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Abstract

We use an overlapping generations setup with two reproductive periods to explore how fertility decisions may differ in response to economic incentives in early and late adulthood. In particular, we analyze the interplay between fertility choices—related to career opportunities—and wages, and investigate the role played by late fertility. We show that young adults only postpone parenthood above a certain wage threshold and that late fertility increases with investment in human capital. The long run trend is either to a low productivity equilibrium, involving high early fertility, no investment in human capital and relatively low income, or to a high productivity equilibrium, where households postpone parenthood to invest in their human capital, with higher late fertility and higher levels of income. A convergence to the

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latter state would explain the postponement of parenthood and the fertility rebound observed in Europe in recent decades.

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1 Introduction

Ongoing changes in fertility, family formation and relationship patterns since the 1960s in developed economies have come to be known as the second demographic transition, SDT (cf. Sobotka (2008)). The SDT is typically described in the economic literature as a sharp decline in total fertility rates—below the replacement rate of 2.1 children per women—and delayed parenthood, *i.e.* an increase in the mean age of mothers. These trends can be explained by improved access for women to tertiary education and the labor market, combined with the widespread availability of efficient contraception and the erosion of marriage. But alongside these seemingly related fertility patterns, trends in age-specific fertility rates (ASFRs) provide additional interesting insights.[1] In European countries, two separate trends emerge: i) a gradual decline in fertility in young women (15 to 29 years of age); ii) an initial decline in fertility (ASFR) followed by an increase over the past three decades for older women (30 to 44 years of age).[2] Furthermore, the last decades have seen a bottoming out of total fertility rates (TFRs), which are now slightly increasing in most European countries (see Figure 1). These recent trends suggest a shift in demographic dynamics in European economies. As well as reflecting a postponement of parenthood, we argue that these ASFR patterns shed light on the fertility rebound, which has been poorly investigated to date.[3] This fertility rebound of particular interest to countries concerned about their low fertility rates (cf. Doepke and Tertilt (2016)).

[1]Our main source is the United Nations 2019 World Population Prospect database, which covers the period 1950–2015 and provides total fertility rates and age-specific fertility rates for 38 European countries. https://population.un.org/wpp/Download/Standard/Fertility/.

[2]The same ASFR trends are observed if different age groups are considered, as shown in Appendix A.

[3]With some recent exceptions, including Yakita (2018) or Ohinata and Varvarigos (2020).

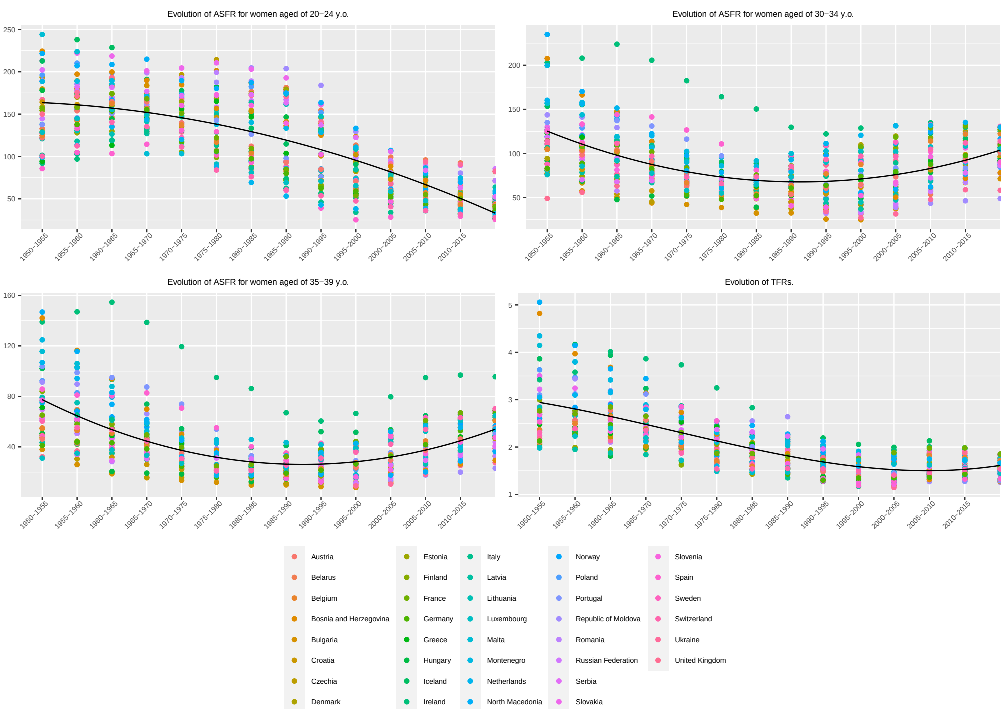

Figure 1: Evolution of three measures of Age-Specific Fertility Rates and Total Fertility Rates in Europe over 1950-2020

The aim of this paper is to provide a theoretical underpinning for these empirical findings. Our main objective is to identify the economic mechanisms underlying recently observed income-fertility relationship in high-income economies (Doepke et al. (2022)). To that end, we analyze the postponement-fertility nexus in a model where individuals choose when and how many children to have. Results suggest possible positive relationships between fertility and both income and human capital, and reproduce the fertility rebound observed in rich industrialized countries.

In an overlapping generations (OLG) setup with two reproductive periods, we explore how fertility decisions differ in response to economic incentives in early and late adulthood. We focus in particular on the interplay between childbearing, career choices (education/work experience), and wages. Both relationships are crucial to understand decisions to postpone parenthood. Young adults can spend their wages on consumption, time-consuming child-rearing activities or investment in their human capital/work experience. Having children at a young age compromises career opportunities, which might translate into a loss of future earnings (wage penalty), loss of skills during job interruptions, and/or loss of experience (Adda et al. (2017)). This theoretical trade-off is consistent with empirical findings that young mothers have a larger wage penalty than older ones, particularly those on the lowest wages. For instance, Miller (2011) has shown that delaying motherhood is associated with an increase in labor markets earnings of around 9% per year of delay, while Budig and England (2001) calculate a wage penalty of 7% per child.

Later in life, households can once again use their income from labor to either consume, save, or raise children. However, pregnancy attempts are now more likely to fail, leading to the co-existence of fertile households who succeed in having children and infertile ones, who fail. Interestingly, we find that early fertility and investment in human capital are substitutes, while investment in human capital and late fertility are complements. Indeed, although career development may appear incompatible at first glance with high late fertility rates, as pointed out by Sobotka et al. (2011), d'Albis et al. (2017) and Nitsche and Brückner (2021), higher-earning, more educated women can nowadays combine late childbearing with continued investment in their professional careers, because they can afford childcare. For instance, Nitsche and Brückner (2021) found that highly educated women in the US born in the 1960s and 70s were more likely to combine family and professional responsibilities with child bearing than their counterparts in previous cohorts, leading them to catch up with the fertility levels of their less-educated counterparts. Similarly, d'Albis et al. (2017) show that young women in Europe are more likely to subsequently start a family if childbearing is postponed for education rather than because of limited access to the labor market. The complementarity of late fertility and human capital is at the core of our model.

In our general equilibrium model, wages are endogenous and outcomes depend on total factor productivity. Two types of stationary equilibria emerge. When productivity is low, the correspondingly low wages prevent career development but favor early fertility, which further limits investments in human capital. When productivity is sufficiently high in contrast, the higher levels of income encourage households to postpone fertility, investing instead in their careers before having children in late adulthood. The higher late fertility in this case more than compensates for the decrease in early fertility, leading to higher total fertility. This scenario is consistent with the fertility rebound mentioned at the beginning of the introduction. We also identify an intermediate long-run configuration where households begin to postpone fertility and invest in careers, but total fertility is lower than in the other regimes.

In addition, these results highlight a new connection between reproductive health and career development, namely that improvements in reproductive health or reproductive technologies imply i) a demographic boom, as expected, ii) a decrease in the capital-to-labor ratio, because of the larger labor force; and iii) increased investment in human capital/work experience. Individuals with better reproductive health will also tend to invest more in career development to cover for future childcare costs. At the society level, this means that better healthcare (technology, practices or knowledge) and higher fertility are associated with higher investment in human capital. On the contrary, since fertility is negatively affected by social externalities such as pollution, poor environmental quality may make career investments less attractive because it pushes down late fertility.

Our paper contributes to the literature on the joint dynamics of economic and demographic variables over time. Introducing endogenous fertility choices into growth models leads to the empirically well-established decrease in fertility with advanced development (Barro and Becker (1989), Galor and Weil (1996, 2000), Bhattacharya and Chakraborty (2012)). In these theoretical models, the accumulation of physical or human capital leads to a reduction in fertility through the so-called quantity-quality trade-off, since greater development implies larger returns on education and higher opportunity costs for childbearing. However, while this literature focuses on the number (*quantum*) of births and its interaction with development, it neglects an equally important feature of demographic dynamics which is the timing of parenthood (the *tempo* of births).

The timing of births and postponement of parenthood have been investigated more recently (d'Albis et al. (2010), Pestieau and Ponthière (2014, 2015), de la Croix and Pommeret (2021), Sommer (2016)), although these studies consider the number of births as given. The question of timing is important because the reproductive period is limited and because fertility decreases with age; the timing of births is therefore a major driver of demographic dynamics. Existing studies mainly highlight the negative effects of postponement on fertility rates. Our contribution is to show instead that postponement can lead to higher fertility rates. Following Iyigun (2000) and d'Albis et al. (2018), households in our model can choose both the timing and the number of children they have, but while d'Albis et al. (2018) and Iyigun (2000) find that total fertility rates continue to decrease, our model reproduces the observed postponement of childbearing and the possibility of a fertility rebound, driven by higher late fertility, as currently observed in Europe.

Our paper is also related to a strand of the literature investigating the emergence of the fertility rebound. Ohinata and Varvarigos (2020), for instance, propose a growth model in which the fertility rebound emerges as the final stage of a three-phase process of demographic change and economic development. This final stage stems from an accumulation of human capital, which leads to a fertility rebound through a strong income effect. Our model also differs from Yakita's (2018), in which external childcare production allows for a fertility rebound as women's wages increase. We contribute to this literature by showing that the rebound can be explained by increased late fertility.

The remainder of the paper is organized as follows. In Section 2, we discuss some stylized facts. Section 3 presents the framework of the model and the choices available to agents. Section 4 presents the different possible regimes depending on parenthood postponement and levels of investment in human capital. In Section 5, we define the intertemporal equilibrium and analyze the dynamics and the existence of a steady state in the different regimes. Section 6 provides a complete picture of the long-term dynamics of the economy and explains the motivations for postponing parenthood. In Section 7, we investigate the effects of potential infertility before concluding in Section 8. Technical details are provided in appendices.

2 Stylized facts

Before presenting the theoretical mechanisms underlying the joint fertility trends mentioned in the introduction (the postponement of parenthood and the fertility rebound), we show how recent patterns in fertility rates are related to a selection of economic variables, focusing on European economies.

In the introduction, we showed in Figure 1 how ASFRs in European countries for women over 30 years of age follow a U-shaped curve over the period 1950–2020, decreasing from 1950 to the mid 1980s and then rising back up. Total fertility rates also seem to be increasing, after reaching a minimum in the mid 2000s. In contrast, the ASFRs of younger women over the same period have continued to decrease. Young European women now have fewer children than their counterparts did in the 1970s and 80s, while older women have more, leading overall to a moderate fertility rebound. The main objective of this paper is to identify the economic mechanisms that have been driving these recent trends.

These demographic changes indeed occurred in the context of a continuous increase in wealth per capita, challenging the well-established relationship between fertility and income in the demographic transition. In this regard, interesting insights can be drawn from Figure 2, where we have plotted age-specific fertility rates against potentially related economic variables in 2017 for a cross section of European countries.[4]

The left and right panels focus respectively on early and late fertility, captured by the ASFRs of 20–24-year-old and 30–34-year-old women. These two measures of fertility are plotted from top to bottom against GDP per capita[5], average monthly earnings for women,[6] and a measure of total factor productivity[7]. The comparison with wages is relevant given the known effect of wages on fertility choices (and of fertility choices on wage profiles) and the importance of wages in our theoretical model. The correlation with total factor productivity is shown because it is used in our model to identify long-term equilibria.

The relationships in the left panel are clearly negative and those on the right, clearly positive. This suggests that the negative fertility–income relationship of the post-Malthusian era still prevails in young women: in Europe in 2017, the richer the economy, the lower the early fertility rate. However, the right panel shows that this relationship does not hold anymore for older women: the more they earn for instance, the higher the late fertility rate is. This means that the relationship between fertility rates and economic factors has changed in recent decades and that differences in wages, income and/or

[4]Data are shown for all 38 of the European countries in Figure 1 minus Bosnia and Herzegovina, Estonia, Ireland, Montenegro, North Macedonia, and Poland because of missing data.

[5]World Bank. GDP per capita current $, The World Bank Group, https://data.worldbank.org/indicator/NY.GDP.PCAP.CD.

[6]International Labour Organization, "ILO modelled estimates database" ILOSTAT, https://ilostat.ilo.org/data/, expressed in units of PPP.

[7]Penn World Tables. International comparisons of production, income and prices 10.0, https://www.rug.nl/ggdc/productivity/pwt/, University of Gröningen, expressed in current PPPs (USA = 1).

