{pt}^{1-a} \quad (1)$$
$$Y_{gt} = A_g K_{gt}^a L_{gt}^{1-a} \quad (2)$$

with $A_p \neq A_g > 0$ and $a \in (0, 1)$. In the following, $k_{it} = K_{it}/L_{it}$ denotes the capital-labor ratio of the sector i.

Let r_t be the interest rate, w_t the wage, p_t the price of the green good, while the polluting good is the numéraire. Profit maximisation in the two sectors give:

$$r_t = aA_p k_{pt}^{a-1} = p_t a A_g k_{gt}^{a-1} \quad (3)$$
$$w_t = (1-a)A_p k_{pt}^a = p_t(1-a)A_g k_{gt}^a \quad (4)$$

Using (3) and (4), we can compute r_t/w_t to find $k_{pt} = k_{gt} \equiv k_t$. This also implies that:

$$p_t = A_p/A_g \equiv p \quad (5)$$

3.2 Households with paternalistic altruism

The economy is populated by individuals whose finite lifespan is divided up into two periods: Youth (inactive period) and adult age (working period). We follow Melindi-Ghidi and Seegmuller (2019) considering that households have preferences for the number of children, with a weight $\epsilon > 0$, and altruism, with a weight $\gamma > 0$. Paternalistic altruism means that households take care about the amount of bequest, capital in our framework, they leave to their children. The size of the generation of adults born in $t-1$ is N_t, growing at an endogenous factor $n_t \in (0, +\infty)$. Therefore, the population size evolves according to $N_{t+1} = n_t N_t$.

When adult, an agent born at time $t-1$ derives utility from consumptions of green c_{gt} and polluting c_{pt} goods, having n_t children and bequest per child through capital holding κ_{t+1}. To ensure that decisions of having children are affected by environmental quality, the preferences of having children depend on an environmental index E_t in a non-separable way. The utility function is given by:

$$\alpha \ln c_{pt} + (1-\alpha) \ln c_{gt} + \epsilon \ln \left[\beta_1 n_t^{\frac{\sigma-1}{\sigma}} + \beta_2 E_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + \gamma \ln \kappa_{t+1} \tag{6}$$

with $\alpha \in (0,1)$, $\beta_1 > 0$, $\beta_2 > 0$, $\sigma > 0$, $\sigma \neq 1$. In particular, the parameter β_2 measures the importance of environmental concerns of households. The parameter σ is the elasticity of substitution between fertility and environmental concerns. Fertility and environmental concerns are substitutes when $\sigma > 1$, whereas they are complements when $\sigma < 1$.

Each young individual induces a rearing time cost $b > 0$ to her parents. Each household supplies $1 - bn_t$ units of labor to firms, earning the competitive wage rate w_t. Moreover, she receives income from capital bequests κ_t coming from her parents remunerated by firms at the rate r_t. This income is shared between family consumptions and capital bequests to children. Therefore, the budget constraint of an adult born in $t-1$ writes as follows:

$$c_{pt} + p_t c_{gt} + n_t(\kappa_{t+1} + bw_t) = R_t \kappa_t + w_t \tag{7}$$

with $R_t \equiv 1-\delta+r_t$ the gross return of capital and $\delta \in (0,1)$ the depreciation rate of capital.

Introducing the aggregate consumption $C_t = c_{pt}^{\alpha} c_{gt}^{1-\alpha}$ which is associated to the aggregate price $P_t = p_t^{1-\alpha}/[\alpha^{\alpha}(1-\alpha)^{1-\alpha}]$, an household maximizes:

$$\ln C_t + \epsilon \ln \left[\beta_1 n_t^{\frac{\sigma-1}{\sigma}} + \beta_2 E_t^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + \gamma \ln \kappa_{t+1} \tag{8}$$

under the budget constraint:

$$P_t C_t + n_t \kappa_{t+1} = R_t \kappa_t + w_t(1 - bn_t) \tag{9}$$

The sharing between the consumptions of green and polluting goods is done in a second step.

One get (see also Appendix A.1):

$$\frac{n_t}{P_t C_t} = \frac{\gamma}{\kappa_t} \tag{10}$$

$$\frac{\kappa_{t+1} + bw_t}{\kappa_{t+1}}\gamma = \frac{\epsilon\beta_1 n_t^{\frac{\sigma-1}{\sigma}}}{\beta_1 n_t^{\frac{\sigma-1}{\sigma}} + \beta_2 E_t^{\frac{\sigma-1}{\sigma}}} \tag{11}$$

$$n_t \kappa_{t+1} = \frac{\gamma}{1+\gamma}[R_t\kappa_t + w_t(1-bn_t)] \tag{12}$$

$$P_t C_t = \frac{1}{1+\gamma}[R_t\kappa_t + w_t(1-bn_t)] \tag{13}$$

$$c_{pt} = \alpha P_t C_t \tag{14}$$

$$p_t c_{gt} = (1-\alpha)P_t C_t \tag{15}$$

To ensure that this solution is an optimum for the household's program, we assume:

Assumption 1 *Either $\sigma \leq 1$ or $\epsilon \geq \gamma$.*

Using this assumption, we show the following lemma:

Lemma 1 *Under Assumption 1, the second order conditions of the household's program are satisfied.*

Proof. See Appendix A.1. ■

Equation (12) can be seen as an expression of the quantity-quality trade-off of having children. Indeed, the level of bequest per child κ_{t+1} represents the quality of children, which implies that:

$$n_t = \frac{\gamma}{1+\gamma}\frac{R_t\kappa_t + w_t(1-bn_t)}{\kappa_{t+1}} \tag{16}$$

Using (11), we also have κ_{t+1}:

$$\kappa_{t+1} = \frac{1 + (\beta_2/\beta_1)(E_t/n_t)^{\frac{\sigma-1}{\sigma}}}{(\epsilon-\gamma)/\gamma - (\beta_2/\beta_1)(E_t/n_t)^{\frac{\sigma-1}{\sigma}}} bw_t \tag{17}$$

When environmental concerns and fertility are substitutes ($\sigma > 1$), a higher quality κ_{t+1} is associated to a lower number of children. In contrast, when environmental concerns and fertility are complements ($\sigma < 1$), a higher quality κ_{t+1} is associated to a higher number of children. Indeed, by inspection of equation (11), when κ_{t+1} increases, the dominant effect goes through a lower marginal utility of bequest, with respect to a higher cost of having children. This requires a decrease of the cost of bequest, which is equal to the number of children, times the marginal utility of having children. When $\sigma > 1$, the first effect dominates explaining that n_t decreases, whereas when $\sigma < 1$, the second effect dominates, which implies a higher level of n_t.

However, the most important is to note, using (11), that when $\sigma > 1$, a higher level of E_t pushes down the marginal utility of having children, which requires a lower marginal utility of bequest, i.e. a higher quality κ_{t+1}. When $\sigma < 1$, a higher level of E_t pushes up the marginal utility of having children, which implies a lower level of the quality of children κ_{t+1}.

Substituting (17) into (16), we highlight the final effect of environmental concern on the decision of having children:

$$n_t = \frac{\gamma}{1+\gamma} \frac{R_t \kappa_t + w_t(1 - bn_t)}{bw_t} \frac{(\epsilon - \gamma)/\gamma - (\beta_2/\beta_1)(E_t/n_t)^{\frac{\sigma-1}{\sigma}}}{1 + (\beta_2/\beta_1)(E_t/n_t)^{\frac{\sigma-1}{\sigma}}} \tag{18}$$

Without environmental concerns, $\beta_2 = 0$, n_t increases with the income associated to bequest over the wage. When β_2 is positive, environmental concerns play a role and adds an effect on fertility decision. First, we observe that n_t decreases with β_2, i.e. with the level of concern. Second, we exploit the effects that go through κ_{t+1}. If $\sigma < 1$, the right-hand side of equation (18) is increasing in E_t and decreasing in n_t. This implies that E_t has a positive effect on n_t. In contrast, if $\sigma > 1$, the right-hand side of equation (18) is decreasing in E_t, and decreasing in n_t if β_2 is low enough. In this case, E_t has a negative effect on n_t. This summarizes the negative effect of environmental quality on fertility.

