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The Prince and Me
A model of Fiscal Credibility

Nicolas End

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Nicolas End†

Email: nicolas.end@univ-amu.fr

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Abstract

Government fiscal actions influence forward-looking private agents' current and future decisions, which, in turn, impact fiscal performance. This paper highlights this expectation channel with a Barro-type endogenous growth model where an impatient government finances growth-enhancing spending through income taxes and public debt. Fiscal and macroeconomic outcomes emerge from the interplay of households and policymakers' preferences for public expenditure and private consumption. I find that the government's maximizing its own utility and facing an endogenous interest spread are sufficient ingredients to yield multiple equilibria, independently of the government's policy intentions. The economy almost always heads to the high public spending equilibrium, emphasizing the importance of fiscal institutions to tame government impatience and bolster fiscal credibility.

JEL *classification*: E60, H30, H11

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†Aix Marseille University, CNRS, Aix Marseille School of Economics (AMSE), Marseille, France.

1 Introduction

In the wake of 2008–09 the Great Financial Crisis (GFC), recovery in many advanced economies remained subdued for almost a decade. One commonly cited reason for this sub-par performance is excessive public indebtedness. Perhaps more harmful than large public debt ratios, governments' inability to provide private agents with a stable and credible fiscal outlook (and convincing path to curb public debt) hampered private growth prospects, undermined investment and consumption, and fueled precautionary savings. Uncertainty about public finances is regularly associated throughout economic history (be it post-war periods or the 1980s–90s for developing economies) with low growth and volatile macroeconomic environment, at times culminating in full-fledged sovereign crises. A similar situation is bound to occur again once the world is done waging its war against CoViD-19.[1]

The empirical relationship between fiscal policy, economic agents' expectations, and macroeconomic instability is not to be proven. Starting from the seminal work by Blanchard and Perotti (2002), the literature has repeatedly emphasized the sizable impact fiscal decisions have on economic output, as well as the difficulty to conclusively estimate it—especially as it depends on many factors, such as the position in the business cycle, the openness to trade and capital flows, the monetary stance, and the nature of the fiscal decision itself. The Keynesian effects of fiscal policy are offset by various crowding out channels (see Blanchard 1991, for a comprehensive exposition).

Several of these links between fiscal and macroeconomic performance operate through expectations. For instance, the fiscal foresight literature shows how private agents anticipate fiscal decisions and adjust their forecasts and plans depending on government announcements (Leeper, Richter, and Walker 2012; Blanchard and Leigh 2013). However, the theoretical underpinnings of such expectation channels remain elusive, apart from the Ricardian equivalence, which posits that rational agents expect a tax hike following a fiscal stimulus. In particular, the possibility of

1. Already, the IMF is calling on governments to clarify their post-CoViD-19 fiscal frameworks (IMF 2021).

a feedback loop between fiscal foresight and fiscal outcomes, and, more importantly, the possibility that such a loop leads to a multiplicity of equilibria have, to the best of my knowledge, never been studied theoretically.

This paper explores these issues, laying down the foundations of a theoretical model that allows for such a feedback loop to operate. It explains how government fiscal actions influence forward-looking private agents' current and future decisions, which, in turn, impact fiscal performance. Eventually, it could contribute to justify the importance of fiscal credibility, a concept on which implicitly rely current practices of imposing fiscal accountability frameworks and medium-term fiscal frameworks—meant to enhance communication around, and oversight of fiscal policy and objectives. Indeed, once established, the feedback loop between government's observed preferences and macroeconomic outcomes could make it possible to model how government's reputation may lead to instability.

The monetary policy literature sets a useful example: it derives from the *ex ante* indeterminacy between the high and low inflation steady states important lessons about the risk of reputational effects, coordination issues between authorities and agents, and expectation-driven fluctuations.[2] The literature has come up with various ways to model these credibility effects. For instance, under game theory approaches, reputation emerges from repeated games, while reputation-building and learning processes can ensue from setups with imperfect or asymmetric information setups (see, for instance, Blackburn and Christensen 1989; Cukierman 1992, and references therein). Another strand of papers introduces Markov-switching mechanisms, where the monetary regime varies stochastically and is unknown to agents (Laxton, Ricketts, and Rose 1994; Cukierman and Meltzer 1986; Jeanne 1997). But most of these models build upon (a) the seminal contribution by Kydland and Prescott (1977), who showed that outcome under discretion might be sub-optimal because the output cost of disinflation is smaller if the policy is credible (the so-called credibility hypothesis); and (b) the New Keynesian model, which proves under sticky prices and rational expectations that monetary policy

2. This is not a normative mechanism; it happens independently of whether the policymaker is virtuous and aims at lowering inflation.

transmits through intertemporal allocations (owing to the forward-looking Phillips curve).

Most of the theoretical literature on fiscal policy and macroeconomic stability has so far focused on the stabilizing role of fiscal policy in the face of cyclical fluctuations, usually by means of counter-cyclical tax instruments. Typically, instability is obtained from exogenous shocks on fundamentals (Kletzer 2006; Moldovan 2010) or volatile expectations (the so-called sunspots; Guo and Lansing 1998; Dromel and Pintus 2008). Besides, most papers rely on the strong assumption that the production technology exhibits increasing returns to scale (Christiano and Harrison 1999; Farmer and Benhabib 1994). To generate multiplicity and indeterminacy, other papers rely on labor market imperfections [3] or some form of segmentation: between consumers (Carli and Modesto, forthcoming), sectors (Brito and Venditti 2010; Mulligan and Sala-i-Martin 1993), or countries (Corsetti et al. 2014). More recent papers have also envisaged rigidities in fiscal policy or specific tax functions as destabilizing and potentially pro-cyclical (Abad, Lloyd-Braga, and Modesto 2020; Nishimura, Seegmuller, and Venditti 2015; Lloyd-Braga and Modesto 2017).[4]

By contrast, this paper focuses on the destabilizing role of the government, thus providing a novel source of instability. It aims at designing a relatively parsimonious setup, so that mechanisms remain relatively intuitive—this ruled out dynamic stochastic general equilibrium models—, based on macroeconomic relations rather than *ad hoc* elements—this eliminated game theory presentations. The framework introduced here is somewhat lean in terms of assumptions: it simply adds a government's optimization program to a standard framework. And the fiscal policy function is captured by a proportional income tax and the aggregate level of public outlays.[5] Yet, it allows some flexibility in modeling government tastes in terms of growth composition and preference for the present. Incidentally,

3. A few examples include Dufourt, Lloyd-Braga, and Modesto (2008), Schmitt-Grohé and Uribe (1997), and Farmer and Benhabib (1994).

4. There is also a substantial amount of research on regime-switching sunspot shocks regarding inflation and monetary policy (starting with Benhabib, Schmitt-Grohé, and Uribe 2001), but the government is usually absent of these models.

5. My results can also be obtained with a standard logarithmic utility function for the consumer, in which the elasticity of intertemporal substitution in consumption is unitary.

I refrain from introducing explicitly the ‘type’ of government, as would a Markov-switching model, in order to show that even in an economy where the government is elected forever and whose preferences are manifest can there be steady state multiplicity.

What matters here is that forward-looking, rational investors and consumers anticipate fiscal policy decisions and adapt their behaviors accordingly. For instance, if households observe that the government is eager to spend and run high public debt in the future, they increase savings and postpone consumption. In turns, such a behavior, bound to yield lesser tax revenues, heightens the need for future tax hikes. By contrast to the classical crowding-out effect that transits through the availability and cost of credit, crowding-out occurs here because households know governments are more impatient and it becomes better for them, in terms of welfare, to smooth out consumption inter-temporally. To get this dynamic and forward-looking perspective, I rely on a growth model *à la* Barro (1990) where the endogenous growth engine is productive government spending, echoing the rich literature on fiscal policy, growth and fluctuations, which often yield multiple equilibria.[6]

Namely, in this paper, two key ingredients are added to an endogenous growth model. First, the government is explicitly modelled as a separate agent—contrary to most of the literature on endogenous growth, for which the government (when modelled explicitly) usually follows a fiscal rule. As such, the government maximizes its own objectives. To represent the short-term electoral pressures it faces (the political economy of myopia), the government is supposed to be more impatient than households. While the government mostly derives utility from its own spending (which positively impacts productivity), it also gets an externality from private consumption. Indeed, any government—whether benevolent or selfish—has reasons to care for public spending and private consumption. Benevolent governments want public spending for the enhancing effect it has on growth, while less virtuous governments like public spending and private consumption for more selfish, electoral motives. By comparison, most of the literature on the im-

6. See for instance Cazzavillan (1996), Turnovsky (1997), Greiner and Semmler (1999), Futagami, Iwaisako, and Ohdoi (2008), Minea and Villieu (2012), and Nishimura et al. (2016); to cite only a few papers.

pact of fiscal policy on growth either relies on some sort of debt target or debt ceiling (Barro 1990; Barro and Sala-i-Martin 1992, 1995) or assumes a benevolent government to derive some normative conclusions about optimal taxation or the optimal financing of public spending (*e.g.*, Lucas Jr 1990; Judd 1985, 1999).

The interactions and frictions between households and the government, seen as two competing, forward-looking agents with different degrees of impatience, are intended to reflect more closely the political economy reality. Moreover, the government is constrained, in the sense that it cannot accumulate assets, and the only financial market it can tap is the sovereign bond market (where it is necessarily a net seller). By comparison, households are more patient and keen to accumulate productive capital.

This alone is not enough to generate instability.[7] I find that there can be only one stationary path—despite the various feedback loops, which are usually found to foster multiple equilibria (Card, Mas, and Rothstein 2008). A key engine of growth is the difference of discount rates between the government and households; however, if that difference is too important, the economy is left without any balanced growth path. Additionally, having heterogeneous preferences is necessary to endogenous growth. In fact, the government's marginal utility should be higher than that of households, and the externality from private consumption in the government's utility function hinders growth. This suggests that worse than an impatient, selfish government is an impatient government that cares a little for the welfare of its electorate.

Second, an imperfection in asset allocation is introduced in the form of an endogenous interest rate spread between private capital and Treasury bonds. This spread exacerbates the tension in the economy between consumption, investment, and public expenditure, in a context where only households can accumulate assets. Government choices affect households through (a) the impact of public spending on productivity; and (b) the quantity of sovereign bonds it issues. Similarly, the consumer's choices feed back into the government utility because of the explicit externality from private consumption, but also because investment decisions change growth prospects, thus tax income. Yet, another channel seems determinant: all

7. This finding confirms and generalizes previous literature, such as Minea and Villieu (2013).

these decisions impact the financing costs of the government. The spread makes productive assets and sovereign bonds imperfect substitutes, in the absence of other financial instruments and with the government's inability to invest in physical assets. It thereby prevents agents from fully smoothing out demand over time—in other words, it prevents them from hedging against each other's choices and preferences. Consequently, growth is necessarily lower with than without the spread.

With this second ingredient, I get a second steady state that is less intensive in public spending. Households can trade off consumption against investment; but these two decisions are not equivalent inter-temporally and impact the government differently. In parallel, the government can either spend or let households consume more. A side finding is that the externality from private consumption in the government's utility function is not necessary, but it makes the occurrence of multiplicity more likely. This seems to suggest that the more governments care about private welfare, the more instability it generates in the economy: a ruthlessly selfish government is more predictable than a somewhat benevolent one.

The steady state that is less intensive in public expenditure is unstable, while the balanced growth path with high public spending (and low consumption) is a saddle point and attracts the only converging dynamic trajectories under rational expectations. This happens as follows: as private agents expect the government to be thrifty and generate (overall) deficits, they increase capital accumulation and reduce their consumption, thereby enticing the government to spend more. The low equilibrium is not attainable except when the economy starts there or if agents are able to credibly coordinate onto it (*i.e.*, changing what each expects about the other's future decisions).

These findings have several policy implications. First, the fact that a multiplicity of stationary trajectories can result from the interplay of a government and citizens optimizing two different goals highlights that macro-fiscal outcomes do not only depend on the government's ability or willingness to implement what is best for the country. Against this risk, the government would need to credibly anchor expectations—like the central banker who commits to a nominal anchor. Fiscal policy, like monetary policy, needs to be clearly and transparently communicated for agents to coordinate towards the preferred equilibrium. Second, even if this is admittedly beyond the framework developed here, which is presented in a context of rational expectations and perfect knowledge about model parameters, similar mechanisms could produce, in situations marred with uncertainty and irrational expectations, swings in private expectations about the government's behavior and preferences can generate fluctuations. This relates to the confidence agents have in their government. Third, a key element of the model is that households and governments have different discount rates. The impatience of a government can be seen as a proxy for its credibility: too impatient a government fails to represent well its citizens and to serve their best interest; it leads households to save more, which is eventually beneficial in terms of capital accumulation but lowers the welfare of private agents. Fiscal rules or a fiscal watchdog can help curb the government's impatience and force it to account for the intertemporal consequences of its actions.

Beyond its contribution to the endogenous growth and instability models, this paper proposes a new facet of the time-consistency issues that face a government, beyond risks of defaulting or deviating from fiscal objectives. In this paper, I highlight that the interactions between sovereign and private decisions may lead to a multiplicity of equilibria. The literature often relies on *ad hoc* costs of default or deviation from prior commitments—either in the form of sanctions from the international community, exclusion from financial markets, or higher risk premia (Eaton and Gersovitz 1981) or because of reputational implications *à la* Bulow and Rogoff (1989). Even without allowing for default, the endogenous growth model developed here involves endogenous macroeconomic channels and costs, and underscores feedback loops between fiscal credibility, macroeconomic performance, and fiscal outcomes. Thus, this paper also relates to the vast literature on the two-way linkages between fiscal and macroeconomic performance. Last, since the model developed here includes an endogenous cost of public debt that responds to how agents perceive the government, it relates to the literature on interest spreads and nonlinear effects of debt accumulation. Last, this model contributes to the strand of literature that studies the interactions between public debt dynamics and fiscal policy, even though I abstract from debt constraints, limits in government ability to tax, strategic default, or sustainability concerns(as in Arellano and Bai 2017; Nishimura, Seegmuller, and Venditti 2015; Collard, Habib, and Rochet 2015).

This paper is organized as follows. In section 2, I present the various constitutive elements of the model. Section 3 highlights some properties of the balanced growth paths and explains the mechanisms at play. I discuss the existence of multiple equilibria and the interplay of preferences between the two agents, without (Section 4) and with (Section 5) an endogenous interest rate spread between private capital and sovereign bonds. I examine the stability of the stationary paths in Section 6, and conclude with some potential extensions in section 7.

