

Working Papers / Documents de travail

Haggling on Values: Towards Consensus or Trouble

Victorien Barbet Noé Guiraud Vincent Laperrière Juliette Rouchier













Haggling on Values: Towards Consensus or Trouble*

VICTORIEN BARBET

Aix-Marseille Univ., CNRS, EHESS, Centrale Marseille, AMSE (France)

Vincent Laperrière

Aix Marseille Univ, Univ Nice Sophia Antipolis, Avignon Université, CNRS, ESPACE, Nice (France)

Noé Guiraud

Aix-Marseille Univ., CNRS, EHESS, Centrale Marseille, AMSE (France)

Juliette Rouchier

LAMSADE (Paris-Dauphine University, France)

March 13, 2019

Abstract: We present a model showing the evolution of an organization of agents who discuss democratically about good practices. This model feeds on a field study we did for a few years in France where we followed Non Profit Organizations, called AMAP (a french short food chain), and observed their construction through time at the regional and national level. Most of the hypothesis we make are here either based on the literature on opinion diffusion or on the results of our field study. By defining dynamics where agents influence each other, make collective decision at the group level, and decide to stay in or leave their respective groups, we analyse the effect of different parameters, like size of organizations, on the stability and representativeness of these organizations. The models proves to be robust and we believe is easy to adapt to other context than AMAP. Moreover the article highlights the tension that exists between stability and representativeness in democratic organizations, along with the negative effect of increasing the number of topics to discuss and the positive effect of having openminded members.

Keywords: democracy, organizations, opinion dynamics, Agent based modeling

^{*}Acknowledgements : Acknowledgements : this work was funded by the Conseil Régional Provence-Alpes-Côte d'Azur (France) through two phD programs.

1 Motivations

Non Profit Organisations (NPO) are part of the daily life of a country, like in France for instance. NPO are created for different purposes, like leisures (sports, arts...), to collect funds, to organise common action to protest against something, to create alternative economical or political systems... They are by the law (Loi 1901 in France) democratic structures, where a board is elected, general assembly has to be scheduled every year with the obligation to invite all the members of the NPO to participate. During this assembly the board is elected, the new budget is voted and the realised one has to be explained. NPO are then democratic structures which organise human, financial and material means in order to fulfil the goals which have motivated their creation. Nevertheless at this atomic stage already, NPO have to face different issues:

Tensions: inside NPO even if members often agree on the goals to achieve, on the values to promote they often disagree on the way to do it, creating tensions and instability.

Local range of actions: most of the time NPO are created locally, and gather few people, limiting their visibility and means for larger actions.

To solve the second issue NPO which fulfil same or similar goals can gather in networks. The network has a larger range of actions but it needs first to harmonize the practices of its members creating a coherent movement to gain in visibility. To do so the network has to impose norms ("good" practices) to its NPO members. Nevertheless in its construction the network can be seen as a bigger NPO gathering as members local NPO. It has its own board made of members of the local NPO, organising its own assembly etc. Unfortunately the network as an NPO do not escape from the first issue: again even if all the local NPO share the same goals and values they can largely differ in the way to implement them, expressed at the network level by the norms. Here the same mechanism which threaten the local NPO stability also threaten the network.

In this article we want to study the evolution of NPO networks due to the disagreement that can arise on the norms to implement (the practices to impose in the network). These norms can evolve through time due to interpersonal interactions between the members of the networks' boards influencing each others opinions on what are the "good" practices to implement. To do so we have created an Agent Based Model (ABM) which is inspired by the particular case of AMAP short food chain (Association pour le Maintien d'une Agriculture Paysanne: french CSA). Nevertheless the model can be easily used for other type of NPO, by simply adapting the scheduling, and the rules of interactions, and this is why we keep a generic name for the NPO we consider all along the paper.

1.1 Questions of values: example of the AMAP networks in France

1.1.1 AMAP values

AMAP (Association pour le Maintient d'une Agriculture Paysanne) are basket scheme short food chains inspired by the Japanese Teikei and the United-States CSA (Community Supported Agriculture). The Japanese Teikei appears in the 70's, following big scandals in the agro-food industry. Mothers living in cities and really concerned with the health of their children, organised them-selves in groups of consumers. These groups contract with cooperatives of producers who are engaged in growing vegetables, with few or no chemical additives and pesticides. The Community Supported Agriculture (CSA) appeared in US in the 80's. It consists in a group of consumers who can pay in advance for a share of the harvest of a producer or several producers. This share is periodically distributed among the consumers, and its quantity may vary depending on the agricultural risks. Most of the time, farmers committed them-selves to grow product in organic or biodynamic way. In France AMAP appear in 2001, created by members of a local antenna of ATTAC, an NPO of popular education created in 1998 in France to promote the Tobin taxation on financial transactions. This NPO also tries among other activities to implement local alternative economic actions. An AMAP consists in (Mundler [2009] and Lamine [2005]) a group of consumers who collectively negotiate an individual contract for the weekly delivery of vegetables baskets at a collective point of distribution in their city. The farmer is in charge of the delivery of vegetables and consumers are in charge of the confection and distribution of baskets. The price of the basket is negotiated and paid in advance at the signature of a contract. A contract lasts for 6 months to 1 year.

As in the Teikei, the producer committed her-self to grow vegetables, with few or no chemical additives and pesticides.

This short food chain differentiates with others in France on two main points. First it is the only one strongly embedded with the notion of risk sharing (Lamine [2005]): consumers take all the financial risks by paying in advanced baskets of variable quantity and composition (due to agricultural risks) but in exchange they can closely look at the agricultural practices of their producer in order to be sure of the quality of vegetables. The second point is how much both consumers and farmer are engaged in the food chain (Mundler [2009]). Indeed the consumer is engaged in the organisation of the daily life of the AMAP by finding the distribution point, preparing baskets, communicating on the events of the AMAP, and they can punctually work in the farm. On the other side the producer is engaged to give information about her products, about the way they are grown, the way to cook them, on how the price of the basket is settled and finally she commits to furnish only vegetables from her own production.

AMAP is an economic action which refers to many principles (not only economics one, see the paragraph on the charter of AMAP below) to defined itself (voir avec No pour les rfrences, je n'ai pas tout compris sur son document). AMAP can be considered as alternative food networks, because their actors shared strong values about what the agriculture and the food distribution system should be, like solidarity, ecology, quality... And they locally interpret these values inside their AMAP through the contract, the rules and the behaviours they implement. Nevertheless as they agree on the values they often disagree on the way to implement them. Then an AMAP's life is sometimes made of conflicts. Indeed the contract in it-self is quite simple and essentially deal with the financial part. It do not rule all the daily life of the AMAP and during these repeated interactions (the daily life) between the group and the farmer, some behaviours of one can be negatively perceived by the other creating conflicts. Here the notion of repeated interactions (during the distribution, the general assembly and the informal discussions) is really important, because tensions do not necessarily appear the first time farmers and consumers met but they will positively or negatively influenced each others behaviours over time. Since the creation of AMAP in 2001 we have identified 5 important conflicting topics:

Organic : Should the farmer have the organic label or not? To which extent farmer can use chemical treatment? etc.