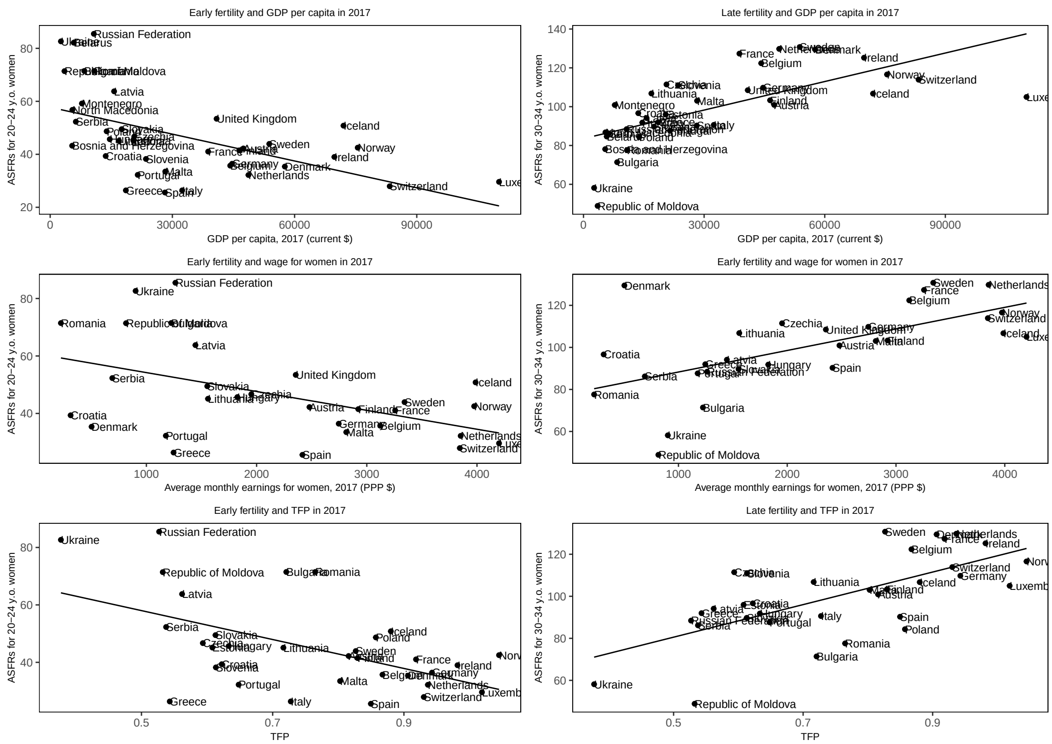

Figure 2: Early and late fertility in European countries as a function of various economic variables in 2017.

productivity may at least partially explain the discrepancies in demographic dynamics between countries observed at the macroeconomic level. Recent economic and demographic studies have highlighted the variance in fertility trends in Europe since the mid 20th century. In particular, total fertility rates have been increasing since the mid 2000s in richer European countries, the so-called high fertility belt (see for instance Frejka and Sobotka (2008), Myrskylä et al. (2009), Luci-Greulich and Thévenon (2014)). This suggests in turn that late fertility is a key driver of this (modest) rebound in total fertility rates. Our theoretical model reproduces these phenomena at a micro level and also reveals different long-run equilibria that provide an overall view of European economies. The first stationary equilibrium is characterized by low income, high early fertility and low late fertility, while the second equilibrium features higher per capita income, higher late fertility and larger career investments.

These trends are illustrated in the following panel analysis of World Bank data on GDP per capita and fertility data from the United Nations 2019 World Population Prospect database. Three ASFRs and the total fertility rate were regressed against a measure of GDP per capita for 38 European countries from the 1980s onward, with country fixed effects.[8] The predicted values of each panel regression are plotted in Figure 3 (regression results provided in Appendix B.).

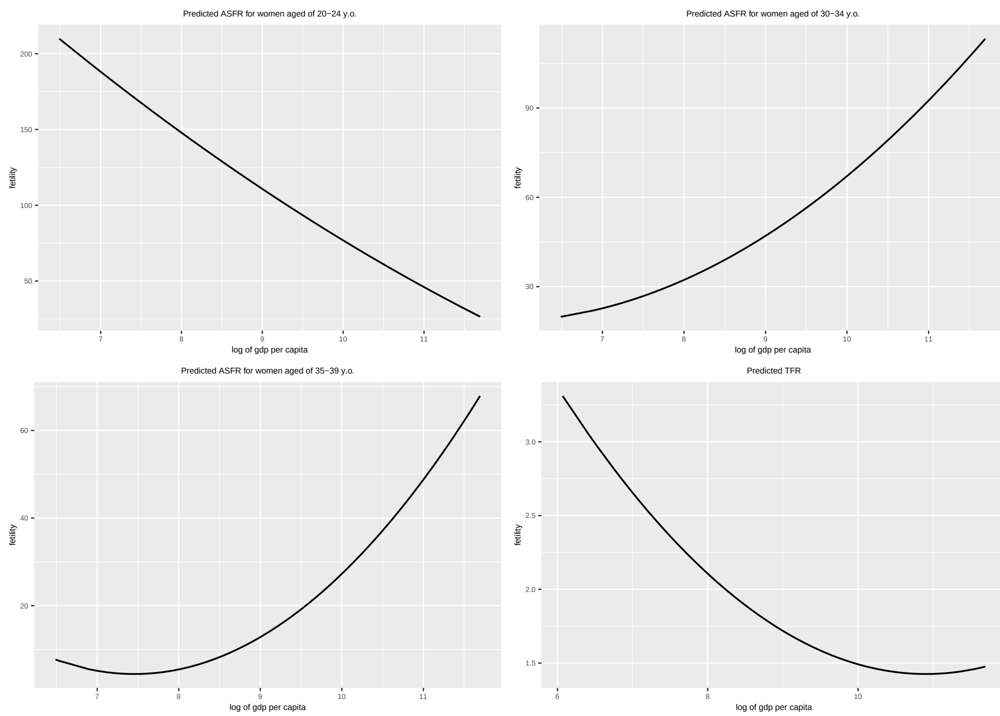

Figure 3: Predicted values.

These empirical results confirm the persistent negative relationship between early fertility and income. They also highlight the reversal of this relationship for late fertility. Increased GDP per capita is associated with higher fertility in older women. Furthermore, at low levels of GDP per capita,

[8]Since the focus here is on periods of moderate growth in Europe, the preceding 30-year period was excluded from the analysis.

the sharp decrease in early fertility as GDP increases dominates the increase in late fertility, so that postponement entails a decrease in total fertility. At higher levels of income, the increase in late fertility offsets and then outweighs the decrease in early fertility, such that total fertility begins to increase at the highest values of GDP per capita. These trends are consistent with recent observations in Europe.

3 The model

We use a dynamic general equilibrium model with accumulation of physical capital, human capital, and endogenous fertility. Time is discrete and indexed by $t = 0, 1, 2....$ There are two types of agents, firms and households. This section focuses on their micro-economic behavior.

3.1 Production

We consider a continuum of firms of unit size producing a final good, Y_t, using both physical capital, K_t, and labor, L_t. For tractability, we assume a Cobb-Douglas technology, *i.e.* $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, with $A > 0$ the total factor productivity and $\alpha \in (0, 1)$ the share of physical capital in the production process. $k_t \equiv K_t/L_t$ is the capital-labor ratio, w_t the wage and r_t the interest rate. Profit maximization gives:

$$w_t = (1-\alpha)Ak_t^{\alpha} \equiv w(k_t) \tag{1}$$

$$r_t = \alpha Ak_t^{\alpha-1} \equiv r(k_t) \tag{2}$$

In the following, $R_t \equiv 1 - \delta + r_t$ represents the interest factor on physical capital, where $\delta \in (0, 1)$ is the depreciation rate of capital.

3.2 Household behavior

We consider a three-period OLG model with generations born at date t of size N_t. In the first period, households (young adults) choose how many children to have and can invest in their careers (human capital) and consume. In the second period, households (older adults) can once again choose to have children, save by accumulating physical capital and consume. In the third period, households are retired and consume their savings. In our model, households have two shots at having children: in early and/or late adulthood.

The timing of fertility is important because delaying parenthood narrows the reproductive period and female fertility naturally decreases with age. We also consider, following de la Croix and Pommeret (2021) or Etner et al. (2020) for instance, that late fertility carries a risk of failure but early fertility does not. Basically, all the mechanisms we highlight capture the choices typically made by women within a household regarding education, careers and fertility.

In this model, a young adult at date t earns the competitive wage w_t and shares their income between consumption, c_{1t}, investment in human capital, h_{t+1} and child-rearing activities if the young adult chooses to have children. In the second period, two types of older adult are considered: those in fertile households (denoted by a superscript F), who are successful in their attempts to have n_{2t+1} children, and those in infertile households (denoted by a superscript I), who fail to have children. The probability of being in a fertile household is given by $\pi \in (0,1)$. Older adults also consume c^j_{2t+1}, save by accumulating physical capital k^j_{t+2} and spend time on child-rearing activities if they have children. Note that the labor income of older adult depends on potential previous investments in human capital. Finally, after retirement, households consume their remunerated savings c^j_{3t+2}, with $j = I, F$.

Households derive utility from consumption and from parenting in the first two periods. Children are not considered perfect substitutes, which generalizes the setup of Iyigun (2000). Following de la Croix and Pommeret (2021), household preferences are represented by an expected utility function, which is additive separable between consumption and parenting:

$$\begin{aligned} \ln c_{1t} + \delta_1 \ln(\mu_1 + n_{1t}) \quad &+ \quad \beta\pi \left[\ln c^F_{2t+1} + \delta_2 \ln(\mu_2 + n_{2t+1}) + \beta \ln c^F_{3t+2}\right] \\ &+ \quad \beta(1-\pi) \left[\ln c^I_{2t+1} + \delta_2 \ln \mu_2 + \beta \ln c^I_{3t+2}\right] \end{aligned} \tag{3}$$

where $\delta_i \geqslant 0$ measures the preference toward having children and $\beta \in (0,1)$ is the discount factor. Also, as in Baudin et al. (2015, 2020), the parameters $\mu_i \geqslant 0$ allow for corner solutions in fertility choices. In early adulthood, the households' budget constraint is:

$$c_{1t} + w_t \phi_1 n_{1t} + h_{t+1} = w_t \tag{4}$$

with $\phi_1 > 0$ the time cost per child. In line with the above-mentioned literature, we assume that child rearing is a time-consuming activity. Note also that investing in human capital is costly in terms of goods. We differ in this regard from d'Albis et al., who ignore the direct cost of education in household budgets, but instead introduce a disutility to investment in human capital. In Iyigun's (2000) model, households do not work in the first period, so choose between spending time on education or raising children. We model household trade-offs more realistically. Based on the literature on the career costs of motherhood, in particular the wage penalty associated with early motherhood, we introduce a work experience variable. Investing in human capital is then equivalent to accumulating work experience and thus increases labor efficiency, captured by a parameter $\varepsilon \geq 0$. Meanwhile, having children when young increases (current) utility but may reduce future labor income. This setup is in line with the empirical literature (e.g. Budig and England (2001), Caucutt et al. (2002), Miller (2011), Olivetti (2006), Herr (2016)), which shows principally that early fertility is associated with a larger wage penalty.

For fertile households, the budget constraints in late adulthood and retirement are given by:

$$c_{2t+1}^F + w_{t+1}\phi_2 n_{2t+1} + k_{t+2}^F = w_{t+1}(1+\varepsilon h_{t+1}) \tag{5}$$

$$c_{3t+2}^F = R_{t+2} k_{t+2}^F \tag{6}$$

while for infertile households, the corresponding budget constraints are:

$$c_{2t+1}^I + k_{t+2}^I = w_{t+1}(1+\varepsilon h_{t+1}) \tag{7}$$

$$c_{3t+2}^I = R_{t+2} k_{t+2}^I \tag{8}$$

where $\phi_2 > 0$, the time cost per child, reduces the income of fertile households. Note that in our setup, the time cost of raising children is only incurred for newborns, infants and toddlers, so that children born in the previous period no longer carry a time cost for their parents. An alternative approach would be to consider that children born in a previous period live with their parents and share consumption spending, as in Pestieau and Ponthière (2014).

Households maximize their utility (3) given their budget constraints (4)-(8) but also the positivity constraints $h_{t+1} \geqslant 0$, $n_{1t} \geqslant 0$ and $n_{2t+1} \geqslant 0$. The choices of young households are governed chiefly by two trade-offs: between consumption and human capital on the one hand, and between consumption and parenthood on the other.

$$\frac{1}{c_{1t}} \geq \beta \left[\frac{\pi}{c_{2t+1}^F} + \frac{1-\pi}{c_{2t+1}^I} \right] w_{t+1}\varepsilon \tag{9}$$

$$\frac{w_t \phi_1}{c_{1t}} \geq \frac{\delta_1}{\mu_1 + n_{1t}} \tag{10}$$

where equation (9) (equation (10)) holds as equality if $h_{t+1} > 0$ ($n_{1t} > 0$). For older adults, the trade-offs for intertemporal consumption smoothing and between consumption and children are:

$$\frac{1}{c^F_{2t+1}} = R_{t+2}\beta\frac{1}{c^F_{3t+2}} \tag{11}$$

$$\frac{w_{t+1}\phi_2}{c^F_{2t+1}} \geq \frac{\delta_2}{\mu_2 + n_{2t+1}} \tag{12}$$

with equality if $n_{2t+1} > 0$. The only trade-off for infertile households is between consumption in the different periods:

$$\frac{1}{c^I_{2t+1}} = R_{t+2}\beta\frac{1}{c^I_{3t+2}} \tag{13}$$

Let us now look at these choices in more detail, starting with those of the older households.