3.3 Environmental index

As we have seen, the utility of households depends on an environmental index E_t. It corresponds to the agents perception of environmental quality. This environmental index linearly increases with the production of the green good over the production of the polluting one:

$$E_t = \theta \frac{Y_{gt}}{Y_{pt}} \tag{19}$$

with $\theta > 0$. Of course, the share of green products in the total production is negatively correlated to carbon emission. Therefore, regarding the stylized facts we have highlighted in Section 2, we keep in mind that the carbon intensity of the economy is inversely correlated to E_t. When E_t is high (low), it corresponds to a low (high) carbon intensity economy.

4 Equilibrium and the dynamic features of fertility

Equations (3) and (4) give:

$$r_t = aA_p k_t^{a-1} \equiv r(k_t) \tag{20}$$

$$w_t = (1-a)A_p k_t^a \equiv w(k_t) \tag{21}$$

Therefore, $R_t = 1 - \delta + r(k_t) \equiv R(k_t)$.

Equilibrium on the labor market is satisfied if $L_{pt} + L_{gt} = N_t(1 - bn_t)$. Equilibrium on the capital market requires $N_t\kappa_t = K_{pt} + K_{gt} = k_t(L_{pt} + L_{gt})$. It implies that $k_t = \kappa_t N_t/[(1 - bn_t)N_t] = \kappa_t/(1 - bn_t)$.

We can rewrite (12) as follows:

$$n_t k_{t+1}(1 - bn_{t+1}) = \frac{\gamma}{1+\gamma}[(1-\delta)k_t + A_p k_t^a](1 - bn_t) \tag{22}$$

with $0 < n_t < 1/b$.

To define the level of environmental quality at equilibrium, we note that $Y_{gt} = N_t c_{gt}$ and $Y_{pt} = N_t c_{pt} + N_{t+1}\kappa_{t+1} - (1-\delta)N_t\kappa_t$. Using (14), (15), the equilibrium condition on the capital market and (22), we obtain:

$$E_t = \theta\frac{Y_{gt}}{Y_{pt}} = \theta\frac{A_g}{A_p}\frac{(1-\alpha)[(1-\delta)k_t^{1-a} + A_p]}{(\alpha+\gamma)A_p - (1-\alpha)(1-\delta)k_t^{1-a}} \equiv E(k_t) \tag{23}$$

where $E'(k_t) > 0$, and $E(k_t) > 0$ for all $k_t < \overline{k}$, with:

$$\overline{k} \equiv \left[\frac{(\alpha+\gamma)A_p}{(1-\alpha)(1-\delta)}\right]^{\frac{1}{1-a}} \tag{24}$$

At the equilibrium, the environmental index is increasing with capital over labor. Indeed, a share of the production of polluting good is used for investment, which reduces with the existing capital stock. This implies that the production of green good, which is not an investment good, over the production of polluting good raises with capital over labor.

Having $E(k_t)$ as an increasing function corresponds to the idea that in countries/States/locations with higher incomes, the production of green over polluting goods is higher, meaning that carbon intensity is lower. This empirical evidence is especially verified for US States. In other words, in our economy, a higher ratio of capital over labor k_t means a higher share of green production in the economy and a lower carbon intensity.

Using (21), (23) and the equilibrium on the capital market, equation (11) rewrites:

$$(1 - bn_{t+1})k_{t+1} = \frac{b\gamma(1-a)A_p k_t^a}{\epsilon\beta_1 n_t^{\frac{\sigma-1}{\sigma}} \left[\beta_1 n_t^{\frac{\sigma-1}{\sigma}} + \beta_2 E(k_t)^{\frac{\sigma-1}{\sigma}}\right]^{-1} - \gamma} \tag{25}$$

Substituting this equation in (22), we get:

$$F(n_t, k_t) = G(n_t, k_t) \tag{26}$$

with:

$$F(n_t, k_t) \equiv \frac{(\epsilon-\gamma)\beta_1 - \gamma\beta_2 \left(E(k_t)/n_t\right)^{\frac{\sigma-1}{\sigma}}}{\beta_1 + \beta_2 \left(E(k_t)/n_t\right)^{\frac{\sigma-1}{\sigma}}} \tag{27}$$

$$G(n_t, k_t) \equiv \frac{(1-a)A_p(1+\gamma)}{(1-\delta)k_t^{1-a} + A_p} \frac{bn_t}{1-bn_t} \tag{28}$$

An intertemporal equilibrium is a sequence (n_t, k_t) satisfying (22) and (26)-(28), with $E(k_t)$ given by (23), $0 < n_t < 1/b$, and $0 < k_t < \overline{k}$.

While (22) is a recursive equation, $F(n_t, k_t) = G(n_t, k_t)$ defines n_t as a function of k_t. Studying this equation, we deduce the relationship between fertility and capital over labor.

Lemma 2 *Under Assumption 1, the following holds. There is $\sigma_{nt} > 1$ such that for all $\sigma \neq \sigma_{nt}$, there exists a function $n_t = n_d(k_t)$ solving $F(n_t, k_t) = G(n_t, k_t)$. In addition, there exists $\sigma_{mt} \in (1, \sigma_{nt})$ such that $n_d'(k_t) > 0$ for $\sigma < \sigma_{mt}$, $n_d'(k_t) < 0$ for $\sigma_{mt} < \sigma < \sigma_{nt}$ and $n_d'(k_t) > 0$ for $\sigma > \sigma_{nt}$.*

Proof. See Appendix A.2. ■

We see that without environmental concerns ($\beta_2 = 0$), n_t increases with k_t. The quantity-quality trade-off is not sufficiently important to reverse the positive income effect on fertility. Income increases more than new bequest, which linearly depends on the wage (see equation (17)).[5] With environmental concerns ($\beta_2 > 0$), this lemma shows that if $\sigma < 1$, there is still a positive link between n_t and k_t. As we have seen previously, E_t positively affects the marginal utility of having children. This implies a lower level of bequest κ_{t+1}, which boosts the income effect on fertility. The environmental index, which increases with capital over labor, is complementary to the number of children, which implies that there is a positive link between these both. Therefore, the environmental concerns channel reinforces the positive link between the number of children and capital over labor.

When σ is higher than 1, the link between n_t and k_t is less clear-cut along a dynamic path. It depends on how σ_{mt} and σ_{nt} evolve with k_t.

Lemma 3 *Under Assumption 1, the following holds. If* $\sigma_{mt} < \sigma < \sigma_{nt}$ *(*$n'_d(k_t) < 0$*), both* σ_{mt} *and* σ_{nt} *are decreasing in* k_t*. If* $1 < \sigma < \sigma_{mt}$ *(*$n'_d(k_t) > 0$*),* σ_{mt} *is decreasing in* k_t*. If* $\sigma > \sigma_{nt}$ *(*$n'_d(k_t) > 0$*),* σ_{nt} *is increasing in* k_t*. Moreover, there exists* $\overline{\sigma} > 1$ *such that* $\sigma_{nt} > \overline{\sigma}$ *for all equilibrium value of* k_t*.*

Proof. See Appendix A.3. ■

To ensure that $n_d(k_t)$ is a function (and not a correspondence), we need to assume that σ never crosses σ_{nt}. Let us assume:

Assumption 2 $\sigma < \overline{\sigma}$.