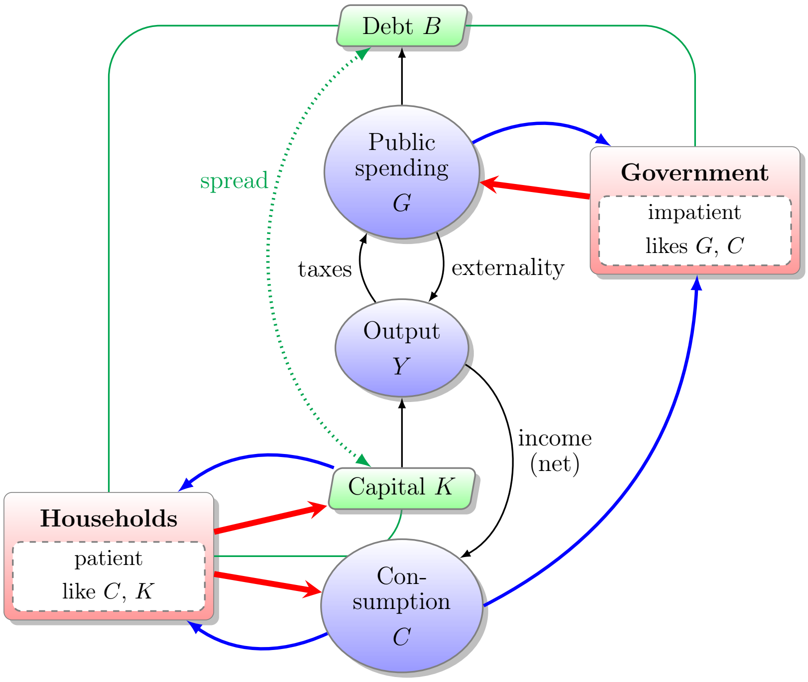
2 The model

I consider an economy in continuous time $t \in \mathbb{R}^+$ that comprised three types of agents: a constant, homogeneous population of households, a large number of identical, competitive firms, and a government. The labor supply is considered fully inelastic. Assuming the economy to be large and developed enough, with deep capital markets, I abstract from modelling international markets.

Since I am interested in how government preferences (and households' response to these preferences) impact macro-fiscal outcomes, I introduce in an endogenous growth model *à la* Barro (1990) a government that is distinct from aggregate households (whence, non-benevolent) and carries out its own maximization program. To represent the short-term electoral pressures it faces, the government is supposed to be more impatient than households; moreover, it cannot accumulate assets, and the only financial market it can tap is the sovereign bond market (where it is a net seller). By comparison, households are more patient and keen to accumulate productive capital.

The model allows for externalities between the private agent's and the government's decisions (Figure 1). Households get direct utility from consumption and equity, the latter for capitalistic reasons. In addition, they indirectly derive utility from capital, through the disposable revenue it generates and that finances

Figure 1. *A model with two utility-optimizing agents*



Note: The color of arrows work as follows: red represents the two agent's decisions in terms of their respective control variables, as part of their optimization program; blue indicates the agent's deriving utility; green shows what financial instruments each has access to; and black stands for economic impacts.

future consumption and investment. Sovereign debt, on the other hand provides additional utility and income through the productive public services it contributes to finance. As for the government, it cares for both private consumption and its own spending.

2.1 Firms

The representative firm relies on a Cobb-Douglas technology to produce the final good. As in Barro (1990), the government contributes to the production function by providing public services and infrastructure:

$$Y_t = AK_t^a(G_tL_t)^{1-a} \tag{1}$$

where Y, $A > 0$, K, L, and G denote the firm's output, total factor productivity (TFP), private capital, labor input, and public spending. $0 < a < 1$ is the capital share of income.

Public spending such as infrastructure, education, health and social insurance, and public services that preserve the rule of law and foster a better business environment (police, effective courts) contribute to make labor more productive. Note that the entire amount of public spending enters the production function in (1), even though some of it might not necessarily enhance growth. For instance, Barro (1990) and Barro and Sala-i-Martin (1992) distinguish between public expenditures that produce a positive externality and non-defense, non-education consumption services. Deficiencies in public financial management systems, as well as political incentives to choose unproductive projects, can also make public spending less efficient. These considerations are implicitly embedded in the TFP factor.[8]

The population size is normalized to one, which is equivalent to considering all variables in a per capita form (*e.g.*, $Y_t = \mathcal{Y}_t/L_t$). The final good is the numeraire; its price is omitted. Therefore, if w and r^k stand for the wage rate and the rental rate of physical capital, profit maximization gives at each time t the usual equality between each factor's marginal cost and return:[9]

$$w_t = (1-a)Y_t/L_t \quad \text{and} \quad r_t^k = aY_t/K_t \tag{2}$$

2.2 Households

The infinite-lived representative consumer supplies at each period an inelastic quantity of labor $L_t = 1$. Starting with initial endowments $K_0 > 0, B_0 \geq 0$, consumers maintain a portfolio of assets composed of productive capital K_t and sovereign bonds B_t. Physical capital depreciates at a constant rate $\delta \in \,]0\,;\,1[$.

8. To account explicitly for this dichotomy in the model, I could introduce a public spending efficiency parameter $\eta \in]0,1]$ and replace G_t in equation (1) by the share ηG_t that is valuable to firms and households. Such an efficiency parameter is implicitly captured by the TFP; resolving the model with η is strictly equivalent to replacing A with $A\eta^{1-a}$ everywhere.

9. And the usual corollary: $Y_t = r_t^k K_t + w_t$.

Consumer preferences are separable over time; instantaneous utility $\mathcal{U}_{ct}(C, K)$ stems from both consumption and capital. Namely, the utility from private consumption is a standard isoelastic function, $\frac{C_t^{1-\theta}-1}{1-\theta}$, with $0 < 1-\theta \leq 1$ the inverse intertemporal elasticity of substitution (IIES) in consumption, which indicates how much households wish to smooth consumption over time. Additionally, I assume that individuals accumulate capital not only to defer consumption, but also for its own sake—the Weberian idea of *capitalism spirit*. As in Kurz (1968a) Zou (1995), and Kamihigashi (2008), owning capital provides an instantaneous utility $v(K) = \kappa\frac{K^{1-\theta}}{1-\theta}$, with $\kappa > 0$, which I assume to be separable from consumption utility.

Under perfect foresight and denoting $\phi > 0$ the discount rate (*i.e.*, the household's degree of impatience), the consumer's intertemporal utility function is:

$$\mathcal{U}_c \equiv \mathbb{E}_0 \int_0^\infty e^{-\phi t} \mathcal{U}_{ct}(C_t, K_t) dt \equiv \mathbb{E}_0 \int_0^\infty e^{-\phi t} \left\{ \frac{C_t^{1-\theta} - 1}{1-\theta} + \kappa \frac{K^{1-\theta}}{1-\theta} \right\} dt \quad (3)$$

Households are mostly interested in their own consumption—by contrast to the government, which I allow in the next subsection to be more or less selfish. However, since they trade off current consumption against investment that will let them consume more in the future (thanks to higher income), the IIES should be at least as high as the share of capital in the economy. Empirically, a is found in the 0.3–0.5 range and θ at around 0.7 (when agents have access to capital markets and with significant heterogeneity; Havranek et al. 2015; Gruber 2013), so that it is not outrageous to assume the following:

Assumption 1 (Consumer's preference)**.** The IIES in consumption is larger than capital intensity: $\theta > a$.

Each household derives income from wage, capital, and sovereign debt. While production and profits are not taxed, all types of income are, with no deduction for depreciation. Hence the budget constraint:

$$C_t + \dot{K}_t + \dot{B}_t \leq R_t^k K_t + R_t^\ell + R_t^b B_t \quad (4)$$

where $R_t^k \equiv (1-\tau) r_t^k - \delta$, $R_t^b \equiv (1-\tau^b) r_t^b$, and $R_t^\ell \equiv (1-\tau) w_t$ stand for the after-tax rates of return on equity, sovereign bonds, and labor—with $0 < \tau < 1$

the *ad valorem* tax rate on income from capital and labor and $0 < \tau^b < 1$ the tax on sovereign bond returns.

Households maximize their utility (3) by choosing a path for C_t, B_t, and K_t, under the budget constraint (4). Under rational expectations and perfect information, it is equivalent to drop the expectation sign $\mathbb{E}$. The resulting optimization problem can be solved with the calculus of variations method.[10] For any continuous-time Lagrange multiplier λ_t that is chosen such that $\lambda_t \neq 0$ if and only if the budget constraint (4) is saturated, the consumer's utility can be written and transformed with an integration by parts as follows:

$$\begin{aligned}\mathcal{U}_c =& \int_0^{\infty} e^{-\phi t}\mathcal{U}_{ct}(C_t, K_t)dt - \int_0^{\infty} e^{-\phi t}\lambda_t \left[C_t + \dot{K}_t + \dot{B}_t - R_t^k K_t - R_t^{\ell} - R_t^b B_t\right] dt \\ =&(K_0 + B_0)\lambda_0 - \lim_{t\to+\infty} e^{-\phi t}\lambda_t(K_t + B_t) + \int_0^{\infty} e^{-\phi t}\mathcal{L}_c dt\end{aligned} \tag{5}$$

where the last integrant is $\mathcal{L}_c \equiv \mathcal{U}_{ct}(C_t, K_t) - \lambda_t \left[C_t - R_t^k K_t - R_t^{\ell} - R_t^b B_t\right] + (\dot{\lambda}_t - \phi\lambda_t)(K_t + B_t)$

The first order conditions can then be derived directly from the equivalent optimization program that maximizes $\mathcal{L}_c$ with no other constraint than non-negativity ones:[11]

$$\frac{\partial \mathcal{L}_c}{\partial C} = C_t^{-\theta} - \lambda_t = 0 \tag{6a}$$

$$\frac{\partial \mathcal{L}_c}{\partial K} = \kappa K^{-\theta} + \lambda_t R_t^k + \dot{\lambda}_t - \phi\lambda_t = 0 \tag{6b}$$

$$\frac{\partial \mathcal{L}_c}{\partial B} = \lambda_t R_t^b + \dot{\lambda}_t - \phi\lambda_t = 0 \tag{6c}$$

10. For another way to reach the same results, one can assume the budget constraint (4) is saturated, replace C_t by $K_t + R_t^{\ell} + R_t^b B_t - \dot{B}_t$ in the utility function, call $u(B, K, \dot{B}, \dot{K}, t)$ the resulting entity under the integral sign, and develop the Euler equations:

$$\forall X \in \{B, K\},\ \frac{\partial u}{\partial X} = \frac{\partial}{\partial t}\frac{\partial u}{\partial \dot{X}}$$

11. I rule out corner solutions *ab initio*; if one of the control variables C, K, or B is nil, then the corresponding first order condition is an inequality.

as well as the transversality conditions:

$$\lim_{t \to +\infty} e^{-\phi t} \lambda_t (K_t + B_t) = \lim_{t \to +\infty} e^{-\phi t} C_t^{-\theta} (K_t + B_t) = 0 \tag{6d}$$

For optimality, the budget constraint is necessarily saturated and λ admits no zero. Thus, the last one of Kuhn-Tucker conditions is $\forall t$, $\lambda_t > 0$ and equations (6a)–(6b) can be combined as follows:

$$\frac{\dot{\lambda}_t}{\lambda_t} = \phi - R_t^k - \frac{\kappa K^{-\theta}}{\lambda_t} = -\theta \frac{\dot{C}_t}{C_t} \tag{7}$$

A no-arbitrage condition stems from equations (6b)–(6c) and the Kuhn-Tucker condition; it takes the form of an endogenous spread χ_t between the rental rate of capital and sovereign yields:

$$\chi_t \equiv R_t^k - R_t^b = -\kappa \left(\frac{C_t}{K_t} \right)^\theta \tag{8}$$

This spread is always negative, indicating that public debt is more expensive than private capital. This is plausible for three reasons. First, χ_t is the after-tax spread; even though sovereign interest rates are usually lower, nominally, than corporate bond yields (for comparable instruments), they can be higher after accounting for effective taxation.[12] Typically, governments grant preferential tax treatments to income from sovereign bonds; moreover, capital is subject to corporate income tax in addition to personal income tax on non-sovereign investment. Second, sovereign bonds are often used in monetary policy operations and enjoy lower risk weights for the purpose of prudential regulations, which reduces their opportunity cost. Third, χ_t is the weighted effective spread and reflects composition effects, while the maturity structure is bound to differ between the two assets. As government debt generally has a lengthier average maturity than corporate securities, the associated risk premium is likely higher, overall.

The model thus allows for spread fluctuations: at a given level and cost of capital, the higher consumption, the more public debt is crowded out in the household's budget constraint (4) and the cheaper it is (*i.e.*, the higher sovereign yields).

12. Except when sovereign credit risk rises substantially, sovereign yields are often found to be a floor for corporate bond yields, because for sovereign securities are usually seen as the risk-less asset of an economy—especially in emerging economies where financial markets are more shallow (Bevilaqua, Hale, and Tallman 2020; Corsetti et al. 2014).

Compared with the existing theoretical literature, introducing this spread between households' and the government's respective costs of borrowing aims at modelling confidence effects in the government's and its bonds. It also plays a determinant role in fostering multiple equilibria, as sections 4 and 5 will demonstrate.

2.3 Government

The government is not a social planner in this paper (as in Acharya and Rajan 2013). It values the consumption of its electorate, but also cares for its own spending. Therefore, the government has its own optimization program, contrary to most of the literature on endogenous growth, for which the government (when modelled explicitly) follows a fiscal rule. Private consumption C_t acts as a positive externality for the government. By contrast with households, the government has no direct utility from the accumulation of capital stock and a different discount factor $\varphi > 0$. I posit the following utility function:[13]

$$\mathcal{U}_g = \int_0^\infty e^{-\varphi t} \frac{G_t^{1-\varsigma} - 1}{1-\varsigma} C_t^{\vartheta} dt \tag{9}$$

The two parameters $0 \leq \varsigma \leq 1$ and $\vartheta \geq 0$ relate to the government's relative preferences for public spending and private consumption, respectively. The inverse intertemporal elasticity of substitution (IIES) in public spending ς indicates how much governments wish to smooth their expenditure over time, while the degree of externality ϑ indicates how much the government's spending decisions are influenced by households' aggregate behavior in terms of consumption. Hence, ϑ represents a perturbing element in public expenditure intertemporal decisions; the marginal utility of public expenditure in the balanced growth path state (that is,

13. In an attempt to make notations a bit easier to remember, I use the same Greek letters for the various parameters of the government's and the consumer's utility functions, but written differently. Namely:

	Household	Government	Mnemonic
C_t	$1-\theta$	ϑ	'th' for thrift (or lack thereof)
G_t	...	$1-\varsigma$	's' for spending
Discount rate	ϕ	φ	'f' for future

the degree of homogeneity of marginal utility) is $\vartheta - \varsigma$ instead of $-\varsigma$, intuitively making convergence more sluggish and fluctuations more likely.

The government finances its spending from two sources: (a) by levying an income tax on wages and interest earnings and (b) by selling bonds to households. Notably, I refrain from assuming that the government is bound by a debt limit. I depart in this from most of the theoretical literature. Blanchard (1984) for instance assumes that there is a limit on the ability to borrow that stems from the government's ability to generate and sustain large surpluses in the future, while Futagami, Iwaisako, and Ohdoi (2008), Minea and Villieu (2013), and Nishimura et al. (2016) rely on explicit debt targets or fiscal rules. Thus, the government can run budget deficits but must comply with the intertemporal budget constraint:

$$G_t + r_t^b B_t \leq \dot{B}_t + \tau r_t^k K_t + \tau w_t + \tau^b r_t^b B_t \tag{10}$$

Incidentally, the government's and consumer's budget constraints (4) and (10) can be combined to yield the product market clearing condition:

$$\dot{K}_t \leq Y_t - \delta K_t - G_t - C_t \tag{11}$$

To solve the government's optimization problem, let me call μ_t the continuous-time Lagrange multiplier associated with the constraint (10) and write out the current-value Hamiltonian:

$$\mathcal{H}_g(B_t, G_t, \mu_t) = \frac{G_t^{1-\varsigma} - 1}{1-\varsigma} C_t^{\vartheta} + \mu_t \left[G_t + R_t^b B_t - \tau r_t^k K_t - \tau w_t\right] \tag{12}$$

under the following transversality condition:

$$\lim_{t \to \infty} e^{-\varphi t} G_t^{-\varsigma} C_t^{\vartheta} B_t = 0 \tag{13}$$

Whenever the budget constraint is saturated, $\mu_t < 0$ and the two first order conditions yield:

$$\frac{\dot{\mu}_t}{\mu_t} = \varphi - R_t^b = \vartheta \frac{\dot{C}_t}{C_t} - \varsigma \frac{\dot{G}_t}{G_t} \tag{14}$$

This equation is key to the government's tradeoff between letting households consume more or spending more itself, while such a decision in turns impact the relative price of assets.