3.2.1 Choices of older households

For infertile older households, equations (8) and (13) lead to $c^I_{2t+1} = k^I_{t+2}/\beta$. Then, the budget constraint (7) yields the levels of consumption and of investment in physical capital:

$$c^I_{2t+1} = \frac{1}{1+\beta}w_{t+1}(1+\varepsilon h_{t+1}) \tag{14}$$

$$k^I_{t+2} = \frac{\beta}{1+\beta}w_{t+1}(1+\varepsilon h_{t+1}) \tag{15}$$

For fertile older households, equations (6) and (11) lead to $c^F_{2t+1} = k^F_{t+2}/\beta$, and the two terms can be obtained from the budget constraint (5):

$$c^F_{2t+1} = \frac{1}{1+\beta}w_{t+1}(1+\varepsilon h_{t+1} - \phi_2 n_{2t+1}) \tag{16}$$

$$k^F_{t+2} = \frac{\beta}{1+\beta}w_{t+1}(1+\varepsilon h_{t+1} - \phi_2 n_{2t+1}) \tag{17}$$

As expected therefore, saving and consumption both increase with income in both types of household. Moreover, $c^F_{2t+1} < c^I_{2t+1}$ if $n_{2t+1} > 0$. As for late fertility, equations (6), (11) and (12) can be combined to show that the number of children is given by $\delta_2 k^F_{t+2} \leq w_{t+1}\beta\phi_2\left(\mu_2 + n_{2t+1}\right)$. Substituting equation (17) into this last inequality gives:

$$\delta_2(1+\varepsilon h_{t+1}) - \phi_2\mu_2(1+\beta) \leqslant n_{2t+1}\phi_2(1+\beta+\delta_2) \tag{18}$$

with equality if $n_{2t+1} > 0$, in which case n_{2t+1} is defined by

$$n_{2t+1} = \frac{\delta_2(1+\varepsilon h_{t+1}) - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \equiv N_2(h_{t+1}) \tag{19}$$

which is an increasing function in h_{t+1}. Note also that $n_{2t+1} > 0$ for all $h_{t+1} \geqslant 0$ if $\delta_2 > \phi_2\mu_2(1+\beta)$. In the following, this inequality is always satisfied, which excludes the possibility of fertile households choosing to remain childless. Equation (19) shows that late fertility does not directly depend on the probability of being fertile. Nevertheless, as will become clear in the following, this health variable affects fertility decisions through its impact on previous investments in human capital. Late fertility is not affected either by the current wage w_{t+1}, because income in later adulthood and the cost of having children both increase linearly with wages. However, late fertility can depend at equilibrium on wages in the previous period, through the level of human capital.

These results indicate that late fertility and investment in human capital are complements. In our model indeed, households have an incentive to invest in human capital to cover the costs associated with having more children in late adulthood. This positive relationship may appear at first sight to contradict the longstanding negative association between fertility and female labor force participation, which is a typical feature of the demographic transition in high-income countries. However, our theoretical result is consistent with recent, mostly empirical studies, which show that the family–career compatibility is a crucial determinant of current fertility trends in high-income countries (Doepke et al. (2022)). In particular, these studies show that the latest cohorts of US and European women tend to postpone fertility to invest in education or their careers, but without necessarily reducing their fertility intentions (see Sobotka et al. (2011), d'Albis et al. (2017), Goldin (2021), Nitsche and Brückner (2021)). These results can be explained in part by better access to childcare services and/or the increased bargaining power of women in households, particularly for more highly educated/skilled women. In our model, investment in human capital is associated with higher late fertility and higher labor income in late adulthood, accurately capturing this phenomenon.

3.2.2 Choices of young households

Let us first consider the fertility choices of young households and determine whether they have children. Using equations (4) and (10), we obtain:

$$\frac{h_{t+1}}{w_t} \geq 1 - \frac{\phi_1}{\delta_1}\mu_1 - \phi_1 \frac{1+\delta_1}{\delta_1} n_{1t} \tag{20}$$

which holds as equality if $n_{1t} > 0$.

It follows that $n_{1t} > 0$ if the share of labor income devoted to investment in human capital is below a certain threshold, i.e. $h_{t+1}/w_t < 1 - \phi_1\mu_1/\delta_1$, which requires $\phi_1\mu_1 < \delta_1$. In contrast, $n_{1t} = 0$ if this share is sufficiently high, i.e. $h_{t+1}/w_t \geqslant 1 - \phi_1\mu_1/\delta_1$, which is always satisfied if $\phi_1\mu_1 \geqslant \delta_1$. When $n_{1t} > 0$, equation (20) can be rearranged to give:

$$n_{1t} = \frac{\delta_1}{\phi_1(1+\delta_1)}\left(1 - \frac{\phi_1\mu_1}{\delta_1} - \frac{h_{t+1}}{w_t}\right) \equiv N_1(h_{t+1}/w_t) \tag{21}$$

which is decreasing with respect to h_{t+1}/w_t, the proportion of income invested in human capital. In the absence of investment in human capital, n_{1t} is constant, because the cost of having children is proportional to w_t, the income of young adults. Otherwise, early fertility decreases as h_{t+1}/w_t increases, meaning that early fertility and investment in human capital are substitutes. We now identify the domains in which investment in human capital (h_{t+1}) is either strictly positive or zero. Substituting equations (4), (14) and (16) into equation (9) yields:

$$\frac{1+\varepsilon h_{t+1} - \phi_2 n_{2t+1}}{1+\varepsilon h_{t+1} - (1-\pi)\phi_2 n_{2t+1}}(1+\varepsilon h_{t+1}) \geqslant \beta(1+\beta)\varepsilon[w_t(1-\phi_1 n_{1t}) - h_{t+1}] \tag{22}$$

which holds as equality when $h_{t+1} > 0$. This is especially the case when the marginal utility of consumption ($1/c_{1t}$) is low (see equation (9)), which happens for sufficiently large values of labor income, w_t. Indeed, when wages are low, the marginal cost of investment in human capital outweighs the marginal benefit, so that $h_{t+1} = 0$. In contrast, the marginal benefit of having children early is sufficiently high for young adults to use all their non-consumed income to raise children.

4 Individual decisions in early and late adulthood

Recall that fertile households always choose to have children in late adulthood, i.e. $n_{2t+1} > 0$ for all $h_{t+1} \geqslant 0$, while young adults can choose to postpone parenthood. We nevertheless assume that n_{1t} is positive for low values of h_{t+1}/w_t:

Assumption 1 *$\delta_1 > \phi_1\mu_1$ and $\delta_2 > \phi_2\mu_2(1+\beta)$.*

This assumption implies that the marginal benefit of having children in late adulthood is sufficiently high for older adults to have children whatever the value of h_{t+1}. In early adulthood, the marginal benefit of having children is assumed to be sufficiently high if the ratio h_{t+1}/w_t is sufficiently low, which is especially true when there is no investment in human capital.

We now present three scenarios arising from early adulthood decisions to have children and/or invest in human capital.

4.1 Early parenthood without investment in human capital ($n_{1t} > 0$ and $h_{t+1} = 0$)

When $h_{t+1} = 0$, equations (19) and (21) yield the levels of early and late fertility:

$$n_{1t} = N_1(0) = \frac{\delta_1 - \phi_1\mu_1}{\phi_1(1+\delta_1)} \equiv \eta_1 \tag{23}$$

$$n_{2t+1} = N_2(0) = \frac{\delta_2 - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \equiv \eta_2 \tag{24}$$

which are both constant and strictly positive under Assumption 1. In addition, using equation (22), this situation ($h_{t+1} = 0$) arises if $w_t \leqslant \underline{w}$, with:

$$\underline{w} \equiv \frac{(1+\delta_1)(1+\phi_2\mu_2)}{\varepsilon\beta(1+\phi_1\mu_1)[(1+\beta)(1+\phi_2\mu_2) + \pi(\delta_2 - \phi_2\mu_2(1+\beta))]} > 0 \tag{25}$$

Below a certain level of income, households choose to have children in early and in late adulthood and do not invest in human capital. Incomes are too low to support investment in human capital and are used instead to raise children and for consumption, because the marginal benefit of having children is higher than the return on human capital.

4.2 Declining early parenthood with investment in human capital ($n_{1t} > 0$ and $h_{t+1} > 0$)

Assuming now that $h_{t+1} > 0$, we have $w_t > \underline{w}$ and equation (22) holds as equality. Under the following assumption:

Assumption 2 $\phi_1\mu_1 > \frac{\delta_1}{1+\beta(1+\beta+\delta_2\pi)}$

we show that

Lemma 1 *Under Assumptions 1-2,*

- h_{t+1} *is a positive and increasing function of* w_t*, with* $w_t = W(h_{t+1})$*, and belongs to* $(0, \tilde{h}(w_t))$ *with* $\tilde{h}(w_t) \equiv (1 + \phi_1\mu_1)w_t$*; Since* h_{t+1}/w_t *is increasing in* w_t*,* h_{t+1} *is even a superior good;*

- h_{t+1} *and* n_{1t} *are substitutes, i.e.* n_{1t} *is decreasing in* h_{t+1} *once the effect of income is taken into account;*

- *There is a finite value of* $\widehat{w}$ *for which* $n_{1t} > 0$ *for* $w_t < \widehat{w}$ *and* $n_{1t} = 0$ *for* $w_t \geqslant \widehat{w}$*.*

Proof. See Appendix C. ■

Human capital investment is an increasing function of income, because higher wages promote consumption in early adulthood, and therefore reduce the marginal cost of investing in human capital. Human capital is even a superior good, *i.e.* it increases more than wages do. Since early fertility is decreasing in the ratio h_{t+1}/w_t, it decreases when w_t increases, because h_{t+1} increases even more. Early fertility and human capital investment are therefore substitutes. As wages increase, the share of income available for raising children decreases, which discourages parents from having children. Indeed, above a certain wage threshold—and the associated level of human capital investment—early fertility vanishes. Households only combine investments in human capital with early and late parenthood at intermediate levels of income.

4.3 Postponement of parenthood with investment in human capital ($n_{1t} = 0$ and $h_{t+1} > 0$)

Finally, let us examine the case in which high incomes push young households to invest in human capital investment without having children.

Lemma 2 *Under Assumptions 1-2, when $w_t \geqslant \widehat{w}$:*

- h_{t+1}*, which is greater than* $\widehat{h} = W^{-1}(\widehat{w})$*, is an increasing function of* w_t*, with* $w_t = \widetilde{W}(h_{t+1})$*, and is a superior good;*
- $n_{1t} = 0$*.*

Proof. See Appendix C. ■

Above a certain level of labor income, young households prefer to postpone parenthood in favor of career development, reducing early fertility to zero. These greater investments in human capital ensure households have higher future incomes to devote to children rearing in late adulthood.

4.4 Total fertility

The above results can be combined to express the total number of children, m_t, households (young, at time t) choose to have in a lifetime:

$$\begin{aligned} m_t = n_{1,t} + n_{2t+1} &= h_{t+1}\left[\frac{\varepsilon\delta_2}{\phi_2(1+\beta+\delta_2)} - \frac{\delta_1}{\phi_1(1+\delta_1)w_t}\right] \\ &+ \frac{\delta_1}{\phi_1(1+\delta_1)}\left(1 - \frac{\phi_1\mu_1}{\delta_1}\right) + \frac{\delta_2 - \mu_2\phi_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \end{aligned} \tag{26}$$

Recall that h_{t+1} is increasing in w_t and note that the expression in brackets is also increasing in w_t but not always positive. Let us now define $\breve{w}$ with $\breve{w} < \frac{\delta_1\phi_2(1+\beta+\delta_2)}{\delta_2\varepsilon\phi_1(1+\delta_1)}$, the income threshold above which total fertility becomes increasing in w_t. First of all, if $\breve{w} < \underline{w}$, total fertility is always an increasing function of income, for all $w_t > \underline{w}$, therefore, the decrease in early fertility is completely offset by an increase in late fertility, which is inconsistent with empirical evidence. If instead $\breve{w} > \underline{w}$, total fertility has a U-shaped relationship with income, as shown in Figure 4. When $\underline{w} < w_t < \breve{w}$, the total number of children and human capital are substitutes. This result is similar to Iyigun's (2000) and total fertility decreases with income. But if $w_t > \breve{w}$, total fertility becomes an increasing function of income because the total number of children and human capital become complements. This implies that the increase in late fertility outweighs the decrease in early fertility. Furthermore, above a certain income level, $w_t > \hat{w}$, households only have children in late adulthood, and this late fertility increases with human capital and income.

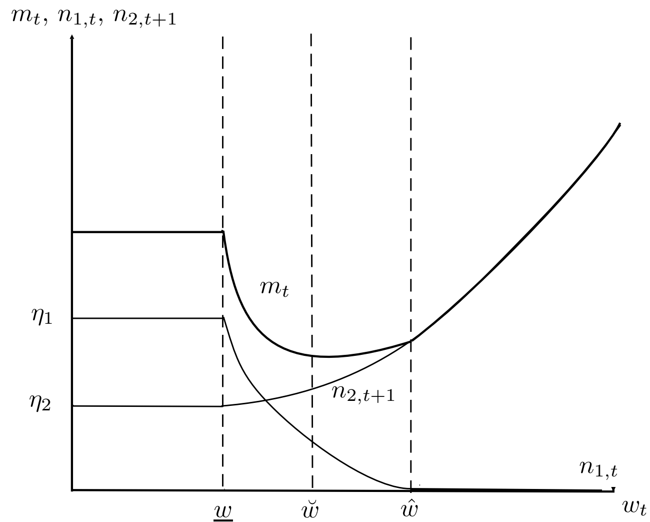

Figure 4: Total (lifetime) fertility as a function of wage levels.

5 Equilibrium analysis of the three different regimes

We start by defining an intertemporal equilibrium. We then investigate the existence, uniqueness and stability of the steady state in the three regimes associated with specific household choices.

