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The Size Distribution of Cities: Evidence from the Lab

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Abstract

In this paper, we bring fresh evidence on the city size distribution from a 'lab' represented by the region of Bukhara observed in the 9$^{\text{th}}$ CE. At that time this region was homogeneous in all respects (technology, amenities, climate, culture, language, religion, etc.) and yet cities had different sizes. We rationalize the city size distribution of this economy in a simple general equilibrium spatial model of which we estimate the parameters using the method of moments. The estimated model predicts very well the 9$^{\text{th}}$ century city size distribution. Spatial centrality is the major determinant of city size. The silk road contributes to explain what centrality cannot. We find little evidence of persistence of the urban structure when comparing the 9$^{\text{th}}$ and the 21$^{\text{st}}$ century. We find instead that centroid of the region has moved towards the economic core of the Uzbek economy.

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1 Introduction

A fundamental fact about people is that they travel between places. A fundamental fact about places is that some are more central than others. These two facts give centrally located places an indisputable location advantage. Can this elementary structure made of places and traveling people explain the size distribution of cities? To answer this question we use data from our archaeological study of the Bukhara region in Uzbekistan.[1] In this study we measure the area of the surface occupied by each settlement in the 9$^{\text{th}}$ century CE. We use the area and the population estimates as alternative measures of city size. We find that spatial centrality explains extremely well the city size distribution.

To show whether geographic centrality gives some locations an advantage over others in an empirical framework, we would like two things to hold in the data. First, centrality should not be endogenous to population size i.e., no reverse causality. Second, other factors should not be affecting population size and spatial centrality i.e., no omitted variable bias. In today's economies, the first scenario of reversed causality could take place because infrastructures built over centuries have changed the effective centrality of places, making some places effectively more or less central than their physical position would entail. Infrastructures are endogenous to population, however, thus providing the ideal situation for reverse causality. The second scenario of omitted variable could also take place because nature could endow centrally located place with a natural advantage (such as fertile land in the middle of a valley). In this case the omitted variable would also trigger the reverse causality if infrastructures are built.

Ideally, one would like to observe a city size distribution as it emerges in the absence of natural advantages, on a perfectly homogeneous geography in the absence of infrastructures, with constant and identical technology in every location, and in the very long run. In this ideal lab, one would observe city size formation subject only to the two unavoidable facts on Earth: some cities are more central than others and centrality is an advantage because traveling is costly. In this paper, we use a unique data set that comes very close to this ideal situation. The data set is the result of fifteen years of archaeological exploration and contains city size and location for the universe of cities in the region of Bukhara in the 9$^{\text{th}}$ century CE. Cities have developed for twelve centuries from 3$^{\text{rd}}$ BCE to 9$^{\text{th}}$ CE without sizable perturbation and in a situation of relative isolation from the rest of the world. This smooth passing of time is a very good approximation of an unperturbed environment where the determinants of city size have had the time to shape the distribution observed in the 9$^{\text{th}}$ century. The technology was constant over these twelve centuries and homogeneous across the entire area. This is a good approximation of the absence of technological change or endogenous productivity heterogeneity due, for instance, to technological spillovers. Transport infrastructure was unnecessary because of the flatness of the land and the absence of natural obstacles to travel between any two

[1]The archaeological mission began in 2009 under the direction of the first author and continues to date. The mission is under the aegis of the Louvre Museum, in collaboration with the French Ministry of Foreign Affairs, the Archaeological Institute of Samarkand, and the Uzbek Academy of Sciences. For the archaeological aspects discussed throughout this paper see Rante and Mirzaakhmedov (2019) and Rante et al. (2022).

points. The silk road was the only factor that affected some cities and not others. These features make the region of Bukhara in those twelve centuries an ideal lab to study the effect of centrality on the city size distribution.

We embed centrality in an elementary general equilibrium spatial model where agents choose where to locate. In the model, centrality is an advantage because travelling is costly. We allow for local market size to influence the city size. In the model this is captured by the advantage of wider exchange opportunities in larger markets. The model is simple to reflect the elementary economy of the 9th century oasis. In spite of its simplicity, the model explains almost perfectly the size of cities observed in the data. The only exogenous factor, the silk road, helps explain what is not explained by centrality. We also investigate whether the location and size distribution of cities of the 9th century has left any trace in the modern economy of that region. We find no strong evidence of persistence. The location of cities and population today is not nearer to that of the 9th century than a random draw would predict. Some persistence is detected at very short distances. We find instead that urban settlements and populations in the 21st century are located North-East with respect to those of the 9th century. This is very interesting given that the region of Bukhara is today completely integrated in the economy of Uzbekistan whose economic core is located North-East (towards Samarkand and Tashkent). It is suggestive to think that, in line with spatial economic models, the integration of the Bukhara region in modern Uzbekistan has attracted places and populations towards the economic core of the country. Consistently with this interpretation, the model estimations based on the 21th century data for the modern economy show a much bigger role of market size in explaining the size distribution of cities.

These results provide an entirely new perspective on the determinants of the city size distribution. To begin with, the city size distribution in the region of Bukhara passes the test of log-normality and the rank-size relationship is concave. This is interesting because concavity and log-normality in the literature are associated to random growth models of city size distribution. Our data and model show instead that these statistical features can be explained by spatial centrality without any reliance on random growth. The second new perspective concerns location fundamentals. A location fundamental is an intrinsic characteristic of a place usually referred to as 'first nature' (such as particularly fertile land, natural harbor, amenities, etc.). The city size distribution resulting only from location fundamentals would be a map from intrinsic characteristics to city size. As noted by Krugman (1996), the explanation of city size distribution based on location fundamentals is not to be excluded but, for it not to be tautological, we need a precise measurement and an accurate definition. This is precisely what we propose: in our interpretation, the centrality of a place is a location fundamental. This simple but new perspective helps make sense of the fact that cities have different sizes even in seemingly homogeneous areas of the world such as the great plains of the U.S. or Russia. Centrality interpreted as a location fundamental sheds also new light on the strong persistence found in Davis and Weinstein (2002). They find that Hiroshima and Nagasaki regained their rank in the city size distribution less than two decades after the nuclear bombing. In our interpretation, this result may be explained not by some intrinsic (and unidentified) characteristics of the cities but rather by the fact that nuclear bombing did not change their centrality.

2 Related Literature

Our explanation for the city size distribution is based on the unavoidable heterogeneous centrality that space generates.[2] This link between space and city size distribution has so far remained unexplored both in the theoretical and empirical literature. Numerous papers have investigated empirically the adherence of the data to the Zip's law or to the log-normality of the city size distribution without using centrality as a possible explanation; see e.g., Zipf (1949), Parr and Suzuki (1973), Rosen and Resnick (1980), Krugman (1996), Eaton and Eckstein (1997), Dobkins and Ioannides (2001), Ioannides and Overman (2003), Gabaix and Ioannides (2004), Soo (2005), Levy (2009), Rozenfeld et al. (2011), Ioannides and Skouras (2013), Lee and Li (2013), Schluter and Trede (2019) Schluter (2021). Two papers use archaeological data. Davis and Weinstein (2002) find closeness to Zipf's law especially for the period that goes from the third century BCE to the third CE. Barjamovic et al. (2019) find adherence to Zipf's law for the reconstructed city sizes of the Bronze Age. Their reconstructed city sizes also exhibit the concavity of the rank-size relationship often found in modern data as well as adherence to log-normality.[3] On the theoretical front, models of city size distribution are typically space-less. A first group of such models is based on stochastic growth and spillovers that give rise to either the Zipf's law or to log-normality. The models developed by Gabaix (1999), Eeckhout (2004), and Duranton (2007) are the most prominent examples. Later papers have departed from random growth to explore other mechanisms. Behrens et al. (2014) develop a model that combines agglomeration economies, sorting of more talented people in larger cities, and selection of more productive firms in large markets. Their model matches a number of stylized facts including Zipf's law for large cities. Davis and Dingel (2019) build a model with complementarity between individual ability and learning opportunities that lead to heterogeneous city size. Their model is able to predict three important features of the data: the positive correlation between skill premia and city size, the constant expenditure share on housing across cities, and the Zipf' law. In these models, cities grow at different rates for various reasons but their geographical position is not taken into account.

At the other extreme, the economic geography literature is all about space but for a long time has been unable to provide explanations for the city size distribution. A famous quote from Krugman (1996) goes that 'we are in the frustrating position of having a striking empirical regularity [the Zipf's law] with no good theory to account for it'. Over a decade later Duranton (2008) remarked 'the often uneasy coexistence between urban systems and the new economic geography.' Quantitative spatial models developed in recent times are in principle able to predict the size of cities but they have never been used for

[2]Heterogeneous centrality of places on Earth is unavoidable. Although it is possible to place points on the surface of a sphere such that each is equally central as any other, it is not possible to place equally central points on any subset of a spherical surface (that is, excluding deserts, oceans, mountains, etc.).