The government in this model implicitly maximizes spending (and debt, too), which is meant to represent realistically the behavior of actual governments. Unlike other papers, my model do not impose a debt ceiling or a debt target; however, the government's preference for spending is still limited by three things: (a) the externality in the government's utility function, to the extent that public spending crowds out private consumption; (b) the cost of sovereign borrowing, which increases if households decide to consume rather than invest; and (c) the transversality constraint (*i.e.*, the sustainability of the government's behavior). The fact that the government pursues its own objectives can be interpreted from a political economy angle: for electoral reasons, governments usually enjoy spending (in ostentatious infrastructure projects or social programs, or simply to "*buy votes*"), while they also care enough about the well-being of their constituencies (which in such a model goes through utility) to be re-elected. Yet, even a benevolent government might have also good reasons to seek public spending. A good social planner should be conscious of the impact of public expenditure on production in equation (1), the social return of public spending being higher than its private return (Barro and Sala-i-Martin 1992).

In this paper, private agents do not necessarily trust the government, partly because it follows its own maximization program and does not act as a benevolent planner. But another credibility issue arises from the government's having a reputedly different discount rate than households. More precisely, I will suppose that government has a stronger preference for the present; because of electoral cycles, it has a shorter time horizon than the infinite-lived households (or successive generations who value the heirs' consumption as their own).[14] Similarly, Aguiar, Amador, and Fourakis (2019) and Acharya and Rajan (2013) argue that governments are more impatient than private agents, as they are motivated by political economy incentives, thereby generating welfare losses. The following assumption establishes the government's relative shortsightedness, which can also be interpreted as a *proxy* for successive governments with a finite time horizon.

Assumption 2 (Impatient sovereign)**.** The government is less patient than households: $\varphi > \phi$.

14. The literature on default often assumes also that governments have limited horizons (*e.g.*, Collard, Habib, and Rochet 2015).

2.4 Intertemporal equilibrium

The resulting dynamic system is the set of equations (7), (10), (11), and (14). The state variables K_t and B_t are predetermined, their values being inherited at each time t from the past history $s < t$. On the other hand, the control variables C_t and G_t are forward-looking. Relying on the non-arbitrage equation (8) and the expressions (2) of the wage and return on investment, I reach the following differential equations for the various variables.

$$\vartheta\frac{\dot{C}_t}{C_t} - \varsigma\frac{\dot{G}_t}{G_t} = \varphi - (1-\tau)a\frac{Y_t}{K_t} + \delta + \chi_t \tag{15a}$$

$$\theta\frac{\dot{C}_t}{C_t} = (1-\tau)a\frac{Y_t}{K_t} - \delta - \phi \tag{15b}$$

$$\dot{B}_t = G_t - \tau Y_t + \left[(1-\tau)a\frac{Y_t}{K_t} - \delta - \chi_t\right] B_t \tag{15c}$$

$$\dot{K}_t = Y_t - \delta K_t - G_t - C_t \tag{15d}$$

Definition 1. With Y_t and χ_t given respectively in (1) and (8), an intertemporal perfect foresight equilibrium is a path $\{C_t, G_t, B_t, K_t\}_{t\in\mathbb{R}_+}$ satisfying the laws of motion (15) for given (K_0, B_0), as well as the transversality conditions (6d) and (13) and the following sign restrictions: $\forall t$, $(C_t,\ G_t,\ B_t,\ K_t) \in \mathbb{R}_+^4$.

Combining equations (15a)–(15c) offers some insights on how the preference parameters impact the pace of debt accumulation: $\frac{\dot{B}_t}{B_t} = \frac{G_t - \tau Y_t}{B_t} + \varphi - \vartheta\frac{\dot{C}_t}{C_t} + \varsigma\frac{\dot{G}_t}{G_t} = \frac{G_t-\tau Y_t}{B_t} + \phi + \theta\frac{\dot{C}_t}{C_t} - \chi_t$. For a given primary surplus, the government tends to accumulate more debt, the higher its preference for the present, the higher its self-interest in growing its own spending, and the less it cares about private consumption. Conversely, debt grows more slowly when households have a higher utility from consumption and a lower direct utility from capital accumulation (which means they are all the more eager to consume rather than save, hence demand higher interest rates on the government's borrowing). It might seem surprising that public debt grows faster when households are more impatient; it is because of the externality of public spending in production: more public spending today provides households with a higher disposable revenue in the future, even though it will need to be financed through taxes later in the future.

As common in the literature, I normalize all variables by the stock of capital. Namely, I let $x_t \equiv \frac{X_t}{K_t}$ denote the shares of total private capital, for $X = C, G, B$, and $\gamma_t \equiv \frac{\dot{K}_t}{K_t}$ the growth rate. With these notations, the spread between private and sovereign yields is $\chi_t = -\kappa c_t^\theta$, and I get:

$$\vartheta \frac{\dot{c}_t}{c_t} - \varsigma \frac{\dot{g}_t}{g_t} = \varphi - (1-\tau) a A g_t^{1-a} + \delta + \gamma_t(\varsigma - \vartheta) - \kappa c_t^\theta \tag{16a}$$

$$\theta \frac{\dot{c}_t}{c_t} = (1-\tau) a A g_t^{1-a} - \delta - \phi - \theta \gamma_t \tag{16b}$$

$$\dot{b}_t = g_t - \tau A g_t^{1-a} + \left[(1-\tau) a A g_t^{1-a} - \delta + \kappa c_t^\theta - \gamma_t\right] b_t \tag{16c}$$

$$\gamma_t = A g_t^{1-a} - \delta - g_t - c_t \tag{16d}$$

The growth rate γ can be negative without implying that the economy altogether is in recession. The output growth is $\hat{y}_t = a\gamma_t + (1-a)\frac{\dot{G}_t}{G_t} = \gamma_t + (1-a)\frac{\dot{g}_t}{g_t}$. The model allows for situation where government spending is the main driver of economic growth (as is the case in some countries where the State is over-bloated, with an overmanned public service and monopolistic state-owned enterprises).

Equation (16c) can be interpreted as a classic debt-accumulation equation. The first two terms form the primary deficit.[15] The square bracket and the last term form the automatic debt dynamics—what the literature often refers to as the interest rate-growth differential.

3 Balanced growth paths

In this section, I examine the steady state(s) of the economy, that is the equilibrium such that c_t, g_t, and b_t are constant over time, under the assumption

15. I call $\tau A g^{1-a} - g$ the primary balance slightly abusively, as it implicitly subtracts from the overall balance the interest bill net of income tax on sovereign yields, rather than the gross interest bill.

that taxation also remains unchanged. On such a balanced growth path, consumption, production, public debt, and public spending are proportional to the stock of capital. In other words, they all grow at the same rate, $\gamma^\star$, so that $\forall X \in \{G, C, B, K\}$, $\forall t \in \mathbb{R}_+$, $X_t = x^\star K_0 e^{\gamma^\star t}$ with $x^\star$ the steady state ratio X/K.[16] Replacing these in the transversality conditions (6d) and (13) yields the following constraints on the growth rate:

$$\gamma^\star < \frac{\varphi}{1-\varsigma+\vartheta} \qquad ; \qquad \gamma^\star < \frac{\phi}{1-\theta} \tag{17}$$

If the accumulation of capital was faster than the ratio of the discount factor and the degree of homogeneity of the utility of an agent, then that utility would diverge.

Definition 2. A balanced growth path are steady state values $(c^\star, g^\star, \gamma^\star, b^\star) \in \mathbb{R}_+^4$ satisfying the inequalities (17) and the following system of equations, where $\xi \equiv \vartheta - \varsigma + \theta$:

$$\xi\gamma^\star = \varphi - \phi - \kappa c^{\star\theta} \tag{18a}$$

$$(1-\tau)aAg^{\star 1-a} = \phi + \delta + \theta\gamma^\star \tag{18b}$$

$$c^\star = Ag^{\star 1-a} - \delta - g^\star - \gamma^\star \tag{18c}$$

$$\tau Ag^{\star 1-a} - g^\star = b^\star \left[\varphi - \gamma^\star(1+\vartheta-\varsigma)\right] \tag{18d}$$

A new parameter emerges: ξ is the divergence between the government and households in terms of the total marginal utility (including the impact of the externality). It is also the gap between the preferences of the two agents and can be decomposed with the various weights involved in the two utility functions: $\xi = [\vartheta - (1-\theta)] + [1-\varsigma]$; the first bracket is the difference between the externality in the government's utility and the household's IIES in consumption, and the second is the government's IIES in public spending (from which households do not derive any utility).

16. For convenience, I use the same notation for capital as for other variables, but obviously $k^\star = 1$.

This parameter ξ plays a crucial role. If the government's and households' utility functions had the same degree of homogeneity ($\xi = 0$), then either $\phi = \varphi + \chi^\star$ and any growth rate $\gamma^\star$ could be solution, or the dynamic system would diverge without any steady state (which is a consequence of the absence of an exogenous debt limit in my model). For the remainder of paper, I will avoid such a situation and assume agents are *heterogeneous enough*. Furthermore, provided that $\xi \neq 0$, the consumption in the steady state is $c^\star = \left[\varphi - \phi - \xi \frac{(1-\tau)aAg^{\star 1-a} - \phi - \delta}{\theta}\right]^{1/\theta} \frac{1}{\kappa^{1/\theta}}$, which imposes a threshold $\varpi \equiv \frac{(\vartheta - \varsigma)(\phi+\delta) + \theta(\varphi+\delta)}{\xi a A(1-\tau)}$ on the admissible values of public spending for a steady state to be exist. Depending on the sign of ξ, this threshold will act either as an upper or a lower bound on $g^\star$. The discriminating condition $\xi \lesseqgtr 0$ thus implies very different steady states. As having an upper bound on public spending is more realistic, I will assume that ξ is non-negative. Appendix B provides the proof that the case $\xi < 0$ is anyway less interesting, as it does not yield multiple equilibria.

Assumption 3. The utility functions of the government and households are distinct, and that of the government has a higher degree of homogeneity: $\xi = \theta + \vartheta - \varsigma > 0$.

The steady-state growth rate of the economy, $\gamma^\star$, is determined by the interplay between the government's and the private agent's respective preferences. However, the government is the only one to influence the productivity of the economy—the output per unit of private capital being $y_t = Ag_t^{1-a}$. Contrary to the standard endogenous growth literature where growth results from savings, technology, and capital decay (*e.g.*, AK models *à la* Romer 1986), it is in this paper's model a function of the deep parameters describing the agents' preferences: $\gamma^\star = (\varphi - \phi + \chi^\star)/\xi$. It can be read as the ratio of preference heterogeneity between the sovereign and households in terms of: (1) their discount factors (adjusted for the relative cost of financing $\chi^\star$) and (2) their propensity to enjoy more spending in the economy (whether theirs or others), as it increases marginally their utility. For growth to be positive, the most impatient agent needs to also have the highest marginal utility.

To better understand how state variables B and K interlink the respective preferences of households and the sovereign, it is useful to extract from equation (18a) a non-arbitrage condition between households and the government in terms of discounted marginal utility:

$$R^b + \gamma^\star(\vartheta - \varsigma) - \varphi = R^k - \theta\gamma^\star - \phi \tag{19}$$

Since $\xi > 0$, the growth rate decreases with the interest rate spread. As a matter of fact, the growth rate is always smaller than in the case without spread $\kappa = 0$, which I will treat specifically in the next section: $\gamma^\star < \frac{\varphi - \phi}{\xi}$; in a sense, the spread thus distorts resource allocation to debt-financed, growth-enhancing public spending. Growth is positive only when the government is sufficiently impatient to compensate the spread it faces.

Preferences interplay through two channels. First, a spread between capital and public debt stems from the households' portfolio decisions, based on private agents' interest in owning capital. Since public debt crowds out consumption and capital accumulation, the larger marginal utility $(-\theta)$ households derive from consumption, the more they can afford to finance private capital at a high cost. Second, there is a tradeoff between agents *via* the level of public debt. When the government is more impatient than households (*i.e.*, $\varphi > \phi$), households need to have a higher marginal utility than the government for growth to be positive. Otherwise, the government takes on more debt, which leads to an unwelcome outcome (namely, an attrition of the capital stock). Only when households have strong views and preferences can they impose some discipline on governments, by rationing its capacity to borrow. Indeed, for a household, the bigger its marginal utility, the lower the price of private capital relative to sovereign yields.

In this model, higher public spending is unconditionally associated with higher growth, thanks to its externality on the production function, but it can crowd out private consumption. Considering $g^\star$ as a variable for a moment, it is straightforward from (18b) that $\gamma^\star$ grows with $g^\star$. By contrast, private consumption benefits from public services only up to a certain point: namely, up to $g^\star = ((1-a)A)^{1/a}$, which is also the level of public expenditure that households would choose if they could (appendix A). Intuitively, public services provide an externality that enhances labor productivity, but their financing weighs on the consumer's purchasing power and crowds private investment out. Beyond a certain level, the cost of public expenditure outweighs its benefits, which is reminiscent of the "*Armey curve*"

—the inverted U-shape relation between the government's size and GDP growth first described by Armey (1995). The tipping point beyond which marginal public spending is counterproductive is higher, the larger the labor intensity in the production function and the higher overall productivity.

The government faces adverse debt dynamics in the steady state. The right-hand side of equation (18d) is always positive, as per the transversality condition (17). As the public debt ratio $b^\star$ cannot be negative, the government ought to generate a primary surplus in the steady state: $\tau A g^{\star 1-a} - g^\star > 0$. Therefore, at the steady state, government spending $g^\star$ will have to be lower than $(\tau A)^{1/a}$.[17] This is not necessarily orthogonal to papers that introduce persistent deficits in growth models (*e.g.*, Minea and Villieu 2012), as these focus on overall (not primary) deficit.

Turning to the household's optimization problem, consuming more is not necessarily a Pareto-improvement. Supposing the economy starts at $t = 0$, and stays on its balanced growth path forever thereafter, the welfare of the representative household, as defined by the utility they get from steady-state consumption and investment, is:[18]

$$\mathcal{U}_c^\star = \frac{K_0^{1-\theta}}{(1-\theta)(\phi - \gamma^\star(1-\theta))}\left[c^{\star 1-\theta} + \kappa\right] - \frac{1}{(1-\theta)\phi} \tag{20}$$

Since the growth rate of capital $\gamma^\star$ decreases with $c^\star$ (equation (18a)), welfare is not necessarily an increasing function of private consumption. I notice that $\frac{\partial \mathcal{U}_c^\star}{\partial c^\star}$ is proportional to $-(2\theta-1)\kappa - \theta\kappa^2 c^{\star\theta-1} - \left[(1-\varsigma+\vartheta)\phi - (1-\theta)\varphi\right] c^{\star -\theta}$ is not always positive. In particular, when the appetite for capital accumulation κ is large, the first terms dominate the square bracket, making welfare higher with a lower level of consumption (which allows for more investment). Similarly, when the government is sufficiently impatient ($(1-\theta)\varphi > (1-\varsigma+\vartheta)\phi$), households are always better off with less consumption; instead, they save and derive utility from building capital.