0 intermediary : Can some times the farmer be helped by an other farmer to complete his baskets and to which extent?

Flexibility: Are consumers compelled to take one basket every weeks or is it possible to have a more adaptive system? How large the offer of baskets should be (for single couple, couple with children, large family)?

Justice in price: What is a fair price for the Farmer? What are the criteria? How is the price of a basket calculated?

Quality evaluation: How can the consumer monitor the "good practices" of their farmer?

Over times some AMAP last, other are dissolved for many reasons, but the main reason is because of conflicts. Often, after the dissolution of their AMAP motivated groups and producers seek for a better match to recreate an AMAP which implements behaviours closer to their ideal. But others definitely quit the system which is a big issue for the AMAP model as a whole.

To summarize, AMAP are local economic action created by engaged actors who retrieve satisfaction to see their points of view on agriculture and food distribution system implemented in the real life through their AMAP. Nevertheless tensions around this point of view arise through repeated interactions, causing the dissolution and recreation of some AMAP in a matching process based on opinions and norms (implemented behaviours). Moreover as we will see these tensions create also troubles at an upper level: the AMAP network level.

1.1.2 AMAP netwoks

As soon as first AMAP were created they gathered into NPO of AMAP called AMAP networks. Motivations behind were to develop the economic model of AMAP (it is the first function of the network) and to

spread and define the values AMAP want to promote (the second function). Networks are highly democratic structures where decisions are taken collectively through haggling and voting. We can distinguish in this democratic process between two main types of members in networks:

the "passive" members who belong to the network but are not implicated in its daily life, and do not come to the general assembly. They rarely participate to the democratic process.

the "active" members who are invested actors. They participate in the daily life and to the decision of the network. They constitute the board/representatives of the network. Our observations shows that active members are a fraction of approximatively 5% of all the members of a network.

Divergences of point of view often arise among the "active" members who take decisions. These divergences sometimes lead to the split of the network. Indeed Alliance Provence the first AMAP network, created in 2002, quickly split in two because of disagreement in its board on the function of the network. The split gave birth in 2004 to Creamap France which concentrates its task on the first function of the network (development of the economic model) and Alliance Provence kept on going bearing the second function of diffusion and definition of the principles of AMAP (vu simpliste peut-tre est-ce que l'on a quelque chose de plus formalis sur les fonctions des rseaux?). Nevertheless this effort of definition was concreted a year ago in 2003 by the edition of the charter of AMAP which was rewritten in 2014 and saying that AMAP are based on 5 principles: small farming, ecological agricultural practices, products of quality financially accessible, popular education and solidarity. Then since 2003 a trademark AMAP was registered by Alliance Provence to prevent the use of the name AMAP for other similar economic system which do not respect the charter.

However, inside the network of Alliance Provence beside the consensus found on the principles of AMAP, tensions appeared around the local interpretations of the charter into norms. These tensions leaded the network to split again, giving birth to Les PAniers MArseillais (PAMA) in 2007. This split occurred around three disagreements. The first one is a structural disagreement on the organisation of the decision process in the network, and the two others are on organic labelling and quality evaluation topics previously exposed in the part on AMAP. Here we can see that the conflicting topics identified at the AMAP level also hold at the network level. Then for an AMAP be part of a network in not only a question of following the principles defined in the charter but also following the specific norms collectively decided by the board of the network, even if these norms differ from the local consensual behaviours already implemented in the AMAP.

Now days AMAP networks are very diverse and numerous, we have counted at least 27 of them organised on 6 different geographic levels from the city level to the national level (cf graphique No inserr). On a same geographic level each network is mainly independent from the other but they can depend on a network from an higher geographic level. For example a regional network can possess departmental networks and itself belong to a national network. Moreover networks also often differ in the function they fulfil like in the previous example of Creamap France and Alliance Provence.

As we have seen networks are not completely stable structures, they often split and are dissolved. We have identified three main factors leading networks to split and sometimes disappear (in bold are the two dynamics we are interested in studying in this paper):

- 1. A geographic factor. It is when for example regional network splits into departmental one
- 2. A disagreement inside the network whether on its function, or on the norms to impose in the network.
- 3. This factor is a case of network disappearance, it is when the network does not make sense any more for its AMAP members, who just leave it. For this third factor we have identified three main cases (dites moi si vous tes d'accord le 2 et 3 sont en quelque sorte additifs):
 - (a) When the functions of two linked network at different geographic levels overlap, the upper network seems not needed any more by its members.
 - (b) When the network does not communicate enough on its actions and "passive" members then do not see the point any more of belonging to the network.
 - (c) When the norms imposed by the network enter too strongly in competition with the locally pre-existing behaviours of its "passive" members pushing them to quit.

To conclude on AMAP networks, we can say that they are highly democratic structures which evolve throught time often because of influence and disagreements on opinions and norms. Disrupting topics in networks are the same as the one inside an AMAP (5 topics) plus one on the network's function.

In this paper we will be interested in the evolution of NPO networks like AMAP ones due to opinions and norms matters. We want to identify the dimensions which drive this evolution and the mechanism behind.

1.2 Question and Hypothesis

We want to answer the following question:

What are the determinants of stability and representativeness of NPO networks?

By stability we mean the number of existing networks, assuming that the more networks there are, the more splits have occurred and then the less stable the system. By representativeness we mean the fraction of all NPO which belong to any existing networks, by opposition to NPO which prefer to stay alone applying only their own norms. Even if networks are numerous if a large fraction of NPO belong to a network, we can consider that as a whole networks are quite representative of the NPO population. This said, we make the hypothesis that determinants of networks evolution are the following:

- **Number of participants:** the number of NPO members of a network. Empirical observations tend to show a negative impact of this determinant on the stability. The larger the number of members is, the greater the chance for the network to split.
- Characteristics of participants: The open-mindedness of agents with respect to other's opinions and haggled norms. we believe that the more agent are open-minded the more stable and representative are networks. The personality of actors matters: if a representative is strongly attached to her opinion she is more likely to split the network to imposed norms close to her point of view. On the other side we have observed other representatives who have their own stable opinion (not easily influenced) but are more likely to discuss in order to find a consensual rule to adopt preserving the network's unity.
- **Number of topics:** The number of topics that are discussed in networks. We expect that an increase in the number of topics will negatively impact stability and representativeness by increasing the number of potential conflicts.
- Organisation of the representatives: Who are the representatives and the shape of the network decision process. In our models this determinant is expressed by the percentage of members of a network taken to be part of the board, the turn-over inside the board and the shape of interactions between the representatives. For the shape of interactions we test the strong hypothesis where there is no influence allowed in the board, in order to see its implication. By eliminating conflicts due to repeated negative interaction we think that the stability of networks will increase (less split in networks) and want to see its implication on the representativeness.