[3]Concavity and log-normality are not discussed in that paper. We base our statement on inspection of their Fig. X and on the analysis of the data in column 2 of their Table III.

this purpose. As an example, Allen and Arkolakis (2014) motivate their seminal paper by the need to 'build a framework suitable to estimate the fraction of spatial inequality that is due to geography' but do not mention the need to explain the size distribution of the city. Previous papers, e.g., Redding and Sturm (2008), Brülhart et al. (2012) had developed simpler versions of quantitative spatial models able to explain important empirical facts but left the issue of city size distribution completely unexplored. Quantitative spatial models have progressed enormously since then, see e.g., Redding (2016), Redding and Rossi-Hansberg (2017), Desmet et al. (2018), Caliendo et al. (2019), Behrens and Murata (2021), Redding et al. (2022) and a thorough review in Redding (2022) but have not been applied to the study of city size distributions, neither theoretically nor empirically.

The models reviewed above rely on a number of key elements that give rise to heterogeneous city size or, in spatial models, to the heterogeneity of economic activity in space. Among these elements there are local externalities (Gabaix, 1999; Eeckhout, 2004), random innovation and large number of industries (Duranton, 2007), human capital accumulation (Eaton and Eckstein, 1997), exogenous and endogenous productivity differences (Allen and Arkolakis, 2014; Desmet et al., 2018; Caliendo et al., 2019), and size-skill complementarity (Davis and Dingel, 2019) to mention only the most common of a long list reviewed and thoroughly discussed in Redding and Rossi-Hansberg (2017). None of these elements are present in the 9$^{\text{th}}$ century economy we observe. Arguably, none of them were present in the entire Antiquity and yet cities had different sizes even then. We therefore focus on centrality and travel costs since they are present in every economy, modern and ancient, and are the only element present in our 9$^{\text{th}}$ century 'lab'.

3 The Oasis of Bukhara: An Ideal 'Lab'

We follow the archaeological tradition and use the term 'oasis' in the sense of a naturally delimited and homogeneously irrigated land. The oasis of Bukhara refers to the geographic area that extends over the delta of the Zerafshan in central Uzbekistan. The delta irrigates with its shallow waters a surface of land whose area measures about 5,100 square kilometers (1,969 sq mi).[4] The oasis is surrounded by a desert. The surface is extremely flat. The altitude difference between the highest and lowest point in the oasis is two hundred meters over more than one hundred kilometers of land that slopes downwards monotonically from North-East to South-West. The time span of our study goes from the 3$^{\text{th}}$ century BCE to the period of maximum expansions of each city, which varies by city but lies between the 9$^{\text{th}}$ and 10$^{\text{th}}$ century CE. Bukhara was among the three largest cities until the end of the 8$^{\text{th}}$ century. Its size boomed from the late 9$^{\text{th}}$ century when it became the capital of the Islamic state that extended from the Oxus River to Ferghana near the border with modern Kyrgyzstan. In our analysis, we used the size of Bukhara before it boomed. Today, Bukhara is the administrative capital of the homonymous region of Uzbekistan which partly overlays with the oasis. To simplify the prose we refer to the upper bound of our time span as to the 9$^{\text{th}}$ century for all cities.

[4]Approximately the size of the French département of Bouches du Rhône (5,087 sq kilometers) and a bit larger than the U.S. state of Rhode Island (1,545 sq mi).

Overview. The oasis characteristics make of it an ideal lab for the study of the size and geographical distribution of cities. Its geographical limits are exogenously determined by the delta of the Zerafshan, its political limits coincided with its geographical limits, and the oasis remained independent from foreign powers for the twelve centuries covered by our study.[5] There was rivalry between major cities and even a war between two cities but no city has ever had supremacy over the oasis in the time span relevant to our study. These features eliminate possible effects on city size due to political organization or supremacy. The oasis was extremely homogeneous in terms of climate, water availability, technology, natural resources, culture, religion, and language. The flatness of the land and the nature of the soil made the natural ramifications of the Zerafshan irrigate homogeneously the entire oasis. Furthermore, canals have been built since the beginning of urbanization. Archaeological evidence shows that six out of the fourteen villages that existed in the first century BCE were not built on the shores of natural water courses but got water from canals. This is not surprising given the clayey nature of the soil. The economy was elementary. It produced agricultural goods and basic manufactures. Due to identical land and climate conditions, the agricultural produces were the same throughout the oasis. No evidence has been found of specialization of a particular area within the oasis in the production of any manufactures or agricultural produces. Manufacturing production consisted of ceramic and metal objects produced by the use of labor, iron, clay, and water. Clay and water were essentially free goods given their vast and homogeneous availability in the oasis. Iron was absent in the entire oasis and was imported from the mountain chain that extends eastbound from the North-East end of the oasis. The transport technology was elementary (horse, donkey, feet). The typical farmer would travel by foot to the nearest marketplace in the company of his donkey. Given the morphology of the land, roads were essentially straight lines between any two points. Bridges were easily built on the shallow waters of the Zerafshan and its ramifications. In this situation, no location fundamental can be imagined in the oasis. The homogeneous unit cost of travel in any direction allows for ruling out the irregularity of the land as a factor influencing the city size distribution. The similar size of ovens found in different cities testifies of an identical technology throughout the oasis and throughout the twelve century under scrutiny. The pottery and utensils found in the oasis exhibit no noticeable difference in quality or style. Technical progress is clearly absent in the twelve centuries under scrutiny. The silk road came to exist and develop at the same time as human occupation of the area. As such, it does not represent a shock to a preexisting economic equilibrium. Silk road merchants used some of the oasis cities for stopovers. They demanded accommodation, market services (such as arranged space for trade with other merchants), food for them, and for livestock. Historic sources give us a sense of how large this additional demand was: caravans often counted dozens of people and hundreds of livestock, and more than a handful of caravans might stop in a city simultaneously

[5]The oasis was situated between two political superpowers: the successive empires that extended over the area of modern Iran (Parthian and Sassanian) and the Chinese empire. With the tacit consent of the superpowers, it enjoyed political independence for many centuries. The migration waves into the oasis that occurred over the centuries often lead to the replacement the political rulers but left the economic, religious, and social structure unchanged.

(Étienne de la Vaissière, 2018). Archaeological excavations by the first author brought to light the remains of structured spaces for merchants, typically made of shops positioned as in the perimeter of a rectangular area that served for the circulation and sampling of the merchandise.

All these features make the Oasis an ideal "lab" for the study of the city size distribution because we can observe such distribution as it arose in the absence of location fundamentals, land irregularity, exogenous amenities, exogenous technological differences, technology progress, and technological spillovers. Of course, these elements matter when they are present (for modern economies they may be important) but the data of the oasis suggests that heterogeneity of city size may arise even in the absence of these elements. In no other part of the world, we find such environmental homogeneity and for such a long time. The silk road is the only external element that possibly came to influence the size of stopover cities.

Three phases of evolution. The urban structures observed in the 9$^{\text{th}}$ century is the result of twelve centuries of evolution that can be subdivided in three periods. In period I (approximately from 3$^{\text{rd}}$ to the 1$^{\text{st}}$ BCE) sites grew from tiny settlements to little villages of about 1 hectare in size and without sizable difference between them. A total of fourteen sites can be dated to this period (Figure 1a). In period II (approximately from 1$^{\text{st}}$ BCE to the end of the 3$^{\text{th}}$ CE) the number of sites increased grandly to reach a total of about three hundred and ten sites (Figure 1b). The increase is big but seemingly not abrupt given that took place over several centuries. In this period most of the pre-existing villages and some of the new villages became cities; the *town* (in the sense described below) with its political center took shape, some sites walled their urban limits and city sizes became more heterogeneous to range from one to three hectares. Importantly, none of the sites had yet a dedicated area for the marketplace and for manufacturing production (the *business district* defined below). In period III (approximately from the 4$^{\text{th}}$ to the 9$^{\text{th}}$ century) new sites appeared but only a few of them became cities, most of them remained in the state of hamlets (Figure 1c). Existing cities expanded at different degrees and some of them made a major structural leap by creating a dedicated area for manufacturing production and for the marketplace (the *business district* described below). Differences in size became bigger and the city size distribution became the one we observe for the 9$^{\text{th}}$ century.

