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Abstract

Recurring statistical issues such as censoring, selection and heteroskedasticity often impact the analysis of observational data. We investigate the potential advantages of models based on quantile regression (QR) for addressing these issues, with a particular focus on willingness to pay-type data. We gather analytical arguments showing how QR can tackle these issues. We show by means of a Monte Carlo experiment how censored QR (CQR)-based methods perform compared to standard models. We empirically contrast four models on flood risk data. Our findings confirm that selection-censored models based on QR are useful for simultaneously tackling issues often present in observational data.

Keywords: Censored Quantile Regression; Contingent Valuation; Flood; Monte Carlo Experiment; Quantile Regression; Selection Model; Willingness to Pay

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1 Introduction

Empirical research in agricultural and resource economics frequently relies on observational data, given that experimental data may either be unavailable for ethical or practical reasons or incompatible with the issue considered. Nature-based data, like quantities or numbers of events, are obtained through the relevant monitoring networks. Human-induced data rely on individual choices and are obtained through surveys, economic markets, or non-market valuation methods that elicit willingness to pay (WTP) for various non-market goods and services. Whatever the origin of these data, they are then analyzed using statistical models to help inform private or public decision-makers by identifying the determinants of the dependent variable of interest.

Effective modeling, however, needs to take several statistical issues into account. First, there is the treatment of censored data, from below or above, for prices, quantities or numbers of events. Second, the impact of individuals' characteristics on the dependent variable is potentially heterogeneous, which may bias estimates. Third, the variance of the variable of interest is likely to be non-independent of some determinants, such as size or income, which generates heteroskedasticity. Fourth, there can be outliers and/or extremely large values related to natural or human factors. Fifth, the selection (or self-selection) issue can bias the results and invalidate statistical inference if it is not the result of a random process. Quantile regression (QR)-based models, which estimate the impact of explanatory variables on conditional quantiles, can help tackle these issues. Yet despite the importance of observational data (and non-market valuation studies), and the increasing use of quantile-based methods, we lack a systematic analysis of their performances in presence of censoring and selection.

We propose to fill this gap by comparing the performances of censored QR (CQR) models and standard censored models, both with and without selection. First, we gather analytical arguments showing how QR-based models can tackle the above-mentioned statistical issues associated with observational data, with a particular focus on WTP-type data. Second, we carry out a Monte Carlo (MC) experiment especially designed to include heterogeneity, censoring and selection in order to compare the statistical performances of the four models in a controlled framework. Finally, we apply these methods to real data from a French contingent valuation (CV) survey on reducing risks associated with flooding.

Our results confirm the advantages of selection censored models w.r.t. standard censored estimates for analyzing observational data. The MC experiment shows that they are less impacted by heteroskedasticity, less biased and more efficient. However, results are mixed on the improvements from CQR estimates over standard censored estimates. Applying models with selection to the flood risk survey confirms their superiority in accounting for motivations for participating in the valuation exercise, while CQR models reveal a strong heterogeneity of coefficients across the conditional distribution.

Our paper contributes to the methodological literature on the treatment of censored observational data (including those elicited with non-market valuation methods) in two main ways. First, CQR has been used in various applications for years to account for null data either observed (like vegetable demand; Gustavsen and Rickertsen, 2006; local precipitation; Friederichs and Hense, 2007; agricultural surface areas; Motamed, McPhail, and Williams, 2016) or revealed (like food safety; Lagerkvist and Okello, 2016). All find heterogeneity in the relationships between the dependent variable and some of the explanatory variables, confirming the interest of using QR-type approaches over standard approaches. However, although the selection issue is likely to arise, only one article accounts for it in QR modeling (Arellano and Bonhomme, 2017a; to explain wage inequality). Second, despite their numerous advantages, QR applications on choice modeling using WTP as dependent variable are still rare, although on the increase. One article alone applies CQR to CV data with zero WTP and finds significant effects (in particular for income) on WTP, whereas Tobit-like models indicate non-significant effects (Krishnamurthy and Kriström, 2016). However, the article does not distinguish valid zero WTP from non valid (i.e. protest) zero WTP. Protest behavior is typically a self-selection process that needs to be accounted for, but we are not aware of any WTP study that has considered it in connection with QR methods. Our study helps fill these gaps by comparing the properties of the four models (censored and CQR, with and without selection), first through an MC experiment and then on real WTP data. Incidentally, we also propose numerical improvements to Arellano and Bonhomme (Arellano and Bonhomme, 2017a) estimator for the CQR model with selection, that allows computation time to be divided by a factor of one hundred on average without significant lack of precision.

2 Quantile regression analysis of censored data with selection

The reasons why (C)QR may perform better than standard models in tackling issues arising from censored data with selection are given below, followed by a specific focus on WTP data.

2.1 Advantages of QR-based methods for observational data

First, QR-based methods are only one of several statistical methods (like nonparametric estimations, latent class, hybrid or random parameter models) that allow for heterogeneity in the coefficients. Each coefficient of a QR corresponds to the coefficient of a regression in which an explanatory variable interacts with an unobserved latent variable that influences the position of observations in the conditional distribution of the dependent variable. It therefore offers a more comprehensive view of the relationship between the dependent variable and the covariates, since the covariates are allowed to have a different impact at each quantile of the conditional distribution of the dependent variable, not only at the mean (Alsayed et al., 2020). This can be useful: for instance, in their CV study, it is only through QR that Furno, Verneau, and Sannino (2016) manage to detect the effect of hypothetical bias on the tails of the distribution. Although QR accounts for unobserved heterogeneity, it is not a total substitute for latent class, hybrid or random parameter models, which account for preference heterogeneity (Ben-Akiva et al., 2002; Boxall and Adamowicz, 2002; Nahuelhual, Loureiro, and Loomis, 2004). The source of the heterogeneity that QR accounts for is, by definition, unobserved, and various sources may be at play.

Second, QR is able to capture the scale shift effects due to heteroskedasticity and reveals how they affect the marginal effects at a given conditional quantile of the dependent variable. Therefore, QR allows us to interpret heteroskedasticity as a special case of heterogeneity. For instance, the scale effect of size or income leads to heterogeneity of the size or income coefficient along the conditional distribution of the dependent variable. In addition, QRs are more robust than ordinary least squares (OLS) regressions to the presence of outliers, fat tails and to non-normal errors (Powell, 1986; O'Garra and Mourato, 2006; Huang and Chen, 2015). Moreover, although OLS is more efficient than QR when the errors are homoskedastic and normally distributed (according to the Gauss Markov theorem), empirical evidence suggests that QR tends to provide more efficient estimates if these assumptions are not met (Hung, Shang, and Wang, 2010; Deaton, 2018).

Third, the presence of censoring of the dependent variable is frequent in observational data, and occurs from below, above or both; and in 0 or any other value. Examples are food or energy consumption, agricultural or natural resource production, meteorological or physical measurements, or limits for insurance coverage, durations or capacities. This censoring induces non-linearities in the relationship between the dependent variable and the explanatory variables. OLS regressions are known to be biased in this case (Amemiya, 1984), calling for the use of specific models (like the Tobit model when censoring is from below in zero) to properly account for censoring. Although QRs have several advantages over OLS regressions, they also lead to biased estimates, so that CQRs still need to be used to properly deal with censoring (Powell, 1986; Kowalski, 2016).

Fourth, (self-) selection is often an issue in empirical applications if statistical inference is based on non-random samples. If the sample is not random with respect to observable and unobservable characteristics, ignoring selection leads to biased estimates and invalidates inference (Heckman, 1979). One solution to the selection issue is using multiple imputation for non-selected observations, which Pennington, Gomes, and Donaldson (2017) find outperformed the Heckman selection model in an MC experiment. But the gold standard in the literature remains the Heckman model, which considers sample selection as a type of omitted-variable bias and adjusts the OLS regression for the probability of being selected in the sample. Recently, Arellano and Bonhomme (2017b) review recent proposals dealing with sample selection in QR models and Arellano and Bonhomme (2017a) propose a model that can be adapted to account for censoring as well.

The main difficulty in applying QR-based models is interpreting the coefficients. (C)QR results express how the coefficients vary, not along the marginal distribution, but rather along the conditional distribution. Hence, the estimates capture the marginal effect of each observed characteristic on the specific quantile of the dependent variable conditional on these characteristics: it is not heterogeneity across the distribution of the dependent variable that is accounted for, but heterogeneity across the distribution of the unobserved determinants of the dependent variable.[1]

2.2 The case of WTP-type data

Stated preference methods used to elicit WTP rely on hypothetical surveys that ask for the fraction of the disposable income that compensates for the change in utility proposed in a hypothetical scenario. WTP-type data perfectly match the observational data framework above and would also benefit from CQR-based models for several reasons.

First, self-selection is due to non-response behavior in the elicitation step, when respondents either refuse to give a WTP or give a WTP that does not correspond to their genuine value for the good proposed. Non-response may be a reaction to:

- the principle of the interview (Chen and Qi, 2018) or of a policy intervention (Wu, Chen, and Liou, 2017), for ethical reasons for instance;
- the subject of the survey: the respondent may feel that it should be someone else's responsibility (Pennington, Gomes, and Donaldson, 2017), or may have extensive knowledge of / interest in the subject and thus judge the information given to be not accurate enough for him/her to set a WTP;
- the methodology / characteristics of the survey (Meyerhoff and Liebe, 2008): the payment vehicle (Chen and Qi, 2018), the way it is conducted, or insufficient information on the good;
- strategic behavior like free-riding or ensuring provision of the good (Strazzera et al., 2003; Wu, Chen, and Liou, 2017; Chen and Qi, 2018).

The first set of reasons is individual-specific, and is founded on beliefs or behaviors unrelated to the survey itself. Whatever its topic, non-response would represent a protest response from these individuals. The second set is good-specific: if the same survey

[1]Some interpretations consider, for instance, that respondents with high WTP are more likely to take into account their resources, attitudes and behaviors, whereas respondents with low WTP may be more affected by budget constraints than by individual preferences (O'Garra and Mourato, 2006; Viscusi, Huber, and Bell, 2012). In our opinion, the heterogeneity of the marginal effects should not be interpreted as heterogeneity across income groups or preference groups. Resources and preferences are observed (although imperfectly for preferences), while the heterogeneity revealed by (C)QR is heterogeneity across the unobserved determinants of the dependent variable.

setting featured another good, these respondents might give a WTP. The third set of reasons is survey-specific: in a different setting, these respondents might express a different (and potentially positive) WTP for the same good. The fourth set is likely to result from combined individual, good and survey characteristics: this combination will lead to WTP lower or higher than the genuine WTP. The reasons underlying non- response are generally identified through debriefing questions after the valuation task, or through attitudinal questions to all respondents.[2] Consequently, when non-response arises, the statistical modeling of WTP data should account for the self-selection process, as for other observational data.