The number of children $n_d(k_t)$ is increasing for $\sigma < \sigma_{mt}$ and decreasing for $\sigma_{mt} < \sigma < \overline{\sigma}$. In addition, following an increasing dynamic path of k_t, σ_{mt} can cross σ. In such a case, $n_d(k_t)$ is an inversely U-shaped function with respect to k_t, as it is illustrated in Figure 5. For $\sigma_{mt} < \sigma < \overline{\sigma}$, the substitution effect between fertility and environmental concerns becomes the dominant effect. As we have already seen, E_t negatively affects the marginal utility of having children. This implies a higher level of bequest κ_{t+1}, which dampens the income effect on fertility. Following an increase of k_t, the environmental index increases. Since n_t and E_t are substitutes, households choose to have a lower number of children.

[5] Such a result can change if, for instance, the household receives a constant endowment as an adding income.

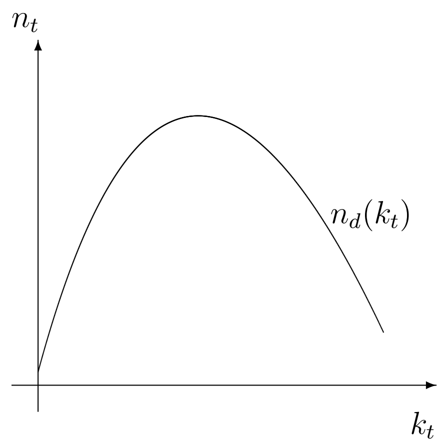


Figure 5: $n_d(k_t)$ inversely U-shaped

5 Steady states

We start by studying the existence of an autarkical steady state, such that $k = 0$. By inspection of equations (22) and (26)-(28), we observe that $k = 0$ and $n = n_a$ is a steady state, if $n_a \in [0, 1/b)$ solves $F(n_a, 0) = G(n_a, 0)$, which is equivalent to:

$$\frac{(\epsilon - \gamma)\beta_1 - \gamma\beta_2 \left(E(0)/n_a\right)^{\frac{\sigma-1}{\sigma}}}{\beta_1 + \beta_2 \left(E(0)/n_a\right)^{\frac{\sigma-1}{\sigma}}} = (1-a)(1+\gamma)\frac{bn_a}{1-bn_a} \tag{29}$$

with $E(0) = \theta A_g(1-\alpha)/[A_p(\alpha+\gamma)]$.

Proposition 1 *Under Assumptions 1 and 2, the following holds:*

(i) *For $\sigma < 1$, there exists a unique steady state $(k, n) = (0, n_a)$ if and only if $\epsilon > \gamma$;*

(ii) *For $\sigma > 1$, there is no steady state with $k = 0$.*

Proof. See Appendix A.4. ■

A steady state with a strictly positive level of capital is a solution $(n, k) \in \mathbb{R}^2_{++}$ satisfying:

$$\begin{aligned} F(n,k) &= G(n,k) \quad (30) \\ n &= \frac{\gamma}{1+\gamma}[1-\delta+A_p k^{a-1}] \equiv n(k) \quad (31) \end{aligned}$$

with $n'(k) < 0$. Since we need to have $n(k) < 1/b$, we impose:

$$k > \left[\frac{A_p \gamma b}{1+\gamma-\gamma(1-\delta)b}\right]^{\frac{1}{1-a}} \equiv \underline{k} \quad (32)$$

We further note that $\underline{k} < \overline{k}$ if and only if $(1-\delta)b < (\alpha+\gamma)/\gamma$, which is satisfied for b and/or γ low enough. Therefore, substituting $n(k)$ in (30), we deduce that a steady state is a solution $k \in (\underline{k}, \overline{k})$ satisfying:

$$\widetilde{F}(k) \equiv F(n(k), k) = G(n(k), k) \equiv \widetilde{G}(k) \quad (33)$$

Using this equation, we show that:

Proposition 2 *Under Assumptions 1 and 2, and* $(1-\delta)b < (\alpha+\gamma)/\gamma$*, the following holds:*

(*i*) *For* $\sigma < 1$*, there exists* $\epsilon_0 > 0$ *such that there is a unique steady state* $k_0 \in (\underline{k}, \overline{k})$ *if and only if* $\epsilon > \epsilon_0$*;*

(*ii*) *For* $\sigma > 1$*, there exists* $\epsilon_1 > 0$ *such that there are (at least) two steady states* k_1 *and* k_2*, such that* $\underline{k} < k_1 < k_2 < \overline{k}$*, if* $\epsilon > \epsilon_1$*.*

Proof. See Appendix A.5. ■

This proposition shows that when households have environmental concerns, there exists a unique steady state with $k > 0$ in case of complementarity, whereas there is multiplicity under substitutability between the number of children and the environmental index. When there is no environmental concern, i.e. $\beta_2 = 0$, $\widetilde{F}(k) = \epsilon - \gamma$ is constant. Since $\widetilde{G}(k)$ is a decreasing function, there is at most one steady state with positive capital k_{NE}. It requires $\epsilon > \epsilon_0$, as in case (*i*) of the proposition.

Because of the existence of a quantity-quality trade-off at stationary equilibria, $n(k)$ is strictly decreasing (see equation (31)). Therefore, when there is a multiplicity of steady states, the levels of fertility rates are clearly ranked. We have $n_1 > n_2$ with $n_1 = n(k_1)$ and $n_2 = n(k_2)$. It also allows us to note that a steady state can be seen as a solution solving $n = n_d(k) = n(k)$. Besides, we can characterize the different steady states according to the slopes of the functions $n_d(k)$ and $n(k)$:

Lemma 4 *Under Assumptions 1 and 2, and* $(1-\delta)b < (\alpha+\gamma)/\gamma$, *we have:*

1. $n'_d(k_0) > n'(k_0)$;
2. $n'_d(k_1) > n'(k_1)$;
3. $n'_d(k_2) < n'(k_2) < 0$.

Proof. See Appendix A.6. ■

Now, we can better understand the mechanism explaining the multiplicity of steady states. Since $n = n(k)$ is a decreasing function, there is a multiplicity of steady states if $n = n_d(k)$ is decreasing at least for some values of k. This explains that there are two steady states when there are environmental concerns and $\sigma > 1$. In contrast, when $\sigma < 1$, we have seen that $n = n_d(k)$ is increasing because of the complementarity between the number of children and the index of environmental quality. This ensures the uniqueness of the steady state.

6 The role of environmental concerns

When β_2 is strictly positive, the environmental index affects the choices of consumers. The question we ask now is how the different steady states evolve according to a modification of the parameter β_2 measuring environmental concerns:

Proposition 3 *Under Assumptions 1 and 2, and* $(1-\delta)b < (\alpha+\gamma)/\gamma$, *the following holds:*

(i) For $\sigma < 1$ *and* $\epsilon > \epsilon_0$, k_0 *increases with* β_2;

(ii) For $\sigma > 1$ *and* $\epsilon > \epsilon_1$, k_1 *increases with* β_2, *while* k_2 *decreases with* β_2.

Following a variation of β_2, $n_i = n(k_i)$ moves in the opposite direction than k_i, for $i = 0, 1, 2$, i.e. n_0 and n_1 decreases with β_2, while n_2 increases with β_2.