[Figure 1a, 1b, and 1c about here]

Data. In the ninth century, the oasis contained 618 sites identified by the presence of mounds resulting from the overlapping of various layers of human occupations over centuries. Sites have specific urban structures that allow grouping them in the following categories: *manufacturing* cities, *agricultural* cities, *hamlets*, and *forts.* A manufacturing city consists of a *town* and a *business district.* The town hosts the political center, which often consists of a fortified building and the village which hosts the population and in some cases administrative buildings.[6] The business district hosts the commercial and manufacturing activities. In some cities, the business district is equipped with dedicated areas for caravan stopovers. We have found evidence that the town of five manufacturing cities was walled. Not surprisingly, three of these were the cities on the border of the oasis. By the analysis of the land slopes at the border of cities, it is likely that many other manufacturing cities were walled but there is no certainty, and in any case, the walls were probably of smaller magnitude. An agricultural city is made of a town structured similarly to that of the manufacturing cities, though smaller in size. Importantly, agricultural cities do not have a business district. A hamlet is an isolated settlement where neither a town nor a business district are present. It typically consisted of a few houses, or a manor and its annexes, or an isolated large farm. Forts, consist of a very small and concentrated settlement hosting only soldiers. In addition to these settlements, in the oasis there are some unstructured sites that exhibit scattered pottery or water points. They do not show evidence of stable human settlement and for this reason we neglect them in our analysis. For clarity of exposition, in the sequel we reserve the term city for the manufacturing and agricultural sites because of their organized urban structure. In conclusion, the universe of 618 settlements consists of 53 manufacturing cities, 284 agricultural cities, 266 hamlets, and 15 forts. Figure 1c refers to the end of Period III and represents the data we use.

Figure 2 shows the layout of the silk road, the position of manufacturing cities, and the main natural water courses. The silk road touches many manufacturing cities but only eleven among these have a dedicated stopover place for caravans.

[Figure 2 about here]

We use three measures of site size. The first is the Residential Area (RA), which is the area of the surface occupied by dwellings and public buildings if any. For manufacturing and agricultural cities this area corresponds to the town area. For hamlets and forts it corresponds to the area occupied by dwellings. The second is the Total Area (TA), which is the area of the total surface occupied by the city. This measure applies to manufacturing cities only and consists of RA (as defined above) plus the area of the business district. The third is Population (POP). We observe the surfaces of towns, business districts, and hamlets with precision.[7] We do not observe population but we can estimate it fairly well by assigning standard population densities used in archaeology to

[6] The term "fortified" should be understood relative to the context. Fortified buildings differed from normal ones only in that they had a lot thicker walls and more robust entrances sometimes architecturally devised to resist a charge of infantry. Otherwise, they were normal constructions built on the ground floor only and whose size would be about ten times that of an average private house in town.

[7] The method of surface identification and measurement is accurately described in Rante and Mirzaakhmedov (2019). Here, it is important to note that we observe the universe of sites and their urban structure, except for the following four cities (city code in parenthesis): Gijduvan (0002), Bukhara (0097) and Vobkent (0116) because the modern urban structure entirely covers the ancient site, and Šargh (0846) because it is completely destroyed. We know the location of these sites but we cannot measure the area covered by the sites.

the areas we have directly measured. Population densities differ by type of settlements. We assign 100 inhabitants per hectare for hamlets and forts, 150 inhabitants per hectare for agricultural cities and 200 inhabitants per hectare for manufacturing cities. The reason for different population densities is that while agricultural cities hosted population only in the town area, manufacturing cities hosted population also in alternative accommodations (soldiers in barracks, workers in the business districts, people in transit in the market area, some dwellings outside the walls, etc.). The population densities we use are standard in archaeology for that time and area of the world and are corroborated by our archaeological excavation where we measure the size of houses, streets, buildings, etc. As robustness checks, however, we use all three measures of size.

Oasis Delimitation. The oasis is naturally delimited by the delta of the Zerafshan. The delta in the 9^{th} century was almost the same as today and is easily identifiable by the green area in the satellite picture of Figure 2. The north-east strip of land along the stretch of the silk road that connects manufacturing cities 104 and 751 deserves some discussion. This stretch of the silk road crosses a quasi-desert. The area was a quasi-desert even in the 9^{th} century and was not inhabited. The distance between city 104 and 751 could be traveled (with some effort) in one day by merchant's caravans and it represented a substantial shortcut towards the heart of the oasis with respect to travelling through the green strip of land. We observe a similar situation at the opposite extreme of the oasis, in the land crossed by the stretch of the silk road that connects city 95 to the rest of the oasis. The other area that looks as a quasi-desert is to the North of the oasis. This area was uninhabitable in the 9^{th} century because it was covered by swamps and it is almost unpopulated even today. Outside the oasis lays the desert. The nearest oasis connected by a fertile strip of land is Samarkand (at 180 km). The other 'nearby' oasis are the Khwarezm (at 320 km) and Karchi (at 160 km) both separated by a desert from the oasis of Bukhara. The only human settlements in the desert were concentrated around sparse water pits along the silk road.

The strong and exogenous isolation of the oasis is very useful for the study of the city size distribution because it assures that there are no data relevant for the study and excluded by the arbitrary delimitation of the area of interest. To see the problem, imagine we wanted to study the city size distribution in France. France would then be the area of interest. The study would encounter the problem that the size of French cities (especially those near the border) is influence by the existence of foreign cities which, by definition, are not taken into account. This is not an easy problem to tackle because there would be no natural end of the chain of influence of some city size on other city size.

4 Aggregation in Urban Systems

The aggregation problem. If we want to predict the city size distribution, the first thing to do is to define what a 'city' is. Some aggregation of urban areas is clearly desirable to define a 'city'. For modern economies, for instance, we do not care to predict the size of a particular suburb or administrative subdivision of an integrated urban area; we care about predicting the size of the urban area as a single social and economic unit. Likewise, for the oasis of Bukhara it would be uninteresting to try to predict the size of every one of the 618 sites. In general, consider the following example where two sites, say a big site A and a small site B are located near to one another (say, New York and Newark, respectively, 8 and 0.3 millions inhabitants). Imagine we want to treat them as distinct 'cities'. Any spatial model used to predict the city size distribution would predict them to have essentially the same size because their centrality as well as other possible exogenous characteristics are essentially the same. The spatial model, by predicting the same size, would underestimate the size of A and overestimate the size of B, thus performing poorly. But this would be a bad use of the model and a bad use of the data because it fails to recognize that A and B are part of the same integrated urban area. It is the size of A+B that we want to explain and not that of A and B separately. Aggregation is therefore useful and desirable to make sense of any model and to use the data appropriately. The remaining issue is the choice of the aggregation method. For modern economies this is not a simple task. The two most popular definitions in the empirical literature for modern economies are census defined cities and metropolitan areas. The difference between these two definitions is huge. For example, according to the first definition New York is made of New York City and counts about eight millions people while according to the second definition New York is made of New York City, Long Island, the Mid and Lower Hudson Valley, Newark, Jersey City, Paterson, Elizabeth, Lakewood, Edison, Bridgeport, New Haven, Stamford, Waterbury, Norwalk, Danbury, as well as other vicinities and counts about twenty million people. Likewise, Paris *intra-muros* counts about 2.2 million people while Grand Paris counts seven million people distributed on 131 municipalities.[8] Similar differences apply, in due proportions, to smaller urban areas. Rozenfeld et al. (2011) provide what is probably the most accurate measure of 'city' size to date. They use high-resolution remote sensing and 'build' cities by clustering urban areas according to the criterion of proximity. They define a 'city' as a maximally connected cluster of populated sites defined at high resolution. Thus, the boundaries of a city are determined by the fall in urbanization density below an arbitrary cut off threshold. Our criterion is similar in the implementation but relies on the exogenous ties between sites.

Urban systems. For the oasis of Bukhara the boundaries of a city are determined exogenously by the transport and storage technologies of the time. The perishable nature of agricultural produces made that they had to be exchanged near the place of production but dedicated market places only existed in manufacturing cities. The need to exchange food for manufactures, animals, clothes, or other food at short distance created a strong interdependence between the manufacturing city (where the dedicated market place was located) and the hinterland made of agricultural cities and hamlets where there were no markets. This interdependence is exogenous since it is due to the perishable nature of food. We use this fact to build our unit of analysis. We aggregate sites into urban systems by assigning each site to the nearest manufacturing city. Our unit of analysis is then the *urban systems*. The criterion is analogous to that of the modern metropolitan areas since

[8] We spare you the list.

we aggregate sites on the basis of their economic and social interdependence but we have the advantage that interdependence is exogenous. Figure 3 shows in different colors the area pertaining to each urban system. The position of the yellow circles coincides with that of the manufacturing cities and the size of the circle represents the population of the urban system. The urban systems in the north-east and south-west extremes of the oasis (resp. site code 104 and 95) are very remote from the rest of the oasis so as to raise the question of whether they should be included in our analysis. The most famous contemporary historian, Al-Narshakhi (943), includes these two cities in the culture and society of the oasis and refers to them as the oasis gateway. However, given the transport technology of the time, the position and size of these two cities might be due to the need for stopover places at the entry of the oasis. To check the robustness of our results we perform the empirical analysis first including and then excluding the gateway cities and their associated sites. We use the term *restricted* oasis for the data-set that excludes the gateway cities and their associated sites, we use the term *archaeological* oasis for the data-set that includes all sites. Fig. 3 represents the archaeological oasis. The restricted oasis obtains by suppressing the extreme South-West and extreme North-East urban systems. In total we have 53 urban systems for the archaeological oasis and 51 for the restricted oasis.