5.1 Intertemporal equilibrium

The population size of the next generation equals the sum of early fertility weighted by the number of young adults plus late fertility weighted by the number of fertile older adults. The population dynamics as a function of time is therefore given by:

$$N_{t+1} = N_t n_{1t} + N_{t-1} n_{2t} \pi \tag{27}$$

If we denote by $n_t = N_{t+1}/N_t$, the population growth rate[9] can be expressed as follows:

$$n_t = n_{1t} + \frac{1}{n_{t-1}} n_{2t}\pi \tag{28}$$

Recall that K_t is the aggregate stock of physical capital used in the production process in period t. It is equal to the sum of the capital held by fertile and infertile households. The market clearing condition satisfies:

$$K_{t+1} = N_{t-1}[\pi k^F_{t+1} + (1-\pi)k^I_{t+1}] \tag{29}$$

As for the labor market, the market clearing condition can be written:

$$L_t = N_t(1-\phi_1 n_{1t}) + N_{t-1}(1+\varepsilon h_t - \pi\phi_2 n_{2t}) \tag{30}$$

while the wage $w_t = w(k_t)$ is given by equation (1). Combining equations (29) and (30) gives:

$$[n_t(1-\phi_1 n_{1t+1}) + 1 + \varepsilon h_{t+1} - \pi\phi_2 n_{2t+1}]k_{t+1} = \frac{1}{n_{t-1}}[\pi k^F_{t+1} + (1-\pi)k^I_{t+1}] \tag{31}$$

Then, substituting equation (19) into equation (17) yields:

$$k^F_{t+1} = \beta\frac{1+\varepsilon h_t + \phi_2\mu_2}{1+\beta+\delta_2} w_t \tag{32}$$

Finally, substituting equations (15) and (32) into equation (31) and using $k_t = [w_t/((1-\alpha)A)]^{1/\alpha}$ leads to:

$$\begin{aligned}[n_t(1 \quad - \quad & \phi_1 n_{1t+1}) + 1 + \varepsilon h_{t+1} - \pi\phi_2 n_{2t+1}]\left[\frac{w_{t+1}}{(1-\alpha)A}\right]^{1/\alpha} \\ &= \frac{w_t}{n_{t-1}}\left[\pi\beta\frac{1+\varepsilon h_t+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\varepsilon h_t)}{1+\beta}\right]\end{aligned} \tag{33}$$

Under Assumptions 1 and 2, equations (28) and (33) define an intertemporal equilibrium, with $n_{1t} = \max\{N_1(h_{t+1}/w_t), 0\}$, $n_{2t} = N_2(h_t)$ and $w_t \leqslant \underline{w}$, $w_t = W(h_{t+1}) \in (\underline{w}, \widehat{w})$ or $w_t = \widetilde{W}(h_{t+1}) \geqslant \widehat{w}$.[10]

There are three possible equilibrium regimes associated with the three household fertility scenarios and governed by the income level w_t:

[9]Note that the population growth rate differs from the total fertility rate, defined as the sum of early and late fertility at a given date ($\mathrm{TFR}_t = n_{1t} + n_{2t}$).

[10]The functions $W(h_{t+1})$ and $\widetilde{W}(h_{t+1})$ are defined in Appendix C.

1. If $w_t \leqslant \underline{w}$, *i.e.* at low income levels, $h_{t+1} = 0$, $n_{1t} = \eta_1$ and $n_{2t+1} = \eta_2$.
2. If $\underline{w} < w_t < \widehat{w}$, *i.e.* at intermediate income levels, $h_{t+1} = W^{-1}(w_t) > 0$, $n_{1t} = N_1(h_{t+1}/w_t) > 0$ and $n_{2t+1} = N_2(h_{t+1}) > 0$.
3. If $w_t \geqslant \widehat{w}$, *i.e.* at high income levels, $h_{t+1} = \widetilde{W}^{-1}(w_t) > 0$, $n_{1t} = 0$ and $n_{2t+1} = N_2(h_{t+1}) > 0$.

The first and third regimes are particularly interesting because they are polar opposites. In the first, low-income households prefer to have children in early adulthood rather than invest in human capital while in the third, high-income households prefer to postpone childbearing to invest in human capital. We investigate these two equilibria first before concentrating on the intermediate income regime.

5.2 Low income regime ($w_t \leqslant \underline{w}$)

When $w_t \leqslant \underline{w}$, equations (28) and (33) become:

$$n_t = \eta_1 + \frac{1}{n_{t-1}}\eta_2\pi \tag{34}$$

and

$$w_{t+1} = \frac{\widetilde{A}w_t^{\alpha}}{\{n_{t-1}[\eta_1(1-\phi_1\eta_1) + 1 - \pi\phi_2\eta_2] + \eta_2\pi(1-\phi_1\eta_1)\}^{\alpha}} \tag{35}$$

with $\widetilde{A} \equiv (1-\alpha)A\left[\pi\beta\frac{1+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta}{1+\beta}\right]^{\alpha}$. These two equations define a two-dimensional dynamic system with two predetermined variables, n_{t-1} and w_t. Indeed, w_t is a function of K_t/L_t, and it follows from equations (15), (29) and (32) along with $L_t = N_t(1-\phi_1\eta_1) + N_{t-1}(1-\pi\phi_2\eta_2)$ that K_t is also predetermined.

A steady state is reached when $n = n_a > 0$ solves $n^2 - n\eta_1 - \eta_2\pi = 0$. This solution is unique and given by:

$$n = n_a = \frac{1}{2}\left(\eta_1 + \sqrt{\eta_1^2 + 4\eta_2\pi}\right) \equiv I_{10}(w) \tag{36}$$

where $I_{10}(w)$ is a constant function. Using equation (34), equation (35) yields:

$$n = \frac{\eta_1\left(\pi\beta\frac{1+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta}{1+\beta}\right) + \pi\eta_2(1-\pi\phi_2\eta_2)\frac{w^{\frac{1-\alpha}{\alpha}}}{[A(1-\alpha)]^{\frac{1}{\alpha}}}}{\pi\beta\frac{1+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta}{1+\beta} - (1-\phi_1\eta_1)\pi\eta_2\frac{w^{\frac{1-\alpha}{\alpha}}}{[A(1-\alpha)]^{\frac{1}{\alpha}}}} \equiv I_{20}(w) \tag{37}$$

Note that $I'_{20}(w) > 0$, $I_{20}(0) = \eta_1 < n_a$. Moreover, $I_{20}(\underline{w}) > n_a$ if and only if $A < A_0$, with:

$$A_0 \equiv \frac{\underline{w}^{1-\alpha}\{n_a[\eta_1(1-\phi_1\eta_1) + 1 - \pi\phi_2\eta_2] + \eta_2\pi(1-\phi_1\eta_1)\}^\alpha}{(1-\alpha)\left[\pi\beta\frac{1+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta}{1+\beta}\right]^\alpha} \tag{38}$$

whereas $I_{20}(\underline{w}) < n_a$ if and only if $A > A_0$.

The conditions governing the existence of a steady state and its stability are summarized in the following proposition (see also Figures 5-7):

Proposition 1 *Let*

$$w_a = \frac{[(1-\alpha)A]^{\frac{1}{1-\alpha}}\left[\pi\beta\frac{1+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta}{1+\beta}\right]^{\frac{\alpha}{1-\alpha}}}{\{n_a[\eta_1(1-\phi_1\eta_1) + 1 - \pi\phi_2\eta_2] + \eta_2\pi(1-\phi_1\eta_1)\}^{\frac{\alpha}{1-\alpha}}} \tag{39}$$

Under Assumptions 1 and 2,

1. *If $A < A_0$, there exists a unique steady state, (w_a, n_a), with $0 < w_a < \underline{w}$ and $h = 0$. (w_a, n_a) is stable and convergent with oscillations.*

2. *If $A > A_0$, there is no steady state.*

Proof. See Appendix D. ■

This proposition shows that if incomes and productivity are low enough, the economy converges to a long-run equilibrium where young adults choose to have children rather than investing in their human capital. Since $h = 0$, the late fertility $n_2 = \eta_2$ is suppressed to a minimum. Individuals do not choose to postpone parenthood to develop their careers and increase future income. Furthermore, fluctuations are dampened because of population growth: this is why the long-run equilibrium is stable and the economy converges with oscillations. Indeed, there is a composition effect within the population growth factor, since fertile households have children in late adulthood ($\eta_2 > 0$). If the population growth factor is high in early adulthood, the population of older fertile adults born two periods ago is relatively low (with respect to the population of young adults), which automatically reduces the current number of children and leads to lower subsequent population growth. The opposite holds when population growth is low in the preceding period.[11]

[11] See Pestieau and Ponthière (2014) for a related result in terms of convergence.

5.3 High income regime ($w_t \geqslant \widehat{w}$)

Under Assumption 2, high levels of income, with $w_t \geqslant \widehat{w}$, imply $h_{t+1} > 0$, $n_{1t} = 0$ and $n_{2t+1} > 0$. As shown above, this also leads to $h_{t+1} \geqslant \widehat{h}$, with $\widetilde{W}(\widehat{h}) = \widehat{w}(= W(\widehat{h}))$. Therefore, when young adults postpone parenthood, they invest more in their human capital and earn more in late adulthood.

Because $n_{1t} = 0$ and $n_{2t} = N_2(h_t)$, equation (28) becomes:

$$n_t = \frac{1}{n_{t-1}} N_2(h_t)\pi \tag{40}$$

and equation (33):

$$\begin{aligned}
&[n_t + 1 + \varepsilon h_{t+1} - \pi\phi_2 N_2(h_{t+1})] \left[\frac{\widetilde{W}(h_{t+2})}{(1-\alpha)A}\right]^{1/\alpha} \\
&= \frac{\widetilde{W}(h_{t+1})}{n_{t-1}} \left[\pi\beta \frac{1+\varepsilon h_t + \phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\varepsilon h_t)}{1+\beta}\right]
\end{aligned} \tag{41}$$

where $\widetilde{W}(h_{t+1})$ is defined in Appendix C by equation (C.8). Equations (40), (41) and $w_t = \widetilde{W}(h_{t+1})$ define a three-dimensional dynamic system, (w_t, h_t, n_{t-1}), which governs the behavior of the economy and where h_t and n_{t-1} are predetermined at time t.

A steady state is reached when $n_{t-1} = n_t = n$ and $h_t = h_{t+1} = h_{t+2} = h$ solve equations (40) and (41). From equation (40):

$$n = \sqrt{N_2(h)\pi} \equiv I_1(h) \tag{42}$$

which is an increasing and concave function of h, given the properties of $N_2(h)$ (see equation (19)). Substituting equation (40) into equation (41), we obtain at the steady state:

$$n = \frac{\Omega_1(h)}{\Omega_2(h)} \equiv I_2(h) \tag{43}$$

with:

$$\Omega_1(h) \equiv \pi N_2(h) \frac{\widetilde{W}(h)^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} (1 + \varepsilon h - \pi\phi_2 N_2(h)) \tag{44}$$

$$\Omega_2(h) \equiv \pi\beta \frac{1+\varepsilon h + \phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\varepsilon h)}{1+\beta} - \pi N_2(h) \frac{\widetilde{W}(h)^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} \tag{45}$$

Let

$$A_1 \equiv \frac{\widetilde{W}(\widehat{h})^{1-\alpha}}{1-\alpha}\left[I_1(\widehat{h})\frac{I_1(\widehat{h})+1+\varepsilon\widehat{h}-\pi\phi_2 N_2(\widehat{h})}{\pi\beta\frac{1+\varepsilon\widehat{h}+\phi_2\mu_2}{1+\beta+\delta_2}+(1-\pi)\frac{\beta(1+\varepsilon\widehat{h})}{1+\beta}}\right]^{\alpha} \tag{46}$$

with $\overline{h}$ defined such that $\Omega_2(h) > 0$ for all $h < \overline{h}$. The existence of a unique and saddle-path stable steady state is defined in the following proposition:

Proposition 2 *Under Assumptions 1 and 2, with δ_2 sufficiently high and π close to 1:*

1. *If $A > A_1$, there exists a unique steady state $h_c \in (\widehat{h}, \overline{h})$ and $n_c = I_1(h_c) > 0$, with $w_c \equiv \widetilde{W}(h_c) > \widehat{w}$. This steady state is also a saddle point if ϕ_2 and α are low enough.*

2. *If A is significantly lower than A_1, there is no steady state for $w > \widehat{w}$.*[12]

Proof. See Appendix E. ■

The results of this proposition are depicted in Figures 5-7. Using $h = \widetilde{W}^{-1}(w)$, a steady state in this regime can be represented in the (w, n) plane as a solution (w_c, n_c) corresponding to the intersection of the following two curves: $n = I_1 \circ \widetilde{W}^{-1}(w) \equiv I_{1w}(w)$ and $n = I_2 \circ \widetilde{W}^{-1}(w) \equiv I_{2w}(w)$, where $I_{1w}(\widehat{w}) > I_{2w}(\widehat{w})$ if and only if $A > A_1$.

Proposition 2 shows that if productivity is high and the economy is sufficiently rich initially, *i.e.* if the initial conditions are such that $w_{t-1} \geqslant \widehat{w}$, it converges to the steady state (h_c, n_c, w_c). Therefore if incomes are high enough, young adults always postpone childbearing, investing instead in their human capital to increase their labor income in late adulthood, and the economy stays in this configuration in the long term.

5.4 Intermediate income regime ($\underline{w} < w_t < \widehat{w}$)

For intermediate levels of income, $\underline{w} < w_t < \widehat{w}$, it follows that $\widehat{h} > h_{t+1} > 0$, $n_{2t+1} = N_2(h_{t+1}) > 0$, which is increasing, and $n_{1t} = N_1[\widetilde{H}(h_{t+1})] \equiv$

[12]Note that when A is lower but close to A_1, the absence of a steady state cannot in principle be verified. Indeed, since $I_1(h)$ is increasing and concave and $I_2(h)$ is increasing and convex, there could be two steady states. We do not investigate this configuration further because it is less relevant to our research question.