Second, censoring is from below and in zero, leading to WTP referred to as "valid" null WTP that does not correspond to non-response behavior. There are three main reasons: a null change in the utility function of the respondent, so that the corresponding compensating income is truly null; a positive change in the utility function, but for which the budget constraint of the respondent does not allow the expression of a positive WTP; a negative change in the utility function, for which the WTP elicitation format does not allow the expression of the corresponding negative WTP.

While most of the goods assessed with stated preference methods are very unlikely to induce a decrease in utility, there are several exceptions, like road speed reductions (Scarpa and Willis, 2006), closure of an airport or protection of privately owned forest (Kriström, 1997), preservation of some wildlife species, like wolves (Broberg and Brännlund, 2008), or development of wind farms (Gudding et al., 2018). Consequently, there is the question of whether negative WTP should be allowed in the statistical modeling (Carson and Hanemann, 2005). If yes, zero WTPs stand as negative WTPs that truly represent a negative utility change but are censored at zero due to an observability issue, and the underlying distributional assumptions of the Tobit model are fulfilled (Sigelman and Zeng, 1999). If no, zero WTPs are strictly null WTP, i.e. 'corner solutions' under an economic rationale that could be modeled separately (a budget constraint or change in utility associated with the good). Although it is standard practice in the literature to consider 'censoring' and 'corner solution' as strictly equivalent (see Krishnamurthy and Kriström, 2016, footnote 3 for instance), the interpretation of the results of a censored regression differs (see the censored model section below).

Third, the relationships between non-zero WTP and its determinants are usually heteroskedastic, especially regarding the income variable. As income increases, WTP is less and less bounded by the budget constraint and the heterogeneity of preferences leads to greater dispersion of WTP values. This kind of feature has already been observed, see for instance the Engel Curves between income and food expenditure in Koenker (2005).

[2]It is indeed admitted that non-answer behavior and the expression of a valid WTP both involve economic and non-economic factors. Hidden behind the former may be positive WTP and behind the latter, motivations other than purely economic (Meyerhoff and Liebe, 2006; Grammatikopoulou and Olsen, 2013; Chen and Qi, 2018).

Fourth, WTP from CV studies may contain many low and/or very high WTPs because no actual out-of-pocket payment is required for the provision of the hypothetical good or service and because the valuation task is difficult. This will hinder the estimation of conditional mean WTP, because strong influence from the upper tail of the WTP distribution potentially leads to mean and median WTPs that significantly differ from each other.

Overall, the following will focus on censored and CQR models, and will explicitly take account of the selection issue in the modeling.

3 Statistical models and estimation procedures

We present the models used both in the MC experiment and the empirical application, relying on conditional means and on conditional quantiles for the specification accounting for censoring and the specification accounting for both selection and censoring.

To cover various types of observational data, the dependent variable is considered continuous, but the statistical models we propose can be applied to non continuous data, with a bivariate Probit or a binary quantile regression with selection for instance (see also Strazzera et al., 2003; for a double-bounded dichotomous choice model with selection) or with a quantile count model (Harding and Lamarche, 2019).

3.1 Models with censoring

3.1.1 Censored model

A **censored model** that accounts for censoring of the dependent variable can be written as:

$$(1) \qquad \begin{cases} Y_i^* = x_i'\beta + e_i \\ \qquad\qquad Y_i = Y_i^* \quad \text{if} \quad Y_i^* > c \\ \qquad\qquad Y_i = c \quad\;\; \text{if} \quad Y_i^* \leq c \end{cases}$$

where Y_i^* is a latent variable (and Y_i its observed counterpart) corresponding to the genuine unobserved Y_i^u, x_i is a matrix of explanatory variables, β a vector of parameters, $e_i \sim N(0, \sigma_e^2)$ is a random term, and c the censoring point from below (the case of censoring from above is straightforward). Under the parametric assumption $Y_i^* \sim N(x_i'\beta, \sigma_e^2)$, the likelihood function of this model is:

$$(2) \qquad L(\beta, \sigma; Y_i, x_i) = \prod_{i=1}^{n} \left(\frac{1}{\sigma_e} \phi \left(\frac{Y_i - x_i'\beta}{\sigma_e} \right) \right)^{I(Y_i > c)} \left(\Phi \left(\frac{c - x_i'\beta}{\sigma_e} \right) \right)^{I(Y_i = c)}$$

with $\phi(.)$ the probability density function of the standard Normal distribution, $\Phi(.)$ the cumulative distribution function (cdf) and $I(.)$ an indicator variable.

Although this model is simple to implement, it is sensitive to incorrect assumptions regarding the error term distribution like homoskedasticity and normality (Wooldridge, 2010). In addition, as mentioned before, this model assumes that the latent variable Y^* corresponding to the genuine unobserved Y^u can be negative, which is not true for consumption, production, quantities, prices, or a CV scenario that rules out negative WTP. Although this issue is often neglected in empirical applications, appropriate procedures would use models based on naturally non-negative distribution like Poisson- lognormal- or Weibull-based regressions (Sigelman and Zeng, 1999; Bateman et al., 2002) and, when 0 corresponds to a 'corner solution', would explicitly model the corresponding decision that leads to censoring (Maddala, 1992; Wooldridge, 2010). Moreover, interpretation of the parameters (and computation of marginal effects) are impacted by whether the censoring is due to data observability or to a 'corner solution'. In the former case, the coefficient provides the effect of the corresponding explanatory variables, whereas in the latter it is a non-linear function of the coefficients (Wooldridge, 2010; Chernozhukov, Fernández-Val, and Kowalski, 2015).

3.1.2 Censored Quantile Regression model

Following Koenker (2005)'s presentation of the **conditional QR model**, the conditional distribution of a random variable Y is denoted $F_{Y|X}(Y|x)$, where X is a set of random explanatory variables. The conditional quantile Q_τ is defined as:

$$Q_\tau(Y|x) = \inf(e : F_{Y|X}(e|x) \geq \tau) = F^{-1}(\tau|x) \tag{3}$$

The **CQR model** was introduced by Powell (1986), and one of its main advantages lies in avoiding the strong parametric assumptions of the Tobit model. The censoring is dealt with using the property of equivariance of quantile functions to monotonic transformations (like censoring). This means, for a linear τ-quantile function in x, and a fixed censoring point c:

$$Q_\tau(Y_i|x_i) = Q_\tau(\max(Y_i^*, c)|x_i) = \max(Q_\tau(Y_i^*|x_i), c) = \max(c, x_i'\beta_\tau) \tag{4}$$

where β_τ is a vector of k parameters associated with the τ-quantile. The CQR estimator of β_τ for a random sample $(Y_i, x_i)_{i=1,\ldots,n}$ is obtained by solving:

$$\min_{\beta_\tau} \sum_{i=1}^{n} \rho_\tau(Y_i - \max(c, x_i'\beta_\tau)) \tag{5}$$

where ρ_τ is the check function defined by:

$$\rho_\tau(u) = u(\tau - I(u < c)) \tag{6}$$

For a given explanatory variable x_k, we interpret the coefficient $\beta_{\tau,k}$ as the change in the quantile of order τ of the conditional distribution for a marginal change in x_k. The

CQR model allows for coefficients' heterogeneity, and Powell (1986) shows that, under some regularity conditions, it is consistent and asymptotically normal whatever the error distribution, which is not true for censored models. As previously, the nature of the censoring (observability or 'corner solution') would affect the computation (Chernozhukov, Fernández-Val, and Kowalski, 2015; Kowalski, 2016). In addition, the interpretation of the partial derivative often requires considerable care as an individual may fall into a different conditional quantile after a marginal change in an explanatory variable, especially when discrete (Koenker, 2005; Cameron and Trivedi, 2010).

Computational difficulties have long prevented an extensive use of the CQR model, but improvements in estimation methods are making things easier. Fitzenberger (1997) state that most algorithms for CQR tend to perform poorly with high censoring and small sample sizes (less than 1000), an issue for observational-type and non-market valuation data. However, Chernozhukov and Hong (2002) propose an estimator and compare it with that proposed by Buchinsky and Hahn (1998) and the Powell (1986) estimator, under a specification with high censoring (45%) and low sample sizes (n=100, 400). Their results show that their algorithm perform better in terms of root-mean-square error (RMSE) and similarly in terms of mean bias (MB). Koenker (2008) review and compare some recent CQR estimators that allow for random censoring (Portnoy, 2003; Peng and Huang, 2008) to the Powell (1986) estimator (with the Fitzenberger, 1997's algorithm). He shows that for fixed censoring (the relevant issue in valuation surveys) these estimators do not perform better than the older Powell (1986) estimator. Overall, the Chernozhukov and Hong (2002) algorithm used with the Powell (1986) estimator is certainly the best existing estimator for observational or WTP-type data, because it performs relatively well in presence of both high censoring and small sample sizes.

3.2 Models with selection and censoring

Because participation may be non-random in the sample with respect to individuals' characteristics, the latter need to be taken into account to derive valid inference on the genuine unobserved Y^u based on observed Y. We thus present below two models that account for selection and censoring.

3.2.1 Censored model with selection

We consider a two-step process in which observations have to be selected before the (censored) value of the outcome is observed. The first step - participation - is based on technical or logical constraints, or relies on an individual decision (like consuming a good or deciding to reveal one's WTP). The second step decides the amount of the outcome (crop surface area; Qualls et al., 2012) or of the WTP (Alvarez-Farizo et al., 1999; Strazzera et al., 2003; Cho et al., 2008).

The first step - participation - can be written as:

$$
\begin{cases} P_i^* = z_i'\alpha + u_i \\ \qquad\qquad P_i = 1 \quad \text{if} \quad P_i^* > 0 \\ \qquad\qquad P_i = 0 \quad \text{if} \quad P_i^* \leq 0 \end{cases} \tag{7}
$$

where P_i^* is a latent variable corresponding to the decision to participate (and P_i its observed counterpart), z_i is a matrix of explanatory variables explaining participation, α a vector of parameters, and $u_i \sim N(0, \sigma_u^2)$ is a random term.