[Figure 3 about here]

Our aggregation is based on the interdependence between manufacturing cities and other sites. This criterion has the implication that non-manufacturing sites should be found more frequently than site of any type around a manufacturing city. We test this implication by use of a distance-based index of geographic co-concentration. Let M be the set of manufacturing cities, let V be the set of other sites, and let N be the set of all sites. In our data (using the same letter for a set and its cardinality) we have $M = 53,\ V = 570,\ N = 618$ for the archaeological oasis while $M = 51,\ V = 560,\ N = 611$ for the restricted oasis. The index of geographic co-concentration we use, denoted $C(r)$, is

$$C(r) = \left(\frac{\sum_{i\in M} v_i(r)}{V}\right)\left(\frac{\sum_{i\in M} n_i(r)}{N-1}\right)^{-1}. \tag{1}$$

where $v_i(r)$ is the number of non manufacturing sites found within a radius r from manufacturing city i; $n_i(r)$ is instead the number of all sites found within the same radius. The maximum distance between any two sites is 131 kilometers for the archaeological oasis and and 85.7 kilometers for the restricted oasis. The index is computed for $r = 1, 2, ..$ to the respective maximum distance. The first term in expression (1) measures the mean percentage of sites belonging to V found within the radius r while the second term measures the analogous mean percentage for all types. Each term is function of r and each, if divided by M, can also be seen as the cumulative distribution of mean frequencies of, respectively, non-manufacturing sites and sites of any type found in the disk of radius r. A value $C(r) > 1$ would indicate that - on average - and within that particular radius r, sites of type V are more frequently found than sites of all type. This would mean that non-manufacturing sites are more concentrated around manufacturing sites than are sites of any type. Vice-versa for $C(r) < 1$. By construction, the index converges to 1 as r approaches the maximum distance. Note that the values of each term for different r are necessarily not independent since V and N are constant. If $v_i(r)$ is high for some r it necessarily has to be low for some other r. Likewise for $n_i(r)$. This dependence across r within each term in parenthesis is not a concern because $C(r)$ measures relative geographic concentration. To detect the statistical significance of the values of the index obtained for the oasis we construct confidence intervals based on simulations of complete spatial randomness. We create a simulated sample by superimposing two homogeneous Poisson point processes, one with 570 sites (that serve as random location of non-manufacturing sites) and one with 53 sites (that serve as random location of manufacturing cities). For the restricted oasis we take 51 and 557 sites respectively. For every radius r from 1 kilometers up to the maximum bilateral distance, we compute co-concentration indices in the sample data. We repeat the draw a thousand times and create a lower and upper local confidence bands of the indices by picking the fifth and the ninety fifth percentile for each r. We also construct a global confidence interval by keeping the percent of rejected index values across all radii equal to 5%, keeping the confidence interval range for each r fixed.

[Figure 4 about here]

Figure 4 plots the results. The concentration of non-manufacturing sites around manufacturing cities exceeds what one would expect from randomness. It is very interesting to note that the two peaks of the distribution are found within disks of radius equal to four and seven kilometers. These peaks testify of the importance of being close to the market place. The peak at a radius of seven kilometers is in remarkable concordance with the standard conjecture made in archaeology that seven kilometers is the maximum distance a farmer could travel daily to go to sell produces to the market place. If archaeologists were right we would expect a drop of the co-concentration index after this limit, which is what the data actually shows. Longer distance could of course be travelled on less frequent bases. Co-concentration remains clearly above confidence intervals for radii between ten and twenty kilometers. In conclusion, this spatial analysis shows that manufacturing cities tend to be surrounded by non-manufacturing sites more than a random draw would predict. This result lends strength to the criterion of aggregation of points in urban system we have used above.

The homogeneity of the oasis is extremely useful in this respect because it allows to rely on the homogeneous Poisson point process. This process scatters randomly points on a surface under the assumption that any sub area of the surface has the same probability of receiving a point. The unique homogeneity of the oasis is very close to this assumption. The inconvenience of dealing with non homogeneous spaces is not only in the complication of constructing a process where the probability that a sub-area receives a point depends on the characteristics of the sub-area. The major difficulty is that the characteristic of the sub-area is not only exogenous (mountains, lakes, etc.) but also endogenous to the city size (infrastructures, urban planning, etc.) and evolves over long time due to technological changes. Most areas of the world are in such situations. In this respect, the oasis of Bukhara is really unique.

Features of the data and summary statistics. Table 1 reports the summary statistics for urban systems in terms of residential area, total area, and population for the archaeological oasis (lines with 53 observations) and for the restricted oasis (lines with 51 observations).

[Table 1 about here]

For visual inspection, Fig. 5a shows the size distribution of urban systems. The distribution passes the test of log-normality predicted by stochastic growth (p-values equal to 0.87, 0.56, and 0.922 for the Skewness-Kurtosis, Shapiro-Wilk and Kolmogorov-Smirnov tests, respectively). Figure 5b shows the log-rank - log-size relationship. This relationship complies with the theoretical prediction in Duranton (2007) for modern economies; namely, that the slope is between zero and -1 on the left-end of the distribution, it is -1 somewhere in the middle of the distribution and it is smaller than -1 at the right-end of the distribution. These two features of the data are interesting because the economy of the 9$^{\text{th}}$ century arguably does not have the features that in modern economies may give rise to log-normality or to concavity.

[Figure 5 about here]

Preliminary Evidence. We start with an OLS regression of the size of the urban systems on a simple and not model-based measure of centrality. Our dependent variable is $\lambda_i = L_i / \sum_i L_i$ where L_i is the size of urban system i and $i = 1, ..M$. As explained above we use three measures of size: Population, Residential area, and Total area. Centrality is measured by the mean distance of urban system i to any other place divided by the sum of such mean distances. This heuristic measure of centrality has the property that if urban systems were equidistant they would have equal size $\lambda_i = 1/M,\ \forall i$. As per the measure of distance (δ_{ij}) we use the geodesic as well as the least cost path based on the Dijkstra (1959) algorithm. We estimate equation (2) where D_{First} and D_{Silk} are dummies that capture, respectively, whether that manufacturing city existed in the first phase of development described above, and whether that city was a stopover point on the silk road. The first term on the right hand side measures centrality.

$$\lambda_i = \frac{\left(\sum_{j=1}^{M} \delta_{ji}\right)}{\sum_{k=1}^{M} \left(\sum_{j=1}^{M} \delta_{ji}\right)} + D_{First} + D_{Silk} + \epsilon_i \tag{2}$$

Results are reported in Tables 2, 3, 4. All three measures of size are strongly correlated with centrality. Close to two-thirds of the variation in urban system sizes is explained by centrality alone as seen by the R^2. When we sequentially add the dummies the fit improves only marginally with the contribution of silk road being slightly larger than that of the other dummy. Interestingly, the silk road dummy becomes strongly significant when size is measured in terms of total area. Total area is made of residential area *plus* plus the area of the surface occupied by the business district. The latter components of the total area are particularly large for stop-over cities since the marketplace hosted merchants and their merchandise. Consistently, wen using this data the performance improves. In conclusion, the OLS regressions hint at the possibly important role of centrality in determining the size distribution of urban systems.

[Table 2,3,4 about here]

5 Model

Our argument is that the spatial distribution of cities determines their size because centrality is an advantage when traveling is costly. To represent this argument we use an elementary nested multinomial logit model of location decisions in the vein of McFadden (1974). This model structure is often used in general equilibrium spatial models.