Proof. See Appendix A.7. ■

By inspection of (27) and (28), we observe that for each level of k, $n = n^d(k)$ decreases with the parameter measuring environmental concerns β_2, whereas $n = n(k)$ does not move (see (31)). Using Lemma 4, we easily deduce the results of Proposition 3.

The comparative statics at the steady states (k_0, n_0) and (k_1, n_1) are of special interest, because as we will see later, these steady states will be stable. In addition, these steady states are characterized buy a lower number of children when environmental concerns are more important. This clearly corresponds to what emerge from the interdisciplinary literature summarized in Section 2.1. This happens because, when the effect of environmental concerns increases, the same level of utility can be reached with a lower number of children. Therefore, higher environmental concerns create an incentive for households to have less children. Using the trade-off between giving bequest and having children (11), we observe that a higher degree of environmental concerns decreases the marginal utility of having children. Therefore, the marginal utility for bequest has to decrease, implying higher bequest. Using (16), this implies a lower number of children.

7 Dynamics under environmental concerns

The dynamics are driven by:

$$k_{t+1}(1 - bn_{t+1}) = \frac{\gamma}{1+\gamma}[(1-\delta)k_t + A_p k_t^a]\left(\frac{1}{n_t} - b\right) \tag{34}$$

where $n_t \equiv n_d(k_t)$ is implicitly defined by $F(n_t, k_t) = G(n_t, k_t)$.

As a starting point, we study the dynamics without environmental concerns, $\beta_2 = 0$. In this case, $F(n_t, k_t) = \epsilon - \gamma$ and equation (26) implies:

$$n_t = \frac{1}{b}\frac{(\epsilon-\gamma)[(1-\delta)k_t^{1-a} + A_p]}{(\epsilon-\gamma)[(1-\delta)k_t^{1-a} + A_p] + (1-a)A_p(1+\gamma)} \tag{35}$$

Substituting this expression in (34), we obtain:

$$\Psi_1(k_{t+1}) = \Psi_2(k_t) \tag{36}$$

with

$$\Psi_1(k_{t+1}) \equiv \frac{k_{t+1}}{(1-a)A_p(1+\gamma)+(\epsilon-\gamma)[(1-\delta)k_{t+1}^{1-a}+A_p]} \tag{37}$$

$$\Psi_2(k_t) \equiv \frac{\gamma b}{(1+\gamma)(\epsilon-\gamma)}k_t^a \tag{38}$$

We note that $\Psi_2(k_t)$ is strictly increasing and concave, with $\Psi_2(0)=0$ and $\Psi_2'(0)=+\infty$. Moreover, we have:

$$\Psi_1'(k_{t+1}) = \frac{\Psi_1(k_{t+1})}{k_{t+1}} \frac{A_p[(1-a)(1+\gamma)+\epsilon-\gamma]+(\epsilon-\gamma)(1-\delta)ak_{t+1}^{1-a}}{A_p[(1-a)(1+\gamma)+\epsilon-\gamma]+(\epsilon-\gamma)(1-\delta)k_{t+1}^{1-a}} > 0 \tag{39}$$

Since $\Psi_1'(k_{t+1})$ is the product of two positive and decreasing functions, it is a decreasing function, meaning that $\Psi_1(k_{t+1})$ is concave. We finally have $\Psi_1(0)=0$ and $\Psi_1'(0)<+\infty$.

We deduce that there is one steady state $k=0$ and we know that there is only one steady state with positive capital, $k_{NE}>0$. Since $\Psi_1'(0)<\Psi_2'(0)$, the steady state k_{NE} is stable because for $k_t \leqslant k_{NE}$, we have $\Psi_1(k_t) \leqslant \Psi_2(k_t) \Leftrightarrow \Psi_1(k_t) \leqslant \Psi_1(k_{t+1}) \Leftrightarrow k_t \leqslant k_{t+1}$. For $k_t \geqslant k_{NE}$, we have exactly the opposite (see also Figure 6).

We now study the dynamics with environmental concerns, i.e. $\beta_2>0$. By inspection of equation (34), we see that the dynamic path $(k_t)_{t\geqslant 0}$ will be monotonic if $n_d(k_t)$ is decreasing (which could happen when $\sigma>1$) or weakly increasing. It could be no more the case if $n_d(k_t)$ strongly increases with k_t (which could a priori happen when $\sigma<1$).

In this paper, we are not interesting in macroeconomic fluctuations. Therefore, when $\sigma<1$, we will assume that β_2 is small enough. In this case, the dynamics are similar than in the model without environmental concerns. In fact, if β_2 is low enough, the left-hand side of equation (34) is still increasing in k_{t+1} and the right-hand side in k_t. In this case, the steady state $k=0$ is unstable and the steady state $k=k_0$ is stable (see also Figure 6). This means that any dynamic paths with $k_t \in (0,\underline{k})$ monotonically converges to k_0. In addition, n_t varies in the same direction than k_t along the dynamic path.

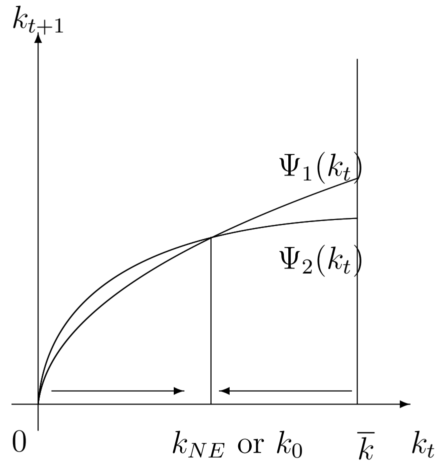


Figure 6: Dynamics when $\beta_2 = 0$ or β_2 low enough

Proposition 4 *Under Assumptions 1 and 2,* $(1-\delta)b < (\alpha+\gamma)/\gamma$ *and, either* $\beta_2 = 0$ *or* β_2 *low enough,* $\sigma < 1$ *and* $\epsilon_0 > 0$*, there is a unique steady state with positive capital,* k_{NE} *or* k_0*, which is globally stable. Any dynamic path* (k_t) *converges monotonically to this steady state. In addition, we always have* $n'_d(k_t) > 0$.

This proposition establishes a result of monotonic convergence to the steady state when $\sigma < 1$. However, k_t and n_t evolves in the same direction. The environmental index reinforces the income effect on fertility. As environmental concerns and the number of children are complementary, they are positively correlated.

We now investigate the dynamics when $\sigma > 1$. As we have seen, this case is interesting because environmental concerns promote a negative relationship between fertility and capital per unit of labor. We first study the stability of the steady state k_2. We show that:

Proposition 5 *Under Assumptions 1 and 2,* $(1-\delta)b < (\alpha+\gamma)/\gamma$ *and* $\sigma > 1$*, the steady state* k_2 *is locally unstable. Any dynamic path* (k_t) *that starts near* k_2 *monotonically diverges.*

Proof. See Appendix A.8. ■

To have a more general picture of the dynamics, we analyze if the economy can converge to the other steady state, k_1, and if this convergence can be characterized by a negative relationship between n_t and k_t.