The second step - Y observation - can be written as a **censored model**, but is now conditional on participation:

$$
\begin{cases} Y_i^* = x_i'\beta + e_i \\ \qquad Y_i = Y_i^* & \text{if} \quad Y_i^* > c \quad \text{and} \quad P_i \ = 1 \\ \qquad Y_i = c & \text{if} \quad Y_i^* \leq c \quad \text{and} \quad P_i \ = 1 \\ \qquad Y_i \text{ not observed} & \text{if} \quad P_i = 0 \end{cases} \tag{8}
$$

We estimate this process as a Heckman selection model with a censored Y equation, assuming a correlation between the two error terms u_i and e_i. We choose a bivariate normal distribution $N_2(z_i'\alpha, x_i'\beta; \Sigma)$ where Σ is the variance covariance matrix of (u_i, e_i) with covariance $\rho\sigma_e$ and $\sigma_u^2 \equiv 1$ (a standard assumption, see Wooldridge, 2010):

$$
\begin{pmatrix} 1 & \rho\sigma_e \\ \rho\sigma_e & \sigma_e^2 \end{pmatrix}
$$

Under these parametric assumptions, the likelihood function of this process is:

$$
\begin{aligned} L(.) = \prod_{P=0} (1 - \Phi(z_i'\alpha)) \times \prod_{P=1, Y=c} \Phi_2(z_i'\alpha, -x_i'\beta/\sigma_e; -\rho) \\ \times \prod_{P=1, Y>c} \left(\frac{1}{\sigma_e} \phi \left(\frac{Y_i - x_i'\beta}{\sigma_e} \right) \right) \times \Phi \left(\frac{z_i'\alpha + \rho(Y_i - x_i'\beta)/\sigma_e}{(1-\rho^2)^{0.5}} \right) \end{aligned} \tag{9}
$$

with $\Phi_2(., .; \rho)$ the cdf of a standard bivariate Normal distribution with correlation ρ. Alternative types of modeling, for use if Y^* cannot be negative, have already been proposed in the censored model.

The estimation procedure recommends that some z are not in x (known as exclusion restriction), to guarantee the identification of all parameters although the non-linearities of the inverse Mills ratio are technically sufficient (Wooldridge, 2010).

3.2.2 CQR model with selection

We adopt the recent approach correcting for sample selection in QR models proposed by Arellano and Bonhomme (2017a), adapted to account for censoring.

The rationale of the first step - participation equation P - and the second step - observation equation Y - as well as notations are similar to the censored model with selection (see equations (7-8)). We assume however that $z = (w, x)$ strictly contains x as in Arellano and Bonhomme (2017a), where w stands for a set of explanatory variables, which constitutes a stronger assumption than the standard exclusion restriction in selection models. We define (u, e) as following a bivariate normal distribution, and model it using a specification with uniform standard marginals and rectangular support.

Let $p(z) = \Pr(P = 1|z)$ be the corresponding propensity score, and assume $\Pr(p(z) > 0) = 1$. With QR models, it is more convenient to use a parametric copula for the bivariate specification, because the copula is equivariant to monotonic transformations, like quantile transformations. We assume as in Arellano and Bonhomme (2017a) that the unconditional copula of (u, e) is indexed by a parameter vector θ. For example, using the normal specification, the unconditional copula of (u, e) may be defined as:

$$C_{u,e}(\tau, p; \theta) = \Phi_2(\Phi^{-1}(\tau), \Phi^{-1}(p); \theta) \tag{10}$$

where Φ_2 is the bivariate normal cdf and Φ^{-1} is the inverse of the standard normal cdf. It follows that we can define the conditional copula $G_x(\cdot,\cdot;\cdot)$ allowing us to compute the next conditional probability (the key to the method) as:

$$\begin{aligned} \Pr(Y^* \leq x'\beta(\tau) | P = 1, Z = z) &= \Pr(u \leq \tau | e \leq p(Z), Z = z) \\ &= C_{u,e}(\tau, p(z); \theta)/p(z) \equiv G_x(\tau, p(z); \theta) \end{aligned} \tag{11}$$

Let $F_{u,e|X}(y, v) \equiv F_X(y, v)$ be the conditional cdf. Suppose that F_X is strictly increasing with respect to u, and that conditional cdf $F_{Y^*|X}(y, x)$ and its inverse are strictly increasing. We assume the τ-quantile of outcomes of participants given z to be linear, and define the censored conditional quantile model as:

$$Q_\tau(\max(Y^*, c)|P = 1, z) = F^{-1}_{\max(Y^*,c)|P=1,z}(\tau, z) = x'\beta_{G_x^{-1}(\tau, p(z,\alpha);\theta)} \tag{12}$$

Estimation is in three steps, partly relying on Arellano and Bonhomme (2017a)'s procedure, and partly on numerical improvements introduced to divide the computation time by about 100.

Arellano and Bonhomme (2017a)'s procedure

Let (Y_i, P_i, z_i), $i = 1, ..., N$, be an i.i.d. sample, with $z_i \equiv (w_i, x_i)$, which strictly contains x, so w are the excluded covariates.

1st step: Maximum likelihood. Arellano and Bonhomme (2017a) use a maximum likelihood approach to obtain $\widehat{\alpha}$, a consistent estimate of the vector of parameters of the participation equation, α. It will be used to define the empirical propensity score that will feature as an instrument for the Generalized Method of Moments (GMM) estimation in the next step:

$$\widehat{p}(z, \alpha) = \widehat{\Pr}(P = 1|z) = \Phi(z'\widehat{\alpha}) \tag{13}$$

2nd step: Grid search method. They use the GMM applied to the empirical version of the theoretical conditional moment restriction:

$$\mathbf{E}[\mathbb{1}(Y \leq x'\beta) - G_x(\tau, \Phi\left(z'\alpha\right); \theta) | P = 1, Z = z] = 0 \tag{14}$$

with $\mathbb{1}(.)$ the indicator function, in order to obtain a consistent estimator of θ, the copula parameter vector of (u, e):

$$\widehat{\theta} = \underset{d \in D}{\arg\min} \left\| \sum_{i=1}^{N} \sum_{l=1}^{L} P_i p(z_i, \widehat{\alpha}) \left[\mathbb{1}\left(Y_i \leq x_i' \widehat{\beta}_{\tilde{\tau}_l}(d) \right) - G_x(\tilde{\tau}_l, p(z_i, \widehat{\alpha}); d) \right] \right\|_2 \tag{15}$$

where $\tilde{\tau}_1 < \tilde{\tau}_2 < ... < \tilde{\tau}_L$ is a finite grid on $(0, 1)$, $|| \cdot ||_2$ is the Euclidean norm, $G_x(\cdot,\cdot;\cdot)$ the conditional copula that measures the dependence between e and u, which is the source of sample selection bias, $p(z_i, \widehat{\alpha})$ are instrument functions with $\dim(p(z_i, \widehat{\alpha})) \geq \dim(\theta)$, and

(16)

$$\widehat{\beta}_{\tilde{\tau}_l}(d) = \underset{b \in B}{\arg\min} \sum_{i=1}^{N} P_i \left[G_x(\tilde{\tau}_l, p(z_i, \widehat{\alpha}); d) \left(Y_i - x_i' b\right)^{+} - G_x(\tilde{\tau}_l, p(z_i, \widehat{\alpha}); d) \left(Y_i - x_i' b\right)^{-} \right]$$

where $\widehat{\beta}_{\tilde{\tau}_l}(d)$ is the CQR coefficient associated with quantile $\tilde{\tau}_l$, $(Y_i - x_i' b)^+ = max(Y_i - x_i' b, 0)$ and $(Y_i - x_i' b)^- = max(x_i' b - Y_i, 0)$.

3rd step: Rotated Quantile Regression. Given $\widehat{\alpha}$ and $\widehat{\theta}$, for any given $\tau \in (0, 1)$ they compute $\widehat{\beta}_\tau$, a consistent estimator of the τ-th quantile regression coefficient:

$$\widehat{\beta}_\tau = \underset{b \in B}{\arg\min} \sum_{i=1}^{N} [\tau(Y_i^* - x_i'\beta)^+ + (1 - \tau)(Y_i^* - x_i'\beta)^-] \tag{17}$$

Here τ is replaced by the selection-corrected individual-specific percentile rank.

Improvements to Arellano and Bonhomme (2017a)'s procedure

Because of the MC experiments, it is crucial to shorten computation time, especially in the second step which accounts for most of it (Arellano and Bonhomme, 2017b). First, we optimize the code by compiling the most computation-intensive part. Then, we replace the gridsearch method of Arellano and Bonhomme (2017a) and estimate $\widehat{\theta}$ by a minimizer algorithm, the estimate being denoted by $\hat{\theta}_m$. The objective function for estimating $\widehat{\theta}$ is not continuous, due to the presence of the indicator function $\mathbb{1}(.)$, which makes it generally non-convex. In practice, when we are in low-dimensional θ (i.e., $\dim(\theta) < 3$, in our case $\dim(\theta) = 1$), the grid search method may be preferable, which is why the initial GMM criterion is built on a double grid with respect to both θ and τ_l, the corresponding percentile rank. However, increasing the density of the grid on τ_l leads to a function locally convex on its support and even globally convex in almost all cases. We use this as a trade-off between precision and computation time and estimate $\hat{\theta}_m$ by maximum likelihood without significant lack of precision w.r.t. a grid search method with a very fine grid on θ (see Figure 1 for an illustration of the change in computation time for two grid densities).

4 Monte Carlo experiment

To characterize the empirical properties of the four models, we carry out an MC experiment especially designed to include censoring, selection and heterogeneity (modeled via a heteroskedastic error term). Specifications that explicitly express the heterogeneity of the coefficients as a function of the quantile (for instance $\beta(\tau) = exp(\tau)$) are possible (Hoshino, 2013), but their interpretation would be less intuitive.

4.1 Design of the Monte Carlo experiment

The data-generating process (DGP) consists of a participation equation, and a linear specification with a censored dependent variable Y_i, that can also be interpreted as a WTP. The dependence between the two equations is accounted for through the correlation parameter ρ of the bivariate distribution of the error term, and the censoring point is set to 0 without loss of generality.

The participation equation is:

$$P_i^* = \alpha_0 + \alpha_1 z_{i,1} + \alpha_2 z_{i,2} + u_i \tag{18}$$

where:

- $z_{i,1}$ is a standard log-normal continuous variable $lnN(0,1)$ that stands for the income variable, for instance.
- $z_{i,2}$ is a standard normal variable $N(0,1)$.
- u_i is i.i.d standard normal.

The outcome equation is:

$$Y_i = \max(\beta_0 + \beta_1 z_{i,1} + \beta_2 x_{i,2} + \beta_3 x_{i,3}^2 + e_i, 0) \tag{19}$$

where:

- $z_{i,1}$ is the same variable as in the DGP equation, with a location shift effect on Y. A scale effect is also accounted for in the error term e_i below: respondents with higher $z_{i,1}$ are also likely to have higher variance in Y.[3]
- $x_{i,2}$ is a standard normal variable $N(0,1)$.
- $x_{i,3}$ is a standard uniform variable $U(0,1)$. This variable is squared in the DGP to allow for non-linearities (Fan and Liu, 2016).

[3]In the revealed and stated preference frameworks, this corresponds to an income effect on WTP: respondents with high incomes are more likely to have higher WTPs.

- e_i is an error term that covers two different heteroskedasticity intensities j ($j = 0, 1$):

$$e_i = (1 + \gamma_j z_{i,1})v_i \tag{20}$$

where v_i is i.i.d standard normal, $\gamma_0 = 0$ and $\gamma_1 = 0.8$.
The first case corresponds to homoskedasticity, while the second produces linear heteroskedasticity in $z_{i,1}$, mimicking the scale effect of income.