The indirect utility derived from being located in urban system i is

$$U_i = u_i + \epsilon_i \tag{3}$$

where u_i and ϵ_i are, respectively, the common and idiosyncratic component of utility. The latter is an *i.i.d.* random variable distributed as a Gumbel with location parameter normalized to zero, scale parameter μ_L, and variance $\mu_L^2\pi^2/6$. The cumulative distribution function is $F(x) = \exp(-\exp(-x/\mu_L))$. Let $f(x)$ denote the associated PDF. An agent will choose city i over any other city k if $u_i + \epsilon_i > u_k + \epsilon_k$ for all $k \neq i$. The probability $\mathbb{P}_i$ that an agent chooses location i when there are N locations is

$$\begin{aligned} \mathbb{P}_i &= \Pr\left[u_i + \epsilon_i = \max\{u_k + \epsilon_k\}\right] && (4) \\ &= \Pr\left[\epsilon_k < \ \epsilon_i + u_i - u_k \quad \forall k \neq i\right] && (5) \end{aligned}$$

For any given ϵ_i, expression (5) is the cumulative distribution evaluated at $\epsilon_i + u_i - u_k$. Given the *i.i.d.* assumption this gives $\mathbb{P}_i|\epsilon_i = \prod_{k\neq i}^N F\left(\epsilon_i + u_i - u_k\right)$. But ϵ_i is a random variable and therefore each of these probabilities must be 'weighted' and 'summed up'. Thus,

$$\begin{aligned} \mathbb{P}_i &= \int_{-\infty}^{\infty} f(x) \prod_{k\neq i}^{N} F\left(u_i - u_k + x\right) dx && (6) \\ &= \int_{-\infty}^{\infty} \frac{1}{\mu_L} e^{-\left(\frac{x}{\mu_L}\right)} e^{-e^{-\left(\frac{x}{\mu_L}\right)}} \prod_{k\neq i}^{N} e^{-e^{-\left(\frac{x+u_i-u_k}{\mu_L}\right)}} dx && (7) \end{aligned}$$

Using the change of variable $y_i = \exp(u_i/\mu_L)$, and $\delta = \exp\left[-\left(\frac{x}{\mu_L}\right)\right]$; noticing that the differential $d\delta = -(1/\mu_L)e^{-(\frac{x}{\mu_L})}dx \leq 0$; and also noticing that as x goes from minus to plus infinity along the support of the Gumbel distribution δ goes from infinity to zero we obtain

$$\mathbb{P}_i = \int_0^\infty e^{-\delta} \prod_{k \neq i}^{N} e^{-\frac{\delta y_k}{y_i}} d\delta = \int_0^\infty \exp\left(\frac{-\delta \sum_{k=1}^N y_k}{y_i}\right) d\delta \tag{8}$$

which, after integration, gives

$$\mathbb{P}_i = \frac{y_i}{\sum_{k=1}^N y_k}. \tag{9}$$

The model has the following properties:

$$\lim_{\mu_L \to \infty} \mathbb{P}_i = \frac{1}{N} \qquad \textit{and} \qquad \lim_{\mu_L \to 0} \mathbb{P}_i = \begin{cases} 0 \text{ if } u_i < \max\{u_k\} \\ 1 \text{ if } u_i = \max\{u_k\} \end{cases} \tag{10}$$

As μ_L (the variance) approaches infinity the idiosyncratic component of utility dominates any other aspect of the location choice and every location has the same probability of being chosen. At the other extreme, as μ_L approaches zero, the idiosyncratic component of utility has negligible impact relative to other aspects in the location decision and every agent will choose the location that gives the highest u_i.

Location decision is taken in a logically distinct time from the consumption and travel decisions. At the time of deciding where to reside, the agent knows she will have stochastic preferences about trip destinations that will manifest themselves after the residential decision has been made. Thus, location decision is based on the expected indirect utility derived from consumption and from travels which we now discuss.

Each individual is endowed with $\bar{l}$ units of labor. Labor is used for production and travel. Let $0 < \alpha < 1$ be the fraction of labor used for production. Then $\alpha\bar{l}$ is the real income of an individual.[9] Indirect utility from consumption equals real income plus a Gumbel distributed stochastic component with parameter μ_C analogous to the parameter μ_L introduced above. The stochastic component represents the idiosyncratic preference for a particular vendor, including oneself. The expected indirect utility from spending one's income in market i is

$$\mathbb{E}\left[c_i\right] = \alpha\bar{l} + \mu_C \ln\left(L_i^m\right) \tag{11}$$

where L_i^m is the number of people present in market place i.[10] L_i^m is proportional to the population of the urban system for market places that do not host silk road merchants, $L_i^m = mL_i$ with $0 < m < 1$. Exchange opportunities in market places that host silk road merchants are larger by a factor $s_i = (L_i + S_i)/L_i$, where S_i is the population of

[9] If we assume Cobb-Douglas preferences with β being the expenditure share on agricultural produces the real income is $\alpha\bar{l}\left(a_A\right)^{-\beta}\left(a_M\right)^{-(1-\beta)}$ where a_A and a_M are unit labor inputs in production (proportional to prices). To save notation we abstract from this multiplicative constant in the sequel.

[10] See Small and Rosen (1981) for the derivation of expected utility from discrete choice models.

merchants stopping over. Indirect utility is increasing in L_i^m because of a wider variety of vendors in the market. Vendors and varieties of goods may be used interchangeably. An equivalent indirect utility would stem from a Dixit-Stiglitz model of monopolistic competition because the number of variety would be proportional to the number of market participants (resident population plus silk merchants if any). In this in interpretation, the term $-\mu_C \ln(L_i^m)$ would be the log of the local price index and μ_C would correspond to $1/(\sigma - 1)$ where $\sigma > 1$ would be the elasticity of substitution between varieties. The intuition would be similar: when the elasticity of substitution approaches infinity varieties are perceived as almost the same, μ_C would approach zero and the availability of many varieties would bring almost no utility. At the other extreme, when the elasticity approaches one, varieties matter a lot, μ_C approaches infinity and the availability of many varieties matters a lot for utility.[11]

There are N trip destination one can chose from if located in urban system i (including i itself). Indirect utility of a trip from i to j is $\tau_{ij} + \zeta_j$. The first addendum is the part of utility common to all agents and the second addendum is the *i.i.d.* Gumbel distributed idiosyncratic destination preference with scale parameter μ_Z. The deterministic component of the indirect utility of a trip from i to j (denoted τ_{ij}) is given by the time spent in j when leaving from i. As a minimal micro-foundation of travel costs we assume that the cost of going from i to j is the time lost in transit. Let, $(1 - \alpha)\bar{l}$ be the time spent traveling and let d_{ij} be the distance between i and j. A return trip would then costs $2d_{ij}/s$ where $s > 0$ is the speed. Then, $\tau_{ij} = (1 - \alpha)\bar{l} - 2d_{ij}/s$. This minimal micro-foundations can be interpreted in the spirit of the Armington model. In this interpretation, each location would produce a distinct variety of such good; then $1 - \alpha$ would be the share of income spent on tradeable goods (say textile) and τ_{ij} would be the indirect utility from consumption of a particular good after paying transport services or, which is the same, after traveling to pick up the good. Nothing hinges on the assumption that all agents travel. An equivalent assumption would be that trips are undertaken by the local ruler and his retinue, or by other public officers (military, ambassadors, etc.). Since these trips are financed by taxes, the cost falls ultimately on individuals. In this perspective, $(1 - \alpha)$ is the time equivalent of taxation raised to finance the trips. In any case, the expected utility obtained from trips is

$$\mathbb{E}[z_i] = \mu_Z \ln\left(\sum_{j=1}^{N} \exp(\tau_{ij}/\mu_Z)\right) \tag{12}$$

The model shares mathematical features with other model structures, notably with Armington, Dixit-Stiglitz, and Eaton-Kortum and can be interpreted in the spirit of these models. Equation (12) for instance is analogous to the concept of market access. Given that we apply the model to the 9$^{\text{th}}$ century economy we prefer to interpret the mechanisms in the context of those times.

[11] See Anderson et al. (1991) for the relationship between discrete choice models and CES preferences.

Total expected utility associated with location i is the sum $\mathbb{E}[c_i]$ plus $\mathbb{E}[z_i]$:

$$\mathbb{E}[u_i] = \bar{l} + \mu_C \ln L_i^m + \mu_Z \ln \left(\sum_{j=1}^{N} \exp\left(-\delta_{ij}/\mu_Z\right) \right) \tag{13}$$

where $\delta_{ij} = 2d_{ij}/s$.

A spatial equilibrium is a geographical distribution of the population such that the probability that a given location is chosen equals the number of individuals who actually have chosen that location. This definition implies that net migration flows are zero in equilibrium. Formally, a spatial equilibrium is the set $\{L_i^*\}$ that satisfies $L_i^* = \sum_{j=1}^{N} \mathbb{P}_i L_j^*$, for $i = 1..N$. Using expressions (9) and (13) and defining $\lambda_i \equiv L_i/\overline{L}$ and $\lambda_i^m = s_i \lambda_i$ we may write the spatial equilibrium as

$$\lambda_i^* = \frac{(\lambda_i^m)^{\frac{\mu_C}{\mu_L}} \left(\sum_j^N e^{\frac{-\delta_{ij}}{\mu_Z}} \right)^{\frac{\mu_Z}{\mu_L}}}{\sum_{k=1}^{N} (\lambda_k^m)^{\frac{\mu_C}{\mu_L}} \left(\sum_j^N e^{\frac{-\delta_{kj}}{\mu_Z}} \right)^{\frac{\mu_Z}{\mu_L}}}, \qquad i = 1..N. \tag{14}$$

It is interesting to examine how the distribution of idiosyncratic preference shocks influences the size distribution of urban systems through illustrative simulations. The variance of the preference shocks is governed by the three key parameters μ_C, μ_Z, and μ_L. For illustrative purposes, we use the coordinates of urban systems in the data and let the model predict the city size distribution for high and low values of each of these parameters keeping constant the other two. The results are plotted in the three panels of Fig. 6 which we discuss in turn.