Proposition 6 *Under Assumptions 1 and 2,* $(1-\delta)b < (\alpha+\gamma)/\gamma$ *and* $\sigma > 1$, *there exist* $\overline{\alpha} > 0$ *and* $\overline{\gamma} > 0$ *such that* $n_d'(k_t) < 0$ *for all* $k_t > \underline{k}$ *if* $\alpha < \overline{\alpha}$ *and* $\gamma < \overline{\gamma}$*. This implies that the dynamics are monotonic for all* $k_t > \underline{k}$ *and the steady state* k_1 *is locally stable.*

Proof. See Appendix A.9. ■

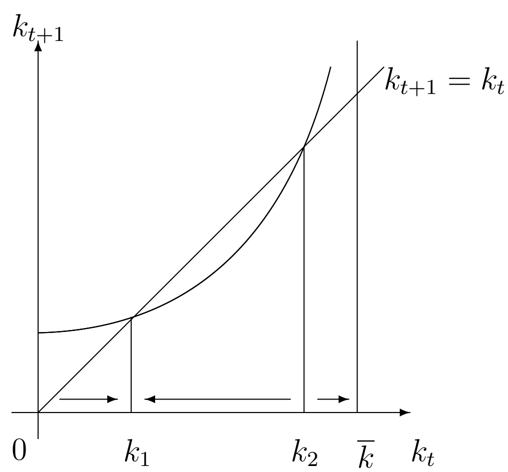


Figure 7: Dynamics when $\beta_2 > 0$ and $\sigma > 1$

Propositions 5 and 6 allow to represent the dynamics when $\beta_2 > 0$ and $\sigma > 1$ as in Figure 7. The main results that we deduce from this analysis is that for all $k_t \in (\underline{k}, k_2)$, the equilibrium converges to k_1 and n_t evolves in the opposite way than k_t.

With the results of Propositions 4-6, we can explain various configurations. Consider two growing economies with different level of capital (per unit of labor) at some date, let say k_a and $k_b (> k_a)$. If $\sigma > 1$, we explain that these economies are characterized by $n_a > n_b$. If $\sigma < 1$, we explain the opposite inequality. If $\sigma > 1$ for economy a and $\sigma < 1$ for economy b, n_a and n_b could both be high and close. On the contrary, if $\sigma < 1$ for economy a and $\sigma > 1$ for economy b, n_a and n_b could both be low and comparable.

We can also refer to the empirical evidence we have highlighted in the introduction and Section 2.2. The environmental index, which represents the share of production of green goods in the economy, increases with capital over labor. This means that carbon intensity is decreasing in k_t. Therefore, the results of Proposition 6 show that when this environmental index and the number of children are substitutable and $k_t > \underline{k}$, fertility evolves as the carbon intensity which means that a State with low carbon intensity is also characterized by a low fertility, and one with high carbon intensity by a high fertility. This is what we observed in Figure 4.

As a direct implication of Proposition 6, if k_t starts below $\underline{k}$, k_t can cross the critical value such that $n_d'(k_t) = 0$. In this case, following an increase of k_t, $n_d(k_t)$ describes an inversely U-shaped curve. This suggests that we explain an inversely U-shaped relationship between fertility and carbon intensity, as described in Figure 1 of the introduction. This is possible because fertility and environmental concerns are substitutable. Following an increase of the capital-labor ratio, the environmental index becomes high enough so that the increase of child quality has a dominant negative effect on the number of children.

8 Concluding remarks

In this paper, we present a model which explicitly introduces an interplay between environmental quality and the quantity-quality trade-off of having children. This idea is illustrated by many studies underlying that environmental concerns have a direct effect on households' fertility decisions. We formalize this idea considering altruistic households having a utility which is non-separable between the number of children and an environmental index, which is strongly correlated with indicators describing the environmental aspects of economic production.

Interesting results occur when environmental concerns and the number of children are substitutable. Fertility can be an inversely U-shaped function of capital over labor, because environmental concerns reinforce the quantity-quality trade-off. At a long run stable equilibrium, a stronger effect of environmental concerns on household's preferences reduces the number of children, as illustrated by the recent literature on this topic. Finally, the dynamics can be described by an inversely U-shaped relationship between fertility and indicators reflecting the environmental impact of economic production, such as the carbon intensity across US States. Hence, our results highlight the importance of accounting for environmental concerns to explain the recent trends in fertility in high income countries reshaping the quantity-quality trade-off.

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A Appendix

A.1 Proof of Lemma 1

The second order conditions for the household problem can be investigated considering the aggregate consumption $C_t = c_{pt}^{\alpha} c_{gt}^{1-\alpha}$ and the aggregate price $P_t = p^{1-\alpha}/[\alpha^{\alpha}(1-\alpha)^{1-\alpha}]$. This means that an household maximizes the utility (8) under the constraint (9). We can introduce the following Lagrangien:

$$L_t \equiv \ln\left[R_t\kappa_t + w_t - n_t(\kappa_{t+1} + bw_t)\right]\frac{1}{P_t} + \epsilon \ln\left[\beta_1 n_t^{\frac{\sigma-1}{\sigma}} + \beta_2 E_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} + \gamma \ln \kappa_{t+1}$$

The first order conditions are given by:

$$\frac{\partial L_t}{\partial n_t} = -\frac{\kappa_{t+1} + bw_t}{P_t C_t} + \frac{\epsilon\beta_1}{\beta_1 n_t + \beta_2 E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}}} = 0 \tag{B.1}$$

$$\frac{\partial L_t}{\kappa_{t+1}} = -\frac{n_t}{P_t C_t} + \frac{\gamma}{\kappa_{t+1}} = 0 \tag{B.2}$$

with $C_t = [R_t\kappa_t + w_t - n_t(\kappa_{t+1} + bw_t)]/P_t$. We deduce the following derivatives:

$$\frac{\partial^2 L_t}{\partial n_t^2} = -\frac{(\kappa_{t+1} + bw_t)^2}{P_t^2 C_t^2} - \frac{\epsilon\beta_1\left(\beta_1 + \frac{\beta_2}{\sigma} E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}-1}\right)}{\left[\beta_1 n_t + \beta_2 E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}}\right]^2} < 0 \tag{B.3}$$

$$\frac{\partial^2 L_t}{\kappa_{t+1}^2} = -\frac{n_t^2}{P_t^2 C_t^2} - \frac{\gamma}{\kappa_{t+1}^2} < 0 \tag{B.4}$$

$$\frac{\partial^2 L_t}{\partial\kappa_{t+1}\partial n_t} = -\frac{1}{P_t C_t} - n_t\frac{\kappa_{t+1} + bw_t}{P_t^2 C_t^2} \tag{B.5}$$

The second order conditions are fulfilled if the Hessian $H_t \equiv \frac{\partial^2 L_t}{\partial n_t^2}\frac{\partial^2 L_t}{\kappa_{t+1}^2} - (\frac{\partial^2 L_t}{\partial\kappa_{t+1}\partial n_t})^2 > 0$. Using (B.1) and (B.2), we get:

$$\begin{aligned} H_t = \; & \frac{n_t(\kappa_{t+1} + bw_t)}{P_t^4 C_t^4}\frac{1+\gamma}{\gamma}\left[n_t(\kappa_{t+1} + bw_t) + P_t C_t \frac{\beta_1 n_t + \frac{\beta_2}{\sigma} E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}}}{\beta_1 n_t + \beta_2 E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}}}\right] \\ & - \left[\frac{P_t C_t + n_t(\kappa_{t+1} + bw_t)}{P_t^2 C_t^2}\right]^2 \end{aligned}$$

We have that:

$$\frac{1+\gamma}{\gamma} n_t(\kappa_{t+1} + bw_t) > P_t C_t + n_t(\kappa_{t+1} + bw_t)$$

because using (B.2), we have $n_t \kappa_{t+1} = \gamma P_t C_t$. Moreover, for $\sigma \leqslant 1$, we have:

$$n_t(\kappa_{t+1} + bw_t) + P_t C_t \frac{\beta_1 n_t + \frac{\beta_2}{\sigma} E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}}}{\beta_1 n_t + \beta_2 E_t^{\frac{\sigma-1}{\sigma}} n_t^{\frac{1}{\sigma}}} \geqslant P_t C_t + n_t(\kappa_{t+1} + bw_t)$$

which ensures $H_t > 0$. For $\sigma > 1$, we use (B.1) to show that:

$$H_t > \frac{[n_t(\kappa_{t+1} + bw_t)]^2}{P_t^4 C_t^4} \frac{1+\gamma}{\gamma} \frac{1+\epsilon}{\epsilon} - \left[\frac{P_t C_t + n_t(\kappa_{t+1} + bw_t)}{P_t^2 C_t^2} \right]^2$$

We deduce that $H_t > 0$ for $\epsilon \geq \gamma$.