Finally, the variance covariance matrix that accounts for the bivariate normal distribution between the error terms of the participation and the outcome equations is:

$$\begin{pmatrix} 1 & \rho\sigma_e \\ \rho\sigma_e & \sigma_e^2 \end{pmatrix}$$

where correlation ρ stands for independence (0) and positive dependence (.5); negative dependence is not accounted for since it is obviously symmetric with positive dependence. The heteroskedasticity of the error term leads to heterogeneity in the relationship between the quantiles of the conditional Y distribution and the covariates. The marginal effect of a covariate on the quantile covariates affects both the location and the scale of the dependent variable:

$$\frac{\partial Q_\tau(Y|z_1)}{\partial z_1} = \beta_1 + \gamma_j F_u^{-1}(\tau) \tag{21}$$

where $F_u^{-1}(\tau)$ is the inverse cdf (i.e. quantile function) of the error term distribution.

We use several specifications for the DGP, varying the sample size n (300 and 1000), the participation rate p and the censoring rate c. We set α_1=2 (to mimic the positive relationship between income and participation; Meyerhoff, Morkbak, and Olsen, 2014), and α_2=-2 (to mimic a negative relationship with participation). We set the participation rate p based on α_0 (100% when α_0=6 and 80% when α_0=-0.2).

We finally set β_1=2 (to mimic a positive relationship with Y, potentially between income and WTP), β_2=-2 (to mimic a negative relationship with Y), and β_3=1 (to mimic a positive quadratic relationship with Y), and control for the censoring rate c based on β_0 (0% when β_0=5 and 40% when β_0=-2).

Overall, we estimate 4 models - Tobit without selection and with (Tobit-S), CQR without selection and with (CQR-S) - and for each of them, we simulate 5000 samples for each of the 2 sample sizes x 2 heteroskedasticity intensities x 2 censoring rates x 2 correlations x 2 participation rates = 32 specifications. We use the three dimensional vector $[0.25, 0.50,\ 0.75]$ to analyze the conditional quantile estimations.

For the CQR model, computations are performed with R and the quantreg (Koenker, 2015) and AER (Kleiber and Zeileis, 2008) packages (codes available upon request). The codes are adapted from the `est_cqrl` function available in the `Counterfactual` package for R.

For the CQR model with selection, the optimization of the code leads to a two-thirds reduction in computation time. Then, we look for the best trade-off regarding the grid density on τ_l and the convexity of the objective function (15). A few trials show that, given the local convexity of the resulting objective function for estimating $\widehat{\theta}$, a denser grid on θ than the default value found in the Matlab algorithm proposed by Arellano and Bonhomme (2017a) (i.e. 0.02 instead of 0.1) is sufficient for a minimizer algorithm to be used and to $\hat{\theta}_m$. This leads to a computation time 15 to 30 times shorter (depending on the sample size in the MC specifications) than the grid search method. Overall, for the MC experiment, we estimate that computation time is divided by about 100 on average. In practice, the best trade-off needs to be determined, because the stepsize for the grids with respect to θ and τ_l may be very sensitive to the parametric model used for the two equations for P and Y respectively.

4.2 Results of the Monte Carlo experiment

The experiment provides extensive information, but we focus on the MB and the RMSE of the outcome equation (see details in Appendix A for reviewers' use only). Our aim here is to determine whether the models can recover the outcome's DGP when there are potential selection and censoring issues which would affect marginal effects (whatever they are) as well as mean and median Y. Table 1 shows these statistics for the slope coefficients, the 2 participation rates, the 2 heterogeneity intensities, the two correlations ρ, censoring and $n = 1000$ and censoring c=40%, but we also comment on the results not shown (see all the specifications in Appendix B for reviewers' use only). To make results clearer, cells are highlighted in gray when the MB differs by less than 10% from the true parameter. For RMSE, cells are highlighted in gray when the RMSE is less than .1, as a rule-of-thumb measurement of the error of each model (see Ferrini and Scarpa, 2007; for similar choices).

[PLEASE INSERT TABLE 1 ABOUT HERE]

Note first that we find a negative MB for β_1 and β_3, and a positive MB for β_2, up to 10% for the former and up to 40% for the two latter. Whatever the specifications, MB is generally larger for the models not accounting for selection than those accounting for it, as expected. RMSE values are also higher in the models without selection. Finally, regarding the two CQR models, we generally find higher RMSE and MB for the 25% and 75% quantiles. This can be explained by two phenomena. Due to censoring, we have less information at the bottom of the conditional distribution, which causes a loss in efficiency (in particular for the 25% quantile). In addition, we are considering MB on coefficients, not on marginal effects, which differ a lot in presence of heteroskedasticity for quantiles other than the 50% quantile (see equation (21)) and depend on the type of censoring.

Let us now consider the effect of sample size (see the specifications with n = 300 in Appendix B for reviewers' use only). As expected, there is no significant change in MB, and RMSE decreases with sample size for all models and all specifications (except for the 75% quantile in both CQR models). Regarding censoring (see Appendix B), we observe consistently that it increases MB and RMSE for all models and all slope coefficients, which was also expected. This effect is stronger with the two models that do not account for selection.

We now turn to the effect of heteroskedasticity on MB and RMSE (see Table 1) for n = 1000 and a censoring rate of 40%. We observe an increase in MB for β_1 and β_2, more marked in the two CQR models, and a less clear increase in β_3. RMSE is consistently found to increase slightly with heteroskedasticity, whatever the specifications, the models and the slope coefficients.

Finally, let us consider the impact of introducing selection. MB and RMSE slightly decrease *ceteris paribus* in the Tobit and CQR models when selection is introduced. This is not observed in the Tobit-S and CQR-S models, where a slight tendency to increase is sometimes found. Note however that both MB and RMSE remain much lower in the selection models than in the non-selection models.

Overall, accounting for selection in the models has an undeniable impact: the Tobit-S and CQR-S models perform better than their counterparts whatever the specifications. In presence of selection, Tobit-S seems to perform better than CQR-S, in terms of both MB and RMSE. These results obviously hold with any type of data supporting censoring, selection and heteroskedasticity, but the next section compares the four models to actual data from a CV study on flood risk.

5 Empirical application

We use data from a CV survey administered in Southeastern France between 26 April and 30 June 2012 via individual face-to-face interviews with respondents both having and not having experience of floods. Decreasing the impact of floods is a major public concern (Reynaud and Nguyen, 2016) and catastrophic river risings or flash floods regularly hit the front pages.

5.1 Method and data

5.1.1 Study design

The questionnaire included eight modules (housing, risk perception, hypothetical monetary choices, personality, Post Traumatic Stress Disorder (PTSD), flood-specific issues, socio-demographic factors and CV scenario). However, we only present in detail the findings relevant to this article: respondents' WTP to reduce their vulnerability and exposure to flooding.

A scenario was proposed to determine respondents' willingness to contribute to the funding of city-level protective devices to assess both the tangible and intangible / psychological gains from prevention and any the corresponding WTP (see Appendix C for the exact wording).[4]

The payment vehicle is a voluntary contribution to a Flood Management Fund. After careful consideration of the various existing elicitation formats, we used the circular payment card (CPC) for its overall performance (Chanel, Makhloufi, and Abu-Zaineh, 2017; Champonnois, Chanel, and Makhloufi, 2018).[5]

5.1.2 Data

The empirical analysis is based on a sample of 599 respondents interviewed at home face-to-face by a specialized survey institute. Four municipalities in Southeastern France were chosen for their varying degrees of exposure to flood risk: at no risk of flooding, at potential risk of flooding, flash-flooded 20 years before the survey (37 deaths and four missing), and two years before the survey (23 deaths, 12 missing). The respondents interviewed had to meet the following inclusion criteria: be older than 18 at the time of the survey, live in one of the four municipalities and, for the two flooded cities, have been physically present and over 18 when flooding occurred.

We consider as protest responses respondents who refuse to contribute to the Flood Management Fund for one of the following reasons: "We need to be sure that everyone pays", "I do not have enough information to choose an amount", "I am not the one who should pay", "I do not want to pay more". Overall, 264 (44.07%) are classified as protesters, 93 (15.53%) give a valid null WTP, and 242 (40.40%) give a strictly positive WTP.

Table 2 presents the summary statistics. The average age of the sample is 51.3; 55.1% are female; 36.2% have at least one child at home; 41.8% have at least a high school certificate; monthly mean respondent income is €1,422; monthly mean household income is €2,106, and 47.6% are homeowners.

[PLEASE INSERT TABLE 2 ABOUT HERE]

5.2 Results

Our choice of WTP determinants is based on the main variables found in previous CV surveys on flooding (Department for Environment Food & Rural Affairs, 2005; Hung, 2005; Abbas et al., 2014; Joseph, Proverbs, and Lamond, 2015; Owusu, Wright, and

[4]We also used a second scenario proposing a contribution to insurance against flood risk. It is not presented here both for the sake of concision and because it makes negative WTP unlikely.

[5]Note that Carson and Groves (2011) conclude that, of the various existing elicitation formats, none stands out as having better statistical and practical properties.

Arthur, 2015: Kuo, 2016; Ren and Wang, 2016), and we choose to limit unobserved heterogeneity by employing the same set of explanatory variables x to explain WTP - even non-significant ones - in all four models (see Tables 3 and 4).

5.2.1 Models not accounting for selection

Regarding the **Tobit** results, all the explanatory variables have a significant effect. Income, wealth, perceived probability of flood, and impatience have positive effects on WTP, which is consistent with theory and argues for the validity of the CV survey. Being subject to PTSD has a positive effect, which suggests that the psychological consequences from a flood event have a significant impact on WTP.

[PLEASE INSERT TABLE 3 ABOUT HERE]

Some coefficients are less intuitive: for example, risk tolerance has a positive effect on WTP, while having previous experience of flooding has a negative effect. The latter may be explained by the fact that having experienced flooding in the past could cause respondents to doubt the risk-mitigation efficacy of the protective action proposed or believe they would be able to relive the same situation and pull through the exact same way.

Regarding the **CQR without selection**, although the signs of the coefficients remain similar to those obtained in the Tobit, the CQR model reveals a strong heterogeneity of coefficients across the conditional distribution. For instance, the marginal effects of income and wealth are increasing, which reveals a higher WTP variance for the richer than for the poorer, a finding frequent in WTP studies (Notaro and De Salvo, 2010). This scale effect is also observed for all other variables and tends to increase in magnitude across quantiles.

Overall, we find clear signs of heterogeneity among the respondents, differing according to the unobserved determinants of WTP (i.e. to their rank in the conditional WTP distribution). Although it is difficult to determine what exactly is embedded in these unobserved components (attitude to the survey, differences in sensitivity to hypothetical bias, *etc.*), CQR shows how they can affect the relation between WTP and observed characteristics.