The parameter μ_C represents the appreciation for large domestic markets. A larger μ_C makes consumers more sensitive to market size because they more strongly appreciate the large exchange opportunities provided by large markets (equation 11). A large μ_C then favors agglomeration of the population in large urban systems. The model predicted size distributions of urban systems for low and high μ_C are represented in Fig. 6a. We see that a bigger μ_C (for given μ_Z and μ_L) translates into a wider range of the distribution especially due to large urban systems becoming larger.

[Figure 6 about here]

The parameter μ_Z is the variance of trip-destination preference shocks. A very large μ_Z means that agents travel to every destination with almost the same probability. Then the expected benefit from traveling depends grandly on centrality. It is almost as if an agent had to travel to all destination with certainty spending in each an equal fraction of her time. Centrally located urban systems will then be highly preferred because the expected (or total) traveling time is low when traveling from them. The resulting city size distribution would be very stretched out. This can also be seen by the fact that $\lim_{\mu_Z \to \infty} \lambda_i^* = (\lambda_i^m)^{\frac{\mu_C}{\mu_L}} \exp\left(-\frac{\sum_j^N \delta_{ij}}{\mu_L N}\right) / \sum_k^N (\lambda_k^m)^{\frac{\mu_C}{\mu_L}} \exp\left(-\frac{\sum_j^N \delta_{ij}}{\mu_L N}\right)$. The population share λ_i^* is increasing in $-\sum_j^N \delta_{ij}$, which is a measure of the centrality of urban system i. At the opposite extreme, when μ_Z is very small, people travel almost certainly to the closest destination. This can be seen by noticing that $\lim_{\mu_Z \to 0} \mathbb{E}[z_i] = \max\{\tau_{ij}\}_i$ where $\{\tau_{ij}\}_i$ is the set of all τ_{ij} pertaining to i. The attractiveness of a location is then given by how close a location is to its closest destination, but the closest destination is home: $\max\{\tau_{ij}\}_i = \tau_{ii} = 1$ for all i. The city size distribution is then only the result of exogeneous heterogeneity created by the silk road. This can be seen by noticing that $\lim_{\mu_Z \to 0} \lambda_i^* = (\lambda_i^m)^{\frac{\mu_C}{\mu_L}} / \sum_k^N (\lambda_k^m)^{\frac{\mu_C}{\mu_L}}$.

The parameter μ_L is the variance of the location preference shocks. A large μ_L reduces the importance of each location specific utility u_i and in the limit, when $\mu_L \to \infty$ annihilates it. A large μ_L makes all urban systems be appreciated almost the same in spite of objectively different u_i. Thus, different urban systems attract approximately the same population and the size distribution becomes more concentrated, as shown in Fig. 6c.

6 Formation of Manufacturing Sites

So far, we have taken the number of manufacturing cities as exogenous. In this section, we make the evolution of sites into manufacturing cities endogenous by use of an elementary model based on the archaeological evidence. The archaeological evidence discussed above suggests an evolution of the urban structure of the oasis in three phases. In particular, organized marketplaces (which define a manufacturing site) appeared only between the 4$^{\text{th}}$ and the 6$^{\text{th}}$ century. This fact and the absence of storage technology for food suggest a natural evolution of sites into urban systems that we model as follows. Let $t(\delta_{ij})$ be a function that measures the fraction of the perishable food that arrives at the destination safe and sound. Of course, this function depends on travel time δ_{ij}. Let $dt(\delta_{ij})/d\delta_{ij} < 0$ and $t(\delta_{ii}) = 1 \ \ \forall i$; also let $\overline{\delta}$ be such that $t(\overline{\delta}) = 0$. As the produce perishes in transit so does the spendable income. The expected utility from spending (non-perished) income in market j for a resident of i is $\alpha \bar{l} t(\delta_{ij}) + \mu_C \ln(L_j^m)$ if $t(\delta_{ij}) > 0$ and zero if $t(\delta_{ij}) = 0$.

Initially (in Period I), when there were only a small number of sites in the oasis it is unlikely that any subset of them were close enough that if food were transported would not entirely perish. That is, initially it is likely that $\min\{\delta_{ij}\}_{i \neq j} > \overline{\delta}$ for all i. Therefore, produce exchange, if at all, took place within every site. Given the scant population in every site, there was no need for dedicated spaces or any form of organized marketplaces. As the number of sites increased (period II), accidental clusters where $\min\{\delta_{ij}\}_{i \neq j} < \overline{\delta}$ for some i and j eventually came to exist and exploration of exchange opportunities beyond one's site began. Initially, proto-markets may emerge in every site of the accidental cluster but the exploration process eventually gives rise to the emergence of a single market within any accidental cluster. The reason is that manufacturers, who are not tied to the land, will choose to reside in the site hosting the largest market. To describe the process in further detail, consider a process where people living in an accidental cluster choose to explore nearby markets to find the one that gives the highest expected utility. The number of people that will travel to the market in location i within an accidental cluster that has K sites in it is

$$L_i^m = \sum_{j \neq i}^{K-1} L_j \mathfrak{I}_j \qquad \forall i, \tag{15}$$

where $\mathfrak{I}_j$ is an indicator function such that

$$\mathfrak{I}_j = \begin{cases} 1 & if \quad \alpha \bar{l} t(\delta_{ji}) + \mu_C \ln(L_i^m) = \min_k \left\{ \alpha \bar{l} t(\delta_{jk}) + \mu_C \ln(L_k^m) \right\} \\ 0 & \quad otherwise \end{cases} \tag{16}$$

Equations (15) give rise to heterogeneous market size even when all sites in cluster K have initially the same resident population. Even when this initial condition is met, markets will not have the same number of participants because places will not, in general, be equidistant. As a consequence the initial distribution of the resident population within the accidental cluster is unstable. The reason is that manufacturers (who are not tied to the land) will move to reside in the site where L_i^m is the largest because of the largest exchange opportunities available there. The concentration of manufacturers will wipe out all other markets within the accidental cluster since exchange with manufacturers can only occur where manufacturers are located. At the end of the process, there will be only one marketplace in any accidental cluster and this market will host the production of manufactures. Note that the emergence of a single market within the accidental cluster is independent of the microfoundation of the exploration decisions. As a matter of fact, one can assume a variety of exploration processes. As long as they give rise to heterogeneous market size, manufacturers will decide to reside in the largest market of the cluster and a single market will eventually emerge. Note also that a single market in a vaster area than an accidental cluster cannot emerge because perishable food would perish before reaching that market.

To check the plausibility of the formation of the manufacturing sites we have just described, we simulate it as follows. We generate a random allocation of 618 points in the archaeological oasis using a homogeneous Poisson point process. Once again the homogeneity of the oasis is extremely useful because it allows relying on a homogeneous Poisson point process. Each of the 618 points may become a manufacturing site but which of them will become one is determined by the accidental clustering of these points. For each point, we compute the number of neighboring points we find within a radius of seven kilometers. We chose seven kilometers because this distance makes the consensus among archaeologists as the maximum distance that can be travelled daily by a farmer to the marketplace for that historical period, that area of the world, and that technology. We also have seen above that within a radius of seven kilometers from a manufacturing city we find one of the highest peaks of co-concentration. The point which has the largest number of neighbors is the first simulated manufacturing site. We then drop this point and its neighbors from the list of potential manufacturing sites and we recount the number of neighbors for each of the remaining points. We identify the point with the largest number of neighbors in this reduced set of points and this will be our second simulated manufacturing site. We reiterate until we have exhausted all points. At this stage, some points will be neighbors of two manufacturing sites. We assign such points to the closest manufacturing site. After this last step, we will have no leftover points and no point being neighbors of more than one manufacturing site. The set of thus-created manufacturing sites and their neighbors constitute the set of simulated urban systems. The objective of this simulation is to verify that the number of urban systems obtained from the simulation is near to the one we observe in the data. We repeat such a simulation one thousand times to obtain a simulation-based distribution of the number of urban systems.

[Figure 7 about here]

Figure 7 shows the result. It is amazing how close simulation results are to the data. The simulation mode for the archaeological oasis is 51 manufacturing sites while the data has 53. For the restricted oasis we have, respectively, 46 and 51. The data is strongly within the simulated confidence intervals for the archaeological oasis while it lies in the upper bound of the confidence interval for the restricted oasis. We are now confident that the unit of analysis we have constructed, the urban systems, constitute a solid base on which we can build our estimations.