17. This condition rules out the level of public spending households would choose in absence of a government, $\tilde{g} = ((1-a)A)^{1/a}$ (see appendix A), as in general $1-a$ is larger than τ.

18. Similarly, the government's welfare is $\mathcal{U}_g^\star = \frac{K_0^{1-\varsigma+\vartheta} g^{\star 1-\varsigma} c^{\star\vartheta}}{(1-\varsigma)(\varphi - \gamma^\star(1-\varsigma+\vartheta))} - \frac{K_0^{\vartheta} c^{\star\vartheta}}{(1-\varsigma)(\varphi - \gamma^\star\vartheta)}$.

4 Role of heterogeneous preferences in the absence of wealth utility

This section examines the role played by the various intra- and inter-temporal preference parameters in the model. To characterize the steady state solution when households derive no utility from capital—at least, not directly—, I temporarily assume away the spread:

Assumption 4 (No spread)**.** Agents do not value the holding of productive assets *per se*: $\kappa = 0$. Therefore, the spread χ_t is nil.

The following system describes the balanced growth path:

$$\gamma^\star = \frac{\varphi - \phi}{\xi} \tag{21a}$$

$$g^\star = \left(\frac{\phi + \delta + \xi^{-1}(\varphi - \phi)\theta}{(1-\tau)aA} \right)^{\frac{1}{1-a}} \tag{21b}$$

$$c^\star = A g^{\star 1-a} - \delta - g^\star - \xi^{-1}(\varphi - \phi) \tag{21c}$$

$$b^\star = \frac{\tau A g^{\star 1-a} - g^\star}{\varphi - \xi^{-1}(\varphi - \phi)(1 + \vartheta - \varsigma)} \tag{21d}$$

Growth sustainability stems for the relative preferences of the heterogeneous agents and their relative level of impatience, as can be observed in equation (21a). In particular, the growth rate $\gamma^\star$ would not be positive without Assumptions 2 and 3. Whomever has the higher discount rate shall benefit spend more, while others shall save.[19] It is actually this very discrepancy in preferences that generates endogenous growth; yet, the smaller the differences in marginal utility (when $\xi \to 0^+$), the higher growth. The tax rate does not impact the growth rate at all.

Public spending is higher the larger the fiscal space, but also the more crucial its role in the production function. As appears from equation (21b), a higher the tax

19. This finding somehow reminiscent of Ramsey (1928)'s "*division of society into two classes, the thrifty enjoying bliss and the improvident at the subsistence level,*" although in an admittedly very different setup where only private agents interact and can accumulate assets.

rate gives the government more space to spend (formally: $\partial g^\star/\partial\tau = \frac{g^\star}{(1-\tau)(1-a)} > 0$). Besides, when capital (through capital intensity a or the depreciation rate δ) or the TFP A contribute more to growth, the government spends less. At a given level of capital accumulation $\gamma^\star$, the government's spending decision is also determined by consumers' preferences: it is higher the more patient households are, and the less marginal utility they derive from consumption. Equation (21c) is mostly an accounting identity; households consume whatever is left once a share of output ($y^\star = Ag^{\star 1-a}$) has been used for government's spending and for gross investment (including depreciation).

Proposition 1 (Unicity without spread)**.** *Under assumptions 1–4, there exists a unique balanced growth path with positive growth and public spending provided that the government is not too impatient:*

$$1-\theta < (1-\varsigma+\vartheta)\frac{\phi}{\varphi} < 1-\varsigma+\vartheta \tag{22}$$

Moreover, there exist $0 < \Phi_b < \Phi_c$ *such that:*

(a) $\forall \varphi-\phi < \xi\Phi_b$, *public debt is positive in the steady state;*

(b) $\forall \varphi-\phi < \xi\Phi_c$, *private consumption is positive in the steady state.*

This unique equilibrium is locally unstable.

■ *Proof of Proposition 1.* Uniqueness of the steady state comes immediately from system (21). Moreover, since $\xi > 0$ and $\varphi > \phi$, all variables are defined; and $g^\star$ and $c^\star$ are positive as soon as they exist. Condition (22) stems from factoring equation (18a) in the transversality conditions; it imposes a ceiling on the degree of homogeneity of the consumer's utility that is slightly stricter than that of Assumption 3. Provided this ceiling is respected, $g^\star > \tau A g^{\star 1-a}$ is enough to ensure that $b^\star > 0$; the *sine qua non* condition is:

$$g^\star = \left(\frac{\phi+\delta+\xi^{-1}(\varphi-\phi)\theta)}{(1-\tau)aA}\right)^{\frac{1}{1-a}} < (\tau A)^{1/a}$$

which leads me to define $\Phi_b \equiv \frac{(1-\tau)aA(\tau A)^{\frac{1-a}{a}}-\phi-\delta}{\theta}$.

For $c^\star = Ag^{\star 1-a} - g^\star - \delta - \xi^{-1}(\varphi-\phi)$ to be positive, I need:

$$g^\star = \left(\frac{\phi+\delta+\xi^{-1}(\varphi-\phi)\theta)}{(1-\tau)aA}\right)^{\frac{1}{1-a}} < \frac{\phi+\delta(1-(1-\tau)a)+\xi^{-1}(\varphi-\phi)(\theta-(1-\tau)a)}{(1-\tau)a}$$

Table 1. *Sensitivity of growth and public spending to preference parameters in the absence of spread*

	Sensitivity to x	
Parameter x	$g^\star$	$\gamma^\star$
Household's IIES θ	$\vartheta - \varsigma$	-1
Externality ϑ in $\mathcal{U}_g$	$-\theta$	-1
Government's IIES ς	θ	1

Note: The two columns provide respectively $\frac{\partial g^{\star 1-a}}{\partial x}$ (abstracting from the positive factor $\frac{\varphi-\phi}{\xi^2(1-\tau)aA}$) and $\frac{\partial \gamma^\star}{\partial x}$ (abstracting from the positive factor $\frac{\varphi-\phi}{\xi^2}$).

The function $f:\ x \mapsto \frac{\phi+\delta(1-(1-\tau)a)+x(\theta-(1-\tau)a)}{(1-\tau)a} - \left(\frac{\phi+\delta+x\theta}{(1-\tau)aA}\right)^{\frac{1}{1-a}}$ is concave, with $f(0) > 0$ and $\lim_{x\to+\infty} f(x) = -\infty$, so there exists a unique Φ_c such that $\forall x \in \mathbb{R}_+,\ f(x) > 0 \Leftrightarrow x < \Phi_c$. The fact that $f(\Phi_b) = \frac{\phi+\delta(1-\theta)}{\theta} + (\tau A)^{1/a}\frac{1-\tau}{\tau} > 0$ proves that Φ_b is smaller than Φ_c. The proof of instability is provided in appendix D. ■

Proposition 1 shows that there is an upper limit on how much more impatient the government can be relatively to households, however this impatience may generate growth. The upper limit is higher when the two agents' preferences are more divergent (*i.e.*, ξ larger), confirming my earlier interpretation that only heterogeneous agents can afford to have different discount rates. Yet, if the government grew too impatient, then it would eventually suppress private consumption altogether.

Apart from discount rates, what role do the various preference parameters play in the model? Table 1 summarizes the sensitivity of growth and public spending to the various preference parameters. The economy grows faster the more the government wants to spend, and the less both agents enjoy private consumption. In particular, instead of contributing to a better outcome, the fact that the government cares for private consumption by the prism of the externality ϑ is detrimental to growth. Public spending, like growth, increases when the government values less private consumption and more its own. The impact of the household's preference on public spending depends on the sign of the government's marginal utility,

$\vartheta - \varsigma$; if $\varsigma > \vartheta$, public spending (at the steady state) is a decreasing function of the household's IIES in consumption. Last, private consumption rises with growth only when the government is not too impatient.[20]

In the absence of the externality from private consumption onto public spending decisions (the source of inefficiency in the model), the economy grows only when the impatient government has stronger preferences than households, meaning that the government's IIES in public spending is higher than the household's IIES in private consumption. If $\vartheta = 0$, the government derives no utility from private consumption—at least *not directly*: there is still, by design, a feedback loop through investment and taxation. This incidentally replicates the standard models found in the literature, although one major difference between this paper and most of the existing literature is the optimizing behavior for the government. By contrast, for instance, Futagami, Iwaisako, and Ohdoi (2008) only have a debt ceiling rule, but I can still obtain a very similar setup to theirs by imposing that the government has the same discount factor as private agents (*i.e.*, $\phi = \varphi$).[21] In this case, I find like them that $b^\star = \frac{\tau A {g^\star}^{1-a} - g^\star}{\phi}$; but since $\gamma^\star(\theta - \varsigma) = 0$, I would need to impose also $\theta = \varsigma$ to ensure that the economy is growing. In a more general case, when the two agents have different preferences for the present, I get $\gamma^\star = \frac{\phi - \varphi}{\varsigma - \theta}$. The most impatient of the two agents needs a larger utility from her own consumption for growth to be positive. And, indeed, public spending $g^\star = \left(\frac{\phi + \delta + \theta\gamma^\star}{(1-\tau)aA}\right)^{\frac{1}{1-a}}$ is larger the larger θ.

5 Role of the endogenous spread

In this section, I return to the general formulation where households derive utility from owning private capital, dropping Assumption 4 (but maintaining As-

20. The derivative $\frac{\partial c^\star}{\partial \gamma^\star} = \frac{[\theta - (1-\tau)a](1-a)A - \theta {g^\star}^a}{(1-\tau)(1-a)aA}$ is positive if and only if $\left(\frac{\phi + \delta + \xi^{-1}(\varphi - \phi)\theta}{(1-\tau)aA}\right)^{\frac{a}{1-a}} < \frac{\theta - (1-\tau)a}{\theta}(1-a)A$.

21. Futagami, Iwaisako, and Ohdoi (2008) use a logarithmic utility function for households: $\mathcal{U}_c = \int_0^\infty \ln C_t e^{-\phi t} dt$, which I can exactly reproduce by doing $\theta \to 1$. Appendix C examines the logarithmic case in further details.

sumptions 1–3). Consequently, they are more willing to smooth out consumption over time to undertake investment, and make spendthrift governments pay more interests.[22] To solve the system, I first use (18b) to express growth as a function of public spending:

$$\gamma^\star = \frac{(1-\tau)aAg^{\star 1-a} - \phi - \delta}{\theta} \tag{23a}$$

I denote $\varpi \equiv \frac{(\vartheta-\varsigma)(\phi+\delta)+\theta(\varphi+\delta)}{\xi aA(1-\tau)}$ and derive from equations (18a)–(18c) two expressions for consumption as a function of $g^\star$:

$$c^\star = c_1(g^\star) \equiv \left(\frac{(1-\tau)aA}{\kappa\theta}\right)^{1/\theta} \left[\xi\varpi - \xi g^{\star 1-a}\right]^{1/\theta} \tag{23b}$$

$$= c_2(g^\star) \equiv \frac{A\left(\theta-(1-\tau)a\right)}{\theta} g^{\star 1-a} - g^\star + \frac{\phi+\delta(1-\theta)}{\theta} \tag{23c}$$

$\varpi^{1/1-a}$ thus appears as the *maximum* admissible value for $g^\star$.[23] Looking at the previous equations, it may seem like the government's preference parameters play but a minor role compared with the consumer's. Yet, they are embedded in ϖ and ξ—in particular, $aA(1-\tau)\varpi = \frac{(\vartheta-\varsigma)(\phi+\delta)+\theta(\varphi+\delta)}{\xi}$ is a centroid of $\phi+\delta$ and $\varphi+\delta$, with weights corresponding to the marginal utilities of the government and the household. In other words, ϖ represents the overall selfishness of the two agents. Once the control variables $c^\star$, $g^\star$ are chosen, public debt stems easily:

$$b^\star = \frac{\tau Ag^{\star 1-a} - g^\star}{\varphi - \gamma^\star(1+\vartheta-\varsigma)} = \frac{\tau Ag^{\star 1-a} - g^\star}{\varphi - \frac{(1-\tau)aAg^{\star 1-a}-\phi-\delta}{\theta}(1+\vartheta-\varsigma)} \tag{23d}$$

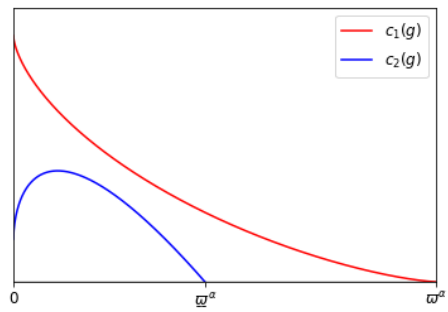
The necessary condition for the economy to grow in the steady state is that $g^{\star 1-a}$ be larger than $h_0 \equiv \frac{\phi+\delta}{(1-\tau)aA} < \varpi$. This means that not only the government cannot be atrophied ($g^\star = 0$ generally failing to satisfy equations (23b) and (23c) simultaneously), but also it ought to play a significant enough role in the economy. This comes directly from the fact that public spending contributes to factor productivity in equation (1).

22. At the steady state, the spread $\chi^\star = \phi - \varphi + \xi \frac{(1-\tau)aAg^{\star 1-a}-\phi-\delta}{\theta}$ is growing with the level of public spending.

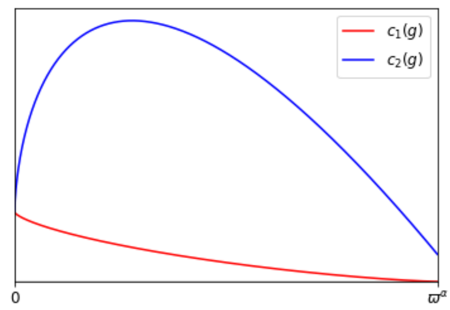
23. One can recognize that $\varpi^{1/1-a}$ is the steady state I found in the previous section, in absence of the spread κ.

Figure 2. *Possible configurations for the two characteristic functions*

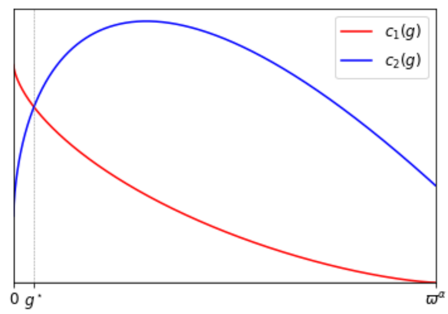
(a) c_1 above c_2



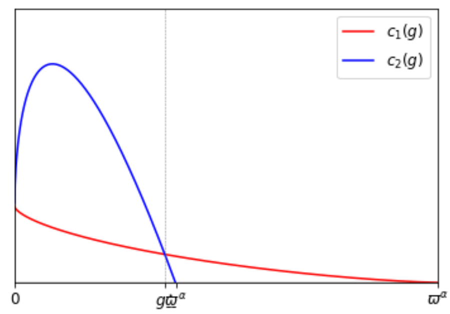
(b) c_2 above c_1



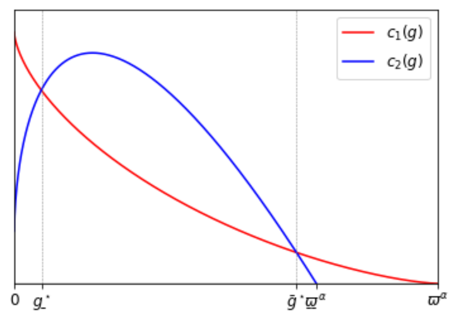
(c) Unique steady state with $c_2'(g^\star) > c_1'(g^\star)$



(d) Unique steady state with $c_2'(g^\star) < c_1'(g^\star)$



(e) Dual steady states



Note: The exponent α is defined as $\alpha \equiv \frac{1}{1-a}$.