5.2.2 Models accounting for selection

We use several exclusion restrictions, adding variables W to the set X of variables to explain selection (i.e., non-protest), in order to guarantee identification and to be consistent with the Arellano and Bonhomme (2017a) estimation procedure. These variables deal with information regarding flood (*Inform*, *NbrInfo* and its square) and the place of residence (3 dummy variables) which can reasonably be considered to influence the decision to participate. Only coefficients are computed and discussed hereafter, not marginal effects. Results of the **Tobit with selection** are more or less similar to the standard Tobit for the WTP equation, with the following exceptions: impatience and past experience of flooding are no longer significant.

[PLEASE INSERT TABLE 4 ABOUT HERE]

The correlation ρ between the unobserved determinants of participation and WTP is negative and very significant. This suggests that there is indeed a bias when not correcting for selection. Since the correlation is negative, unobserved factors that make participation more likely tend to be associated with lower WTP (and *vice-versa*).

Regarding the participation equation (i.e. on the probability of non-protesting), respondents with high perceived likelihood of being flooded (*ProbaFlood*), who are familiar with a large number of media for information about flood risk ($NbrInfo^2$) or who are impatient are less likely to participate. On the other hand, those who have PTSD or are tolerant to risk are more likely to participate.
In the **CQR with selection**, as with the standard CQR, increased magnitude of effect is observable for all variables, suggesting that all variables have a heteroskedastic effect. Again, signs are similar, and the results regarding the selection equation are also similar to those obtained for the Tobit, although those living in a recently flash-flooded place (*Draguignan*) are more likely to participate.

6 Conclusion

This article confirms the advantages of selection censored models w.r.t. censored models for analyzing observational data, especially CQR in presence of heteroskedasticity, first through an MC experiment and second by applying selection models to a CV study on flood-risk protection. Their use appears relevant with any data where an underlying selection process is likely to be non-random and where heteroskedasticity and outliers arise. Non-economic examples are the number of occurrences of an event or the quantities consumed in fields such as agriculture, energy, climate, environment and health. Other applications include economic data, whether directly observed on markets (prices, rates, taxes), indirectly revealed (shadow prices), or stated in surveys (WTP). CQR models should also have advantages for policy makers, providing a picture of how the effects of WTP determinants are distributed across the population and not only on their conditional mean, which can clearly be misleading.
There are a number of possible extensions to this work.

First, exploring the performances of the four models in the MC experiment on a larger set of parameters would be easy, although time-consuming. Varying patterns of heteroskedasticity, using different censoring, selection and correlation values or introducing interaction variables would help us assess their respective advantages and limitations.

Second, our initial intention was to test in the MC experiment how a heteroskedastic Tobit (Messner, Mayr, and Zeileis, 2016) would perform w.r.t. CQR models in presence of controlled heteroskedasticity. Because the Tobit model is known to be sensitive to departure from the homoskedasticity assumption, accounting for it explicitly could be expected to improve its performance. A heteroskedastic version of the censored models we propose (with and without selection) can be obtained easily by replacing σ_e by σ_{e_i}, and expressing σ_{e_i} as a positively defined function of individual characteristics, whose parameters are estimated. However, although we did obtain better MB and RMSE with the heteroskedastic Tobit, we faced computational issues in estimating the model with selection that prevent us from adding it to this article.

Third, we suggest a way, compatible with our modeling, to identify and account for negative WTP, in the spirit of Gudding et al. (2018). Briefly, a first question asks the respondent whether s/he considers that the good proposed decreases his/her welfare level. If the answer is positive, s/he is asked his/her WTP to avoid deterioration; otherwise, s/he is asked his/her WTP to benefit from the good. Then, the WTP is elicited as well as the reasons for null WTP, to discriminate between true zero (no interest, budget constraint) and protest. If a high enough share of respondents have a negative utility for the good, two independent statistical models chosen among the four models we proposed can be estimated, respectively on the positive and the negative WTP. The average predicted WTP can be computed to properly account for gainers and losers, in cost benefit analysis for instance. If the proportion is too low, a censoring selection model chosen from the two we proposed, which allow for negative WTP, is estimated on the whole sample.

Finally, an interesting avenue of research is suggested by the fact that despite many models can account for heterogeneity in the coefficients (QR-based, nonparametric, latent class, hybrid or random parameter models), they have not been comprehensively compared to date. A study comparing these models and defining the kind of heterogeneity accounted for by each would be useful when choosing the proper model to analyze nature- or human-based observational data, including non-market valuation studies.

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DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
β_1									
p=100%, $\gamma_0 = 0$, $\rho = 0$	MB	-0.212	-0.113	-0.125	-0.119	-0.001	-0.044	-0.064	-0.037
	RMSE	0.108	0.060	0.065	0.062	0.000	0.001	0.000	0.000
p=100%, $\gamma_0 = 0$, $\rho = 0.5$	MB	-0.212	-0.114	-0.125	-0.119	-0.002	-0.052	-0.038	-0.054
	RMSE	0.108	0.060	0.065	0.061	0.000	0.001	0.000	0.001
p=100%, $\gamma_1 = 0.8$, $\rho = 0$	MB	-0.213	0.171	-0.196	-0.573	-0.119	-0.227	-0.200	-0.073
	RMSE	0.111	0.128	0.102	0.227	0.020	0.024	0.010	0.001
p=100%, $\gamma_1 = 0.8$, $\rho = 0.5$	MB	-0.213	0.172	-0.194	-0.572	0.026	-0.154	-0.244	-0.305
	RMSE	0.111	0.129	0.102	0.226	0.019	0.011	0.015	0.015
p=80%, $\gamma_0 = 0$, $\rho = 0$	MB	-0.185	-0.077	-0.092	-0.099	-0.001	-0.047	-0.067	-0.046
	RMSE	0.095	0.042	0.049	0.052	0.000	0.001	0.001	0.001
p=80%, $\gamma_0 = 0$, $\rho = 0.5$	MB	-0.189	-0.087	-0.100	-0.105	-0.250	-0.053	-0.040	-0.056
	RMSE	0.097	0.047	0.052	0.055	0.007	0.001	0.000	0.001
p=80%, $\gamma_1 = 0.8$, $\rho = 0$	MB	-0.19	0.228	-0.159	0.555	-0.120	-0.239	-0.214	-0.112
	RMSE	0.101	0.165	0.085	0.220	0.021	0.027	0.012	0.002
p=80%, $\gamma_1 = 0.8$, $\rho = 0.5$	MB	-0.194	0.221	-0.167	-0.562	0.031	-0.159	-0.251	-0.334
	RMSE	0.102	0.161	0.089	0.222	0.021	0.012	0.158	0.173
β_2									
p=100%, $\gamma_0 = 0$, $\rho = 0$	MB	0.888	0.282	0.417	0.601	0.040	0.120	0.146	0.133
	RMSE	0.444	0.146	0.211	0.303	0.001	0.004	0.005	0.005
p=100%, $\gamma_0 = 0$, $\rho = 0.5$	MB	0.887	0.284	0.416	0.599	0.029	0.138	0.052	0.089
	RMSE	0.444	0.146	0.211	0.302	0.001	0.005	0.001	0.002
p=100%, $\gamma_1 = 0.8$, $\rho = 0$	MB	0.900	1.101	0.603	0.110	0.106	0.851	0.277	-0.313
	RMSE	0.451	0.436	0.304	0.092	0.010	0.112	0.019	0.046
p=100%, $\gamma_1 = 0.8$, $\rho = 0.5$	MB	0.899	1.097	0.598	0.109	0.065	0.863	0.103	-0.3712
	RMSE	0.450	0.434	0.302	0.092	0.008	0.116	0.003	0.06
p=80%, $\gamma_0 = 0$, $\rho = 0$	MB	0.787	0.205	0.315	0.492	0.037	0.125	0.160	0.150
	RMSE	0.394	0.109	0.161	0.249	0.001	0.004	0.006	0.006
p=80%, $\gamma_0 = 0$, $\rho = 0.5$	MB	0.768	0.188	0.296	0.469	0.044	0.132	0.058	0.095
	RMSE	0.385	0.101	0.152	0.237	0.008	0.004	0.001	0.023
p=80%, $\gamma_1 = 0.8$, $\rho = 0$	MB	0.805	0.989	0.496	0.019	0.104	0.855	0.313	-0.291
	RMSE	0.404	0.392	0.252	0.061	0.010	0.114	0.025	0.040
p=80%, $\gamma_1 = 0.8$, $\rho = 0.5$	MB	0.786	0.964	0.470	-0.008	0.054	0.837	-0.121	-0.355
	RMSE	0.394	0.382	0.239	0.059	0.100	0.101	0.004	0.060
β_3									
p=100%, $\gamma_0 = 0$, $\rho = 0$	MB	-0.429	-0.142	-0.203	-0.285	-0.015	-0.067	-0.029	-0.032
	RMSE	0.445	0.223	0.255	0.332	0.022	0.033	0.020	0.041
p=100%, $\gamma_0 = 0$, $\rho = 0.5$	MB	-0.431	-0.140	-0.205	-0.291	-0.008	-0.074	-0.045	-0.016
	RMSE	0.446	0.219	0.256	0.338	0.021	0.035	0.022	0.030
p=100%, $\gamma_1 = 0.8$, $\rho = 0$	MB	-0.439	0.267	-0.293	-0.854	-0.053	0.315	-0.037	-0.568
	RMSE	0.465	0.743	0.349	0.570	0.217	1.149	0.093	0.210
p=100%, $\gamma_1 = 0.8$, $\rho = 0.5$	MB	-0.441	0.265	-0.296	-0.857	-0.035	0.330	-0.060	-0.552
	RMSE	0.469	0.741	0.353	0.573	0.205	1.117	0.112	0.197
p=80%, $\gamma_0 = 0$, $\rho = 0$	MB	-0.377	-0.098	-0.149	-0.230	-0.015	-0.061	-0.031	-0.032
	RMSE	0.465	0.743	0.349	0.570	0.217	1.149	0.093	0.210
p=80%, $\gamma_0 = 0$, $\rho = 0.5$	MB	-0.370	-0.087	-0.140	-0.223	-0.028	-0.057	-0.041	-0.020
	RMSE	0.393	0.207	0.221	0.292	0.304	0.030	0.025	0.035
p=80%, $\gamma_1 = 0.8$, $\rho = 0$	MB	-0.392	0.321	-0.247	-0.807	-0.057	0.340	-0.043	-0.564
	RMSE	0.434	0.897	0.337	0.547	0.231	1.219	0.121	0.227
p=80%, $\gamma_1 = 0.8$, $\rho = 0.5$	MB	-0.376	0.331	-0.226	-0.791	-0.038	0.356	-0.053	-0.561
	RMSE	0.418	0.913	0.320	0.537	0.214	1.181	0.130	0.203