1.3 An agent-based opinion model

To answer our research questions we have made an Agent Based Model (ABM). We believe that in addition to an empirical study, agent based modelling is well suited to answer our questions because it allows us to test our hypothesis in a virtual environment which we are able to stress as we want. ABM also allows us to observe and better understand the mechanism behind the results. Moreover opinions and norms dynamics have been extensively studied in agent based modelling which gives us robust tools to build up our model.

The majority of the ABM in opinions dynamics are model who study the emergence of clusters of opinions among a closed set of agents who repeatedly influenced each others opinions by random paired interactions. There are mainly two ways of modelling opinions. The first is to consider discrete opinions, in a finite set like in Epstein [2001], and Rouchier and Tanimura [2012] (binary opinions). The second way is to consider continuous opinions, taken in [0,1], [-1,1] or even in \mathbb{R} like in Hegselmann and Krause [2002], Weisbuch et al. [2001], Deffuant et al. [2002], Jager and Amblard [2005] ... Most of the models are unidimensional, interested in the convergence, polarisation or fragmentation of opinions in clusters.

Few of them are multi dimensional, and most of the time those are discrete models. Multi dimensionality is used only if the subject treated by the model imposed it like in Axelrod [1997] and in Rouchier and Tanimura [2012].

The general term of "opinions" dynamics cover all this literature but inside papers terms used and theirs definitions may vary depending on the object studied and the aims of authors. For example in Axelrod [1997] which is a spatial multi dimensional discrete opinions dynamics model, each dimensions is called a trait, the possible values each trait can take are called features and when agent of a same area shared the same feature on a trait (due to influence) this feature is then called a culture or a cultural feature. In Epstein [2001], he only deals with norms (not opinions), agents can choose to behave with respect to two different norms, and they have to choose which one depending on the influence of their neighbours. In Deffuant et al. [2002] opinions are called the points of view of the agents which can be influenced, and clusters of opinions are called norms, it is the same in Jager and Amblard [2005], but opinions there, are identified to individual attitudes of agents. In our model opinions, are the points of view of agents about the good practices to implement in their NPO or network. Agents can haggled to construct norms based on their opinions. Norms are then imposed practices collectively negotiated and not necessarily emerging opinions. Norms can be assimilated to rules for example. In our model, opinions are in [-1,1], and are multi dimensional as we have identified at least 5 dimensions of disagreements on norms in the empirical observations. A dimension can be considered as a topic and a specific dimension discussed by actors at a point in time as the hot topic of the moment.

All opinions dynamics models rely on what is called the uncertainty or the confidence interval which is a characteristic of agents. It represents how much agents are convinced about their opinion and takes the form of a distance. If the opinions of an other agent is more distant from mine than my confidence threshold, then she can not manage to influence me. This is the basic idea. Depending on the model this uncertainty/confidence threshold, can be heterogeneous among agents and allowed to vary through interactions (Hegselmann and Krause [2002], Deffuant et al. [2002], ...). The influence model we have chosen here for our model is the Social Judgement Theory (SJT) of Jager and Amblard [2005]. As many other model it has the uncertainty/confidence threshold, but it also has a rejection threshold. Both are fixed over time and interactions. The rejection threshold works as follow: if the opinions of other agent is more distant from mine than this threshold then we will enter in conflict and our opinions will negatively influence each other (repellent opinions).

To conclude on opinions dynamics models we can highlight that they are regularly successfully used in more applied model on different field. For example, in Deffuant et al. [2005], the bounded confidence model of relative agreement (Deffuant et al. [2002]) is used in an innovation diffusion model, where social utility matter. In Rouchier and Tanimura [2012] the relative agreement is used again to study the transmission of opinions and norms to newcomer in an institution. In Rouchier and Tanimura [2012] they studied the effect of over confident agent in the learning process of a group using a multidimensional binary model of opinions. In Casilli et al. [2014], the Social Judgement Theory is used to study the evolution of norms in eating disorder internet forum.

Our paper is organised as follow. In Section 2 assumptions and principles of our models will be presented. In section 3 we describe the model and explore its results. In section 4 we give the interpretation of the results and the validation of the model. Finally we conclude in section 7.

2 Model

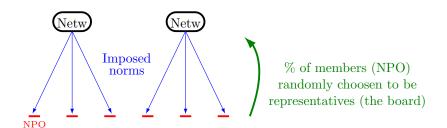
2.1 General overview of the models

Our model is based on collective norms construction and on organisations which are articulated around these norms. The model is constituted of two main types of agents:

- "Basic" agents, characterized by their vector of opinions. They can:
 - influence each others on their opinions
 - haggle norms from their opinions
 - take decisions depending of their degree of agreement with the haggled norms

• "Structure" agents, which are constituted of "basic" agents (their members) and characterized by a vector of norms haggled by their members. They properly do not take decisions, their evolution only depends on decisions taken by their basic agents.

Our model is called the NPO Network Model (NNM), the "basic" agents are NPO, making the simplification here that NPO are agents which hold opinions, can influenced each other and haggled. The "Structure" agents of the NNM are called Networks, and represent NPO networks. NPO are members of Networks and some of these NPO are taken to be part of the board of Networks. These representatives influence each other and haggled norms to imposed to all NPO of the Network. Disagreements with these haggled norms lead the Network to evolve: losing members, splitting, disappearing. The general scheme of the NNM is given in the following picture:



2.2 Assumptions

- 1. There is no cost for agents to enter, quit, split or destroy a Network. This is definitely not completely true, but here we are interested in the mechanism of norms creation which leads networks to split so we put aside other economic considerations which are not as important for actors engaged in NPO than norms considerations
- 2. For null utility Agents always prefer to be involved in a Network rather than to stay alone. As we can observe in the introduction NPO Networks seems to be a natural and wanted way of collective organisation of NPO, then for a null utility agent will prefers to belong to a network rather than to stay alone.
- 3. Agents opinions can be influenced through the Social Judgement Theory Model Jager and Amblard [2005]. influence take place everywhere between representatives in networks: phone call, mailing, regular meeting for the daily life of the network, general assembly. We have chosen the SJT for three main reasons:
 - (a) The rejection grasp the tensions described in the empirical observations, and well replicates the conflicts that arise in NPO and Network on hot topics due to repeated interactions on disruptive subjects.
 - (b) The existence of two thresholds allows us to model a larger set of personalities for our agents. If the two thresholds are close together, then their is an hight ego involvement Jager and Amblard [2005], giving birth to segregative agents, not open to discussion: either you are with me or against me. A larger gap between confidence and rejection allows us to model agent more open to discussion and consensus: agent who can be really confident in her position (little uncertainty) but who will not easily enter in conflict with others (large rejection) and who are able to seek for and find consensus. Modelling these types of personalities makes sense because we have encountered this two types of behaviours among the representatives of networks during the field observations.
 - (c) The fact that rejection and confidence are fixed thresholds eases the definition of a simple utility function on norms for agents, based on this two parameters.
- 4. NPO members of the board (the representatives) are randomly taken, and stay in the board unless they leave the Network during a split. In the specific case of AMAP, NPO willing to be part of the representatives do not show off specific shape of opinions and are not so numerous. Then every motivated AMAP can be part of the board without restriction. Indeed as there is often less

applicants than places offered in board the elections are most of the time a simple formality. That is why we can take randomly the representatives among members of the Network. Moreover, still in the specific case of AMAP we can observe that there is almost no turn-over in board, and most of the representatives are here since the beginning.