Proposition 2 (Multiplicity)**.** *Let* $\varpi = \frac{(\vartheta-\varsigma)(\phi+\delta)+\theta(\varphi+\delta)}{\xi a A(1-\tau)}$. *Under Assumptions 1–3, the system* (23) *admits at most two solutions. Furthermore, there exist critical values* $\underline{\varpi} > 0$, $0 < \underline{\kappa} < \overline{\kappa}$, *and* $\hat{\varphi} > \phi$ *such that the following is true:*

(a) If $\kappa > \overline{\kappa}$ *and* $\varpi < \underline{\varpi}$*, there is no stationary solution (Figure 2b).*

(b) If $\kappa > \overline{\kappa}$ *and* $\varpi > \underline{\varpi}$*, there is one stationary solution,* $\overline{g}^\star$*, such that* $c_2'(\overline{g}^\star) < c_1'(\overline{g}^\star)$*, which is associated with positive growth* $\overline{\gamma}^\star$ *provided that* $\varphi < \hat{\varphi}$ *(Figure 2d).*

(c) If $\kappa < \overline{\kappa}$ *and* $\varpi < \underline{\varpi}$*, there is one stationary solution,* $\underline{g}^\star$*, such that* $c_2'(\underline{g}^\star) > c_1'(\underline{g}^\star)$*, which is associated with positive growth* $\overline{\gamma}^\star$ *provided that* $\varphi > \hat{\varphi}$ *(Figure 2c).*

(d) When $\kappa < \overline{\kappa}$ *and* $\varpi > \underline{\varpi}$*, there are either no (Figure 2a) or two solutions (Figure 2e). A necessary and sufficient condition for two steady states to co-exist is that* $\kappa > \underline{\kappa}$*. In such as case, I denote* $0 < \underline{g}^\star < \overline{g}^\star < \varpi^{\frac{1}{1-a}}$ *the two steady states and* $\forall x \in \{c, b, \gamma, \chi\}$, $\underline{x}^\star$, $\overline{x}^\star$ *the associated variables. Imposing* $\varphi > \hat{\varphi}$ *ensures that the economy grows in both steady states. The lower (*i.e., *less intensive in public spending) steady state is richer in private consumption than the higher one (*$\underline{c}^\star > \overline{c}^\star$*) but grows more slowly (*$\underline{\gamma}^\star < \overline{\gamma}^\star$*) and carries more debt (*$\underline{b}^\star > \overline{b}^\star$*).*

Last, if $\varpi < (\tau A)^{1/a}$*, the government runs a primary surplus in all existing steady states.*

■ *Proof of Proposition 2.* To simplify the calculus, I define $C_1(h) \equiv S\left[\varpi - h\right]^{1/\theta}$; $C_2(h) \equiv \frac{AW}{\theta}h - h^{\frac{1}{1-a}} + V$; $V \equiv \frac{\phi+\delta(1-\theta)}{\theta}$; $W \equiv \theta - (1-\tau)a$; and $S \equiv \left(\frac{(1-\tau)aA\xi}{\kappa\theta}\right)^{1/\theta}$. The system (23b–23c) is then equivalent to $c^\star = C_1(g^{\star 1-a}) = C_2(g^{\star 1-a})$. Since $g \mapsto g^{1-a}$ is a one-to-one transformation of $\mathbb{R}_+$, the number of balanced growth paths equals that of intersections between $C_1(\cdot)$ and $C_2(\cdot)$.

The function C_1 is defined, positive, decreasing, convex, and C^∞ on $]0\,;\varpi[$, with $C_1(0) > 0$ and $C_1(\varpi) = 0$ (Figure 2). Besides, since $W > 0$ under Assumption 1, C_2 is concave, increasing on the interval $[0\,; h_2']$, with $h_2' \equiv \left(\frac{AW(1-a)}{\theta}\right)^{\frac{1-a}{a}}$, and decreasing thereafter, thereby admitting $C_2(h_2') = \frac{a}{1-a}\left(\frac{AW(1-a)}{\theta}\right)^{\frac{1}{a}} + V$ as its global *maximum.*

Because $C_2(0) = V > 0$ and $\lim_{+\infty} C_2 = -\infty$, C_2 admits a unique zero $\underline{\varpi} > h_2'$, which constitutes an upper bound to the set of acceptable solutions, since negative levels of private consumption are of no interest to this paper.

Existence of steady states. The difference $C_1 - C_2$ is a strictly convex function. This in itself proves that there cannot be more than two stationary solutions. If $C_1(0) < C_2(0)$ and $C_1(\varpi) = 0 < C_2(\varpi)$, there is none. Posing:

$$\overline{\kappa} \equiv \frac{(\vartheta - \varsigma)(\phi + \delta) + \theta(\varphi + \delta)}{\theta} \left[\frac{\theta}{\phi + \delta(1-\theta)}\right]^{\theta} \tag{24}$$

the former is equivalent to $\kappa > \overline{\kappa}$, and the latter to $\varpi < \underline{\varpi}$. If $\kappa < \overline{\kappa}$ and $\varpi < \underline{\varpi}$, or else if $\kappa > \overline{\kappa}$ and $\varpi > \underline{\varpi}$, $C_1(\cdot)$ and $C_2(\cdot)$ cross each other only once, and there is a unique solution.

Let me assume now that $\kappa < \overline{\kappa}$ and $\varpi > \underline{\varpi}$. Either C_1 remains above C_2 $\forall h \in [0\,;\,\varpi]$, or they cross twice.[24] Since $C_1'(0) < C_2'(0)$ (from Assumption 1) and $C_1'(\varpi) = 0 > C_2'(\varpi)$ (from $\varpi > \underline{\varpi} > h_2'$), $C_1 - C_2$ is continuous, decreases between 0 and a certain $\tilde{h} > h_2'$, and increases on $\left[\tilde{h}\,;\,\varpi\right]$. Hence, there are exactly two solutions to $C_1(h) = C_2(h)$ if and only if $C_1(\tilde{h}) < C_2(\tilde{h})$. At this stage, I need to step back and explicit the dependence of all these entities on the parameter κ: $\tilde{h}(\kappa)$ is the unique h such that $C_1'(\tilde{h}(\kappa), \kappa) = C_2'(\tilde{h}(\kappa))$ and I am looking for a discriminating criterion for $\nu(\kappa) = C_1(\tilde{h}(\kappa), \kappa) - C_2(\tilde{h}(\kappa)) \lesseqgtr 0$. Because $\forall 0 < \kappa < \overline{\kappa}$, $\nu'(\kappa) = \frac{\partial C_1}{\partial \kappa}(\tilde{h}(\kappa)) + \frac{\partial \tilde{h}}{\partial \kappa}\left[C_1'(\tilde{h}(\kappa)) - C_2'(\tilde{h}(\kappa))\right] = \frac{\partial C_1}{\partial \kappa}(\tilde{h}(\kappa)) < 0$, there is a unique $\underline{\kappa} \in \,]0\,;\,\overline{\kappa}[$ such that $\forall \kappa \in \,]0\,;\,\overline{\kappa}[\,,\ \nu(\kappa) < 0 \iff \kappa > \underline{\kappa}$.

Positive growth. Owing to equation (23a), I want to make sure that any steady state be such that $g^{\star 1-a} < h_0 = \frac{\phi+\delta}{(1-\tau)aA} < \varpi$. I first study cases such that there is a *low* steady state such that $c_1'(\underline{g}^\star) < c_2'(\underline{g}^\star)$ (this happens when $\kappa < \overline{\kappa}$ and $\varpi < \underline{\varpi}$, or else when $\underline{\kappa} < \kappa < \overline{\kappa}$ and $\varpi > \underline{\varpi}$). That low steady state is higher than $h_0^{\frac{1}{1-a}}$ if and only if $C_1(h_0) > C_2(h_0)$ and $C_1'(h_0) < C_2'(h_0)$, the second condition being superfluous in the case of a single steady state—it might help to refer to Figure 2 to better visualize. Given that $C_1(h_0) = \left(\frac{\varphi - \phi}{\kappa}\right)^{1/\theta}$ and $C_2(h_0) = \frac{\phi+\delta}{(1-\tau)a} - \delta - h_0^{\frac{1}{1-a}}$, the inequality $C_1(h_0) > C_2(h_0)$ is equivalent to $\varphi > \hat{\varphi}$, with:

$$\hat{\varphi} \equiv \phi + \kappa \left[\frac{\phi + \delta}{(1-\tau)a} - \delta - \left(\frac{\phi + \delta}{(1-\tau)aA}\right)^{\frac{1}{1-a}}\right]^{\theta}$$

24. Technically, there is a rare, tangent case, too, with the two functions touching only once without crossing.

Meanwhile, under Assumption 2, $C_1'(h_0) < C_2'(h_0)$ is equivalent to $\varphi > \underline{\varphi}$, with:

$$\underline{\varphi} \equiv \phi + \kappa^{\frac{1}{1-\theta}} \left[\frac{\max\left\{0;\ \theta^2 \left(\frac{\phi+\delta}{(1-\tau)aA}\right)^{\frac{a}{1-a}} - A(1-a)\theta^2 + (1-\tau)(1-a)a\theta\right\}}{(1-\tau)(1-a)aA\xi} \right]^{\frac{\theta}{1-\theta}}$$

However, this second condition is superfluous as $\hat{\varphi} > \underline{\varphi}$. Indeed, when $\underline{\varphi}$ is strictly larger than ϕ, it is such that if $\varphi = \underline{\varphi}$, $C_1'(h_0) = C_2'(h_0)$, so $\underline{g}^{\star 1-a} < h_0$ ($< \overline{g}^{\star 1-a}$ when the latter is defined); consequently, $C_1(h_0) < C_2(h_0)$, which proves that $\underline{\varphi} < \hat{\varphi}$. Now, when there is only a *high* steady state $\overline{g}^{\star}$(*i.e.*, when $\kappa < \overline{\kappa}$ and $\varpi < \overline{\varpi}$), $C_1(h_0) - C_2(h_0)$ has to be negative for h_0 to be lower than $\overline{g}^{\star 1-a}$, or equivalently, $\varphi < \hat{\varphi}$.

Compatibility of the various conditions. One can notice that $\varphi > \hat{\varphi} \iff \kappa > \hat{\kappa}$ with $\hat{\kappa} \equiv (\varphi - \phi)\left[\frac{\phi+\delta}{(1-\tau)a} - \delta - \left(\frac{\phi+\delta}{(1-\tau)aA}\right)^{\frac{1}{1-a}}\right]^{-\theta}$. Assume $\kappa = \hat{\kappa}$, meaning that $C_1(h_0) = C_2(h_0)$—in other words, $h_0^{1/(1-a)} \in \left\{\underline{g}^{\star}, \overline{g}^{\star}\right\}$. If h_0 corresponds to the low steady state, then necessarily $C_1(0) < C_2(0)$—meaning that $\kappa = \hat{\kappa} < \overline{\kappa}$ (since $\overline{\kappa}$ is the lowest κ such that $C_1(0) \geq C_2(0)$). Plus, h_0 has to be smaller than $\tilde{h}$, hence $C_1(\tilde{h}) < C_2(\tilde{h})$ by Rolle's theorem—meaning that $\hat{\kappa} > \underline{\kappa}$ (since $\underline{\kappa}$ is the biggest κ such that $C_1(\tilde{h}) \geq C_2(\tilde{h})$). On the other, hand, if h_0 is the high steady state, $C_1(\tilde{h}) < C_2(\tilde{h})$—meaning $\hat{\kappa} > \underline{\kappa}$ as well.

Characteristics of the steady states. In all cases, the primary balance $\tau A g^{1-a} - g$ is a surplus at the steady state as soon as $g^{\star} < (\tau A)^{1/a}$; since $g^{\star} < \varpi^{1/1-a}$, $\varpi < (\tau A)^{1/a-1}$ is a sufficient condition. Since $c_1(\cdot)$ is decreasing, it comes immediately that, when there are two steady states, $\underline{c}^{\star} = c_1(\underline{g}^{\star}) > c_1(\overline{g}^{\star}) = \overline{c}^{\star}$. Besides, $\frac{\partial \gamma^{\star}}{\partial g^{\star}} > 0$ from equation (23a), so the high steady state is associated with more growth. Last, equation (23d) yields that the sign of $\frac{\partial b^{\star}}{\partial g^{\star}}$ is the same as $(\theta\varphi + (\phi+\delta)(1+\vartheta-\varsigma))[\tau A(1-a)g^{\star -a} - 1] + (1-\tau)a^2 A(1+\vartheta-\varsigma)g^{\star 1-a}$, which is positive under realistic choices of parameters.[25] *QED.* ■

The multiplicity of equilibria stems from the interference of the concurrent maximization programs of two uncooperative agents. First, agents have direct and indirect externalities on each other. On top of the explicit externality from household's consumption decisions in the government's utility function (quantified by ϑ), public spending and investment contribute to growth, with a feedback effect

25. Indeed, $\frac{\partial b^{\star}}{\partial g^{\star}}$ is a U-shaped function of $g^{\star}$; it suffices that it be positive at its minimum, which leads to the following condition: $\theta\varphi + (\phi+\delta)(1+\vartheta-\varsigma) < A^{\frac{1}{a}}(1+\vartheta-\varsigma)(1-\tau)a\tau^{\frac{1-a}{a}}$, which is in general true.

on the satisfaction of both agents—through disposable income and tax revenue. Second, the spirit of capitalism distorts the allocation of savings between sovereign bonds and investment—households do not have access to loans and only have those two assets at their disposal to smooth out their consumption. Because of the household's preference for capital, these two assets are imperfect substitutes, giving rise to an interest rate spread.

More precisely, multiplicity comes mostly from the tradeoff in the consumer's maximization program between consuming and accruing capital, while only the latter has a positive impact on production and tax revenues. Schematically, since both consumption and capital appear in their utility function, households can at each period trade off consumption against investment (especially when κ is sufficiently large). But these two decisions have different intertemporal implications: consumption crowds out productive public spending, while investment fosters more taxable income in the future and lowers sovereign yields. In parallel, at a given time t, the government can accept to spend less if households consume sufficiently more; but here again, this is not equivalent on an intertemporal basis (as consumption is less useful for future growth).

This multiplicity of stationary states could lead to indeterminacy, as each agent may expect the other to settle on either balanced growth path. Starting close to the *low* equilibrium (*i.e.*, with lower public expenditure), assume for instance that households expect an increase in public spending, thus an increase in output growth. Because of their financing constraint, they need to decide whether to cut consumption or investment. Since they not only expect the return on their investment to allow them to consume more in the future, but also get a direct satisfaction from owning capital, they are then more inclined to save and invest, rather than consume—fueling future growth further, as well as prospects for tax revenue. As a result, the larger public spending can be sustained.

I find that the multiplicity of balanced growth paths in the model hinges on three main elements. First, Proposition 2 highlights the primordial role of the sensitivity κ of the spread $\chi^\star$ to households' choices (deriving from their appetite for capital), If κ is too small, the multiplicity is not ensured; neither is it when it is too large. If households get a strong utility from owning capital, they charge a prohibitive interest rate on sovereign bonds; yet, in parallel, the economy grows faster, and, because households value consumption relatively less, the government ends up in the high public spending equilibrium. Conversely, if they do not attach so much direct value to capital, they generate less growth, and the competition between the government's and household's preferences prevents the existence of any steady state (provided that $\varpi < \underline{\varpi}$). This is what happened in Section 4. The spread is thus an artefact enabling households to make their voices heard, which leads to the emergence of a low public spending steady state.