Note: Cells highlighted in gray when Mean Bias differs by less than 10% from true parameter or RMSE<.1

Table 1: Mean Bias and RMSE for n=1000 and censoring rate=40%

Variable	Label	Mean	Std. Dev.	Min.	Max.	N
Dependent						
P	Participation (=1)	0.550	0.496	0	1	599
WTP	Willingness to pay for protective devices (in €)	93.46	145.54	0	1500	335
Sociodemographic						
Gender	Gender (Male=1)	0.449	0.497	0	1	599
Age	Age (in years)	51.293	17.003	16	94	593
Child	Has at least one child (=1)	0.362	0.481	0	1	599
Education	Education (ordinal variable)	1.853	1.14	1	4	599
Owner	Is the owner of the housing (=1)	0.476	0.5	0	1	599
X variables						
Income	Monthly income of the respondent (in €)	1423.478	904.531	0	8000	575
HousingRisk	Living on the ground floor or in a house (=1)	0.605	0.489	0	1	593
PastExperience	Already experienced a flood (=1)	0.521	0.5	0	1	593
ProbaFlood	Perceived likelihood of being flooded in the next 10 years (in %)	9.353	14.958	0	100	593
Impatience	Preference for the present score (1-7 score)	2.974	2.756	0	7	568
RiskTolerance	Loss lover score (1-4 score)	1.56	0.86	1	4	593
Happy	Declared subjective well-being (0-10 score)	6.772	2.043	0	10	593
PTSD	Post-Traumatic Stress Disorder (=1)	0.105	0.306	0	1	593
W variables						
Draguignan	Living in Draguignan, flash-flooded 2 years before the survey (=1)	0.256	0.436	0	1	593
Vaison	Living in Vaison-la-Romaine, flash-flooded 20 years before the survey (=1)	0.251	0.434	0	1	593
Berre	Living in Berre l'Etang, at potential risk of flooding (=1)	0.238	0.426	0	1	593
Inform	Looked for information about flood risk (=1)	0.14	0.347	0	1	593
NbrInfo	Number of media known for information about flood risk (integer)	2.526	1.422	0	8	593

Table 2: Summary Statistics (n=599)

	Tobit	CQR		
		0.25	0.50	0.75
Intercept	-130.307***	-58.168**	-70.612**	-59.227*
	(0.002)	(0.039)	(0.048)	(0.088)
Income	0.030***	0.011*	0.015	0.019***
	(0.004)	(0.100)	(0.141)	(<.001)
PastExperience	-76.041***	-11.013	-28.045**	-38.154***
	(<.001)	(0.129)	(0.049)	(0.003)
ProbaFlood	3.247***	0.966**	1.521**	3.825***
	(<.001)	(0.014)	(0.028)	(<.001)
Impatience	-14.214***	-5.643***	-10.259***	-17.711***
	(<.001)	(<.001)	(<.001)	(<.001)
RiskTolerance	33.433***	5.470	23.737***	33.767***
	(<.001)	(0.446)	(0.002)	(0.006)
Happy	15.285***	8.943**	12.146***	12.245***
	(0.003)	(0.012)	(0.002)	(0.002)
Wealth	0.127***	0.019	0.098	0.208***
	(0.004)	(0.550)	(0.118)	(<.001)
PTSD	68.880**	30.030	62.205	110.732***
	(0.02)	(0.472)	(0.149)	(<.001)
Log Likelihood	-1,479.044			
N	310	310	310	310

P-values in brackets: *** if p-value<.01, ** if p-value<.05, * if p-value<.1 .

Table 3: Standard Tobit and CQR

	Tobit with selection		CQR with selection			
	Participation	WTP	Participation		WTP	
				0.25	0.50	0.75
Intercept	0.291	-37.847	0.234	-4.710	-25.863**	-34.723**
	(0.301)	(0.196)	(0.428)	(0.128)	(0.040)	(0.048)
Income	0.016	0.027**	-0.215	0.002*	0.013*	0.012
	(0.712)	(0.050)	(0.200)	(0.090)	(0.074)	(0.136)
PastExperience	-0.012	-5.199	-0.051	-5.755**	-27.229	-41.157
	(0.882)	(0.521)	(0.709)	(0.028)	(0.104)	(0.218)
ProbaFlood	-1.112**	2.370***	-1.545***	0.184***	0.921***	2.676***
	(0.020)	(<.001)	(0.009)	(0.010)	(0.008)	(0.002)
Impatience	-0.890**	0.194	-1.361**	0.012	0.037	-0.269
	(0.045)	(0.506)	(0.020)	(0.830)	(0.584)	(0.370)
RiskTolerance	0.487***	22.862**	0.413**	0.380	7.693*	27.571**
	(0.010)	(0.015)	(0.017)	(0.234)	(0.068)	(0.024)
Happy	0.004	8.403**	0.004	0.450**	4.122**	5.848*
	(0.847)	(0.025)	(0.8387)	(0.038)	(0.018)	(0.078)
Wealth	<.001	0.106***	<.001	0.007	0.012	0.061*
	(0.667)	(0.005)	(0.711)	(0.214)	(0.124)	(0.056)
PTSD	0.714*	67.188**	0.991*	-0.099	23.932	36.834*
	(0.100)	(0.030)	(0.087)	(0.818)	(0.240)	(0.052)
Inform	0.01		0.006			
	(0.120)		(0.107)			
NbrInfo	-0.001		-0.001			
	(0.566)		(0.768)			
NbrInfo^2	-0.440**		-0.398**			
	(0.030)		(0.039)			
Draguignan	0.089		0.126*			
	(0.125)		(0.054)			
Vaison	-0.0261		-0.012			
	(0.411)		(0.666)			
Berre	-0.001		<.001			
	(0.782)		(0.920)			
Rho	-0.572***		-0.680***			
	(<.001)		(<.001)			
N	593	310	593	310	310	310

P-values in brackets: *** if p-value<.01, ** if p-value<.05, * if p-value<.1.

Table 4: Tobit with selection and CQR with selection

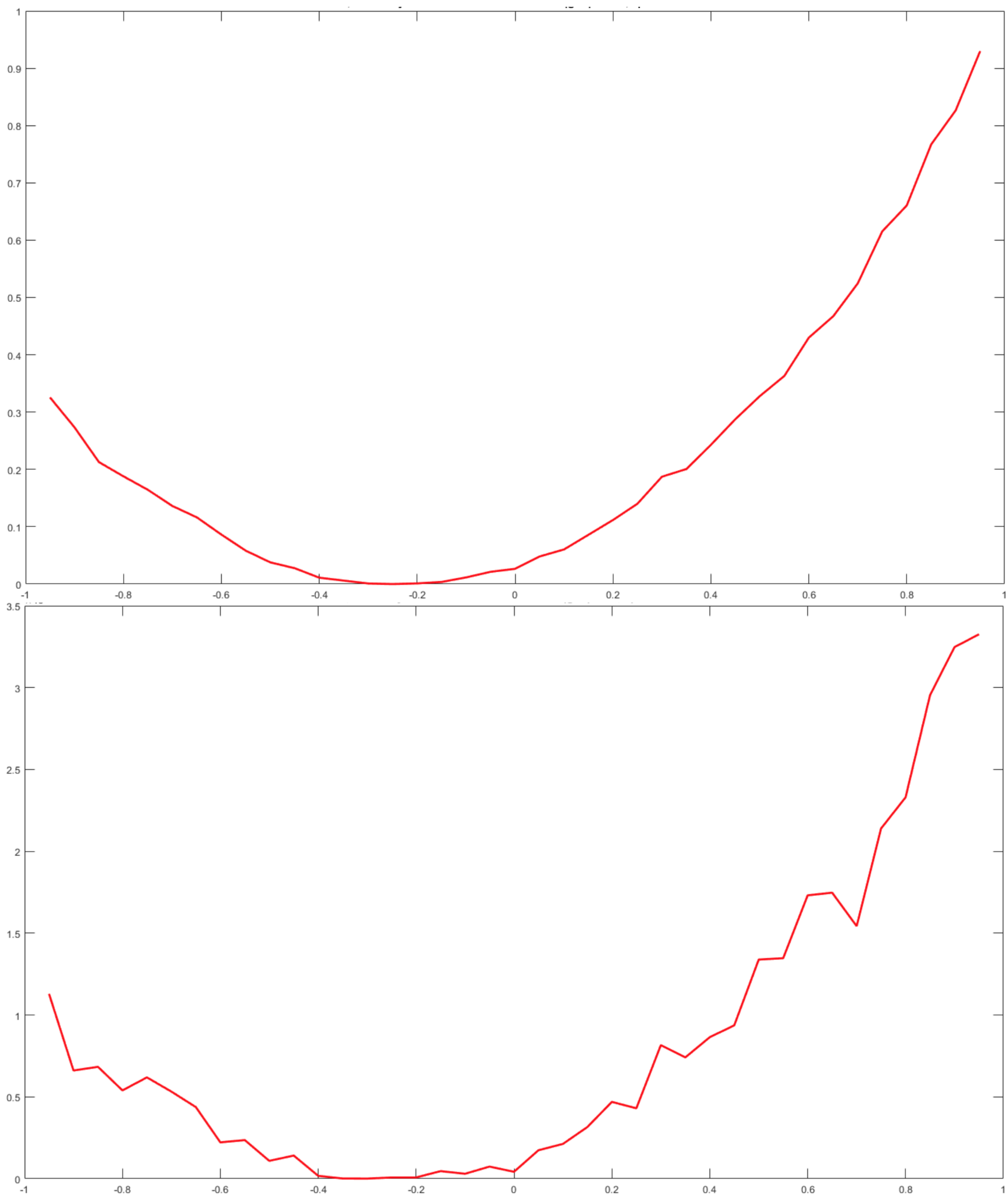

Figure 1: GMM objective function for two grid densities, $n = 1000$. Upper panel: θ=0.01, CPU time = 9.202 s. Lower panel: θ=0.2, CPU time = 0.684 s).

Appendix A: Details on MC experiment

Two standard criteria used in the MC and QR literature to compare performance of different models are Mean Bias (MB) and Root-Mean-Square Error (RMSE). They both measure the magnitude of the deviations of MC estimates from the true estimate (Paarsch, 1984; Buchinsky and Hahn, 1998; Chernozhukov et al., 2015).

For a given specification, the MB is defined as: $\frac{1}{R}\sum_{r=1}^{R}(\hat{b}_r - b)$ and the RMSE is defined as: $\sqrt{\frac{1}{R}\sum_{r=1}^{R}\left(\frac{\hat{b}_r - b}{b}\right)^2}$ where b is the true value of the marginal effect of x on WTP and $\hat{b}_r$ the estimation of the marginal effect of x_r on WTP_r for the r^{th} of the R MC replications.