$\widetilde{N}_1(h_{t+1}) > 0$, which is decreasing. Equation (28) then becomes:

$$n_t = \widetilde{N}_1(h_{t+1}) + \frac{1}{n_{t-1}} N_2(h_t)\pi \tag{47}$$

and equation (33):

$$\begin{aligned} [n_t(1 - \phi_1 \widetilde{N}_1(h_{t+2})) + 1 + \varepsilon h_{t+1} - \pi\phi_2 N_2(h_{t+1})] \left[\frac{W(h_{t+2})}{(1-\alpha)A}\right]^{1/\alpha} \\ = \frac{W(h_{t+1})}{n_{t-1}} \left[\pi\beta \frac{1+\varepsilon h_t + \phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\varepsilon h_t)}{1+\beta}\right] \end{aligned} \tag{48}$$

where $W(h_{t+1})$ is defined in Appendix C by equation (C.2). Equations (47) and (48) and $w_t = W(h_{t+1})$ define a three-dimensional dynamic system that governs the dynamics of (h_t, w_t, n_{t-1}), where h_t and n_{t-1} are predetermined at time t.

A steady state is reached when $n_{t-1} = n_t = n$ and $h_t = h_{t+1} = h_{t+2} = h$ solve equations (47) and (48). According to equation (47):

$$n = \frac{1}{2}\left(\widetilde{N}_1(h) + \sqrt{\widetilde{N}_1(h)^2 + 4N_2(h)\pi}\right) \equiv \widehat{I}_1(h) \tag{49}$$

and substituting equation (47) into equation (48) yields:

$$n = \frac{\widehat{\Omega}_1(h)}{\widehat{\Omega}_2(h)} \equiv \widehat{I}_2(h) \tag{50}$$

with

$$\begin{aligned} \widehat{\Omega}_1(h) \equiv\ & \pi N_2(h)\frac{W(h)^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}}(1+\epsilon h - \pi\phi_2 N_2(h)) \\ & + \widetilde{N}_1(h)\left[\pi\beta\frac{1+\epsilon h+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\epsilon h)}{1+\beta}\right] \end{aligned} \tag{51}$$

$$\begin{aligned} \widehat{\Omega}_2(h) \equiv\ & \pi\beta\frac{1+\epsilon h+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\epsilon h)}{1+\beta} \\ & - \pi N_2(h)\frac{W(h)^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}}(1-\phi_1\widetilde{N}_1(h)) \end{aligned} \tag{52}$$

Then, we claim that:

Lemma 3 *Under Assumptions 1 and 2, if δ_1 is sufficiently high, π is close to 1 and ϕ_1 is sufficiently low, $\widehat{I}_1(h)$ is decreasing and $\widehat{I}_2(h)$ is increasing for all $h \in (0, \widehat{h})$.*

Proof. See Appendix F. ■

Since we assume that δ_1 is sufficiently high and ϕ_1 sufficiently low, $\widehat{I}_1(h)$ is decreasing because the negative impact of h on fertility predominates. If for some reason,[13] the stationary value of h increases, the decrease in early fertility leads to reduced population growth despite the increase in late fertility.

Using $h = W^{-1}(w)$, we can define $\widehat{I}_{1w}(w) \equiv \widehat{I}_1(h) \circ W^{-1}(w)$, which is decreasing, and $\widehat{I}_{2w}(w) \equiv \widehat{I}_2(h) \circ W^{-1}(w)$, which is increasing. We further have $\widehat{I}_{1w}(\underline{w}) = I_{10}(\underline{w})$, $\widehat{I}_{1w}(\widehat{w}) = I_{1w}(\widehat{w})$, $\widehat{I}_{2w}(\underline{w}) = I_{20}(\underline{w})$, and $\widehat{I}_{2w}(\widehat{w}) = I_{2w}(\widehat{w})$. This means that $\widehat{I}_{1w}(\underline{w}) < \widehat{I}_{2w}(\underline{w})$ is equivalent to $A < A_0$ and $\widehat{I}_{1w}(\widehat{w}) > \widehat{I}_{2w}(\widehat{w})$ is equivalent to $A > A_1$. This allows us to deduce the following proposition (see also Figures 5-7)[14]:

Proposition 3 *Under Assumptions 1 and 2, if δ_1 is sufficiently high, π is close to 1 and ϕ_1 is sufficiently low:*

1. *If $A < A_0$ and $A > A_1$, there is no steady state for $w \in (\underline{w}, \widehat{w})$;*

2. *If $A_0 < A < A_1$, there is a unique steady state (w_b, n_b) with $w_b \in (\underline{w}, \widehat{w})$*

This proposition shows that in principle, the economy does not remain in this intermediate income regime in the long term if productivity, A, is too low or too high. For intermediate levels of productivity on the other hand, there is a long-run equilibrium with $h > 0$, $n_1 > 0$ and $n_2 > 0$, where households invest in human capital and partially postpone parenthood.[15]

6 What makes households choose or choose not to postpone parenthood?

We aim to identify the underlying factors driving one economy to converge toward a long-run equilibrium where young adults postpone parenthood, and

[13] Without modifying the expressions for $\widetilde{N}_1(h)$ and $N_2(h)$) of course.
[14] This analysis also immediately shows that $A_0 < A_1$.
[15] Continuity with respect to the configuration where $A > A_1$ or $A < A_0$ suggests that this steady state may also be a stable saddle point.

another to a steady state in which young households choose to have children at the expense of human capital investment. The preceding sections suggest that the productivity parameter A plays a key role.

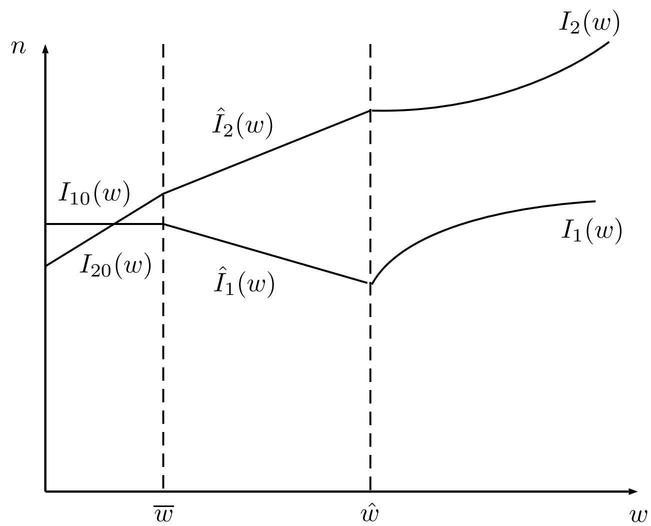

Figure 5: Total fertility as a function of income with $A < A_0$

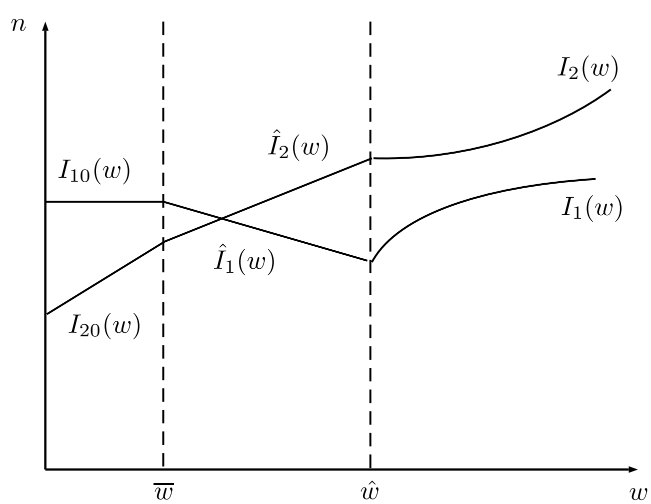

Figure 6: Total fertility as a function of income with $A_0 < A < A_1$

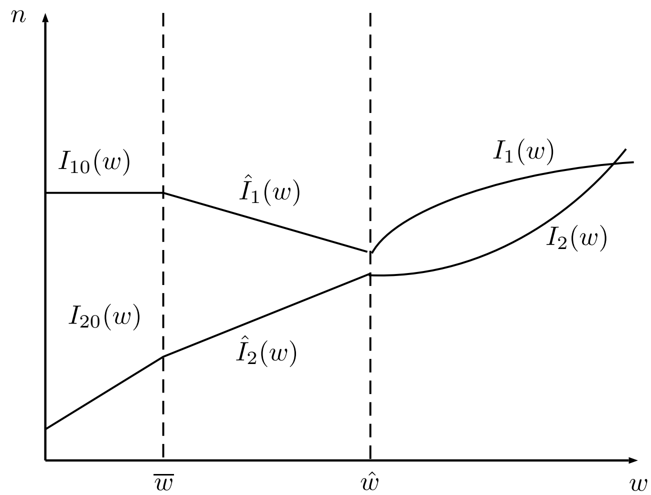

Figure 7: Total fertility as a function of income with $A > A_1$

It follows from Propositions 1-3 that:[16]

Proposition 4 *Under Assumptions 1 and 2, if δ_1 and δ_2 are sufficiently high, π is close to 1, and ϕ_1, ϕ_2 and α are low enough:*

1. *If $A < A_0$, a steady state (w_a, n_a) exists, with $w_a < \underline{w}$;*
2. *If $A \in (A_0, A_1)$, a steady state (w_b, n_b) exists, with $\underline{w} < w_b < \widehat{w}$;*
3. *If $A > A_1$, a steady state (w_c, n_c) exists, with $w_c > \widehat{w}$.*

The three different scenarios outlined in Proposition 4 are depicted in Figures 5-7. If productivity is low, the economy cannot converge to a long-run equilibrium with high wages where young adults postpone having children to increase their human capital. The only steady state involves low wages, zero investment in human capital, and maximal early fertility. With high productivity in contrast, the regime in which young households neglect investment in human capital is no longer stable. The economy might converge toward a long-term equilibrium with higher wages, partial or complete postponement

[16]Recall that for $A < A_1$, the existence of steady states such that $w > \widehat{w}$ cannot be excluded, especially if A is close to A_1, because $I_1(h)$ is concave and $I_2(h)$ convex. These scenarios are not the focus of the study and are not investigated further.

of childbearing, and investment in human capital to increase future labor income.

Another way to interpret this proposition is to consider a positive productivity shock, such that A passes above A_0. The steady state then shifts from the low to a higher income regime and fertility behaviors change. An economy initially with relatively high early fertility and no investment in human capital may finally end up in an equilibrium where young adults prefer to have children later in life after investing in their careers.

To further examine what happens following an increase in total factor productivity (A), note firstly that all the curves in Figures 5-7 ($I_{20}(w)$, $\widehat{I}_{2w}(w)$ and $I_{2w}(w)$) shift downward. Indeed, in the low income regime ($w < \underline{w}$), an increase in productivity induces an increase in saving, which increases capital and thus the equilibrium wage. Because wages remain below $\underline{w}$, investment in human capital is still neglected and the early and late fertility rates do not change. In the intermediate income regime ($\underline{w} < w < \widehat{w}$), an increase in productivity has a similar positive effect on the equilibrium wage, but this now affects fertility behaviors. Because early fertility and human capital are substitutes, early fertility decreases. And because late fertility and human capital are complements, late fertility increases. Total fertility decreases because under our assumptions, the first effect dominates. Finally, in the high income regime ($w > \widehat{w}$), any increase in productivity leads to an increase in late fertility (as in the intermediate regime), because late fertility and human capital are complements, and thus total fertility also increases because households do not have children in early adulthood.

These results highlight the importance of a general equilibrium framework in revealing the crucial role played by wages and by productivity. Any increase in productivity promotes capital accumulation, which leads to higher wages and higher per capita GDP, and to greater investments in human capital if $w > \underline{w}$. For $A_0 < A < A_1$, an increase in A leads to a long-run equilibrium with lower population growth, because early fertility is lower. In contrast, if $A > A_1$, an increase in productivity leads to a steady state with higher wages and higher GDP per capita, and larger population growth because late fertility increases. This corresponds basically to a fertility rebound.

These results also provide a theoretical explanation for the empirical facts described in Section 2. In particular, Proposition 4 accounts for the diversity of fertility trends in European countries over the past few decades (Frejka and Sobotka (2008); Myrskylä et al. (2009)). While northern and western

European countries form a so-called "high fertility belt", with relatively low early fertility and high late fertility rates supported by relatively high income levels, eastern and southern European countries continue to have relatively high early fertility and low late fertility, with lower total fertility rates. These trends match the features of the intermediate income equilibrium for southern and eastern Europe and of the high income equilibrium for northern and western Europe, albeit with non-zero early fertility. Our results are also consistent with the fertility rebound highlighted by Frejka and Sobotka (2008); Myrskylä et al. (2009); Sobotka (2017); Yakita (2018); Ohinata and Varvarigos (2020). In fact, this fertility rebound is a consequence of total fertility being higher at the high income equilibrium than at the intermediate equilibrium, due to the complementary nature of late fertility and career development.

7 The role of fertility decline

An important feature of our model is the possible failure of pregnancy attempts in late adulthood. Young adults who postpone childbearing risk being unable to have children in later life, with a probability $(1-\pi)$. However, π depends on several factors. It can be affected positively by health technologies, practices or knowledge, or negatively by social externalities such as pollution. In any case, variations in π alter the risk of being infertile and presumably, a lower risk should have a positive effect on population growth. We further investigate the effects of an increase in π on household investments in human and physical capital. We focus on the low and high income steady states, characterized respectively by zero investment in human capital and early fertility, and by postponement of parenthood and substantial investment in human capital.

In the low income steady state:

Proposition 5 *Under Assumption 1 and with $A < A_0$, an increase in π leads to an increase in n_a, while w_a decreases if ϕ_1 is low enough.*

Proof. See Appendix G. ■

On the one hand, population growth increases because the proportion of fertile households increases through an extensive margin effect: more households have children in late adulthood. On the other hand, the overall effect on wages, and therefore on the capital-labor ratio, depends on two mechanisms. First, an increase in the proportion of fertile households reduces savings since more households have to raise children. This is a negative effect of the cost of children. There is also a dilution effect due to the larger labor force. Indeed, when ϕ_1—the unit time cost per child—is low, early fertility and late fertility both increase, as mentioned before. The positive effect on population growth then outweighs the negative effect on the labor supply of older adults (because children are time consuming), which contributes to the increase in the labor force. These two mechanisms mean that wages and the capital-labor ratio both decrease when π increases. The low income equilibrium thus involves a larger but poorer population. In other words, the larger the risk of infertility is for households (the lower π is), the higher the capital-labor ratio (or GDP per hour worked) is, because households invest more in physical capital and work less.