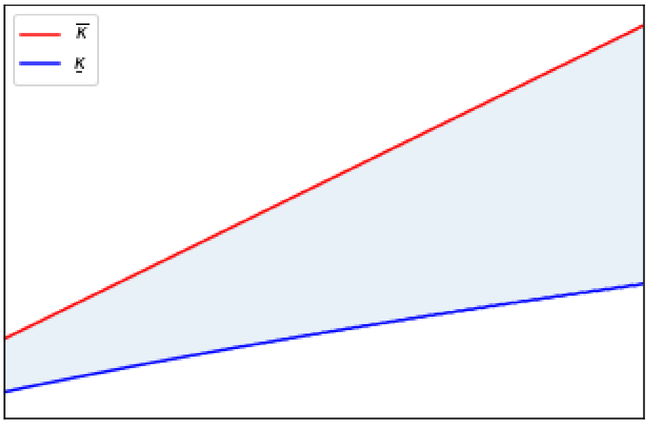
Second, the steady state growth rate depends on how much more impatient the government is relative to households, similarly to section 4. More precisely, for the economy to admit a stationary path with positive growth, the government needs to be sufficiently impatient (especially when the spirit of capitalism κ is strong), but not excessively.

The third condition on ϖ, is subtler; it requires ϖ to be larger than a certain threshold, which means that the government's preference for the present and for its spending should be relatively strong compared with consumers' inclination. Note that $\underline{\varpi}$ is implicitly defined by the function $c_2(\cdot)$; therefore, it depends only on the household's IIES and discount rate and the broad structure of the economy (a, A, δ) and the tax rate. As a result, the condition $\varpi > \underline{\varpi}$ is one about the government's parameters—for instance, it can be seen as a lower bound on its discount rate φ: the government needs to be sufficiently more impatient than households. Surprisingly, the externality ϑ from private consumption on the government's utility does not play such a determinant role—it intervenes only through the government's marginal utility $\vartheta - \varsigma$. As a matter of fact, it is not indispensable to the multiplicity of equilibria, as the following corollary proves.

Corollary 1. *Even absent the externality ($\vartheta = 0$), there can be multiple equilibria, even though in rarer cases. Namely, the interval of admissible κ is narrower the smaller ϑ, as illustrated on Figure 3. Appendix F provides some calibration examples.*

■ *Proof of Corollary 1.* As ϑ decreases, ϖ becomes higher—this is intuitive as the latter quantifies the total degree of selfishness in the economy; thus, it is easier to satisfy $\varpi > \underline{\varpi}$

Figure 3. *Wealth utility factors κ that ensure multiplicity when varies the private consumption externality ϑ on the government's utility*



Note: The shaded area represents the possible values of κ, delimited by $\overline{\kappa}$ and $\underline{\kappa}$ from Proposition 2.

in Proposition 2, since the threshold does not depend on ϑ. From equation (24), the smaller ϑ, the lower $\overline{\kappa}$. More precisely (owing to the fact that $C_1(0)|_{\underline{\kappa}} = C_2(0)|_{\underline{\kappa}} = \frac{\phi+\delta(1-\theta)}{\theta}$, by the very definition of $\underline{\kappa}$):

$$\frac{\partial \overline{\kappa}}{\partial \vartheta} = \frac{\phi+\delta}{\theta} \, C_1^{-\theta}(0)\Big|_{\underline{\kappa}} > 0$$

Similarly, using the implicit definitions of $\tilde{h}$ and $\underline{\kappa}$ given in the proof of Proposition 2 and the implicit function theorem, I find that $\frac{\partial \underline{\kappa}}{\partial \vartheta} = \frac{-\,\partial \nu/\partial \vartheta\,|_{\kappa=\underline{\kappa}}}{\nu'(\underline{\kappa})} = \frac{-\,\partial C_1/\partial \vartheta\,|_{h=\tilde{h}(\underline{\kappa})}}{\nu'(\underline{\kappa})} > 0$; in other words, $\underline{\kappa}$ is also lower, the smaller the externality. After replacing ν The exact derivative is as follows:

$$\frac{\partial \underline{\kappa}}{\partial \vartheta} = \frac{\phi+\delta - a(1-\tau)A\tilde{h}(\overline{\kappa})}{\theta} \, C_1^{-\theta}(\tilde{h}(\kappa))\Big|_{\overline{\kappa}} > 0$$

The size of the band in which κ needs to reside for the model to admit two solutions reduces, too. More precisely, the partial derivative of the ratio $\overline{\kappa}/\underline{\kappa}$ with respect to ϑ is positive. Indeed, for the difference of growth rates to be positive:

$$\frac{1}{\overline{\kappa}}\frac{\partial \overline{\kappa}}{\partial \vartheta} > \frac{1}{\underline{\kappa}}\frac{\partial \underline{\kappa}}{\partial \vartheta} \iff \frac{\phi+\delta - a(1-\tau)A\tilde{h}(\overline{\kappa})}{\varpi - \tilde{h}(\overline{\kappa})} > \frac{\phi+\delta}{\varpi}$$

it suffices that $\phi+\delta$ be greater than $a(1-\tau)A\varpi$, which can comes immediate, as $a(1-\tau)A\varpi$ is a centroid of $\phi+\delta$ and $\varphi+\delta$. *QED.* ■

Even though the externality ϑ is not essential, it makes multiplicity more likely. It is because the externality makes the government more ambivalent in its choices than when it only pursues its own spending. Eventually, what underpins the existence of multiple equilibria in this model is the interplay between the two agents through the spread χ and their distinct goals, and their relative impatience. Incidentally, I could also choose a logarithmic utility from consumption (*i.e.*, set $\theta \to 1$) and still get multiple equilibria (a proof is provided in appendix C.

The remainder of this section investigates how changes in preference parameters move the steady state(s), with an illustration on Figure E.1.

Proposition 3 (Comparative statics)**.** *Under the same assumptions as Proposition 2, the following results hold:*

(a) The balanced growth path the more intensive in public spending contains even more public spending:

- *when the government's impatience φ or its* IIES *ς decrease;*
- *when the externality ϑ, the households' impatience ϕ, their taste for capital ownership κ, the tax rate τ, or the total factor productivity A increase.*

(b) The lower balanced growth path $\underline{g}^\star$ behaves symmetrically; factors that increase the high balanced growth path also push the two stationary paths further apart.

(c) Growth in the high steady state $\overline{\gamma}^\star$ increases when ϑ or κ increase or when ς or φ decrease; but this makes consumption $\overline{c}^\star$ shrink.

(d) Growth in the high steady state $\underline{\gamma}^\star$ increases when ϑ, κ, τ, or ϕ decrease or when ς or φ increase; the impact on $\underline{c}^\star$ depends on whether $c_2'(\underline{g}^\star) \lessgtr 0$.

■ *Proof of Proposition 3.* Since all the functions involved are sufficiently smooth, I can invoke the implicit function theorem to assess how the balanced growth paths $g^\star$, defined by $c_1(g^\star) = c_2(g^\star)$, respond to a change in a given parameter x: $\frac{\partial g^\star}{\partial x} = \left.\frac{-\partial(c_1-c_2)/\partial x}{\partial(c_1-c_2)/\partial g}\right|_{g=g^\star}$. Since Proposition 2 proved the denominator is positive for the high balanced growth path, and negative for the low balanced growth path, a factor that pushes one up drives the other one down. All the partial derivatives involved are computed in Table E.1; this is enough to prove the assertions (a) and (b) in the proposition. Regarding the influence of a change in parameters on $c^\star$ and $\gamma^\star$, I rely on equations (23a) and (23c), respectively (see Table E.2). The issue is that, while $c_2'(\overline{g}^\star)$ is necessarily negative, the sign of $c_2'(\underline{g}^\star)$ is unknown *a priori*. ■

While distinct government preferences are at the foundation of multiplicity, I find that stronger preferences—when the government is more impatient (relative to households) or has a stronger IIES—tend to make the two steady state converge towards each other. Eventually, when there is too much tension between the two maximization programs, the multiplicity is rescinded.

This shows that putting in place fiscal institutions to dull government's impatience and steer the economy to the preferred steady state might have the opposite effect. What Proposition 3 shows is that a less impatient government might spend even more, unless its impatience changes enough to get the economy out of the situation of multiplicity. The same goes for institutions intended to increase policymakers' preference for private over public consumption.

6 Dynamics

In this section, I examine the dynamic stability of the steady states found with Proposition 2. The dynamic system of equations (16) can be reduced to a three-dimensional autonomous system with respect to c_t, g_t, and b_t, the latter being predetermined:

$$\dot{c}_t = L(c_t, g_t)c_t \tag{25a}$$

$$\dot{g}_t = M(c_t, g_t)g_t \tag{25b}$$

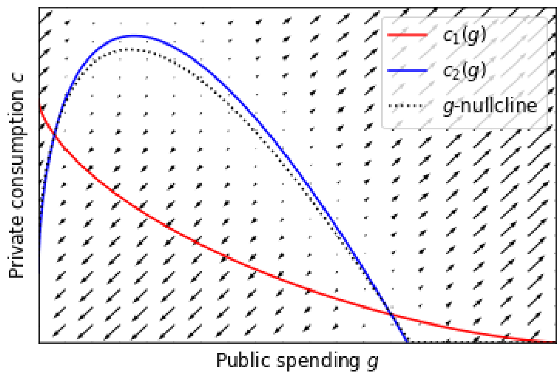
$$\dot{b}_t = g_t - \tau A g_t^{1-a} + N(c_t, g_t)b_t \tag{25c}$$

where $L(\cdot), M(\cdot), N(\cdot)$ are defined as follows:

$$L(c,g) \equiv g + c + \left[\frac{(1-\tau)a}{\theta} - 1\right] Ag^{1-a} - \frac{\phi + (1-\theta)\delta}{\theta}$$
$$M(c,g) \equiv L(c,g) + \frac{1}{\varsigma}\left[\kappa c^{\theta} + \frac{\xi(1-\tau)Ag^{1-a}}{\theta} - \varphi + \phi - \frac{\xi(\delta+\phi)}{\theta}\right] Ag^{1-a}$$
$$N(c,g) \equiv g + c + \kappa c^{\theta} - (1 - a + a\tau)Ag^{1-a}$$

The three equations (25) formulate an autonomous dynamic system with respect to the two forward-looking, control variables c_t and g_t and the predetermined state variable b_t. The evolution over time of these three variables determine the entire paths of all endogenous variables, for a given size and composition of balance sheets (K_0, B_0). For reference, the growth rate of the economy is simply $\gamma_t = Ag_t^{1-a} - \delta - g_t - c_t$.

Figure 4. *Phase diagram in the two steady state case*



Note: The arrows indicate the dynamics in the various regions of the (g, c) plan. The dotted line is the g-nullcline where $\dot{g} = 0$, while the c-nullcline is the blue line $c = c_2(g)$.

The dynamic adjustment of b_t is always unstable. By contrast, papers relying on some sort of debt anchor (either a cap on indebtedness or a debt objective) find that debt is associated with stable local dynamics Futagami, Iwaisako, and Ohdoi (2008). As a matter of fact, the very nature of debt dynamics generates divergent paths, unless the economy starts at the debt-stabilizing primary balance.

What drives the dynamics for private consumption and public expenditure is the spread and the market clearance condition on the product market. The first equation in system (25) says that $\frac{\dot{c}}{c} = c - c_2(g)$.[26] It means that consumption grows whenever it is above $c_2(g)$, which comes from the market clearance condition once public spending and investment have been determined. Besides, $\frac{\dot{g}}{g} = \frac{\dot{c}}{c} + \frac{\kappa}{\varsigma}(c^\theta - c_1^\theta(g))$; public spending grows when consumption is higher than both $c_1(g)$ and $c_2(g)$ and shrinks whenever consumption is smaller than both $c_1(g)$ and $c_2(g)$. The g-nullcline is the locus defined by $c + \frac{\kappa}{\varsigma}c^\theta = c_2(g) + \frac{\kappa}{\varsigma}(c^\theta - c_1^\theta(g))$, which is located between the c_1 and c_2 curves (closer to one or the other depending on the strength of household's appetite for investment and the government's IIES in spending), as illustrated on Figure 4. The function $c_1(\cdot)$ comes from the endogenous spread functional form $\chi = -\kappa c^\theta$, so $c > c_1(g)$ signifies that the spread is too small compared with fundamentals; intuitively, public spending tends to grow more than consumption when the spread is small. The fact that $c - c_2(g)$ boosts both control variables shows how the government and households compete for any slack in the product market.

Proposition 4. *In the context of Proposition 2, all possible steady states are hyperbolic. Moreover:*

(a) If $\kappa > \overline{\kappa}$ and $\varpi > \underline{\varpi}$, the unique stationary path $\overline{g}^\star$ is a locally determinate saddle-point.

(b) If $\kappa < \overline{\kappa}$ and $\varpi < \underline{\varpi}$, the unique stationary path $\underline{g}^\star$ is an unstable node (a source).

(c) If $\kappa \in]\underline{\kappa}\,; \overline{\kappa}[$ and $\varpi > \underline{\varpi}$ the two stationary paths have distinct local dynamic properties: the lower steady state $\underline{g}^\star$ is unstable and the higher $\overline{g}^\star$ is a saddle.

■ *Proof of Proposition 4.* Classically, the local stability properties of a given steady state are determined looking at the sign of the (real parts of the) eigenvalues associated to the Jacobian matrix of system (25), which is:

$$\mathcal{J}(g) = \begin{pmatrix} \frac{\partial L}{\partial c}c + L & \frac{\partial L}{\partial g}c & 0 \\ \frac{\partial M}{\partial c}g & \frac{\partial M}{\partial g}g + M & 0 \\ \frac{\partial N}{\partial c}b & 1 - \tau(1-a)Ag^{-a} + \frac{\partial N}{\partial g}b & N \end{pmatrix} \tag{26}$$

26. As a reminder, $c_1(\cdot), c_2(\cdot)$ have been defined in equations (23b) and (23c).

There is an obvious eigenvalue associated with the debt ratio. At the steady state, it can be expressed thanks to equation (18d) as: $N(c^\star, g^\star) = \frac{\tau A {g^\star}^{1-a} - g^\star}{b^\star} = N^\star \equiv \varphi - (1 + \vartheta - \varsigma)\gamma^\star$. It has to be positive for the transversality condition (17) to hold.