7 Estimations

In this section we estimate the model using data for the Bukhara oasis of the 9^{th} century and for the same geographic area using data for the 21^{st} century. The comparison between the 9^{th} and the 21^{st} century is of great interest as it allows us to compare the change in the strength of the mechanisms that give rise to the city size distribution.

The equation to be estimated is the log of (14), which is

$$\ln \lambda_i^* = \frac{\mu_C}{\mu_L - \mu_C} \ln s_i + \frac{\mu_Z}{\mu_L - \mu_C} \ln \sum_{j=1}^{N} e^{\frac{-\delta_{ij}}{\mu_Z}} - \ln \sum_{k=1}^{N} (s_k \lambda_k^*)^{\frac{\mu_C}{\mu_L}} \left(\sum_{j=1}^{N} e^{\frac{-\delta_{kj}}{\mu_Z}} \right)^{\frac{\mu_Z}{\mu_L}} \tag{17}$$

We jointly estimate μ_L, μ_Z and μ_C by matching three data moments: variance, skewness, and range. We use geodesic distances to measure $\{\delta_{ij}\}$.

Estimation using 9^{th} century data. We estimate two different versions of equation (17). In the fist one we estimate equation (17) as is. In the second one we remove the silk road effect by setting $s_i = 1 \; \forall i$. This allows us to assess the goodness of the model as well as the role of the silk road in determining the size distribution of urban systems. For the first estimation we have to measure $s_i = (S_i + L_i)/L_i$ when manufacturing city i is a stopover place for silk road merchants as explained above. This requires measuring S_i. To do so we use the information we have on the size of the business district of each manufacturing city. We regress the size of the business district on population and calculate the residuals. Such residuals, when belonging to the stopover cities, measure the excess size of the business district correlated with the status of the stopover city. As such it proxies the excess market size (the excess exchange opportunities) arising from the status of a stopover city. Thus, for stopover cities, we replace S_i in the estimating equation (17) with the residual values. As usual, we perform the empirical analysis using the three measures of size: population, residential area, total area. The results of moment matching are shown in Tables 5, 6, and 7. The model matches extremely well the targeted moments for both definitions of the oasis and for the three measures of size. The model performs slightly less well in matching two of the three moments when we do not account for the silk road. When the silk road effect is removed from the estimation the model overestimates the variance and underestimates the range. This is precisely what to expect because, without the silk-road effect, the model is less able to account for very large and very small urban systems. If we overlay the kernel density plot predicted by the model versus the 9$^{\text{th}}$ century data, we find that the model performs quite well in terms of matching the spread of the distribution (Fig. 8a). This is very interesting because the oasis is very close to an ideal lab where cities have had the time to evolve in the absence of technical progress, endogenous infrastructures, intrinsic location fundamentals, etc.

[Figure 8 about here]

[Table 5,6, and 7 about here]

Comparison with 21$^{\text{st}}$ century data. How much of the modern city size distribution the simple model above can explain? We check this by estimating the model on Uzbek census data of 2018 for the regions of Uzbekistan that coincide with the oasis of Bukhara. The data is provided by the Uzbek Statistic Agency and contains the population count for every city, town, and rural settlement for the modern administrative districts that include the 9$^{\text{th}}$ century oasis. The denomination of 'city', 'town', and 'rural settlement' is assigned by the Statistical Agency based on administrative criterion. Unless otherwise specified we shall refer to any of them as a 'places' regardless of the administrative denomination. The oasis of Bukhara overlaps with the central part of the modern region of Bukhara and with the southern part of the modern region of Navoi. The data does not contain the geographic coordinates. We have geo-localized the places by matching names (in Uzbek, Russian, or English) on Google map available in English and Russian. We have matched places to coordinates one by one because of a number of spelling mistakes in the transliteration contained in the census data. We were not able to geolocalize about sixteen per cent of the smallest paces. Since some of the modern district delimitation lays outside the 9$^{\text{th}}$ century oasis some of the unidentified places are likely to be outside the area of interest, which reduces the problem of missing geolocalizations. Furthermore, to the extent that the non geolocalized places are randomly distributed across the districts the missing data would not alter substantially the measured size distribution. The cleaned data contains 1146 places located in the area covered by the oasis of Bukhara.

As discussed above, some aggregation is necessary to construct a suitable unit of analysis. While the aggregation criterion was based on the the transport technology for the 9^{th} century oasis such exogenous determination of the unit of analysis does not exist for the 21^{st} century data. We then arbitrarily assign to the K largest places the role of main city of an urban aggregation. This role is analogous to that of the manufacturing cities of the 9^{th} century. We assign every other site to the closest of these K largest places to form an urban system analogous to that of the 9^{th} century. To anchor the number K to the data we chose it in a way to replicate the partition of population between the manufacturing city and the other sites observed in the 9^{th} century. We compute the average manufacturing city share of the population for the urban system (9^{th} century) and we choose K such that the average population share of the K largest places of the 21^{st} century is the same as the average share of manufacturing city in the ancient urban systems. This gives us $K = 90$ urban system for the modern oasis. Table 8 reports the estimation results. The first interesting finding is that the model doesn't perform as well in matching moments of the 21^{th} than in matching those of the 9^{th} century. The deterioration of the model performance is measured by the γ statistics, which falls from 1.1 in the 9^{th} to -2.19 in the 21^{st} century, and is visualised in Fig. 8b[12]. This deterioration is to be expected since the industrial structure, the transport technology, infrastructures, and the political geography of the area have changed substantially. The last two aspects are particularly important; differently from the 9^{th} century, the oasis of Bukhara is now fully integrated with the other regions of Uzbekistan. As a consequence, the effective distances have changed and they have changed endogenously.

[Table 8 about here]

In addition to the model fit we also want to measure the importance of each mechanism and whether the estimated parameters have changed between the 9^{th} and the 21^{st} century. Table 9 reports the estimated values of μ_C, μ_Z, and μ_L by using equation (14) for the two centuries. The results suggest that the market size effect is an order of magnitude higher today than historically as μ_C goes from 0.18 to 2.78. This explains the occurrence of on average larger city sizes than before and also slightly fatter tails. Importance of geographic centrality increased while location fundamentals mattered less in explaining city size distribution in the 21^{st} century. These results are in line with the hypothesis that in an industrial economy (albeit moderately industrial) the agglomeration mechanisms related to city size are far more important than in an agricultural economy. Although this is to be expected it is very interesting to see it confirmed in the 'lab'. This interpretation is to be taken with caution, however, since the overall fit for the 21^{st} century is rather poor as quantified by the γ statistics reported in the last column of Table 9.

[12] The parameter γ is the coefficient in the regression of population shares observed in the data on population shares predicted by the model. The estimating equation is $\lambda_i^{\text{data}} = \gamma \lambda_i^{\text{model}} + \epsilon_i$.

[Table 9 about here]

Lastly, Figure 9 shows the kernel density approximation of the urban systems size distribution for the two centuries. We can see how the distribution, which was earlier close to log normality, starts to have thicker tails indicating few very large cities in the distribution.

[Figure 9 about here]

8 Persistence

How much of the spatial structure of the 9$^{\text{th}}$ century has remained in the economy of the 21$^{\text{st}}$ century? Anecdotal evidence is suggestive since some modern cities are located very close to the ancient cities and have kept the same name through the centuries. However, as we shall see, not much of the ancient structure has remained today. We measure two aspects of persistence. The first concerns the proximity of new sites to old sites (relative persistence). The second one concerns the overall position of sites relative to the oasis (absolute persistence).

Relative persistence. We compute the co-concentration index between the 53 manufacturing cities of the 9$^{\text{th}}$ century and the 90 largest cities of the 21$^{\text{st}}$ century. The result is shown in Fig. 9. We see that the data lays within the 90% local confidence bands (blue dotted line). However the data is outside the 95% global confidence intervals which gives us only weak evidence of persistence in city locations today relative to their 9$^{\text{th}}$ century locations. Evidence in favour of persistence is stronger for very short distance (less than five kilometers) but we cannot exclude that such persistence is the result of a random draw. Overall, the interpretation of this result is that perhaps some large cities of the 21$^{\text{st}}$ century are nearer to the manufacturing cities of the 9$^{\text{th}}$ century than a random draw would predict. But the attractiveness of history fades away almost certainly beyond the tenth kilometer.

[Figure 10 about here]

Absolute persistence. We find instead is a clear shift of the spatial structure towards the North-East. In Fig. 11, we plot the cumulative shares of places which are within k kilometers from the North-East border of the 9$^{\text{th}}$ century Bukhara oasis. In red we have the shares for the of 21$^{\text{st}}$ century and in blue the shares for the 9$^{\text{th}}$ century.