A.2 Proof of Lemma 2

Using (27) and (28), we get:

$$\frac{\partial G(n_t, k_t)}{\partial n_t} - \frac{\partial F(n_t, k_t)}{\partial n_t} = \frac{G(n_t, k_t)^2}{b n_t^2} \frac{(1-\delta)k_t^{1-a} + A_p}{(1-a)A_p(1+\gamma)}$$
$$- \frac{\epsilon \beta_1 \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}}{\left[(\epsilon-\gamma)\beta_1 - \gamma \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right]^2} \frac{\sigma-1}{\sigma} \frac{F(n_t, k_t)^2}{n_t}$$

Using equation (26), this is equivalent to:

$$\frac{\partial G(n_t, k_t)}{\partial n_t} - \frac{\partial F(n_t, k_t)}{\partial n_t} = \frac{\beta_1 + \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}}{(\epsilon-\gamma)\beta_1 - \gamma\beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}} \frac{1}{1-bn_t} \frac{G(n_t, k_t)^2}{n_t}$$
$$- \frac{\epsilon \beta_1 \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}}{\left[(\epsilon-\gamma)\beta_1 - \gamma \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right]^2} \frac{\sigma-1}{\sigma} \frac{G(n_t, k_t)^2}{n_t}$$

This is strictly positive if and only if:

$$\left[\beta_1 + \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right] \left[(\epsilon-\gamma)\beta_1 - \gamma\beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right]$$
$$> \epsilon \beta_1 \beta_2 (E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}} (1-bn_t)(\sigma-1)/\sigma \qquad \text{(B.6)}$$

which is equivalent to:

$$\frac{\sigma-1}{\sigma} < \left[(\epsilon-\gamma)\beta_1 - \gamma\beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right] \frac{\beta_1(E(k_t)/n_t)^{\frac{1-\sigma}{\sigma}} + \beta_2}{\epsilon\beta_1\beta_2(1-bn_t)} \equiv \frac{\sigma_{nt}-1}{\sigma_{nt}} \tag{B.7}$$

Since $F(k_t, n_t) > 0$, the right-hand side of this inequality is strictly positive. Then, there exists $\sigma_{nt} > 1$ such that inequality (B.7) is satisfied for all $\sigma < \sigma_{nt}$ and if the right-hand side of inequality (B.7) is higher than 1, we will say by convention that $\sigma_{nt} = +\infty$. It is obvious to see that σ_{nt} decreases with β_2 and $\sigma_{nt} = +\infty$ when β_2 tends to 0.

Now, using (27) and (28), we get:

$$\begin{aligned}\frac{\partial F(n_t,k_t)}{\partial k_t} - \frac{\partial G(n_t,k_t)}{\partial k_t} &= \frac{1-bn_t}{bn_t}\frac{(1-\delta)k_t^{-a}}{A_p(1+\gamma)}G(n_t,k_t)^2 \\ &- \frac{\epsilon\beta_1\beta_2(E(k_t)/n_t)^{-\frac{1}{\sigma}}}{\left[(\epsilon-\gamma)\beta_1 - \gamma\beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right]^2}\frac{E'(k_t)}{n_t}\frac{\sigma-1}{\sigma}F(n_t,k_t)^2\end{aligned}$$

Using (23), we compute the following derivative:

$$E'(k_t) = \theta\frac{A_g}{A_p}(1-\alpha)\frac{(1-\delta)A_p(1+\gamma)(1-a)k_t^{-a}}{[(\alpha+\gamma)A_p - (1-\alpha)(1-\delta)k_t^{1-a}]^2}$$

We deduce that:

$$\begin{aligned}\frac{\partial F(n_t,k_t)}{\partial k_t} - \frac{\partial G(n_t,k_t)}{\partial k_t} &= \frac{G(n_t,k_t)^2(1-\delta)(1-a)k_t^{-a}}{\left[(\epsilon-\gamma)\beta_1 - \gamma\beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right]^2} \\ &\frac{N_t}{[(\alpha+\gamma)A_p - (1-\alpha)(1-\delta)k_t^{1-a}][(1-\delta)k_t^{1-a} + A_p]}\end{aligned}$$

with

$$\begin{aligned}N_t = &\left[\beta_1 + \beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right]\left[(\epsilon-\gamma)\beta_1 - \gamma\beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\right][(\alpha+\gamma)A_p \\ &-(1-\alpha)(1-\delta)k_t^{1-a}] - \epsilon\beta_1\beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}\frac{\sigma-1}{\sigma}A_p(1+\gamma)\end{aligned}$$

This expression is positive if and only if:

$$\frac{\sigma-1}{\sigma} < \frac{(\alpha+\gamma)A_p - (1-\alpha)(1-\delta)k_t^{1-a}}{A_p(1+\gamma)}(1-bn_t)\frac{\sigma_{nt}-1}{\sigma_{nt}} \equiv \frac{\sigma_{mt}-1}{\sigma_{mt}} \tag{B.8}$$

Since $k_t < \overline{k}$, there exists $\sigma_{mt} \in (1, \sigma_{nt})$ such that $\frac{\partial F(n_t,k_t)}{\partial k_t} - \frac{\partial G(n_t,k_t)}{\partial k_t} > 0$ for all $\sigma < \sigma_{mt}$. If the right-hand side of inequality (B.8) is higher than 1, we will say by convention that $\sigma_{mt} = +\infty$.

Using (26), we deduce that:

$$\frac{dn_t}{dk_t} = n_d'(k_t) = \frac{\frac{\partial F(n_t,k_t)}{\partial k_t} - \frac{\partial G(n_t,k_t)}{\partial k_t}}{\frac{\partial G(n_t,k_t)}{\partial n_t} - \frac{\partial F(n_t,k_t)}{\partial n_t}}$$

is positive for all $\sigma < \sigma_{mt}$, is negative for $\sigma_{mt} < \sigma < \sigma_{nt}$ and positive for $\sigma > \sigma_{nt}$.

A.3 Proof of Lemma 3

If $\sigma_{mt} < \sigma < \sigma_{nt}$ which implies that $n_d'(k_t) < 0$, we note that $E(k_t)/n_t$ is increasing in k_t and $1/(1 - bn_t)$ is decreasing in k_t. Using (B.7), we deduce that σ_{nt} is decreasing in k_t.

Using (B.7), we substitute $(\sigma_{nt} - 1)/\sigma_{nt}$ in (B.8). We easily deduce that σ_{mt} is decreasing in k_t.