To look further into how reproductive health interacts with career investment, we turn our attention to how the high income equilibrium is affected by variations in the risk of being infertile.

Proposition 6 *Under Assumptions 1 and 2, if $A > A_1$, δ_2 is sufficiently high, π is close to 1, and ϕ_2 and α are low enough, an increase in π leads to an increase in h_c and n_c and to a decrease in w_c.*

Proof. See Appendix H. ■

When the proportion of fertile households increases, population growth naturally accelerates. The high income levels mean that households only have children in late adulthood and the fertile share of the population increases. The effect on wages and thus on the capital-labor ratio is ultimately similar to the one in the low income scenario. An increase in the proportion of fertile older adults reduces savings because more adults have to cover the costs of raising children, and there is once again a dilution effect due to the larger labor force. Indeed, above a certain fertility rate, the positive effect of a higher share of fertile households on population growth outweighs the negative effect of reduced labor supply from older adults. Since the dilution effect predominates, wages and the capital-labor ratio decrease when π increases. However, the improved reproductive health of older adults also affects levels of investment in human capital. Adults with children consume less than infertile adults, so the expected marginal utility of consumption is higher when π is closer to one, hence the expected marginal benefit of investing in human capital increases. When the proportion of fertile households increases, the number and the total cost of children in late adulthood both increase. The increased human capital of older adults pushes up their income, allowing them to cover this additional cost more easily. Finally, this positive effect on human capital investment mitigates the negative effect of π on the capital-labor ratio by increasing levels of saving in late adulthood, since household incomes are higher, reinforced through the growth of the labor force. In any case, an increase in the risk of infertility (a decrease in π), implies a decrease in human capital investment but an increase in the capital-labor ratio and thus in GDP per hour worked. This is clearly due to the timing of investments: contrary to career investments, investments in physical capital are made once agents known whether they are fertile or not.

8 Conclusion

This paper investigates the relationship between fertility decisions and economic variables including earnings, productivity and human capital. In our model, households choose both how many children to have (the quantum of births) and when to have them (the tempo of births). It emerges that early fertility and human capital are substitutes, but that late fertility and human capital are complements, leading to two forms of long-run equilibrium. If productivity is low, incomes are lower and households have children in early adulthood instead of investing in their human capital. If productivity is high in contrast, incomes are higher and households choose to postpone childbearing to invest in human capital. Our analysis provides an explanation for the fertility rebound currently observed in Europe, which is driven by an increase in late fertility. Finally, reducing the risk of infertility in late adulthood encourages investment in human capital, as a means of covering the increased cost of raising children in late adulthood.

References

Adda, J., Dustmann, C., and Stevens, K. (2017). The career costs of children. *Journal of Political Economy*, 125(2):293–337.

Barro, R. J. and Becker, G. S. (1989). Fertility choice in a model of economic growth. *Econometrica: journal of the Econometric Society*, pages 481–501.

Baudin, T., de la Croix, D., and Gobbi, P. (2020). Endogenous childlessness and stages of development. *Journal of the European Economic Association*, 18(1):83–133.

Baudin, T., de La Croix, D., and Gobbi, P. E. (2015). Fertility and childlessness in the united states. *American Economic Review*, 105(6):1852–1882.

Bhattacharya, J. and Chakraborty, S. (2012). Fertility choice under child mortality and social norms. *Economics Letters*, 115(3):338–341.

Budig, M. J. and England, P. (2001). The wage penalty for motherhood. *American sociological review*, pages 204–225.

Caucutt, E. M., Guner, N., and Knowles, J. (2002). Why do women wait? matching, wage inequality, and the incentives for fertility delay. *Review of Economic Dynamics*, 5(4):815–855.

d'Albis, H., Augeraud-Véron, E., and Schubert, K. (2010). Demographic-economic equilibria when the age at motherhood is endogenous. *Journal of Mathematical Economics*, 46(6):1211–1221.

d'Albis, H., Greulich, A., and Ponthière, G. (2017). Education, labour, and the demographic consequences of birth postponement in europe. *Demographic Research*, 36:691–728.

d'Albis, H., Greulich, A., and Ponthière, G. (2018). Development, fertility and childbearing age: A unified growth theory. *Journal of Economic Theory*, 177:461–494.

de la Croix, D. and Pommeret, A. (2021). Childbearing postponement, its option value, and the biological clock. *Journal of Economic Theory*, 193:105–231.

Doepke, M., Hannusch, A., Kindermann, F., and Tertilt, M. (2022). The economics of fertility: A new era. Technical report, National Bureau of Economic Research.

Doepke, M. and Tertilt, M. (2016). Families in macroeconomics. In *Handbook of macroeconomics*, volume 2, pages 1789–1891. Elsevier.

Etner, J., Raffin, N., and Seegmuller, T. (2020). Reproductive health, fairness, and optimal policies. *Journal of Public Economic Theory*, 22(5):1213–1244.

Frejka, T. and Sobotka, T. (2008). Overview chapter 1: Fertility in europe: Diverse, delayed and below replacement. *Demographic research*, 19:15–46.

Galor, O. and Weil, D. N. (1996). The gender gap, fertility, and growth. *American Economic Review*, 86(3):374–387.

Galor, O. and Weil, D. N. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *American economic review*, 90(4):806–828.

Goldin, C. (2021). *Career and Family: Women's Century-Long Journey toward Equity*. Princeton University Press.

Herr, J. L. (2016). Measuring the effect of the timing of first birth on wages. *Journal of Population Economics*, 29(1):39–72.

Iyigun, M. F. (2000). Timing of childbearing and economic growth. *Journal of Development Economics*, 61(1):255–269.

Luci-Greulich, A. and Thévenon, O. (2014). Does economic advancement 'cause'a re-increase in fertility? an empirical analysis for oecd countries (1960–2007). *European Journal of Population*, 30(2):187–221.

Miller, A. R. (2011). The effects of motherhood timing on career path. *Journal of Population Economics*, 24(3):1071–1100.

Myrskylä, M., Kohler, H.-P., and Billari, F. C. (2009). Advances in development reverse fertility declines. *Nature*, 460(7256):741–743.

Nitsche, N. and Brückner, H. (2021). Late, but not too late? postponement of first birth among highly educated us women. *European Journal of Population*, 37(2):371–403.

Ohinata, A. and Varvarigos, D. (2020). Demographic transition and fertility rebound in economic development. *The Scandinavian Journal of Economics*, 122(4):1640–1670.

Olivetti, C. (2006). Changes in women's hours of market work: The role of returns to experience. *Review of Economic Dynamics*, 9(4):557–587.

Pestieau, P. and Ponthière, G. (2014). Optimal fertility along the life cycle. *Economic Theory*, 55(1):185–224.

Pestieau, P. and Ponthière, G. (2015). Optimal life-cycle fertility in a barro-becker economy. *Journal of Population Economics*, 28(1):45–87.

Sobotka, T. (2008). Overview chapter 6: The diverse faces of the second demographic transition in europe. *Demographic research*, 19:171–224.

Sobotka, T. (2017). Post-transitional fertility: childbearing postponement and the shift to low and unstable fertility levels. Technical report, Vienna Institute of Demography Working Papers.

Sobotka, T., Zeman, K., Lesthaeghe, R., Frejka, T., and Neels, K. (2011). Postponement and recuperation in cohort fertility: Austria, germany and switzerland in a european context. *Comparative Population Studies*, 36(2-3).

Sommer, K. (2016). Fertility choice in a life cycle model with idiosyncratic uninsurable earnings risk. *Journal of Monetary Economics*, 83:27–38.

Yakita, A. (2018). Female labor supply, fertility rebounds, and economic development. *Review of Development Economics*, 22(4):1667–1681.

Appendices

A Additional stylized facts

This section presents additional empirical data on fertility trends in Europe from 1950 to 2020. Figure 8 shows how ASFRs have evolved in four age groups over this period.

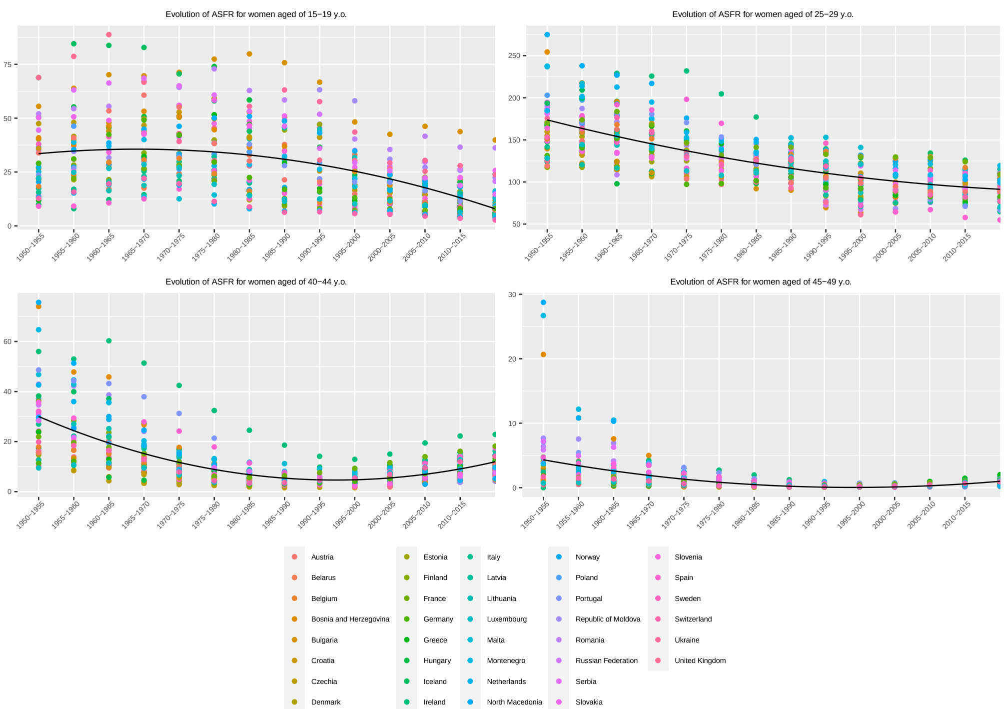

Figure 8: Evolution of age-specific fertility rates in Europe from 1950 to 2020

Those trends are consistent with those presented in the introduction: an uninterrupted decrease in early fertility but a uptick in late fertility.

B Empirical Analysis

The predicted values plotted in Figure 3 are derived from the following panel regression model:

$$Fert_{i,c,t} = \alpha_c + \beta \log GDP_{c,t} + \delta(\log GDP_{c,t})^2 + \varepsilon_{c,t} \tag{B.1}$$

where $Fert_{i,c}$ is a measure of fertility in country c, with i =ASFR 20–24, ASFR 30–34, ASFR 35–39, or TFR, α_c accounts for country fixed-effects and $\varepsilon_{c,t}$ is the error term. The results of the model are presented in the following table:

	ASFR 20–24	ASFR 30–34	ASFR 35–39	Total fertility
log gdp	4.84	-185.55^{***}	-145.12^{***}	-2.21^{***}
	(11.35)	(11.55)	(8.02)	(0.18)
log gdp^2	-2.05^{**}	10.23^{***}	7.93^{***}	0.10^{***}
	(0.62)	(0.63)	(0.44)	(0.01)
R^2	0.89	0.59	0.65	0.80
Adj. R^2	0.88	0.55	0.61	0.78
Num. obs.	199	199	199	199

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 1: Results of the panel regression

C Optimal level of investment in human capital

Proof of Lemma 1

Substituting equations (19) and (21) into (22) yields:

$$w_t = W(h_{t+1}) \equiv \frac{1}{1+\phi_1\mu_1} \left[\frac{(1+\beta)(1+\varepsilon h_{t+1}) + \phi_2\mu_2(1+\beta)}{(1+\beta+\delta_2\pi)(1+\varepsilon h_{t+1}) + (1-\pi)\phi_2\mu_2(1+\beta)}(1+\varepsilon h_{t+1})\frac{1+\delta_1}{\beta(1+\beta)\varepsilon} + h_{t+1}\right] \quad \text{(C.2)}$$

with $h_{t+1} < (1+\phi_1\mu_1)w_t \equiv \widetilde{h}(w_t)$ to ensure $c_{1t} > 0$. Equation (C.2) can be written as a second degree polynomial in $1+\varepsilon h_{t+1}$:

$$\begin{aligned}\Gamma(1+\varepsilon h_{t+1}) \;\equiv\;& (1+\varepsilon h_{t+1})^2[1+\delta_1+\beta(1+\beta+\delta_2\pi)] + (1+\varepsilon h_{t+1})[\phi_2\mu_2(1\\ &+\delta_1+\beta(1+\beta)(1-\pi)) - \beta(1+\beta+\delta_2\pi)(w_t\varepsilon(1+\phi_1\mu_1)\\ &+1)] - \beta(1+\beta)(w_t\varepsilon(1+\phi_1\mu_1)+1)(1-\pi)\phi_2\mu_2 = 0 \quad \text{(C.3)}\end{aligned}$$

Since $\Gamma(1) = \varepsilon\beta(1+\phi_1\mu_1)[2(1+\beta)+\delta_2\pi](\underline{w} - w_t) < 0$ and $\Gamma(1+\varepsilon\widetilde{h}(w_t)) > 0$, there is a unique solution h_{t+1} in $(0, \widetilde{h}(w_t))$. This solution defines an increasing function of w_t, because $\partial\Gamma(1+\varepsilon h_{t+1})/\partial h_{t+1} > 0$ and $\partial\Gamma(1+\varepsilon h_{t+1})/\partial w_t < 0$, implying that $w_t = W(h_{t+1})$ exists and is an increasing function. Note also that equation (C.2) is equivalent to:

$$\begin{aligned}&\frac{(1+\beta)(1+\varepsilon h_{t+1})+\phi_2\mu_2(1+\beta)}{(1+\beta+\delta_2\pi)(1+\varepsilon h_{t+1})+(1-\pi)\phi_2\mu_2(1+\beta)}\left(\frac{1}{h_{t+1}}+\varepsilon\right)\\ &=\frac{\beta(1+\beta)\varepsilon}{1+\delta_1}\left[\frac{w_t}{h_{t+1}}(1+\phi_1\mu_1)-1\right]\end{aligned} \tag{C.4}$$

Since the left-hand side of this equation is decreasing in h_{t+1} and the right-hand side is decreasing in h_{t+1}/w_t, this implicitly defines a function, $h_{t+1}/w_t = \widetilde{H}(h_{t+1})$, that is increasing in h_{t+1}. Since $h_{t+1} = W^{-1}(w_t)$ is an increasing function, h_{t+1}/w_t is also increasing in w_t, meaning that human capital is a superior good.