For the other two eigenvalues, I focus on the upper-left 2×2 sub-matrix of $\mathcal{J}(g)$, which I call $\tilde{\mathcal{J}}(g)$. I notice that $L(c, g) = c - c_2(g)$ and $M(c, g) = c - c_2(g) + \frac{\kappa}{\varsigma}\left(c^\theta - (c_1(g))^\theta\right)$ and remember the balanced growth path equations (18), in order to express the sub-Jacobian evaluated at a steady state as follows:[27]:

$$\tilde{\mathcal{J}}(g^\star) = \begin{pmatrix} c^\star & -c_2'(g^\star)c^\star \\ g^\star + \frac{\kappa\theta}{\varsigma} g^\star {c^\star}^{\theta-1} & -c_2'(g^\star)g^\star - \frac{\kappa\theta}{\varsigma} c_1'(g^\star) g^\star {c^\star}^{\theta-1} \end{pmatrix} \tag{27}$$

The characteristic polynomial is: $\tilde{p}(X) \equiv \det(X\mathbb{I}_2 - \tilde{\mathcal{J}}) = X^2 - \operatorname{Tr}\tilde{\mathcal{J}} X + \det\tilde{\mathcal{J}}$. The trace of the sub-Jacobian matrix equates the sum of its two eigenvalues, which appears to always be positive, indicating that at least one of the eigenvalues is positive:

$$\operatorname{Tr}\tilde{\mathcal{J}}(g^\star) = c^\star - c_2'(g^\star)g^\star - \frac{\kappa\theta}{\varsigma} c_1'(g^\star) g^\star {c^\star}^{\theta-1} \tag{28}$$

$$= \frac{\phi + (1-\theta)\delta}{\theta} + \frac{aA{g^\star}^{1-a}}{\theta}\left[\theta - (1-\tau)a + \frac{(1-\tau)(1-a)\xi}{\varsigma}\right] > 0 \tag{29}$$

And the product of the two eigenvalues is:

$$\det\tilde{\mathcal{J}}(g^\star) = \frac{\kappa\theta}{\varsigma} g^\star {c^\star}^{\theta}\left\{c_2'(g^\star) - c_1'(g^\star)\right\} \tag{30}$$

Recall now the topology of the two curves C_1 and C_2 in the proof of Proposition 2 (and on Figure 2). It comes immediately that $\det\tilde{\mathcal{J}}(\underline{g}^\star) > 0$ while $\det\tilde{\mathcal{J}}(\overline{g}^\star) < 0$.

In sum, none of the three eigenvalues of $\mathcal{J}$ has a zero real part at the steady state. At the lower steady state $\underline{g}^\star$, the Jacobian matrix has one positive eigenvalue and two eigenvalues whose real parts are positive; therefore, the lower steady state is unstable.[28] By contrast, at the higher steady state $\overline{g}^\star$, all three eigenvalues are real and two out of three are positive—this is a saddle point.[29] ■

27. Since all other steady state variables are function of $g^\star$ in system (18), I simply call $g^\star$ the corresponding steady state.

28. It is an unstable node or focus-node, depending on whether the roots are all real or not.

29. To know whether the dynamical properties of the lower steady state are an unstable node or an unstable focus-node with oscillations, I compute the discriminant:

$$\Delta = \left[c^\star - c_2'(g^\star)g^\star - \frac{\kappa\theta}{\varsigma} c_1'(g^\star) g^\star {c^\star}^{\theta-1}\right]^2 - 4\frac{\kappa\theta}{\varsigma} g^\star {c^\star}^{\theta}\left\{c_2'(g^\star) - c_1'(g^\star)\right\} \tag{31}$$

From simulations, it appears this discriminant is always positive.

The local dynamics of this model are locally determinate. Proposition 4 shows that there is no sink: no steady state is able to attract all trajectories that come into its neighborhood in the (c, g) plane—which would be necessary for local indeterminacy, given that public debt dynamics are locally unstable. Instead, the steady state with low public spending is a repeller, while the high steady state is a saddle-point (with a stable manifold of dimension 1). Therefore, While the steady state multiplicity found in last section opened the door to indeterminacy, dynamic trajectories that respect the positivity and transversality constraints (under rational expectations) necessarily follow the same path that leads to the high steady state $\overline{g}^{\star}$. Starting points outside of this trajectory would generate divergent trajectories.

This is somewhat similar to a poverty trap mechanism *à la* Kurz (1968b): no matter the initial intentions of the government, the economy is more likely to move away from the low public spending and high consumption steady state, and fall into a steady state with a large government size. Luckily, this also ensures that the economy grows faster, because large public spending forces consumers to save (hence invest) more.

Why should the government and households fail to coordinate on the high consumption steady state? Starting on the low balanced growth path, assume private agents expect a future increase in public spending or that a shock puts the economy off the steady state. Since public expenditure crowds out household's spending, households decide to smooth out their consumption plan over time, which has two effects: (a) it lowers the sovereign bond spread and (b) it brings out more savings which households invest in productive capital—promise of growth (and future tax revenue). Both of these effects in turn opens up more fiscal space: the government can indeed spend more. This is sustainable, owing to the multiplier effect of government spending on growth, which makes it possible to finance a larger level of public debt. From the government's perspective, if policymakers expect households to cut consumption to finance more investment, they will be able to bank on the associated stream of future tax revenues to increase immediately public expenditure (any debt sustainability analysis is based on medium-term macro-fiscal projections). This is even more true than the cut in consumption would otherwise mean a slight decrease in government utility, due to the externality.

By contrast, at the higher balanced growth path, while the mechanisms are identical, their respective strengths are not. A marginal change in consumption when consumption is initially lower translates into a larger impact on household's utility, for which households make the government pays more. Moreover, the production technology gets decreasing returns from government spending, so a marginal change public expenditure when starting from an already high level does not impact much expected tax revenues. In sum, sovereign financing costs change more radically than around the lower steady state, with lesser prospects to pay them with future growth, thereby impacting more forcefully the government's debt servicing capacity. The economy likely goes back to the high steady state.

The economy gets trapped in an equilibrium with more public spending and lower private consumption. This happens in the model even in the absence of information asymmetry, strategic decisions, or uncertainty. Incidentally, in this model, reaching the high public spending steady state is not necessarily bad for households: provided they have a sufficient appetite for capital accumulation, their welfare can improve when consuming less (section 3). However, this is not necessarily the case, and there might be social preferences for less government expenditure (for instance because public debt overhang is seen as risky). Typically, such situations would call for commitment mechanisms: clarity, transparency, and accountability on the objectives of the government, and possibly some correction mechanisms to handle deviations. But this paper highlights how such mechanisms are bound to fail.

7 Conclusion

I have considered a Barro-like economy where endogenous growth is due to the impact of government spending on productivity. I depart from the previous literature by allowing the government to maximize its own utility function, with a distinct set of preferences and a higher level of impatience compared with households. The interplay between the household's and the government's respective decisions, in a context where these two agents do not have the same access to financial instruments and the only two available assets are imperfect substitutes, implies several feedback loops between the two and contributes to the emergence of multiple stationary paths.

With a relatively simple setup, this model thus illustrates how accounting explicitly for a government's preferences and level of myopia can lead to multiplicity of stationary paths. Using a relatively parsimonious setup, the model yields two balanced growth paths, one being unstable, and the other being a saddle point. The unstable one produces low public spending and high private consumption, but is unlikely to be reached. The saddle point is characterized by large public spending, and puts households in a situation where they have to save and invest more. This could potentially be extended and proven to be a novel mechanism of instability, while with further assumptions equilibria multiplicity could lead to expectation-driven shocks.

In this paper's model, agents are all rational and they all perfectly know each other's preference parameters. However, a natural interpretation of my findings is the following. If at the beginning of time $t = 0$, households were clueless about their government's preferences, had to make a guess, and adjust theirs accordingly to ensure that the economy nonetheless grew, failing to do so correctly could generate an even more sub-optimal outcome. Going one step further, these priors about government's preferences could be governed by the observation of past performances and the announcement of intentions (*e.g.*, an *electoral program*), with a learning process. In such a setup, the outcome would thus be influenced by the credibility of the government and its past performance—this is what I find empirically in End (2020)—and vulnerable to sunspot disturbances. Were the government unable to anchor expectations, the existence of multiple equilibria associated with different expectations could easily lead to a Markov-switching rational expectation path. This is especially likely under adaptive learning: Grandmont (1998) shows that uncertainty about the local stability of the economy leads agents to wrongly extrapolate past observed deviations from equilibrium, thereby making the learning dynamics (locally) diverge.

The model developed here could be extended in several ways to yield indeterminacy and even more than two balanced growth paths. First, paralleling the externality in the government's utility function and considering public spending and private consumption partial substitutes (Ni 1995; Balducci 2006), households could derive a direct utility from public expenditure. For instance, with an instantaneous utility that looks like $\frac{C_t^{1-\theta}-1}{1-\theta}G_t^{\sigma}$, the model could yield up to three stationary solutions. Second, increasing returns to scale (possibly through a stronger productive externality from public spending) or an IIES in consumption larger than unity are often associated with indeterminacy and poverty traps (Brito and Venditti 2010)—such cases could be interesting extensions. Third, a (possibly endogenous) Laffer curve in tax revenue or state-contingent tax rates (which could depend on the position in the business cycle or on the debt ratio) could be added. Fourth, the spread could be made dependent on the level of debt, by assuming households value the holding of sovereign securities, as a different class of assets (as in Modesto et al. 2020).[30]

The fact that instability may emerge from the government's maximizing its own, myopic goals, carries policy implications. First, a too impatient government may fail to represent well its citizens and doesn't have their best interests at heart. Therefore, it might deliver a sub-optimal outcome (multiple equilibria). Curbing the government's relative impatience and forcing it to pay more attention to the intertemporal consequences of its actions thus appear as sufficient reasons to put in place fiscal rules or a fiscal watchdog. Second, under indeterminacy, public policies are insufficient to drive the economy to high growth solutions during the transition to long-term equilibrium. Agents decisions (private and government) will place the economy towards one or another converging path, independently of initial conditions or other fundamentals. Against this, the government needs to credibly anchor expectations for the economy to reach the best equilibrium—similar the central banker who commits to a nominal anchor in order to alleviate indeterminacy. This happens independently of actual type of the government

30. They could derive an instantaneous utility from their stock of sovereign bonds, for the same capitalism spirit reason as private capital, but also for a rational diversification purpose. As a matter of fact, aggregate portfolios are even empirically found to favor government paper over productive capital compared with what a classical Capital Asset Pricing Model (CAPM) would prescribe.

and only involves how view it (credible or not). This paper therefore provides a theoretical justification to the importance of fiscal credibility, on which implicitly rely current practices of imposing fiscal accountability frameworks and medium-term fiscal frameworks—meant to enhance communication around fiscal policy and objectives.

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Appendices

A Benevolent government

As a benchmark, it is interesting the consider an economy from which the government as a separate agent with its own preferences is absent—or equivalently an economy in which the government chooses public spending and debt so as to maximize household utility. In this appendix, the production technology is assumed similar to that of subsection 2.1, with an externality from public spending. Household choose at once how much they intend to consume, how much they want to invest in capital, and how much public spending they mandate their government to undertake.

The maximization program for households is only subject to the market clearing condition (11) and writes:

$$\max_{C_t,K_t,G_t} \mathcal{U}_c \qquad \text{such that} \dot{K}_t + \delta K_t + G_t + C_t \leq Y_t \tag{A.1}$$

As in subsection 2.2, this optimization problem can be solved with the calculus of variations method. Any continuous-time Lagrange multiplier such that $\forall t \lambda_t \neq 0 \iff$ the constraint is saturated verifies:

$$\mathcal{U}_c = K_0\lambda_0 - \lim_{t\to+\infty} e^{-\phi t}\lambda_t K_t + \int_0^{\infty} e^{-\phi t}\left\{\frac{C_t^{1-\theta}-1}{1-\theta} + \kappa\frac{K^{1-\theta}}{1-\theta} + \lambda_t[Y_t - \delta K_t - G_t - C_t] + (\dot{\lambda}_t - \phi\lambda_t)K_t\right. \tag{A.2}$$

which leads to the following Kuhn-Tucker conditions (after normalizing as in subsection 2.4):

$$\lambda_t = C_t^{-\theta} > 0 \tag{A.3a}$$

$$g_t = (A(1-a))^{1/a} \tag{A.3b}$$

$$\theta\frac{\dot{c}_t}{c_t} = Aag_t^{1-a} + \kappa c_t^{\theta} - \phi - \delta - \theta\gamma_t \tag{A.3c}$$

$$\lim_{t\to+\infty} e^{-\phi t}\lambda_t K_t = 0 \tag{A.3d}$$

B Negative ξ case

Proposition 5. *If $\xi < 0$, and under Assumptions 1–2, there is a $\overline{\varpi}$ such that there exists a balanced growth path if and only if $\varpi = \frac{(\vartheta-\varsigma)(\phi+\delta)+\theta(\varphi+\delta)}{\xi a A(1-\tau)} \leq \overline{\varpi}$, in which case that solution is unique.*

■ *Proof of Proposition 5.* I use similar simplifying notations than in the proof of Proposition 2: $c^\star = C_1(g^{\star 1-a}) = C_2(g^{\star 1-a})$ with $C_1(h) \equiv S\left[\xi\varpi - \xi h\right]^{1/\theta}$; $C_2(h) \equiv \frac{A(\theta-(1-\tau)a)}{\theta}h - h^{\frac{1}{1-a}} + \frac{\phi+\delta(1-\theta)}{\theta}$; and $S \equiv \left(\frac{(1-\tau)aA}{\kappa\theta}\right)^{1/\theta}$. ξ plays no role in C_2, so the analysis in the proof of Proposition 2 is still valid; in particular, I can define $\overline{\varpi}$ as I defined $\underline{\varpi}$ before, that is as the only $h > 0$ such that $C_2(h) = 0$. What changes with $\xi < 0$ is that now, $C_1(\cdot)$ exists and is positive and non-decreasing on $[\varpi\,;\,+\infty[$, with $C_1(\varpi) = 0$ and $C_1(+\infty) = +\infty$.[31]

There cannot be a solution if $\varpi > \overline{\varpi}$, so I assume that $\varpi \leq \overline{\varpi}$ and focus the analysis on $[\varpi\,;\,\overline{\varpi}]$. Since $C_1(\varpi) = 0 < C_2(\varpi)$ and $C_2(\overline{\varpi}) = 0 < C_1(\overline{\varpi})$, the two functions necessarily interact an odd number of times. Given that $\forall h$, $C_1''(h) = \frac{\xi^2 S(1-\theta)}{\theta^2}\left[\xi\varpi - \xi h\right]^{1/\theta - 2} > 0$, C_1 is convex on $[\varpi\,;\,+\infty[$, while C_2 is concave; so there cannot be more than a unique solution. *QED.* ■

C Logarithmic utility from consumption

Proposition 6. *If $\theta \to 1$, the utility derived from consumption by the household in* (3) *becomes logarithmic and the direct utility from capital ownership becomes constant $v(K) = \kappa$. Then, under Assumptions 1–3, one can find $\underline{\kappa}$, $\overline{\kappa}$, and $\underline{\varpi} > 0$ such that two steady states coexist if and only if $\underline{\kappa} < \kappa < \overline{\kappa}$ and $\varpi > \underline{\varpi}$.*

■ *Proof of Proposition 6.* According to Assumption 1, and going back to the notations introduced in the proof of Proposition 2, then C_1 becomes linear ($\forall h$, $C_1(h) = S\,[\varpi - h]$), while C_2 is still concave. Thus, there is exactly one point $\tilde{h} > 0$ in which the two functions have the same slopes—namely, $\tilde{h} = ((AW + S)(1-a))^{\frac{1-a}{a}}$ (with $W = (1 - (1-\tau)a)$).