Note that this marginal effect is not equal to β for CQR when the quantile is different from 0.5. For the homoskedastic case, it is equal to β. For the heteroskedastic case, it is equal to $\beta + \gamma_{j=1,2}F_u^{-1}(\tau)$, with $F_u^{-1}(\tau) = 0$ if and only if $\tau = 0.5$ for a symmetric, zero-centered distribution (as the standard normal).

Moreover, for symmetric (zero-centered) distributions and for conditional mean models, the mean is equal to the median (and equals zero for zero-centered distributions) and $b = \beta$.

Appendix B: Additional Monte Carlo results

β_1

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, $\rho = 0$	MB	0	0.002	0.001	-0.001	-0.0102	-0.1034	-0.0485	-0.041
	RMSE	0.014	0.02	0.018	0.02	0.0005	0.0027	0.0006	0.0004
p=100%, γ_0=0, $\rho = 0.5$	MB	0.001	0.002	0.001	-0.001	-0.0113	-0.0845	-0.1278	-0.065
	RMSE	0.014	0.02	0.018	0.02	0.0005	0.0018	0.0042	0.0011
p=100%, γ_1=0.8, $\rho = 0$	MB	0	0.401	0	-0.401	-0.0124	-0.0459	-0.2362	-0.1246
	RMSE	0.042	0.283	0.045	0.163	0.0241	0.0018	0.0144	0.0026
p=100%, γ_1=0.8, $\rho = 0.5$	MB	0	0.4	0	-0.402	-0.0173	-0.1195	-0.1394	-0.2429
	RMSE	0.042	0.282	0.045	0.163	0.0246	0.0079	0.0054	0.0098
p=80%, γ_0=0, $\rho = 0$	MB	0	0.002	0	-0.002	-0.0102	-0.1034	-0.0485	-0.041
	RMSE	0.015	0.02	0.019	0.021	0.0005	0.0027	0.0006	0.0004
p=80%, γ_0=0, $\rho = 0.5$	MB	-0.016	-0.016	-0.015	-0.016	-0.0113	-0.0845	-0.1278	-0.065
	RMSE	0.017	0.023	0.021	0.022	0.0005	0.0018	0.0042	0.0011
p=80%, γ_1=0.8, $\rho = 0$	MB	-0.001	0.407	-0.001	-0.408	-0.0123	-0.0459	-0.2362	-0.1246
	RMSE	0.044	0.288	0.048	0.166	0.0241	0.0018	0.0144	0.0026
p=80%, γ_1=0.8, $\rho = 0.5$	MB	-0.016	0.383	-0.02	-0.427	-0.0173	-0.1195	-0.1394	-0.2429
	RMSE	0.045	0.272	0.049	0.173	0.0246	0.0079	0.0054	0.0098

β_2

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, $\rho = 0$	MB	-0.001	-0.001	0.000	0.001	0.101	0.178	0.203	0.248
	RMSE	0.029	0.043	0.040	0.044	0.004	0.008	0.010	0.016
p=100%, γ_0=0, $\rho = 0.5$	MB	-0.001	-0.002	-0.001	0.001	0.104	0.258	0.161	0.164
	RMSE	0.029	0.044	0.040	0.044	0.004	0.017	0.007	0.007
p=100%, γ_1=0.8, $\rho = 0$	MB	0.000	0.537	0.002	-0.534	0.130	0.866	0.333	-0.252
	RMSE	0.045	0.217	0.058	0.375	0.012	0.116	0.028	0.031
p=100%, γ_1=0.8, $\rho = 0.5$	MB	-0.001	0.536	0.001	-0.534	0.130	0.994	0.220	-0.378
	RMSE	0.045	0.217	0.058	0.376	0.011	0.153	0.013	0.068
p=80%, γ_0=0, $\rho = 0$	MB	0.001	0.001	0.000	0.001	0.101	0.178	0.203	0.248
	RMSE	0.033	0.048	0.044	0.048	0.004	0.008	0.010	0.016
p=80%, γ_0=0, $\rho = 0.5$	MB	0.002	-0.001	-0.001	-0.002	0.104	0.258	0.161	0.164
	RMSE	0.032	0.048	0.043	0.048	0.004	0.017	0.007	0.007
p=80%, γ_1=0.8, $\rho = 0$	MB	-0.004	0.532	-0.003	-0.539	0.130	0.866	0.333	-0.252
	RMSE	0.052	0.217	0.066	0.382	0.012	0.116	0.028	0.031
p=80%, γ_1=0.8, $\rho = 0.5$	MB	-0.002	0.535	-0.002	-0.540	0.130	0.994	0.220	-0.378
	RMSE	0.051	0.218	0.066	0.382	0.011	0.153	0.013	0.068

β_3

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, $\rho = 0$	MB	0.004	0.008	0.002	-0.002	-0.018	-0.060	-0.050	-0.106
	RMSE	0.190	0.268	0.244	0.267	0.042	0.043	0.043	0.098
p=100%, γ_0=0, $\rho = 0.5$	MB	0.000	-0.001	0.001	-0.004	-0.025	-0.067	-0.084	-0.091
	RMSE	0.193	0.273	0.249	0.278	0.046	0.054	0.089	0.069
p=100%, γ_1=0.8, $\rho = 0$	MB	0.003	0.548	-0.001	-0.547	-0.015	0.394	-0.104	-0.670
	RMSE	0.292	1.450	0.347	0.431	0.267	1.321	0.141	0.299
p=100%, γ_1=0.8, $\rho = 0.5$	MB	0.006	0.558	0.006	-0.539	-0.033	0.425	-0.164	-0.655
	RMSE	0.301	1.475	0.357	0.430	0.260	1.675	0.276	0.319
p=80%, γ_0=0, $\rho = 0$	MB	0.001	0.003	-0.001	-0.001	-0.018	-0.060	-0.050	-0.106
	RMSE	0.217	0.305	0.278	0.298	0.042	0.043	0.043	0.098
p=80%, γ_0=0, $\rho = 0.5$	MB	0.001	0.002	0.001	-0.006	-0.025	-0.067	-0.084	-0.091
	RMSE	0.215	0.301	0.275	0.298	0.046	0.054	0.089	0.069
p=80%, γ_1=0.8, $\rho = 0$	MB	0.002	0.545	0.005	-0.538	-0.015	0.394	-0.104	-0.670
	RMSE	0.342	1.518	0.403	0.452	0.267	1.321	0.141	0.299
p=80%, γ_1=0.8, $\rho = 0.5$	MB	0.000	0.549	0.001	-0.544	-0.032	0.425	-0.164	-0.655
	RMSE	0.343	1.519	0.402	0.456	0.259	1.675	0.276	0.319

Note: Cells are highlighted in gray when the Mean Bias differs by less than 10% from the true parameter. For the RMSE, cells are highlighted in gray when the RMSE is less than .1.

Table B-1:Mean Biasand Root-Mean-Square Error for n=300 and censoring rate=0%

β_1

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, ρ = 0	MB	-0.228	-0.120	-0.131	-0.125	0.000	-0.071	-0.055	-0.080
	RMSE	0.120	0.070	0.072	0.069	0.001	0.001	0.001	0.002
p=100%, γ_0=0, ρ = 0.5	MB	-0.228	-0.120	-0.132	-0.125	-0.001	-0.073	-0.090	-0.088
	RMSE	0.120	0.070	0.073	0.069	0.001	0.001	0.002	0.002
p=100%, γ_1=0.8, ρ = 0	MB	-0.232	0.163	-0.202	-0.580	-0.080	-0.019	-0.339	-0.157
	RMSE	0.127	0.149	0.114	0.232	0.040	0.002	0.029	0.004
p=100%, γ_1=0.8, ρ = 0.5	MB	-0.229	0.166	-0.199	-0.579	0.009	-0.213	-0.065	-0.350
	RMSE	0.126	0.152	0.113	0.231	0.042	0.022	0.002	0.019
p=80%, γ_0=0, ρ = 0	MB	-0.202	-0.085	-0.099	-0.107	0.000	-0.072	-0.062	-0.079
	RMSE	0.107	0.054	0.058	0.061	0.001	0.001	0.001	0.002
p=80%, γ_0=0, ρ = 0.5	MB	-0.206	-0.093	-0.107	-0.112	-0.001	-0.078	-0.121	-0.087
	RMSE	0.109	0.057	0.061	0.063	0.001	0.002	0.004	0.002
p=80%, γ_1=0.8, ρ = 0	MB	-0.203	0.228	-0.163	-0.561	-0.081	-0.046	-0.345	-0.162
	RMSE	0.114	0.184	0.098	0.225	0.045	0.003	0.030	0.004
p=80%, γ_1=0.8, ρ = 0.5	MB	-0.210	0.214	-0.171	-0.567	0.014	-0.270	-0.087	-0.363
	RMSE	0.118	0.176	0.102	0.227	0.044	0.035	0.003	0.021

β_2

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, ρ = 0	MB	0.889	0.292	0.427	0.607	0.040	0.118	0.115	0.042
	RMSE	0.447	0.160	0.222	0.310	0.002	0.004	0.004	0.001
p=100%, γ_0=0, ρ = 0.5	MB	0.887	0.291	0.424	0.606	0.030	0.073	0.019	0.034
	RMSE	0.445	0.159	0.221	0.310	0.002	0.002	0.000	0.001
p=100%, γ_1=0.8, ρ = 0	MB	0.900	1.110	0.608	0.109	0.101	0.801	0.162	-0.547
	RMSE	0.453	0.443	0.313	0.123	0.019	0.100	0.007	0.141
p=100%, γ_1=0.8, ρ = 0.5	MB	0.905	1.111	0.610	0.118	0.066	0.703	-0.016	-0.608
	RMSE	0.455	0.444	0.314	0.125	0.019	0.077	0.001	0.175
p=80%, γ_0=0, ρ = 0	MB	0.788	0.213	0.323	0.498	0.038	0.079	0.099	0.038
	RMSE	0.397	0.126	0.174	0.258	0.002	0.002	0.003	0.001
p=80%, γ_0=0, ρ = 0.5	MB	0.767	0.195	0.302	0.470	0.029	0.055	-0.011	0.031
	RMSE	0.387	0.118	0.163	0.244	0.002	0.001	0.000	0.001
p=80%, γ_1=0.8, ρ = 0	MB	0.805	0.993	0.499	0.020	0.092	0.726	0.124	-0.548
	RMSE	0.406	0.399	0.263	0.110	0.021	0.083	0.005	0.142
p=80%, γ_1=0.8, ρ = 0.5	MB	0.789	0.974	0.476	-0.005	0.065	0.618	-0.040	-0.638
	RMSE	0.399	0.392	0.252	0.108	0.021	0.060	0.001	0.194