- 5. At each turn representatives can influence each other and haggle new norms only around one specific topic randomly pick-up. we have observed in our field study on AMAP that some dimensions become hot topics through time and began the core of the discussions in the network. Moreover all the dimensions are rarely discussed at the same time.
- 6. Network can split. Splits occurs only when representatives disagree on a newly haggled norms on one specific topic. The two new Networks resulting from a split inherit the norms of the old one except on the disrupting topic.
- 7. Representatives of the split Network are automatically representatives of the new Network they have chosen to belong to. As said before representatives are motivated members and are few in our case. We observe in spite of the successive splits that representatives of the new networks are often the representatives of the former one and many of them are here since the creation of the first network.
- 8. NPO which are not involved in a Network join the existing Network which gives them the maximum positive utility. It's the same during a split, NPO of the former Network join the new one which gives them the maximum positive utility. we assume it's quite easy and not much time consuming for NPO to gather data on the norms of networks (you can find them on their websites), and then to join the best for them.

2.3 Agents

The "basic" agents of the NNM are NPO. NPO are characterized by their vectors of opinions (\overrightarrow{a}) uniformly distributed in $[-1,1]^d$ where d is the number of topics) and their uncertainty (U) and rejection (T) thresholds.

The rationality of NPO is built on three submodels for the choices of agents:

- 1. **An opinions dynamic model (the SJT)** which describes the way "basic" agents influenced each other opinions.
- 2. An haggling model which describes how "basic" agents construct norms from their opinions.
- 3. A decision model composed of :
 - (a) an **utility function** which computes the utility derived by a "basic" agent to belong to some "structure" agents
 - (b) decision rules which define actions taken by the agent depending on her utility

2.3.1 The Social Judgement Theory

As previously exposed in the assumptions subsection we will use the Social Judgement Theory of Jager and Amblard [2005]. Influence occurs between pairs of "basic" agents, who hold vectors of opinions of dimension d ($\overrightarrow{a} = (a^1, a^2, ..., a^d)$) with $a^k \in [-1; 1], \forall k \in \{1, 2, ..., d\}$). They also have their own characteristics an uncertainty/confidence (U) and a rejection (T) thresholds. We first assume that U and T are the same for all agents and for all topics (dimensions). When two agents i and i' influence each others they modified their opinions as follow:

Influence of agent i' on agent i on the topic k:

$$\left\{ \begin{array}{ll} \text{if} & |a_i^k - a_{i'}^k| < U & \text{then} & a_i^k = a_i^k + \mu(a_{i'}^k - a_i^k) \\ \text{if} & U \leqslant |a_i^k - a_{i'}^k| \leqslant T & \text{then} & \text{no influence} \\ \text{if} & T < |a_i^k - a_{i'}^k| & \text{then} & a_i^k = a_i^k + \mu(a_i^k - a_{i'}^k) \end{array} \right.$$

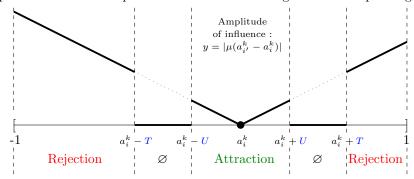
Influence of agent i on agent i' on the topic k:

$$\begin{cases} \text{ if } & |a_i^k - a_{i'}^k| < U & \text{ then } & a_{i'}^k = a_{i'}^k + \mu(a_i^k - a_{i'}^k) \\ \text{ if } & U \leqslant |a_i^k - a_{i'}^k| \leqslant T & \text{ then } & \text{no influence} \\ \text{ if } & T < |a_i^k - a_{i'}^k| & \text{ then } & a_{i'}^k = a_{i'}^k + \mu(a_{i'}^k - a_i^k) \end{cases}$$

with a_i^k and $a_{i'}^k$ constrain to be in [-1;1].

If opinions are less distant than U, they attract each other. If opinions are more distant than T, then they repel each other. If opinions distance is between U and T, no influence occurs, agents are indifferent. μ denotes the strength of influence, it is the same for all agents and is settled to 0.10 as in Jager and Amblard [2005].

The following picture shows the amplitude of the influence of agent i' on i depending on $a_{i'}^k$:



In a group of n agents the influence takes place asynchronously. A first pair of agents is randomly taken and influence each other, then a second pair is randomly taken ... The influence process stops after n/2 pairs have interacted. In this process some agent will perhaps not interact at all when others can be taken several times. In a group of 2 agents, they automatically interact and only once.

2.3.2 The Haggling Model

In our model when "basic" agents haggled they simply compute the arithmetic mean of their opinions. They are only able to haggled norm on one topic at a time. For a board \mathcal{B} (composed by n "basic" agents) which haggled a norm on the k^{th} topic (b^k) the formula is given by:

$$b^k = \frac{\sum\limits_{i \in \mathcal{B}} a_i^k}{n}$$

2.3.3 The Decision Model

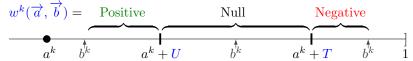
"Basic" agents, the NPO, are able to influence each other and haggled norms for their "structure" agents the Network, they only miss now to position themselves with respect to these norms and take the adequate actions.

First a "basic" agent needs to compute her utility thanks to her opinions $(\overrightarrow{a}=(a^1,a^2,...,a^d))$ and the norms of her "structure" agent $(\overrightarrow{b}=(b^1,b^2,...,b^d))$. Let U be the uncertainty and T the rejection thresholds of the "basic" agent, the utility retrieves by this "basic" agent in her "structure" agent is denote W and

$$W(\overrightarrow{a}, \overrightarrow{b}) = \sum_{k=1}^{d} w^{k}(\overrightarrow{a}, \overrightarrow{b})$$
 with $w^{k}(\overrightarrow{a}, \overrightarrow{b}) = \begin{cases} U - |a^{k} - b^{k}| \geqslant 0 & \text{if } 0 \leqslant |a^{k} - b^{k}| \leqslant U \\ 0 & \text{if } U \leqslant |a^{k} - b^{k}| \leqslant T \\ T - |a^{k} - b^{k}| \leqslant 0 & \text{if } T \leqslant |a^{k} - b^{k}| \end{cases}$

 w^k is the utility obtain on the k^{th} dimension. It is positive if the norm on feature k is closed enough to the agent opinion: the "basic" agent is quiet comfortable with the norm imposed by her "structure" agent.

After, if the norms is far enough but not too far (between U and T) the "basic" agent is indifferent. And finally if the norm imposed is too far the utility becomes negative, modelling the disagreement of the "basic" agent. So w^k is decreasing in the distance between the norm and the actual opinion of the agent. The following picture shows the principle of w^k computation:



The global utility is the sum of the utility derived on each dimension. This is an important point, because negative utilities (disagreements) on some topics can be compensate by an high positive utility on one ore more other topics. A "basic" agent can agree (retrieve a positive utility) with the haggled norms of her "structure" agent even if she disagrees on some topics. The agreement with norms is then a global notion.