If either $1 < \sigma < \sigma_{mt}$ or $\sigma > \sigma_{nt}$, we have $n_d'(k_t) > 0$. Using (26), (27) and (28), we observe that $F(k_t, n_t)$ is decreasing in $E(k_t)/n_t$. To understand how $E(k_t)/n_t$ evolves with k_t, we need to analyse the derivative of $G(k_t, n_d(k_t))$ with respect to k_t. Using the proof of Lemma 2, we have:

$$\begin{aligned} \frac{dG(k_t, n_d(k_t))}{dk_t} &= \frac{\partial G}{\partial k_t} + \frac{\partial G}{\partial n_t} n_d'(k_t) \\ &= \frac{-\frac{\partial G}{\partial k_t}\frac{\partial F}{\partial n_t} + \frac{\partial G}{\partial n_t}\frac{\partial F}{\partial k_t}}{\frac{\partial G}{\partial n_t} - \frac{\partial F}{\partial n_t}} \end{aligned} \tag{B.9}$$

Using (28), we have:

$$\frac{\partial G/\partial k_t}{\partial G/\partial n_t} = -\frac{(1-\delta)(1-a)k_t^{-a}}{(1-\delta)k_t^{1-a} + A_p}(1 - bn_t)n_t \tag{B.10}$$

Using (23) and (27), we have:

$$\frac{\partial F/\partial k_t}{\partial F/\partial n_t} = -n_t \frac{(1-\delta)A_p(1+\gamma)(1-a)k_t^{-a}}{[(\alpha+\gamma)A_p - (1-\alpha)(1-\delta)k_t^{1-a}][(1-\delta)k_t^{1-a} + A_p]} \tag{B.11}$$

Since $\partial G/\partial n_t > 0$ and $\partial F/\partial n_t > 0$, the numerator of (B.9) is negative if and only if:

$$1 - bn_t < \frac{A_p(1+\gamma)}{(\alpha+\gamma)A_p - (1-\alpha)(1-\delta)k_t^{1-a}}$$

which is always satisfied. In this case, the sign of (B.9) is given by the opposite sign of the denominator. Using the proof of Lemma 2, we deduce that $E(k_t)/n_t$ is increasing in k_t for $\sigma < \sigma_{nt}$ and $E(k_t)/n_t$ is decreasing in k_t for $\sigma > \sigma_{nt}$.

Therefore, for $\sigma > \sigma_{nt}$, $E(k_t)/n_t$ is decreasing in k_t and $1/(1-bn_t)$ is increasing in k_t. This implies that σ_{nt} is increasing in k_t.

For $1 < \sigma < \sigma_{mt}$, $E(k_t)/n_t$ is increasing in k_t because $\sigma_{mt} < \sigma_{nt}$. This implies that σ_{mt} is decreasing in k_t.

Finally, using (26)-(28), we note that at an equilibrium, the expression $(\epsilon - \gamma)\beta_1 - \gamma\beta_2(E(k_t)/n_t)^{\frac{\sigma-1}{\sigma}}$ is always strictly positive. Using (B.7), this implies that σ_{nt} is strictly higher than 1. Therefore, there exists $\overline{\sigma} > 1$ such that $\sigma_{nt} > \overline{\sigma}$ for all equilibrium value of k_t.

A.4 Proof of Proposition 1

A steady state $n = n_a$ with $k = 0$ should be a solution of the equation $G(n_a, 0) - F(n_a, 0) = 0$. Whatever the value of σ, we have $\lim_{n\to 1/b} G(n,0) - F(n,0) = +\infty$. Moreover, for $\sigma < 1$, $G(0,0) - F(0,0) = \gamma - \epsilon < 0$ if and only if $\epsilon > \gamma$, which ensures the existence of a solution n_a. For $\sigma > 1$, $G(0,0) - F(0,0) = \gamma > 0$.

Using the proof of Lemma 2 and Assumption 2, we have $\frac{\partial G(n,0)}{\partial n} - \frac{\partial F(n,0)}{\partial n} > 0$ at each possible steady state. We easily deduce that there is no steady state with $k = 0$ for $\sigma > 1$ and there is a unique one for $\sigma < 1$. Otherwise, we should have in both cases at least one solution such that $\frac{\partial G(n,0)}{\partial n} - \frac{\partial F(n,0)}{\partial n} < 0$, which is ruled out by Assumption 2.

A.5 Proof of Proposition 2

Substituting (24) and (32) in equation (23), we get:

$$E(\underline{k}) = \theta \frac{A_g}{A_p} \frac{1-\alpha}{\alpha + \gamma - \gamma b(1-\delta)} > 0 \tag{B.12}$$

and $E(\overline{k}) = +\infty$. Using (31), we also have $n(\underline{k}) = 1/b$ and:

$$n(\overline{k}) = (1-\delta)\frac{\gamma}{\alpha+\gamma} \tag{B.13}$$

Using these expressions, we deduce that $\widetilde{G}(\underline{k}) = +\infty$ and:

$$\widetilde{G}(\overline{k}) = \frac{(1-a)(1-\alpha)b(1-\delta)}{\alpha+\gamma-\gamma b(1-\delta)} > 0 \tag{B.14}$$

We also have:

$$\widetilde{F}(\underline{k}) = \frac{\epsilon\beta_1}{\beta_1 + \beta_2 b^{\frac{\sigma-1}{\sigma}}\left[\theta\frac{A_g}{A_p}\frac{1-\alpha}{\alpha+\gamma-\gamma b(1-\delta)}\right]^{\frac{\sigma-1}{\sigma}}} - \gamma \tag{B.15}$$

and $\widetilde{F}(\overline{k}) = \epsilon - \gamma$ if $\sigma < 1$ and $\widetilde{F}(\overline{k}) = -\gamma$ if $\sigma > 1$.

Since $n'(k) < 0$, we easily deduce from (28) that $\widetilde{G}'(k) < 0$. Using (27), we have:

$$\widetilde{F}'(k) = \frac{\epsilon\beta_1\beta_2}{\left[\beta_1 + \beta_2\left(\frac{E(k)}{n_1(k)}\right)^{\frac{\sigma-1}{\sigma}}\right]^2}\frac{1-\sigma}{\sigma}\left(\frac{E(k)}{n_1(k)}\right)^{\frac{-1}{\sigma}}\frac{E'(k)n(k)-n'(k)E(k)}{E(k)^2} \tag{B.16}$$

Therefore, $\widetilde{F}'(k) > 0$ if $\sigma < 1$ and $\widetilde{F}'(k) < 0$ if $\sigma > 1$.

We can conclude that if $\sigma < 1$, there exists a unique steady state $k_0 \in (\underline{k}, \overline{k})$ if $\widetilde{F}(\overline{k}) > \widetilde{G}(\overline{k})$, i.e.

$$\epsilon > \gamma + \frac{(1-a)(1-\alpha)b(1-\delta)}{\alpha+\gamma-\gamma b(1-\delta)} \equiv \epsilon_0 \tag{B.17}$$

If $\sigma > 1$, both $\widetilde{F}(k)$ and $\widetilde{G}(k)$ are decreasing with $\widetilde{F}(\underline{k}) < \widetilde{G}(\underline{k})$ and $\widetilde{F}(\overline{k}) < \widetilde{G}(\overline{k})$. Let us consider $\widehat{k} = (XA_p)^{\frac{1}{1-a}}$, with $X \in (\frac{\gamma b}{1+\gamma-\gamma b(1-\delta)}, \frac{\alpha+\gamma}{(1-\alpha)(1-\delta)})$. This ensures that $\widehat{k} \in (\underline{k}, \overline{k})$. To show the existence of two steady states k_1 and k_2, we will prove that $\widetilde{F}(\widehat{k}) > \widetilde{G}(\widehat{k})$. Using (23) and (31), we have:

$$E(\widehat{k}) = \frac{\theta A_g}{A_p}\frac{(1-\alpha)[(1-\delta)X+1]}{\alpha+\gamma-(1-\alpha)(1-\delta)X} \tag{B.18}$$

$$n(\widehat{k}) = \frac{\gamma}{1+\gamma}\frac{(1-\delta)X+1}{X} \tag{B.19}$$

Using these expressions, we deduce that:

$$\widetilde{G}(\widehat{k}) = \frac{b\gamma(1+\gamma)(1-a)}{(1+\gamma)X - b\gamma[(1-\delta)X+1]} \tag{B.20}$$

$$\widetilde{F}(\widehat{k}) = \frac{\epsilon\beta_1}{\beta_1 + \beta_2\left[\frac{\theta A_g}{A_p}\frac{1+\gamma}{\gamma}\frac{(1-\alpha)X}{\alpha+\gamma-(1-\alpha)(1-\delta)X}\right]^{\frac{\sigma-1}{\sigma}}} - \gamma \tag{B.21}$$

Since X does not depend on ϵ, we conclude that there is a value $\epsilon_1 > 0$ such that, for $\epsilon > \epsilon_1$, we have $\widetilde{F}(\widehat{k}) > \widetilde{G}(\widehat{k})$.