Using equation (21), we deduce that $n_{1t} = N_1[\widetilde{H}(h_{t+1})] \equiv \widetilde{N}_1(h_{t+1})$ is decreasing in h_{t+1}. Equation (C.2) shows that h_{t+1} tends to $+\infty$ if w_t tends to $+\infty$. Then, using equation (C.4) and L'Hospital's rule, we show that h_{t+1}/w_t tends to κ_∞ when h_{t+1} tends to $+\infty$, with:

$$\kappa_\infty \equiv \frac{\beta(1+\beta+\delta_2\pi)(1+\phi_1\mu_1)}{1+\delta_1+\beta(1+\beta+\delta_2\pi)} < \frac{\widetilde{h}(w_t)}{w_t} = 1+\phi_1\mu_1 \tag{C.5}$$

Using equations (21) and (C.5), this implies that n_{1t} tends to $n_{1\infty}$ when h_{t+1} tends to $+\infty$, with:

$$n_{1\infty} = \frac{\delta_1}{\phi_1(1+\delta_1)}\left[1-\frac{\phi_1\mu_1}{\delta_1}-\frac{\beta(1+\beta+\delta_2\pi)(1+\phi_1\mu_1)}{1+\delta_1+\beta(1+\beta+\delta_2\pi)}\right] \tag{C.6}$$

Finally, we can claim that $n_{1\infty} < 0$ if and only if:

$$\phi_1\mu_1 > \frac{\delta_1}{1+\beta(1+\beta+\delta_2\pi)} \tag{C.7}$$

If this last inequality is satisfied, there are finite values $\widehat{h}$ and $\widehat{w} = W(\widehat{h})(> \underline{w})$ such that $n_{1t} > 0$ for $w_t < \widehat{w}$ and $n_{1t} = 0$ for $w_t \geqslant \widehat{w}$.

Proof of Lemma 2

Under Assumption 2, $n_{1t} = 0$ because $w_t \geqslant \widehat{w}$. Equation (22) then becomes:

$$\begin{aligned}&w_t = \widetilde{W}(h_{t+1}) \equiv \\ &\frac{(1+\beta)(1+\varepsilon h_{t+1})+\phi_2\mu_2(1+\beta)}{(1+\beta+\delta_2\pi)(1+\varepsilon h_{t+1})+(1-\pi)\phi_2\mu_2(1+\beta)}\frac{1+\varepsilon h_{t+1}}{\beta(1+\beta)\varepsilon}+h_{t+1}\end{aligned} \tag{C.8}$$

Note that this equation is equivalent to equation (C.2) with $\delta_1 = \phi_1\mu_1 = 0$. Thus, the results of Lemma 1 apply, with $\delta_1 = \phi_1\mu_1 = 0$, $n_{2t+1} = N_2(h_{t+1})$ and $n_{1t} = 0$. In particular, $w_t = \widetilde{W}(h_{t+1})$ is a well-defined increasing function of h_{t+1} and h_{t+1} is a superior good.

D Proof of Proposition 1

The existence and uniqueness of the steady state (w_a, n_a) follow from the above analysis. Differentiating (34) and (35) in the neighborhood of the steady state (w_a, n_a), immediately shows that the two eigenvalues of the Jacobian matrix are given by the following elasticities:

$$\begin{aligned}\frac{dn_t}{n} &= -\frac{\frac{1}{n}\eta_2\pi}{\eta_1 + \frac{1}{n}\eta_2\pi}\frac{dn_{t-1}}{n} \\ \frac{dw_{t+1}}{w} &= \alpha\frac{dw_t}{w}\end{aligned}$$

This means that one eigenvalue belongs to $(-1, 0)$ and the other to $(0, 1)$. Hence, the steady state (w_a, n_a) is stable and convergent with oscillations.

E Proof of Proposition 2

E.1 Existence and uniqueness of the steady state

A steady state is reached when (n, h) solves $n = I_1(h) = I_2(h)$. Note also that $\widehat{h}$ can be defined by substituting $w_t/h_{t+1} = \delta_1/(\delta_1 - \phi_1\mu_1)$ into equation (C.4). It immediately follows that $\widehat{h}$ does not depend on A. Equations (19) and (C.8) show that neither $N_2(h)$ nor $\widetilde{W}(h)$ depend on A. Thus, we claim that $I_2(\widehat{h}) < I_1(\widehat{h})$ for $A > A_1$, where A_1 is given by equation (46).[17]

Note that $N_2(h)$ is a linear function of h and $\widetilde{W}(h)$ tends to linearity when π tends to 1. Indeed, if $\pi = 1$, from equation (C.8), we have:

$$\widetilde{W}(h) = \frac{(1+\beta)(1+\varepsilon h) + \phi_2\mu_2(1+\beta)}{(1+\beta+\delta_2)\beta(1+\beta)\varepsilon} + h \tag{E.9}$$

We also have:

$$1 + \varepsilon h - \pi\phi_2 N_2(h) = \frac{(1+\beta+\delta_2(1-\pi))(1+\varepsilon h) + \pi\phi_2\mu_2(1+\beta)}{1+\beta+\delta_2} \tag{E.10}$$

[17] Of course, $A > A_1$ ensures that $\Omega_2(\widehat{h}) > 0$.

This implies that $\Omega_1(h)$ is increasing and convex, and $\Omega_2(\widehat{h}) > 0$. If δ_2 is sufficiently high and π is close to 1, $\Omega_2(h)$ is decreasing. Indeed, from equations (19) and (45):

$$\Omega_2'(h) < \pi \frac{\beta\epsilon}{1+\beta+\delta_2} + (1-\pi)\frac{\beta\epsilon}{1+\beta} - \pi \frac{\widetilde{W}(\widehat{h})^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} \frac{\delta_2\epsilon}{\phi_2(1+\beta+\delta_2)}$$

which is negative for $\delta_2 \left[\frac{\pi}{\phi_2} \frac{\widetilde{W}(\widehat{h})^{\frac{1-\alpha}{\alpha}}}{[(1-\alpha)A]^{\frac{1}{\alpha}}} - (1-\pi)\frac{\beta}{1+\beta} \right] > \beta$. Moreover, $\Omega_2(h)$ is concave because $N_2(h)$ increases linearly with h and $\widetilde{W}(h)$ also increases with h.

Therefore, there is a value $\overline{h} > \widehat{h}$ for which $\Omega_2(h)$ is positive for $h \in (\widehat{h}, \overline{h})$ and tends to 0 when h tends to $\overline{h}$. This value $(\overline{h})$ is an an upper bound for h since $\Omega_2(h)$ would become negative if h were to become larger than $\overline{h}$. This also implies that $I_2(h)$ is increasing and convex.

Hence, we have $I_2(\overline{h}) = +\infty > I_1(\overline{h})$. Since $I_2(\widehat{h}) < I_1(\widehat{h})$ for $A > A_1$ and $I_1(h)$ is increasing and concave, we deduce that a unique steady state $h_c \in (\widehat{h}, \overline{h})$ exists, if π is close to 1 and δ_2 is sufficiently high. This also explains why there is no such steady state if A is substantially lower than A_1. Of course, this last conclusion may not hold if A is smaller but close to A_1 because $I_1(h)$ is concave and $I_2(h)$, convex. In this case, the existence of two steady states cannot be excluded, but this possibility is not investigated further.

E.2 Stability

Using $w_t = \widetilde{W}(h_{t+1})$, equations (C.8), (40) and (41) define a three-dimensional dynamic system governing the dynamics of (h_t, n_{t-1}, w_t), with two predetermined variables. To study the local stability of the steady state (h_c, n_c, w_c), we focus on the limit case where $\pi = 1$. By continuity, our result still holds when π is sufficiently close to 1. When $\pi = 1$, the dynamic system (C.8), (40) and (41) can be written:

$$h_{t+1} = \frac{w_t\beta\epsilon(1+\beta+\delta_2) - (1+\phi_2\mu_2)}{\epsilon[1+\beta(1+\beta+\delta_2)]} \tag{E.11}$$

$$n_t = \frac{1}{n_{t-1}} \frac{\delta_2(1+\varepsilon h_t) - \phi_2\mu_2(1+\beta)}{\phi_2(1+\beta+\delta_2)} \tag{E.12}$$

$$w_{t+1} = (1-\alpha)A \left\{ \frac{w_t\beta(1+\epsilon h_t+\phi_2\mu_2)[1+\beta(1+\beta+\delta_2)]}{D(h_t, n_{t-1}, w_t)} \right\}^{\alpha} \tag{E.13}$$

with:

$$D(h_t, n_{t-1}, w_t) \equiv [(1+\epsilon h_t)\delta_2/\phi_2 - \mu_2(1+\beta)][1+\beta(1+\beta+\delta_2)]$$
$$+n_{t-1}(1+\beta)[w_t\beta\epsilon(1+\beta+\delta_2)+(1+\phi_2\mu_2)\beta(1+\beta+\delta_2)] > 0 \quad \text{(E.14)}$$

Differentiating this dynamic system in the neighborhood of the steady state, we obtain:

$$\frac{dh_{t+1}}{h} = j_{13}\frac{dw_t}{w} \quad \text{(E.15)}$$
$$\frac{dn_t}{n} = j_{21}\frac{dh_t}{h} + j_{22}\frac{dn_{t-1}}{n} \quad \text{(E.16)}$$
$$\frac{dw_{t+1}}{w} = j_{31}\frac{dh_t}{h} + j_{32}\frac{dn_{t-1}}{n} + j_{33}\frac{dw_t}{w} \quad \text{(E.17)}$$

with

$$j_{13} = \frac{w_c\beta\epsilon(1+\beta+\delta_2)}{w_c\beta\epsilon(1+\beta+\delta_2)-(1+\phi_2\mu_2)} > 1 \quad \text{(E.18)}$$
$$j_{21} = \frac{\delta_2\epsilon h_c}{\delta_2(1+\varepsilon h_c)-\phi_2\mu_2(1+\beta)} \in (0,1)\ ,\ j_{22} = -1 \quad \text{(E.19)}$$
$$j_{31} = \alpha\left[\frac{\epsilon h_c}{1+\epsilon h_c+\phi_2\mu_2} - \frac{\epsilon h_c(1+\beta(1+\beta+\delta_2))\delta_2/\phi_2}{D(h_c,n_c,w_c)}\right] \quad \text{(E.20)}$$
$$j_{32} = -\alpha\frac{n_c(1+\beta)[w_c\beta\epsilon(1+\beta+\delta_2)+(1+\phi_2\mu_2)\beta(1+\beta+\delta_2)]}{D(h_c,n_c,w_c)} < 0 \quad \text{(E.21)}$$
$$j_{33} = \alpha\left[1 - \frac{n_c(1+\beta)w_c\beta\epsilon(1+\beta+\delta_2)}{D(h_c,n_c,w_c)}\right] \in (0,\alpha) \quad \text{(E.22)}$$

The characteristic polynomial of to this linearized system can be written $P(\lambda) \equiv \lambda^3 - T\lambda^2 + M\lambda - D = 0$, where the trace T, the sum of principal minors M and the determinant D of the associated Jacobian matrix are given by:

$$T = j_{22} + j_{33} = -(1-\alpha) - \alpha\frac{n_c(1+\beta)w_c\beta\epsilon(1+\beta+\delta_2)}{D(h_c,n_c,w_c)} < 0 \quad \text{(E.23)}$$
$$M = -j_{13}j_{31} - j_{33} \quad \text{(E.24)}$$
$$D = j_{13}(j_{21}j_{32} + j_{31}) \quad \text{(E.25)}$$

j_{31} can be expressed using equations (E.14) and (E.20):

$$j_{31} = \frac{\alpha\epsilon h_c}{(1+\epsilon h_c+\phi_2\mu_2)D(h_c,n_c,w_c)\phi_2}[\phi_2 n_c\beta(1+\beta)(w_c\epsilon+1+\phi_2\mu_2)(1$$
$$+\beta+\delta_2) - (\phi_2+1+\beta)(1+\beta(1+\beta+\delta_2))\mu_2\delta_2]$$

When ϕ_2 tends to 0, equation (45) shows that $\overline{h}$ remains finite, implying that this is also the case for h_c and w_c. We also know that $\phi_2 n_c$ tends to 0. We deduce that $j_{31} < 0$ if ϕ_2 is sufficiently low. This implies that $D < 0$ and $P(0) = -D > 0$. Moreover, (E.23)-(E.25), we get:

$$\begin{aligned} P(1) &= 1 - T + M - D = 2(1 - j_{33}) - j_{13}(j_{21}j_{32} + 2j_{31}) > 0 \\ P(-1) &= -1 - T - M - D = -j_{13}j_{21}j_{32} > 0 \end{aligned}$$

Since $P(+\infty) = +\infty$ and $P(-\infty) = -\infty$, this implies that one eigenvalue is negative and smaller than -1 ($\lambda_1 < -1$). Since $P(1) > 0$, $P(-1) > 0$ and $P(0) = -D > 0$, the two other eigenvalues, λ_2 and λ_3, have the same sign and are both either larger or smaller than 1 (in absolute value terms).