The utility is maximum if $\overrightarrow{a} = \overrightarrow{b}$ and $W(\overrightarrow{a}, \overrightarrow{b}) = W(\overrightarrow{a}, \overrightarrow{a}) = dU$. It is the case if the "structure" agent apply norms exactly equal to the opinions of the agents or if the "structure" agent imposed no norms to its members (*ie* each member is free to apply their own rules). The second case is true when the *Network* have not haggled any norms yet.

Second, depending on the utility the "basic" agent needs to decide which action to take:

 $\mathbf{W}(\overrightarrow{a}, \overrightarrow{b}) < \mathbf{0}$: If the utility is negative then the agent disagree with the norms and choose to split or leave the "structure" agents depending if she is respectively part of the board or not.

 $\mathbf{0} \leq \mathbf{W}(\overrightarrow{a}, \overrightarrow{b})$: If the utility is null or positive then the agent is satisfied and stay in or join the "structure" agent

2.4 Networks

The "Structure" agents of the NNM are *Networks*, they are characterized by their members, their boards (\mathcal{B} a set of *NPO* randomly taken among their members) and their vector of norms (\overrightarrow{b}) of dimension d resulting from the haggling between representatives. An *NPO* can belong to only one *Network* at a time.

2.5 Scheduling

- Initialisation: creation of NPO and creation of the primary Network gathering all the NPO. A
 fraction of NPO are randomly taken to be the representatives constituting the board of this primary
 Network.
- 1. For each Network, a specific topic (a dimension of the vector of opinions) is randomly picked-up:
 - (a) **Influence**: If allowed by the user (parameter *influence-board?* turned TRUE), the *NPO* of the board of the *Network* influence each other on this specific dimension of their vectors of opinions.
 - (b) **Haggling**: *NPO* of the board haggled a new norm for the specific topic. This new norm replace the old one in the *Network* vector of norms:
 - If all the board agree with this new vector of norms, then it is imposed to all the *NPO* of the *Network*. If some other *NPO* (not in the board) disagree they leave the *Network*.
 - If at least one NPO of the board disagree, then the board is split into two parts by affinity on the specific topic giving birth to two new MeatStructures. Then all the NPO of the previous Network have to join one or the other new MeatStructures or to leave. The new Networks inherit the vector of norms of the old one except on the disrupting topic.
- 2. **NPO alone inscription**: *NPO* alone join the existing *Network* which gives them the maximum positive utility.
- 3. **Completion of boards**: boards of *Networks* for which the fraction of representatives is less than parameter *%-in-board* are completed.

4. **Next time step**: back to point 1.

2.5.1 Details

the primary Network: the vector of norms of the primary Network is initialized to NA^d . In the case d = 5, we have $\overrightarrow{b} = (NA, NA, NA, NA, NA)$. When a topic is turn to NA, no norm is imposed by the *Network*, and each *NPO* applied its own rule on this topics, that is its own opinion.

Splitting of the board by affinity: If the board \mathcal{B} disagree with the new vector of norms after having haggled a new norm on the hot topic k $(b_{\mathcal{B}}^k)$, then representatives which have opinions on the left side of $b_{\mathcal{B}}^k$ constitute the board \mathcal{B}_l $(\mathcal{B}_l = \{i \in \mathcal{B}, \text{ st } a_i^k \leq b_{\mathcal{B}}^k\})$ and the others constitute the board \mathcal{B}_r . Each board computes a new haggled norm on the k^{th} topics $(b_{\mathcal{B}_l}^k)$ giving birth to two new *Networks* which inherit the norms of the old one except on the dimension k.

2.6 Simulation protocol

The parameters of the NNM are the following :

nb.NPO: number of *NPO* created for the simulation.

nb.topics: number of topics, *ie* dimension of vectors of opinions and norms.

U: Uncertainty threshold of the SJT, homogeneous among NPO.

T: Rejection threshold of the SJT, homogeneous among NPO.

strength.of.influence: denotes the speed of attraction or rejection of opinions in the SJT.

%.in.board: define the size of the board of a Network as a percentage of its number of members.

influence?: allows the user to cut off the influence process among the NPO of the board.

The behaviour space will be explored following the next table:

Behavior space Network evolution model				
$nb.NPO^* = \{100, 200, 400\}$	$U \in \{0.1, 0.2,, 1, 1.1\}$			
$\%.in.board \in \{5, 10, 20, 50, 100\}$	$T \in \{0.1, 0.2,, 1, 1.1\}$			
$nb.topics \in \{1, 2, 5\}$	$influence? \in \{true, false\}$			
strength.of. influence = 0.10				

^{*:} nb.NPO = 100 and 400 were only run with influence? = true

A simulation for a given set of parameters of the NNM is made of 100 time steps, time for the system to reach a steady state where all Networks are stable and NPO alone are not able to find a corresponding Network any more. Each set of parameters is run 30 times.

From the data we have observed in the field, our baseline scenario will be the following:

- nb.NPO = 200
- nb.topics = 5
- %.in.board = 5
- $\bullet \ \ influence? = true$

We will use it most of the time as as a reference in our result section.

3 Results

3.1 Indicators

As it has been previously exposed in our research questions subsection, we want to look at two specific dimensions in the NNM:

- the stability of *Networks*. As there is only one *Network* at the beginning, and that each new one comes from a split, the number of *Networks* alive at the end of the simulation (*nb.NETWs*) is a good proxy for measuring the stability of the system. A *Network* is considered as alive when it is composed of at least 2 *NPO*.
- the representativeness of *Networks*: do *NPO* decide to stay alone or to belong to a *Network*? The fraction of all *NPO* engaged in an alive *Network* at the end of the simulation (*fraction.NPOs.in.NETWs*) is then a good proxy for measuring the representativeness.

Our indicators are measured at the end of each simulation when the steady state is reached. For our results we compute and draw for each set of parameters the mean of each indicator on the 30 runs.

3.2 Observation

To obtain our result we have plotted our two indicators on a graphs which represent a mapping of the characteristics of agents. The horizontal axis represents the rejection threshold (T) and the vertical axis the uncertainty threshold U. Each square of the mapping correspond to a unique couple (U,T) and the mapping is triangular because by definition $U \leq T$. The colour of a square represents the mean on the 30 runs for a given couple (U,T) of nb.NETWs. Similarly the size of points in the middle of each square represents the mean of the representativeness obtained for the given couple on the 30 runs. Two examples are given in Figure 1.

Alone, a graph only gives information on the effect of U and T on indicators and for a given set of parameters. To see the effect of other parameters we need to compare graphs two by two.