A.6 Proof of Lemma 4

Using (33), we see that $\widetilde{F}'(k) > \widetilde{G}'(k)$ is equivalent to:

$$\begin{aligned} & \frac{\partial F}{\partial n}n'(k) + \frac{\partial F}{\partial k} > \frac{\partial G}{\partial n}n'(k) + \frac{\partial G}{\partial k} \\ \Leftrightarrow\ & \frac{\partial F}{\partial k} - \frac{\partial G}{\partial k} > n'(k)\left(\frac{\partial G}{\partial n} - \frac{\partial F}{\partial n}\right) \end{aligned}$$

Since $\sigma < \sigma_{nt}$, $\frac{\partial G}{\partial n} - \frac{\partial F}{\partial n} > 0$. Therefore, the last inequality is equivalent to:

$$n_d'(k) = \frac{\frac{\partial F}{\partial k} - \frac{\partial G}{\partial k}}{\frac{\partial G}{\partial n} - \frac{\partial F}{\partial n}} > n'(k)$$

Using the proof of Proposition 2, the lemma immediately follows.

A.7 Proof of Proposition 3

Using (33), we deduce that:

$$\frac{dk_i}{d\beta_2} = \frac{-\partial\widetilde{F}(k)/\partial\beta_2}{\widetilde{F}'(k) - \widetilde{G}'(k)} \tag{B.22}$$

Since we have $\partial\widetilde{F}(k)/\partial\beta_2 < 0$, the sign of $dk_i/d\beta_2$ is given by the sign of $\widetilde{F}'(k)-\widetilde{G}'(k)$. Using the proof of Proposition 2, we deduce that $dk_0/d\beta_2 > 0$, $dk_1/d\beta_2 > 0$ and $dk_2/d\beta_2 < 0$.

The effects on the corresponding values of $n_i = n(k_i)$ come from the fact that the function $n(k)$ is decreasing and does not depend on β_2.

A.8 Proof of Proposition 5

To analyze the local stability of the steady state k_2, we differentiate equation (34) in its neighborhood. We get:

$$\frac{dk_{t+1}}{dk_t}\left(\frac{1-bn}{bn}-\frac{n_d'(k)k}{n_d(k)}\right)=\frac{(1-\delta)k+aA_pk^a}{(1-\delta)k+A_pk^a}\frac{1-bn}{bn}-\frac{1}{bn}\frac{n_d'(k)k}{n_d(k)}$$

Using Lemma 4, we know that $n_d'(k)<0$ for $k=k_2$. This means that $dk_{t+1}/dk_t>0$. Moreover, $dk_{t+1}/dk_t>1$ is equivalent to:

$$\frac{n_d'(k)k}{n_d(k)}<\frac{(a-1)A_pk^{a-1}}{(1-\delta)+A_pk^a}$$

Using (31), this last inequality is equivalent to:

$$\frac{n_d'(k)k}{n_d(k)}<\frac{n'(k)k}{n(k)}$$

Using Lemma 4, we deduce that this is satisfied at the steady state $k=k_2$.

A.9 Proof of Proposition 6

Using (B.7) and (B.8), we can evaluate $(\sigma_{mt}-1)/\sigma_{mt}$ for $k_t=\underline{k}$ given by (32):

$$\begin{aligned}\frac{\sigma_{mt}-1}{\sigma_{mt}} &= \frac{\alpha+\gamma-\gamma b(1-\delta)}{1+\gamma-\gamma b(1-\delta)}\left[(\epsilon-\gamma)\beta_1-\gamma\beta_2(E(\underline{k})/n_t)^{\frac{\sigma-1}{\sigma}}\right]\\ &\quad \frac{\beta_1(E(\underline{k})/n_t)^{\frac{1-\sigma}{\sigma}}+\beta_2}{\epsilon\beta_1\beta_2}\end{aligned}$$

This expression is increasing in n_t, which is lower than $1/b$, and using (23), we have:

$$E(\underline{k})=\theta\frac{A_g}{A_p}\frac{1-\alpha}{\alpha+\gamma-\gamma b(1-\delta)}$$

We deduce that:

$$\frac{\sigma_{mt}-1}{\sigma_{mt}}<\frac{\alpha+\gamma-\gamma b(1-\delta)}{1+\gamma-\gamma b(1-\delta)}\frac{\epsilon-\gamma}{\epsilon}\left[\frac{\beta_1}{\beta_2}\left(\frac{A_p}{A_g\theta}\frac{\alpha+\gamma-\gamma b(1-\delta)}{(1-\alpha)b}\right)^{\frac{\sigma-1}{\sigma}}+1\right]$$

This last expression is lower than $(\sigma - 1)/\sigma$ if α and γ are sufficiently low. Therefore, there exist $\overline{\alpha} > 0$ and $\overline{\gamma} > 0$ such that $(\sigma_{mt} - 1)/\sigma_{mt} < (\sigma - 1)/\sigma$ when $k_t = \underline{k}$ if $\alpha < \overline{\alpha}$ and $\gamma < \overline{\gamma}$. Since σ_{mt} is decreasing in k_t, $(\sigma_{mt} - 1)/\sigma_{mt} < (\sigma - 1)/\sigma$ for all $k_t > \underline{k}$, which implies that $n'_d(k_t) < 0$ for $k_t > \underline{k}$.

By inspection of equation (34), we deduce that $dk_{t+1}/dk_t > 0$ for all $k_t > \underline{k}$, meaning that the dynamics are monotonic. Moreover, since $n'_d(k_1) > n'(k_1)$, we can use the proof of Proposition 5 to conclude that $0 < dk_{t+1}/dk_t < 1$ at the steady state k_1.

B Data description

The US Energy Information Administration (EIA) provides annual data on the carbon intensity of each State in the US. The EIA defines a carbon intensity (economy): *"The amount of carbon by weight emitted per unit of economic activity - most commonly gross domestic product (GDP) (CO2 emissions/GDP). The carbon intensity of the economy is the product of the energy intensity of the economy and the carbon intensity of the energy supply. We currently express this value as the full weight of the CO2 emitted, rather than the weight of just carbon"*. The data on the two measures of environmental concerns, i.e. the percentage of Americans thinking that stricter environmental laws and regulations are worth the cost *versus* considering they cost too many jobs and hurt the economy, come from the PEW research center: Survey conducted on a sample size of more than 20000 Americans from all 50 states in 2014. *https://www.pewresearch.org/religion/religious-landscape-study/compare/views-about-environmental -regulation/by/state.* Data on total fertility rate in 2020 come from the National Center for Health Statistics, National Vital Statistics Reports, Vol. 70, No. 17, February 7, 2022, while per-capita income data come from the Federal Reserve Bank of St. Louis Economic Data (FRED).