β_3

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, ρ = 0	MB	-0.429	-0.136	-0.202	-0.286	0.004	-0.043	-0.054	-0.073
	RMSE	0.479	0.344	0.352	0.426	0.074	0.070	0.113	0.099
p=100%, γ_0=0, ρ = 0.5	MB	-0.427	-0.136	-0.199	-0.284	0.012	-0.011	-0.067	-0.072
	RMSE	0.478	0.339	0.348	0.418	0.069	0.074	0.132	0.144
p=100%, γ_1=0.8, ρ = 0	MB	-0.447	0.250	-0.304	-0.862	0.008	0.414	-0.107	-0.613
	RMSE	0.530	1.016	0.463	0.611	0.594	2.200	0.469	0.322
p=100%, γ_1=0.8, ρ = 0.5	MB	-0.437	0.267	-0.294	-0.859	0.011	0.500	-0.104	-0.639
	RMSE	0.520	1.034	0.453	0.610	0.593	3.239	0.491	0.413
p=80%, γ_0=0, ρ = 0	MB	-0.379	-0.094	-0.156	-0.239	-0.001	-0.050	-0.057	-0.074
	RMSE	0.451	0.362	0.352	0.418	0.078	0.092	0.117	0.106
p=80%, γ_0=0, ρ = 0.5	MB	-0.372	-0.089	-0.143	-0.223	0.003	-0.010	-0.066	-0.078
	RMSE	0.446	0.357	0.342	0.412	0.073	0.081	0.130	0.141
p=80%, γ_1=0.8, ρ = 0	MB	-0.393	0.321	-0.249	-0.820	-0.004	0.388	-0.115	-0.621
	RMSE	0.515	1.248	0.479	0.606	0.668	2.476	0.473	0.321
p=80%, γ_1=0.8, ρ = 0.5	MB	-0.382	0.326	-0.227	-0.801	-0.034	0.494	-0.107	-0.639
	RMSE	0.509	1.245	0.471	0.595	0.631	3.372	0.487	0.415

Note: Cells are highlighted in gray when the Mean Bias differs by less than 10% from the true parameter. For the RMSE, cells are highlighted in gray when the RMSE is less than .1.

Table B-2: Mean Bias and Root-Mean-Square Error for n=300 and censoring rate=40%

β_1

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, ρ = 0	MB	0.000	0.000	0.000	0.000	-0.011	-0.051	-0.071	-0.051
	RMSE	0.008	0.010	0.010	0.011	0.000	0.001	0.001	0.001
p=100%, γ_0=0, ρ = 0.5	MB	0.000	0.000	0.000	-0.001	-0.012	-0.054	-0.056	-0.073
	RMSE	0.007	0.010	0.009	0.010	0.000	0.001	0.001	0.001
p=100%, γ_1=0.8, ρ = 0	MB	0.000	0.401	-0.001	-0.400	-0.029	-0.200	-0.178	-0.065
	RMSE	0.024	0.277	0.025	0.159	0.011	0.019	0.008	0.001
p=100%, γ_1=0.8, ρ = 0.5	MB	0.000	0.400	0.001	-0.399	-0.026	-0.110	-0.235	-0.263
	RMSE	0.024	0.277	0.025	0.159	0.012	0.006	0.014	0.011
p=80%, γ_0=0, ρ = 0	MB	0.000	0.000	0.000	-0.001	-0.011	-0.051	-0.071	-0.051
	RMSE	0.008	0.011	0.010	0.011	0.000	0.001	0.001	0.001
p=80%, γ_0=0, ρ = 0.5	MB	-0.015	-0.016	-0.015	-0.014	-0.009	-0.054	-0.056	-0.073
	RMSE	0.011	0.014	0.013	0.013	0.000	0.001	0.001	0.001
p=80%, γ_1=0.8, ρ = 0	MB	0.000	0.403	0.000	-0.403	-0.029	-0.200	-0.178	-0.065
	RMSE	0.025	0.279	0.026	0.160	0.011	0.019	0.008	0.001
p=80%, γ_1=0.8, ρ = 0.5	MB	-0.014	0.382	-0.019	-0.422	-0.028	-0.110	-0.235	-0.263
	RMSE	0.027	0.265	0.029	0.168	0.011	0.006	0.014	0.011

β_2

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, ρ = 0	MB	0.000	0.000	0.000	0.000	0.103	0.158	0.172	0.179
	RMSE	0.016	0.024	0.022	0.024	0.003	0.006	0.007	0.008
p=100%, γ_0=0, ρ = 0.5	MB	-0.001	0.000	-0.001	-0.001	0.103	0.227	0.110	0.164
	RMSE	0.016	0.024	0.022	0.025	0.003	0.013	0.003	0.007
p=100%, γ_1=0.8, ρ = 0	MB	-0.001	0.535	0.000	-0.535	0.132	0.869	0.281	-0.323
	RMSE	0.024	0.212	0.032	0.370	0.008	0.117	0.020	0.049
p=100%, γ_1=0.8, ρ = 0.5	MB	0.000	0.536	0.002	-0.534	0.128	0.956	0.180	-0.320
	RMSE	0.025	0.213	0.032	0.369	0.011	0.142	0.008	0.048
p=80%, γ_0=0, ρ = 0	MB	0.000	0.000	0.000	0.000	0.103	0.158	0.172	0.179
	RMSE	0.018	0.026	0.024	0.026	0.003	0.006	0.007	0.008
p=80%, γ_0=0, ρ = 0.5	MB	0.000	-0.004	-0.002	-0.003	0.079	0.227	0.110	0.164
	RMSE	0.018	0.026	0.025	0.026	0.002	0.013	0.003	0.007
p=80%, γ_1=0.8, ρ = 0	MB	0.000	0.537	0.000	-0.535	0.132	0.869	0.281	-0.323
	RMSE	0.028	0.214	0.036	0.370	0.008	0.117	0.020	0.049
p=80%, γ_1=0.8, ρ = 0.5	MB	-0.002	0.533	-0.004	-0.540	0.130	0.956	0.180	-0.320
	RMSE	0.028	0.212	0.036	0.374	0.011	0.142	0.008	0.048

β_3

DGP	Statistics	Tobit	CQR 25%	CQR 50%	CQR 75%	Tobit-S	CQR-S 25%	CQR-S 50%	CQR-S 75%
p=100%, γ_0=0, ρ = 0	MB	0.001	0.005	0.001	-0.001	-0.040	-0.065	-0.059	-0.085
	RMSE	0.106	0.148	0.136	0.149	0.015	0.023	0.016	0.031
p=100%, γ_0=0, ρ = 0.5	MB	-0.002	-0.003	-0.002	-0.005	-0.040	-0.064	-0.069	-0.084
	RMSE	0.106	0.148	0.135	0.149	0.015	0.020	0.019	0.031
p=100%, γ_1=0.8, ρ = 0	MB	0.001	0.543	0.001	-0.542	-0.059	0.344	-0.087	-0.654
	RMSE	0.161	1.262	0.192	0.376	0.102	0.876	0.048	0.211
p=100%, γ_1=0.8, ρ = 0.5	MB	-0.002	0.537	-0.002	-0.545	-0.056	0.375	-0.107	-0.639
	RMSE	0.163	1.249	0.193	0.379	0.117	0.910	0.082	0.197
p=80%, γ_0=0, ρ = 0	MB	0.001	0.000	0.001	0.002	-0.040	-0.065	-0.059	-0.085
	RMSE	0.118	0.163	0.152	0.165	0.015	0.023	0.016	0.031
p=80%, γ_0=0, ρ = 0.5	MB	-0.002	0.002	0.000	-0.002	-0.037	-0.064	-0.069	-0.084
	RMSE	0.119	0.163	0.150	0.165	0.013	0.020	0.019	0.031
p=80%, γ_1=0.8, ρ = 0	MB	0.000	0.546	0.000	-0.546	-0.059	0.344	-0.087	-0.654
	RMSE	0.187	1.295	0.221	0.387	0.102	0.876	0.048	0.211
p=80%, γ_1=0.8, ρ = 0.5	MB	0.000	0.545	0.000	-0.544	-0.064	0.375	-0.107	-0.639
	RMSE	0.185	1.293	0.216	0.386	0.105	0.910	0.082	0.197

Note: Cells are highlighted in gray when the Mean Bias differs by less than 10% from the true parameter. For the RMSE, cells are highlighted in gray when the RMSE is less than .1.

Table B-3: Mean Bias and Root-Mean-Square Error for n=1000 and censoring rate=0%

Appendix C: Hypothetical scenario

A translation of the questions and the scenario presented to respondents and relevant to this study is reproduced below. Sentences in italics are for the reader and were not read to respondents.

Introduction by Interviewer

"You are going to be the main actor in our fictitious scenario. You will have to take the best decision regarding your housing. Only your opinion matters, there is no wrong or right answer. Not everyone is fully aware of the way the flood insurance system works, so we present it briefly. In France, every third-party liability insurance policy regarding fire or damage includes a mandatory contribution known as CatNat. To benefit from this type of compensation in the event of flood, the flood event must have been declared a 'natural catastrophe' by joint ministerial decree and the goods (property and belongings) must be insured. Compensation will be subject to a €380 deductible. Personal injuries are not covered by the CatNat system. They are covered either by a personal insurance policy, or by the national government if a civil servant (administrative or elected) can be held responsible for the occurrence of the flood event."

Protective devices scenario

"Let us imagine that the CatNat insurance still covers the flood-related events. Your current insurance contract still covers all other types of events, and your premium remains unchanged. Imagine that the national government creates a Flood Management Fund to finance protective devices against flood. Building dikes, water retention ponds or improving rain water evacuation networks would reduce the height and speed of water and would completely eliminate the risk of flood in your commune. This work will only be realized if the population involved contributes to the Flood Management Fund. We would like to know how much maximum you would be willing to pay per year to this Fund."

Note to the interviewer: *If the respondent asks for details on the level of protection, the cost of the protective devices or the way they would be funded, please give the following answer:*

"This survey is part of a research project that involves several communes. What we are considering here is a fictitious situation, so that the exact way the protective devices would be implemented is not yet decided. When answering, however, imagine that everybody covered by this protective devices pays, like the household waste removal tax, for instance."

"We remind you that you have previously declared that the probability of being flooded during the coming year is ...." *(remind the respondent of his/her previous answer to question L16-1).*

WTP Question 1. "Would you be willing to contribute to the Flood Management Fund to finance protective devices against flood?".

Note to the interviewer:
If the answer to question WTP Question 1 is "No", then ask for the reasons.

If the answer to WTP Question 1 is "Yes", then ask the following:

WTP Question 2. "How much maximum would you be willing to pay per year? To help you, here is a card with several amounts."

Note to the interviewer: *[Present the circular payment card].*

"Do not forget that this money will be drawn from your household's budget! You will therefore have less money at the end of the month for consumption or savings."