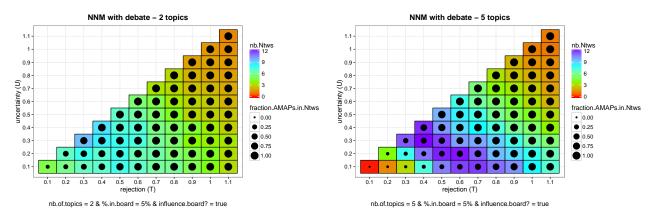


Figure 1: The graph on the right represents the mapping for the baseline scenario, and the graph on left is the mapping for a scenario close to the baseline one except on the number of topics (two instead of five).

Nevertheless comparisons between graphs are not always easy and do not tell us more about whether the effect is significant or not. That is why we have chosen to plot a second mapping more qualitative, which summarises the differences between the two graphs and gives an idea of whether they are significant or not. The following graph is a example, which shows the effect of changing the number of topics from 2 to 5, exploiting the two previous graphs. Again the colour of square indicates the effect of the change in number of topics on the stability, and the shape of the dots indicates the effect on representativeness.

To obtain the effect of switching from the set of parameters 1 to the set of parameters 2 on the indicator I, we proceed as follow. We have a series of 30 observations of I for the set of parameters 1 and for a given couple (U,T) denoted by $I^1_{(U,T)}$, and an other series of 30 observations for the set

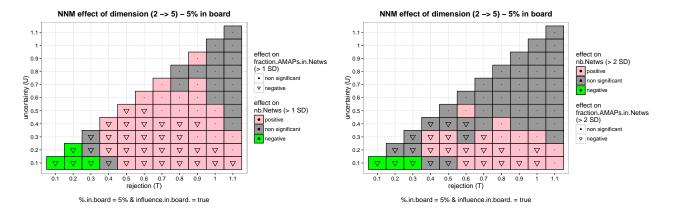


Figure 2: The two graphs summarize the effect of switching from 2 to 5 topics in the baseline scenario. The graph on the left detects smaller effects than the graph on left because of its calibration.

of parameters 2 on the same couple (U,T) denoted by $I^2_{(U,T)}$. We make the difference between the two means of these series and divide it by the standard deviation for the set of parameters 1:

$$\Delta_{(U,T)}^{1\to 2} = \frac{\overline{I_{(U,T)}^2} - \overline{I_{(U,T)}^1}}{SD(I_{(U,T)}^1)}$$

 $\Delta_{(U,T)}^{1\to 2}$ represents the effect of switching from 1 to 2 in number of standard deviation of I on the first set of parameters. Then depending on an arbitrary threshold k, we consider that the effect of the switch between 1 and 2 is significant if $|\Delta_{(U,T)}^{1\to 2}| \geq k$. Most of the time we have used k=2 in this article which means that the effect is superior or equal to 2 standard deviation with respect to the first 30 observations.

From studying these graphs we have deduced the following results summarized in the next table. Results are exposed more widely just after.

Results	in		nutch	٠11
Resilits	ın	а	nutsne	-11

		Stability	Representativeness	
∕ nb of NPO		7	7	
topics discussed	→ nb of topics	×	×	
	preference for the first topic	7	7	
characteristics of agents	open-minded (large U and T)	7	7	
	narrow-minded (small U and T)	\searrow	\searrow	
	≯ %.in.board	×	7	
organisation of the board		>	7	
	no debate	7	\searrow	

3.3 Characteristics of NPO

3.3.1 Effect of the open-mindedness (U and T)

Increasing U and T increases the stability (few Networks at the end) and the representativeness (Networks gather lots of NPO). Decreasing U and T implies less stability (lots of Networks) and less representativeness. Continuing decreasing U and T, representativeness keeps on shrinking and the number of Networks begins to decrease too.

The mechanism behind the result is quite simple as agents are open-minded (large U and T) they more easily retrieve positive utility of an haggled norms and often agree with it. But as they are narrow-minded they retrieve less positive utility on haggled norms and the rejection tends to move away their opinions decreasing again their utility. In this second context Networks split more often (decreasing stability) and NPO alone are more exigent when it comes to join a Networks, decreasing the representativeness. When agents are extremely narrow-minded, Networks splits a lot and loose their members until they are dissolved, then at the end there is few Networks alive gathering few NPO. Here the small number of Networks does not imply high stability, it is exactly the opposite.

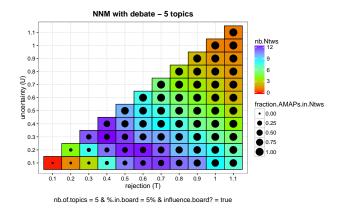


Figure 3: Mapping in U and T, for the baseline scenario.

We have stopped the exploration of the behaviour space of U and T at 1.1 because for T > 1.1 the model ends up generally with the following pattern: one big Network gathering a large majority of NPO.

3.4 Topics

3.4.1 Effect of the number of topics

The effect of the number of topics depends on the mapping U and T. If they are large enough nothing really significant seems to happens. If they are not too large, increasing the number of topics have a negative effect on the representativeness and stability. Again here for U and T very small the diminishing number of Networks does not mean more stability but more dissolution of Networks.

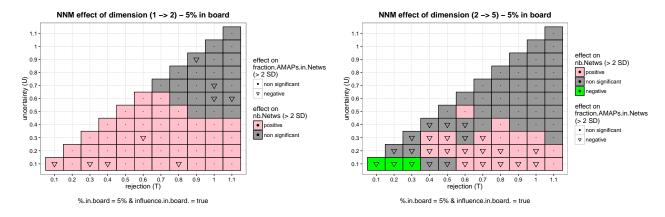


Figure 4: On the left the effect of switching from 1 to 2 topics in the baseline scenario. On the right the effect of switching from 2 to 5 topics in the baseline scenario.

If U and T are not too large, adding topics increases the number of potential disagreements inside Networks leading to more splits and then to less stability. Moreover if agents are too narrow-minded then Networks split very often and lots of NPO quit leading them to disappear. That is why the number of Networks decreases by adding new topics for very small U and T (green area on the right graph in Figure 4). The effect on representativeness is a bit more complex: there is two opposite effects. The first one, which is positive, is that as the number of Networks is increasing there is a larger offer of Networks to join for NPO alone, improving the representativeness. The second effect, which is negative, is that for U and T not too large the addition of a new topics is more likely to generate negative utility and then it is harder for NPO alone to find a corresponding network which gives them a positive utility. This effect increases the number of NPO which stay alone decreasing the representativeness. In ours case the second effect is stronger than the first one explaining the result.

3.4.2 Effect of preference for the first topic

Here we want to introduce a preference on one common topics for all agents. The idea behind comes from our field study: some topics are often considered by the majority of actors as more important than others. To model this fact we have introduced a preference for the first topics. Which means that agents weight more the utility retrieved one the first dimension than on others. For n topics, the utility on the first dimension is weighted (n-1) and utilities on other topics are weighted only 1. The first dimension is now as important as all the others together.

The introduction of such a preference improves stability. The representativeness is improved too for small U and T. The effect observed is significant for only one standard deviation (see Figure 5).