By convexity of $C_1 - C_2$, there are at most two stationary solutions; there are exactly two of them if and only if the three following conditions are verified:

- $C_2(0) = V < C_1(0) = S\varpi$, which is equivalent to $\kappa < \overline{\kappa}$ with $\overline{\kappa} \equiv \frac{\varphi + \delta + (\vartheta - \varsigma)(\phi + \delta)}{\phi}$;
- $C_2(\varpi) < C_1(\varpi) = 0$, which is equivalent to $\varpi > \underline{\varpi}$ where $\underline{\varpi}$ is the unique zero of C_2 on $\mathbb{R}_+$.
- $C_2(\tilde{h}) > C_1(\tilde{h})$, which is equivalent to $\mu(S) > 0$, where $\mu : s \mapsto a(1-a)^{\frac{1-a}{a}}(AW + s)^{\frac{1}{a}} - s\varpi +$ is a convex function, whose minimum is reached in $s = S' \equiv \frac{\varpi^{\frac{a}{1-a}}}{1-a} - AW$. Since $\mu(0)$ is positive and $\mu(S') = C_2(\varpi)$ is negative whenever $\varpi > \underline{\varpi}$, there exist two values $S_1 < S' < S$ such that $\forall s$, $\mu(s) > 0 \iff s < S_1 ors > S_2$, or

31. ϖ is potentially negative when $\xi < 0$.

equivalently that there exist $\underline{\kappa} < \hat{\kappa}$, such that $\forall \kappa > 0$, $\mu(\frac{(1-\tau)aA\xi}{\kappa}) > 0 \iff \underline{\kappa} < \kappa < \hat{\kappa}$.

I finish by noticing that $\mu(\frac{(1-\tau)aA\xi}{\overline{\kappa}}) = a(1-a)^{\frac{1-a}{a}} \left(AW + \frac{\phi}{\varpi}\right)^{\frac{1}{a}} > 0$, thanks to $\overline{\kappa}$'s definition, which means that $\kappa < \overline{\kappa}$ is more restrictive than $\kappa < \overline{\kappa}$. *QED.* ■

D Local dynamics in the absence of wealth utility

Proposition 7. *Let assumptions 1–4 be verified, as well as the conditions provided in Proposition 1 for a stationary equilibrium to exist and comply with positivity constraints. This unique equilibrium is locally unstable.*

■ *Proof of Proposition 7.* This is a particular case of Proposition 4 with $\kappa = 0$, so I will draw from the notations and results of section 6. The eigenvalue associated with the debt ratio b is the same: $N^\star = \varphi - (1 + \vartheta - \varsigma)\gamma^\star$; it is necessarily positive given the transversality condition (17). For the other two dimensions, the Jacobian matrix is:

$$\tilde{\mathcal{J}} = \begin{pmatrix} c^\star & -c_2'(g^\star)c^\star \\ g^\star & -c_2'(g^\star)g^\star + \frac{(1-\tau)aA\xi(1-a)}{\varsigma\theta} g^{\star 1-a} \end{pmatrix}$$

with $c_2(\cdot)$ defined as in (23c). The trace and determinant of this matrix can be expressed as follows:

$$\operatorname{Tr}\tilde{\mathcal{J}} = \frac{\phi + (1-\theta)\delta}{\theta} + \frac{aAg^{\star 1-a}}{\theta}\left[\theta - (1-\tau)a + \frac{(1-\tau)(1-a)\xi}{\varsigma}\right] > 0 \tag{D.1}$$

$$\det \tilde{\mathcal{J}} = \frac{(1-\tau)aA\xi(1-a)}{\varsigma\theta} c^\star g^{\star 1-a} > 0 \tag{D.2}$$

This means that the other two eigenvalues are also positive (or their real parts are). Whence, the steady state is a source, which is strongly unstable. *QED.* ■

E Comparative statics

Table E.1. *Sensitivity analysis*

x	$\left.\frac{\partial c_1}{\partial x}\right\vert_{g=g^\star}$	$\left.\frac{\partial c_2}{\partial x}\right\vert_{g=g^\star}$	$\left.\frac{\partial (c_1-c_2)}{\partial x}\right\vert_{g=g^\star}$
g	$\frac{-\xi(1-\tau)aA(1-a)g^{-a}}{\kappa\theta^2}c^{1-\theta} < 0$	$\frac{A(\theta-(1-\tau)a)(1-a)}{\theta}g^{-a} - 1$	> 0 in $\overline{g}^\star$, < 0 in $\underline{g}^\star$
A	$\frac{-\xi(1-\tau)aAg^{1-a}}{\kappa\theta^2}c^{1-\theta} < 0$	$\frac{\theta-(1-\tau)a}{\theta}g^{1-a} > 0$	< 0
a	$\frac{\xi(1-\tau)A(a\ln g-1)g^{1-a}}{\kappa\theta^2}c^{1-\theta}$	$\frac{-(1-\tau)-A(\theta-(1-\tau)a)\ln g}{\theta}g^{1-a}$	
δ	$\frac{\xi}{\kappa\theta^2}c^{1-\theta} > 0$	$\frac{1-\theta}{\theta} > 0$	
τ	$\frac{-\xi aAg^{1-a}}{\kappa\theta^2}c^{1-\theta} > 0$	$\frac{aA}{\theta}g^{1-a} < 0$	< 0
θ	$\frac{(\vartheta-\varsigma)(1-\tau)aAg^{1-a}-(\phi+\delta)(1-\theta)}{\kappa\theta^3}c^{1-\theta} - \frac{c\ln c}{\theta}$	$\frac{\gamma}{\theta} > 0$	
ϑ	$\frac{-\gamma c^{1-\theta}}{\kappa\theta} < 0$	0	< 0
ς	$\frac{\gamma c^{1-\theta}}{\kappa\theta} > 0$	0	> 0
φ	$\frac{1}{\kappa\theta}c^{1-\theta} > 0$	0	> 0
ϕ	$\frac{\vartheta-\varsigma}{\kappa\theta^2}c^{1-\theta} < 0$	$\frac{1}{\theta} > 0$	< 0
κ	$\frac{-c}{\kappa\theta} < 0$	0	< 0

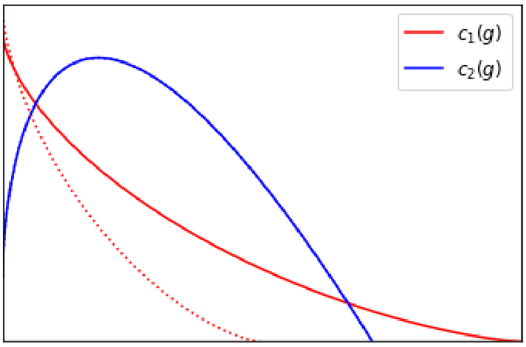
Note: All the stars that should otherwise designate the stationary variables have been removed, for the sake of readability.

Table E.2. *Sensitivity analysis (part 2)*

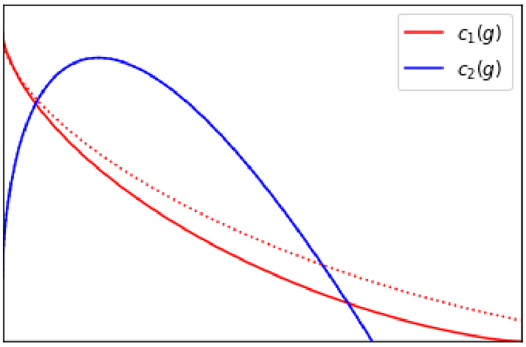
x	$\frac{\partial\gamma^\star}{\partial x}$	$\frac{\partial c^\star}{\partial x}$
A	$\frac{(1-\tau)ag^{\star 1-a}}{\theta}+\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial A}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial A}+\frac{(\theta-(1-\tau)a)}{\theta}g^{\star 1-a}$
a	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial a}+\frac{(1-\tau)A(1-a\ln g^\star)g^{\star 1-a}}{\theta}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial a}-\frac{A(\theta-(1-\tau)(1-a\ln g^\star))}{\theta}g^{\star 1-a}$
δ	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \delta}-\frac{1}{\theta}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \delta}+\frac{1-\theta}{\theta}$
τ	$\frac{-aAg^{\star 1-a}}{\theta}+\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \tau}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \tau}+\frac{Aa}{\theta}g^{\star 1-a}$
θ	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \theta}-\frac{\gamma^\star}{\theta}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \theta}+\frac{\gamma^\star}{\theta}$
ϑ	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \vartheta}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \vartheta}$
ς	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \varsigma}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \varsigma}$
φ	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \varphi}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \varphi}$
ϕ	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \phi}-\frac{1}{\theta}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \phi}+\frac{1}{\theta}$
κ	$\frac{(1-a)(1-\tau)aAg^{\star -a}}{\theta}\frac{\partial g^\star}{\partial \kappa}$	$c_2'(g^\star)\frac{\partial g^\star}{\partial \kappa}$

Figure E.1. *Parameter sensitivity of the two characteristic functions*

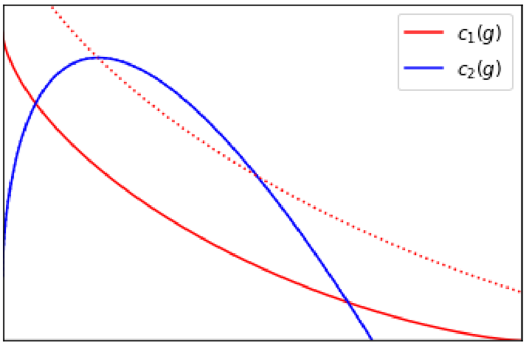
(a) Consumption externality ϑ



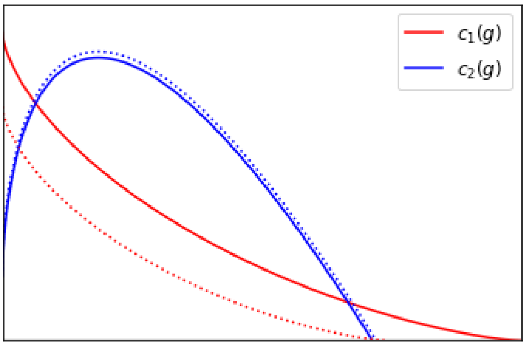
(b) Government's elasticity ς



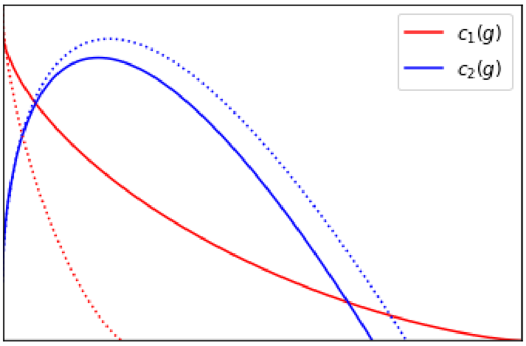
(c) Government's impatience φ



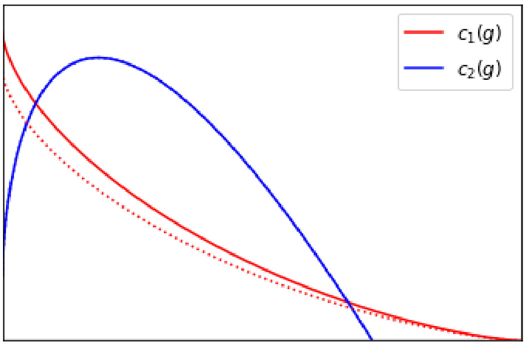
(d) Household's impatience ϕ



(e) Household's elasticity θ

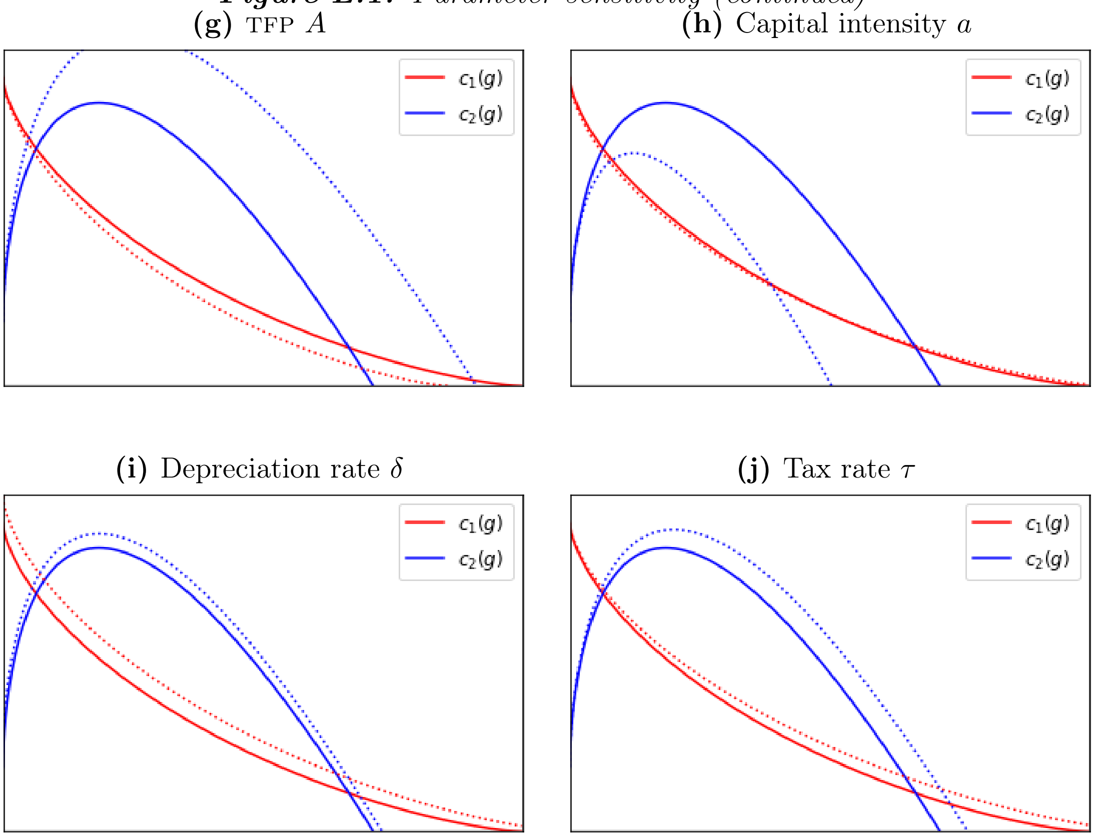


(f) Spirit of capitalism κ



Note: The dotted lines illustrate how a 10 percent increase in the considered parameter impacts the characteristic functions $c_1(\cdot)$ and $c_2(\cdot)$.

Figure E.1. *Parameter sensitivity (continued)*



Note: The dotted lines illustrate how a 10 percent (50 percent for δ) increase in the considered parameter impacts the characteristic functions $c_1(\cdot)$ and $c_2(\cdot)$.

F Calibration

The table below provides parameters that yield one or two steady states, with and without the externality ϑ in the government's utility function.

Table F.1. *Examples of parameters*

		Two steady states		One steady state (low)		One steady state (high)	
	Description	$\vartheta > 0$	$\vartheta = 0$	$\vartheta > 0$	$\vartheta = 0$	$\vartheta > 0$	$\vartheta = 0$
θ	Consumption weight for household	0.70	0.70	0.70	0.70	0.72	0.72
ϑ	Consumption weight for government	0.20	-	0.20	-	0.19	-
ς	Public spending weight for government	0.87	0.67	0.87	0.67	0.87	0.68
ϕ	Household's discount rate	0.60	0.60	0.70	0.70	0.60	0.60
φ	Government's discount rate	0.90	0.90	0.90	0.90	0.90	0.90
τ	Tax rate	0.40	0.40	0.40	0.40	0.40	0.40
κ	Utility from capital	0.12	0.12	0.10	0.10	0.39	0.39
δ	Depreciation rate	0.05	0.05	0.05	0.05	0.05	0.05
A	TFP	4.00	4.00	4.00	4.00	3.10	3.10
a	Capital intensity	0.35	0.35	0.35	0.35	0.35	0.35