Using (E.19)-(E.21), we have $j_{21}j_{32} + j_{31} > j_{32} + j_{31} > -\alpha$. This means that $D > -\alpha j_{13}$. Under weak values of α, $\overline{h}$ remains finite, therefore so do h_c and j_{13}. We deduce that if α is sufficiently low, $D > -1$. Since $\lambda_1 < -1$, $\lambda_2\lambda_3 < 1$. If all the eigenvalues are real, λ_2 and λ_3 belong to $(-1, 1)$ and have the same sign. If λ_2 and λ_3 are complex conjugates, their absolute value is smaller than 1. Since the dynamic system is characterized by two predetermined variables, the steady state (h_c, n_c, w_c) is a saddle point.

F Proof of Lemma 3

Since $\widetilde{N}_1(h_{t+1}) = N_1(\widetilde{H}(h_{t+1}))$, it follows that $\widetilde{N}_1'(h_{t+1}) = N_1'(h_{t+1}/w_t)\widetilde{H}'(h_{t+1})$, where

$$N_1'\left(\frac{h_{t+1}}{w_t}\right) = -\frac{\delta_1}{\phi_1(1+\delta_1)} < 0 \tag{F.26}$$

and differentiating (C.4), $\widetilde{H}'(h_{t+1})$ satisfies:

$$\begin{aligned} &\widetilde{H}'(h_{t+1})\frac{\beta\epsilon(1+\phi_1\mu_1)}{1+\delta_1}w_t^2[(1+\beta+\delta_2\pi)(1+\epsilon h_{t+1}) + (1-\pi)\phi_2\mu_2(1+\beta)]^2 \\ &= (1+\epsilon h_{t+1})[(1+\epsilon h_{t+1})((1+\beta)(1+\pi\phi_2\mu_2) + \pi\delta_2(1+\phi_2\mu_2)) \\ &+2\phi_2\mu_2(1+\beta)(1-\pi)] + (\phi_2\mu_2)^2(1-\pi)(1+\beta) \end{aligned} \tag{F.27}$$

which implies that $\widetilde{H}'(h_{t+1}) > 0$.

Using (49), we obtain:

$$\widehat{I}_1'(h) = \frac{1}{2}\left(\widetilde{N}_1'(h) + \frac{2\widetilde{N}_1'(h)\widetilde{N}_1(h) + 4N_2'(h)\pi}{2\sqrt{\widetilde{N}_1(h)^2 + 4N_2(h)\pi}}\right) \tag{F.28}$$

The inequality $\widehat{I}_1'(h) < 0$ is satisfied for all $h \in (0, \widehat{h})$ if:

$$-\widetilde{N}_1'(h)\sqrt{N_2(h)\pi} > \pi N_2'(h) \tag{F.29}$$

Using (F.27), when π tends to 1, we have:

$$\widetilde{H}'(h) = \frac{1+\phi_2\mu_2}{1+\beta+\delta_2}\frac{1+\delta_1}{\beta\epsilon(1+\phi_1\mu_1)w^2} \tag{F.30}$$

Since $W(h)$ and $\widetilde{W}(h)$ are increasing, $w < \widetilde{W}(\overline{h})$, where $\widetilde{W}(\overline{h})$ does not depend on ϕ_1 (see the proof of Proposition 2).

Using (19), (F.26) and (F.30), inequality (F.29) is satisfied for $\pi = 1$ if:

$$\frac{\delta_1}{\phi_1(1+\phi_1\mu_1)} > \frac{\delta_2\beta\epsilon^2\widetilde{W}(\overline{h})^2}{\sqrt{\phi_2}(1+\phi_2\mu_2)}\sqrt{\frac{1+\beta+\delta_2}{\delta_2-\phi_2\mu_2(1+\beta)}} \tag{F.31}$$

which holds if ϕ_1 is low enough. This implies by continuity that $\widehat{I}_1'(h) < 0$ of ϕ_1 is sufficiently low and π is close to 1.

Let us focus now on $\widehat{I}_2(h)$. Note that $1 - \phi_1\widetilde{N}_1(h)$ is increasing in h. By direct inspection of equations (45) and (52), if $\Omega_2(h)$ is decreasing in h (see the proof of Proposition 2), this implies that $\widehat{\Omega}_2(h)$ is decreasing in h. Moreover,

$$\widehat{\Omega}_1'(h) \quad = \quad \Omega_1'(h) + X_1(h) \tag{F.32}$$

with $\Omega_1'(h) > 0$ (see the proof of Proposition 2) and:

$$\begin{aligned} X_1(h) \quad = \quad & \widetilde{N}_1'(h)\left[\pi\beta\frac{1+\epsilon h+\phi_2\mu_2}{1+\beta+\delta_2} + (1-\pi)\frac{\beta(1+\epsilon h)}{1+\beta}\right] \\ & +\widetilde{N}_1(h)\left[\pi\beta\frac{\epsilon}{1+\beta+\delta_2} + (1-\pi)\frac{\beta\epsilon}{1+\beta}\right] \end{aligned} \tag{F.33}$$

Let us consider in the following that π tends to 1. Note that $w_t > \underline{w}$, with:

$$\underline{w} = \frac{(1+\delta_1)(1+\phi_2\mu_2)}{\epsilon\beta(1+\phi_1\mu_1)(1+\beta+\delta_2)} \tag{F.34}$$

Using (21), (F.26), (F.30) and (F.34), equation (F.33) allows us to deduce that:

$$\begin{aligned} X_1(h) \quad > \quad & -\frac{\delta_1\beta(1+\phi_1\mu_1)(1+\epsilon h+\phi_2\mu_2)}{\phi_1(1+\delta_1)^2(1+\phi_2\mu_2)} \\ & +\frac{\delta_1\beta\epsilon}{\phi_1(1+\delta_1)(1+\beta+\delta_2)}\left(1-\frac{\phi_1\mu_1}{\delta_1}-\frac{h}{w}\right) \end{aligned} \tag{F.35}$$

From equation (C.4):

$$\frac{h}{w} = \frac{(1+\phi_1\mu_1)(1+\beta+\delta_2)\beta\epsilon h}{(1+\epsilon h+\phi_2\mu_2)(1+\delta_1)+\epsilon h\beta(1+\beta+\delta_2)} \tag{F.36}$$

Substituting this expression into inequality (F.35), we obtain:

$$X_1(h) > -\frac{\delta_1\beta(1+\phi_1\mu_1)(1+\epsilon h+\phi_2\mu_2)}{\phi_1(1+\delta_1)^2(1+\phi_2\mu_2)}$$
$$+\frac{\delta_1\beta\epsilon}{\phi_1(1+\beta+\delta_2)}\frac{(\delta_1-\phi_1\mu_1)(1+\phi_2\mu_2)+\epsilon h[\delta_1-\phi_1\mu_1(1+\beta(1+\beta+\delta_2))]}{(1+\epsilon h+\phi_2\mu_2)(1+\delta_1)+\epsilon h\beta(1+\beta+\delta_2)}$$

This last inequality is strictly positive if:

$$\frac{\epsilon(1+\delta_1)^2}{1+\beta+\delta_2}\frac{\delta_1(1+\phi_2\mu_2+\epsilon h)-\phi_1\mu_1[1+\phi_2\mu_2+\epsilon h(1+\beta(1+\beta+\delta_2))]}{(1+\epsilon h+\phi_2\mu_2)(1+\delta_1)+\epsilon h\beta(1+\beta+\delta_2)}$$
$$> \frac{(1+\phi_1\mu_1)(1+\epsilon h+\phi_2\mu_2)}{1+\phi_2\mu_2}$$

Since h is finite, this implies that there is a lower bound $\underline{\delta}_1$ such that this last inequality is satisfied if $\delta_1 > \underline{\delta}_1$ and ϕ_1 is low enough. By continuity, if π is lower but close to 1, δ_1 is sufficiently high and ϕ_1 is sufficiently low, $X_1(h) > 0$, implying that $\widehat{\Omega}_1'(h) > 0$. Since $\widehat{\Omega}_2(h)$ is decreasing, we deduce that $\widehat{I}_2'(h) > 0$.

Therefore, if π is close to 1, δ_1 is high enough and ϕ_1 is sufficiently small, $\widehat{I}_1(h)$ is decreasing and $\widehat{I}_2(h)$ is increasing for all $h \in (0, \widehat{h})$.

G Proof of Proposition 5

Since η_1 and η_2 do not depend on π, inspecting equation (36) shows that n_a increases with the proportion of fertile households, π. Using (39), we deduce that the elasticity of w_a with respect to π has the same sign as:

$$-\frac{\phi_2\frac{\beta}{1+\beta}}{\pi\beta\frac{1+\phi_2\mu_2}{1+\beta+\delta_2}+(1-\pi)\frac{\beta}{1+\beta}}+\frac{\phi_2}{n_a(1-\phi_1\eta_1)+1-\pi\phi_2\eta_2}$$
$$-\frac{2n_a(1-\phi_1\eta_1)+1-\pi\phi_2\eta_2}{n_a(2n_a-\eta_1)[n_a(1-\phi_1\eta_1)+1-\pi\phi_2\eta_2]}$$

Under Assumption 1, the first term is lower than $-\phi_2$. The second term is lower than ϕ_2 if:

$$n_a(1-\phi_1\eta_1)+1-\pi\phi_2\eta_2 > \frac{(\delta_1-\phi_1\mu_1)(1-\phi_1\mu_1)}{\phi_1(1+\delta_1)^2}+1-\pi\frac{\delta_2-\phi_2\mu_2(1+\beta)}{1+\beta+\delta_2} > 1$$

which is satisfied if ϕ_1 is sufficiently low. In this a case, w_a decreases with π.

H Proof of Proposition 6

Equation (42) shows that $I_1(h)$ increases with h and π. Equation (C.8) yields:

$$\frac{\partial\widetilde{W}(h)}{\partial\pi} = -\frac{\delta_2(1+\varepsilon h)-\phi_2\mu_2(1+\beta)}{(1+\beta+\delta_2\pi)(1+\varepsilon h)+(1-\pi)\phi_2\mu_2(1+\beta)}\left(\widetilde{W}(h)-h\right) < 0 \tag{H.37}$$

This implies that $\Omega_1(h)/\pi$ decreases with π and from equations (45) and (H.37), we have:

$$\begin{aligned}\frac{\partial(\Omega_2(h)/\pi)}{\partial\pi} &= -\frac{1}{\pi^2}\frac{\beta(1+\epsilon h)}{1+\beta}\\ &+\frac{N_2(h)}{[(1-\alpha)A]^{1/\alpha}}\frac{1-\alpha}{\alpha}\widetilde{W}(h)^{\frac{1-2\alpha}{\alpha}}\frac{[\delta_2(1+\varepsilon h)-\phi_2\mu_2(1+\beta)]\left(\widetilde{W}(h)-h\right)}{(1+\beta+\delta_2\pi)(1+\varepsilon h)+(1-\pi)\phi_2\mu_2(1+\beta)}\end{aligned}$$

Under the assumptions of Proposition 2, in particular that δ_2 be sufficiently high, π is close to 1 and ϕ_2 is sufficiently low, α can be set low enough for the above expression to be positive, taking into account that $h \in (\widehat{h}, \overline{h})$ remains finite. In this case, for a given value of h, $I_2(h)$ decreases with π. Since $I_2'(h_c) > I_1'(h_c)$, we deduce that:

$$\frac{dh_c}{d\pi} = \frac{\partial I_1(h_c)/\partial\pi - \partial I_2(h_c)/\partial\pi}{I_2'(h_c)-I_1'(h_c)} > 0$$

and $dn_c/d\pi > 0$ because $n_c = \sqrt{N_2(h_c)\pi}$.

Using equations (19) and (C.8), equation (41) can be rewritten:

$$\begin{aligned}&\left[n_t+\frac{(1+\beta+\delta_2(1-\pi))(1+\varepsilon h_{t+1})+\pi\phi_2\mu_2(1+\beta)}{1+\beta+\delta_2}\right]\left[\frac{w_{t+1}}{(1-\alpha)A}\right]^{1/\alpha}\\ &=\frac{w_t}{n_{t-1}}\beta\frac{(1+\beta+\delta_2(1-\pi))(1+\varepsilon h_t)+\pi\phi_2\mu_2(1+\beta)}{(1+\beta)(1+\beta+\delta_2)}\end{aligned}$$

At steady state, using (42), this equation becomes:

$$\varpi(h,\pi) \equiv \left[\frac{w^{1-\alpha}}{(1-\alpha)A}\right]^{1/\alpha} \frac{1+\beta}{\beta}$$
$$= \frac{(1+\beta+\delta_2(1-\pi))(1+\varepsilon h)+\pi\phi_2\mu_2(1+\beta)}{\pi N_2(h)(1+\beta+\delta_2)+\sqrt{\pi N_2(h)}[(1+\beta+\delta_2(1-\pi))(1+\varepsilon h)+\pi\phi_2\mu_2(1+\beta)]}$$

where $N_2(h)$ is given by equation (19). This leads to $\partial\varpi(h,\pi)/\partial\pi < 0$ and, if ϕ_2 is low and δ_2 is high, $\partial\varpi(h,\pi)/\partial h < 0$. We deduce from previous results that w_c decreases with π.