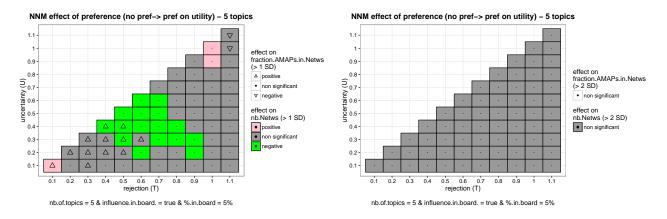


Figure 5: On the left the effect of introducing preference for the first topics in the baseline scenario, effects are detected if they are superior to one standard deviation. On the right the graph shows that there is no effect superior to two standard deviation.

Here adding preference for one common topics counter the negative effect of adding new topics. In the example even if there are 5 topics, thanks to the preference, at the utility function level it is almost like there is only 2 of same weight. Then the situation obtained at the end is in between the case of 2 and 5 topics without preferences (see Figure 6).

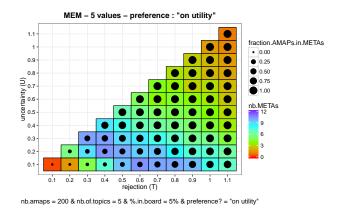


Figure 6: Mapping for the baseline scenario with 5 topics and with preference. The graph appears to be a sort of median case between the case with two topics and the one with five without preference (see Figure 1).

3.5 Organization of the board

3.5.1 Effect of the size of board

Increasing the percentage of members taken to be representatives decreases the stability but enhances the representativeness (see Figure 7).

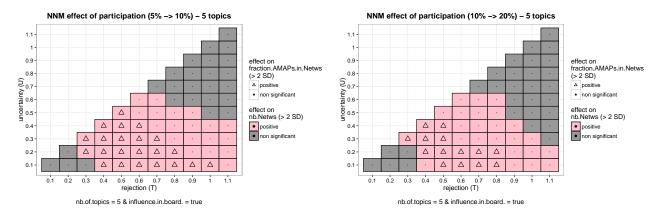


Figure 7: On the left the effect of switching from the baseline scenario to a scenario with 10% in board. On the right the effect of switching from 10% to 20% in board.

Indeed as the number of representatives increases, the distribution of opinions in the board is wider and a wider distribution increases the chance of disagreement and then of splits. But then as the number of *Networks* is higher the choice for *NPO* alone is larger and it is easier for them to find a corresponding *Network* to join, improving representativeness.

3.5.2 Effect of the turnover in board

In our model we have a structural turnover in our boards only due to the mechanism of splitting and board completion, but without splits, no turnover. Nevertheless in the real world we have observed a small turnover, most of the time due to active members who have no more time or energy for the network. Then they quit the board but keep on belonging to the network. To model this fact we have added a probabilistic turnover in boards. Each turn NPO of board have a percentage of chances to quit the board and remain in the network as passive members. After this if the number of NPO in the board is lower than the minimum authorized by %.in.board then new members are randomly picked up among the passive members to become representatives.

The results for the introduction of turnover are the following. Increasing the turnover in board decreases the stability but enhances the representativeness. The effect of the turnover is sharp when we switch from none to some turnover, but then tends to be less significant.

The mechanism behind results are the following. The turnover threatened the stability of boards leading to more splits, but in exchange these splits gave birth to new *Networks* improving the offer of *Network* for *NPO* alone, increasing the representativeness (as in the case of the effect of participation).

3.5.3 Effect of the number of agents

Increasing the number of NPO in the primary Network again decreases stability but improves representativeness.

The mechanism behind is the same as the one for the size of board because if the number of NPO increase for a constant %.in.board the size of boards increases too, making the distribution of opinions among representatives wider...

3.5.4 Effect of the influence

Cutting the influence in board improves stability but decreases representativeness.

As there is no more influence among representatives, there is no more rejection, decreasing the probability of disagreement and then improving stability. But as the number of *Networks* is lower *ceteris*

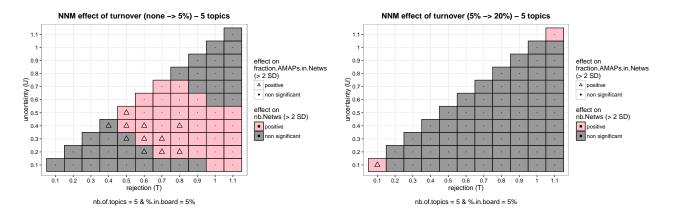


Figure 8: On the left the effect of switching from no turnover to 5% in the baseline scenario. On the right the effect of switching from 5% of turnover to 20% in the baseline scenario. The small increase of 5% in the left graph has huge effect whereas the large increase of 15% in the second graph has almost no effect.

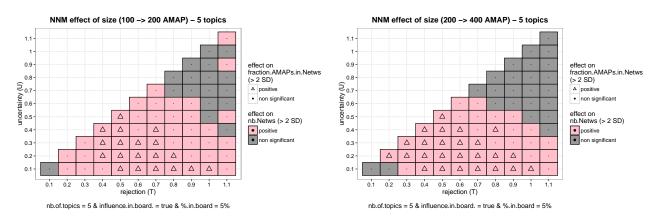


Figure 9: On the left the effect of switching from 100 NPO in the baseline scenario to 200 NPO. On the right the effect of switching from 200 NPO in the baseline scenario to 400 NPO.

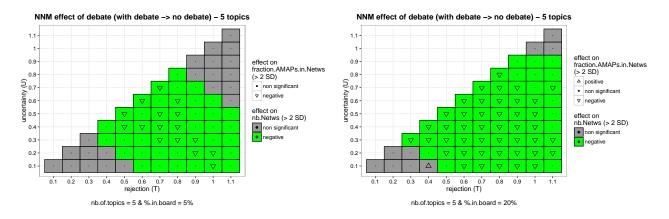


Figure 10: On the left the effect of cutting influence in the baseline scenario. On the right the effect of cutting influence in the baseline scenario with 20% in board. The effect of influence is naturally sharper if the size of the board is bigger.

paribus there is a smaller offer for NPO alone decreasing their chances to find a corresponding Network and then diminishing the representativeness.

4 Validations & Interpretations

In this section we propose a validation of our model by showing how the effect we have observed hold in the reality. When we have no data to validate or invalidate our model , ie when the model tells more than our observations have been able to we propose a interpretation of these results.

4.1 Negative Effect of the Size of the Board on Network Stability

Our model tells us that the percentage of members taken to be part of the board of a network has a negative effect on its stability, leading networks to split more often. This point is particularly interesting because during our field observations, representatives of networks often compelled about the low rate of participation of passive members in the network's daily life and decision process. But according to our model this low percentage of participation helps the constitution of a strong group of representatives sharing common vision on norms to implement, stabilizing the network. Nevertheless the results of our model need to be lowered: in reality whereas in our model, all disagreement in the board do not lead to the systematic split of the network. Indeed a representative alone who disagree rarely causes the network to split, it depends on her personality and on her opportunity to find other representatives who share her opinions and agree to split. These factors are not taken into account in our model where split occurs as soon as at least one representative disagrees. All this said, we can always conclude that an increase of the percentage of participants creates a favourable playground for disagreements threatening the stability.

4.2 Negative Effect of the Number of Topics on the Stability and Representativeness of Network, and the counter effect of preference

According to our model the number of normed topics is really important, because its the main factor which is able to decrease both in the same time stability and representativeness. On the field representatives are already well aware of its negative effect and take some counter measures. First representatives can play on the level of application (more or less strictly) of their norms to relax tensions in the network. Actually for example their is no direct penalty taken against an AMAP which is not respecting the rules but corrective actions are collectively decided with this AMAP and time scheduled in the long run. Second counter measure: representatives can takes the decision of imposing no norm on a too disrupting topic which threaten their unity. In fact, this decision is linked with the importance representatives gives to the topic. There are mainly two cases:

- If the topics is really important, which means they acknowledge that this dimension has to be normed, they can simply buy some time by post-pounding the decision in order to discuss a little bit more. In this case our model due to the rejection will predict that a second round of deliberation (influence) will produce even more disagreement leading the network to split for sure. Then post-pounding the decision, according to our model do not save the network but only buy time to prepare a well organised split.
- If the topic discussed is considered by the representative as not much important, they can simply decide not to norm it. They consider their is no need to threat the network unity for such a "weak" topic.

Even if its mechanism is not descriptive our model of preferences introduced in the NNM tries to grasp these two points by saying: their is topics which are considered consensually as more important than others, what is the impact of modelling such preferences in the utility function? The results obtained are quite realistic, indeed the introduction of a common hierarchy in topics improves the stability of the networks compared to the case where all topics have the same importance: it well replicates the effect of the second points exposed.

4.3 Negative Effect of the Influence on the Stability of Networks

We have seen that the less representatives can influence each others the more stable the network. Indeed in our model as in the reality influence is a double edge knife, on one side if opinions of representatives are already close influence will tightened the board and then the network. On the other side, if the distribution of their opinions is wide the influence process is more likely to create two cohesive groups of

opposite opinions (polarization observed in the SJT) leading the network to split. Nevertheless cutting all influence in the real world is impossible and preventing debates in the board does not fit with the strong democratic will of actors. That is why through years networks have developed animation skills in order to allow everyone to express their points of view while preventing violent clashes (rejection feature of the Social Judgement Theory) which conduct to the polarization of opinions and to splits.

4.4 The Negatively Correlated Impact on Stability and Representativeness

Instability is not necessarily a negative aspect for networks. For example an increase in the size of board or allowing the influence in board sure increase the instability but in return the number of different networks increases, giving more chances to NPO alone to find a corresponding network, enhancing representativeness of networks as a whole. On the one hand the relative weights of networks decreases but on the other hand less NPO are left alone. The paradox is that the instability of networks help them to be more efficient at the global level because they reach more NPO, but less efficient at their individual level assuming that bigger networks are able to mobilise more means to help their NPO.

Moreover this negatively correlated impact we are discussing here occurs most of the time when we are playing on parameters which implies more democracy :

the number of NPO: Networks have to conceal peacefully opinions of more and more members.

the participation rate to boards: More people are motivated to participate together to the daily life of networks.

the organisation of debates: Organisation of democratic debates in board where every representative can express her own point of view to others.

turnover in boards: As boards are no longer closed group of representatives who impose their norms as they want to their networks.

5 Conclusion

In this article we address the question of the determinants of evolution of organizations like action oriented groups. The proposed model is inspired by a field study on a french short food chain called AMAP. The field observations help us in different ways by providing us:

- the stylized fact we have replicated, which is the evolution by successive splits of an organization due to disagreements on the norms to impose
- important characteristics of the system we have integrated in the assumption of the model see Section 2.2:
- values to test for certain parameters like the number of agents or the fraction of members in boards.
- hints to validate the behaviour of the model.

The model shows a strong negative effect of the number of topics discussed on both stability and representativeness. Then in a context where lots of topics are at stake with no possibility to reduce their number, the only way to counter these negative effects is to play on the open-mindedness of members. Playing on this last parameter reveals to be complicated but it also highlight the key role of the way people are able to receive and integrate others' point of view in the stability and representativeness of democratic structures.

An others important result of the model is the tension that appears between stability and representativeness in democratic settings. This is a real issue for organizations promoting democracy in their functioning but needing lot of members to fulfill their missions.

The proposed model proves to be robust and we believe is easy to adapt to other context than AMAP. The model could also be used to test for the efficiency of different tools, like communication, in relaxing the tension between stability and representativeness in democratically structured organizations.

References

- Robert Axelrod. The dissemination of culture: A model with local convergence and global polarization. *The Journal of Conflict Resolution*, (2):203, 1997. ISSN 00220027.
- Antonio A. Casilli, Juliette Rouchier, Paola Tubaro, and Juliette Rouchier Paola Tubaro Antonio, A. Casilli. How to build consensus in a health-oriented online community: Modeling a 'pro-ana' forum. Revue francise de sociologie, (4):731, 2014. ISSN 9782724633771.
- Guillaume Deffuant, Frederic Amblard, and Gerard Weisbuch. How can extremism prevail? a study based on the relative agreement interaction model. *Journal of Artificial Societies and Social Simulation*, (4), 2002.
- Guillaume Deffuant, Sylvie Huet, and Frdric Amblard. An individual-based model of innovation diffusion mixing social value and individual benefit. *American Journal of Sociology*, 110(4):1041 1069, 2005. ISSN 00029602.
- Joshua M. Epstein. Learning to be thoughtless: Social norms and individual computation. *Computational Economics*, (1):9, 2001. ISSN 0927-7099.
- Rainer Hegselmann and Ulrich Krause. Opinion dynamics and bounded confidence models, analysis and simulation. *Journal of Artificial Societies and Social Simulation*, (3), 2002.
- W. Jager and F. d. Amblard. Uniformity, bipolarization and pluriformity captured as generic stylized behavior with an agent-based simulation model of attitude change. COMPUTATIONAL AND MATH-EMATICAL ORGANIZATION THEORY, 10(4):295 – 303, 2005. ISSN 1381298X.
- Claire [b1] (analytic) Lamine. Settling shared uncertainties: Local partnerships between producers and consumers (english). Sociologia ruralis, 45(4):324 345, 2005. ISSN 00380199.
- Patrick Mundler. Les associations pour le maintien de l'agriculture paysanne : solidarit, circuits courts et relocalisation de l'agriculture. *Pour*, (2):155, 2009. ISSN 0245-9442.
- Juliette [b1] (analytic) Rouchier and Emily [b2] (analytic) Tanimura. When overconfident agents slow down collective learning (english). Simulation (San Diego, Calif.), 88(1):33 49, 2012. ISSN 00375497.
- Gerard Weisbuch, Jean Pierre Nadal, Frederic Amblard, and Guillaume Deffuant. Interacting agents and continuous opinions dynamics. 2001.