[Figure 11 about here]

The red line is always above the blue line indicating that modern places are situated more to the North-East relative to ancient sites. This is the case for the location of places relative to sites (Fig. 11a) and for population (Fig. 11b). Figure 12 gives us a sense of the magnitude of the North-East move by tracking the centroid of the oasis. Between the 9^{th} and the 21^{st} century the centroid has moved North-East by 7.19 kilometers for population and by 11.8 kilometers for location of places relative to sites. We can also track the shifts of the centroid for two other point in time, the late 19^{th} century and the 1970s. This is done by manually locating points using the maps reported in the online appendix. Both maps have been compiled for military purposes. The first map dates of 1893 while the second dates of the 1970s. Since population data is not reported we can only relay on the location of points on the map. The 19^{th} century map counts 606 points while the 20^{th} century map counts 857 points. The centroid in the 1893 map is near to that of the 9^{th} century while the centroid of the 1970s is near to that of the 21^{st} century. The positions of the four centroids are remarkably in line with the landmark of the oasis historic evolution. The oasis of Bukhara was an isolated area in the 9^{th} century; it was annexed to the Russian empire in the mid of the 19^{th} century as part of the Russian Turkestan (a much vaster region than today's Uzbekistan); in 1924 the Uzbek Soviet Socialist Republic was created and the oasis became one of the Republic's regional administrative subdivisions; the oasis kept that same collocation in today's Uzbekistan. The centroids moved accordingly: the 19^{th} century aggregation in the Russian Turkestan did not move the centroid towards North-East. This may be due to the fact that the agglomeration mechanisms of the industrial revolution were still very week or non existent. Only after the industrialization (albeit modest) and the integration of the oasis in a country whose economic center is situated North-East (toward the capital city of Tashkent) the centroid moved North-East. It is suggestive to hypothesize that infrastructures that linked the oasis to the rest of the country and the agglomeration mechanisms of industrialization have caused the 20^{th} century shift. The North-East economic shift (in population, sites, and centroid) is reminiscent of similar shifts documented by Redding and Sturm (2008) for Germany and by Brülhart et al. (2012) for Austria. Here we observe such shifts occurring over a leap of eleven centuries from the 9^{th} to the beginning of the 21^{st}.

[Figure 12 about here]

9 Conclusion

In this paper, we use a unique archaeological data set that provides the size and location of the universe of settlements in the oasis of Bukhara for the 9^{th} century. This data set is ideal for the study of the city size distribution because it pertains to an extremely homogeneous and isolated area where cities have developed without any sizable perturbation. We find that spatial centrality explains extremely well the city size distribution. This is an absolute novelty in the literature. Previous empirical and theoretical studies have not considered spatial centrality as a possible explanation for the city size distribution. We find little evidence of persistence over centuries of the location of sites. Interestingly we find that the centroid of the region (both in population and site location) moved North-East in modern times relative to its position before the 20$^{\text{th}}$ century. Such shift is possibly explained by the industrialization and by the integration of the region into Uzbekistan.

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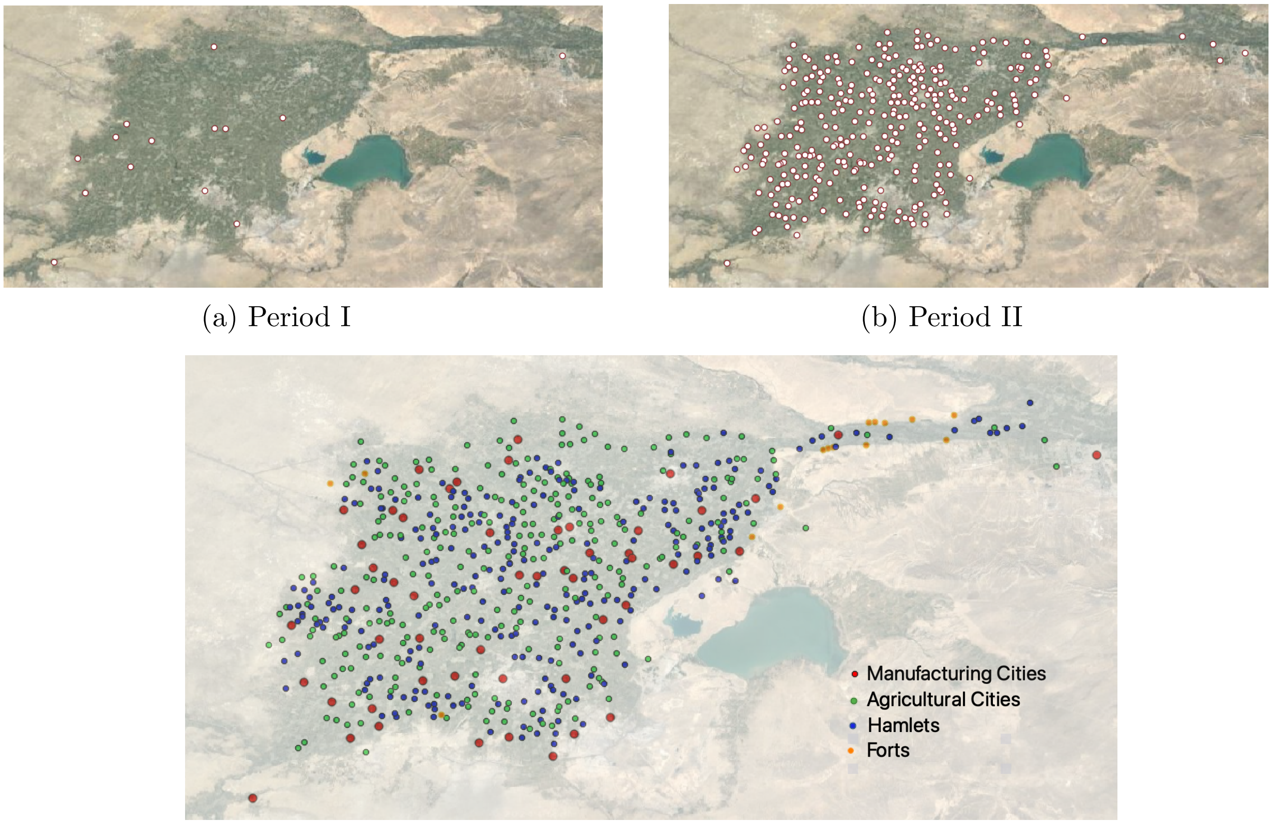
Figure 1: The Oasis of Bukhara



(a) Period I

(b) Period II

(c) Period III

Notes. Panel (a) plots the 14 earliest sites which came up between the 3rd century BCE and 1st century CE. Panel (b) plots the sites which came up between the 1st and the 4th century CE. Panel (c) shows the 618 points that came up between the 4th and 9th century CE.

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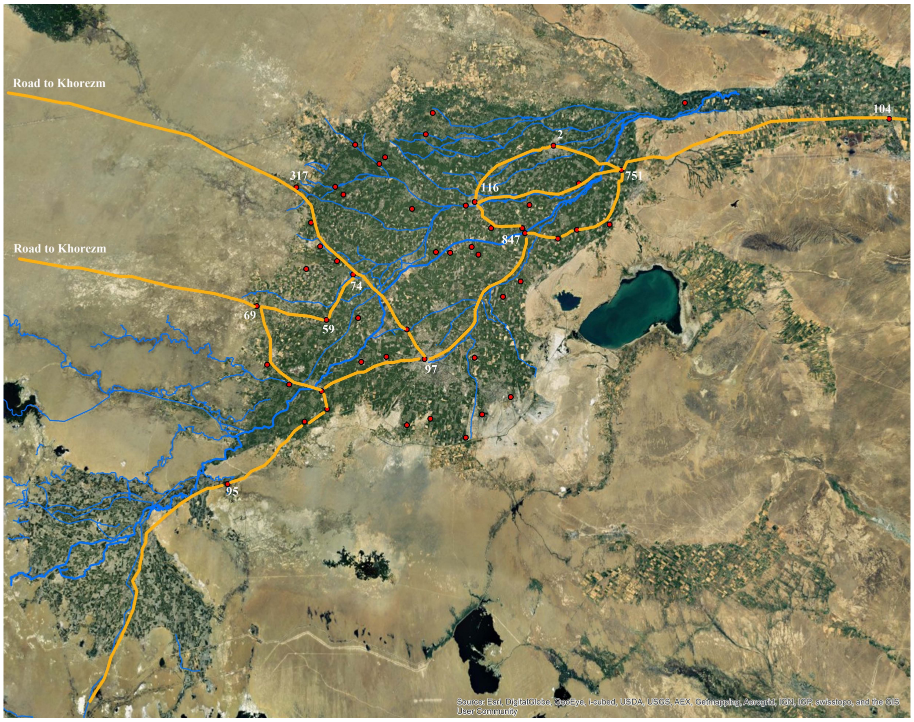
Figure 2: Silk Roads and Stopover cities



Notes. The orange lines mark the routes of the silk roads. Numbers indicate city codes of the stopover cities. Blue lines indicate the waterways of the Zerafshan River and its ramifications.

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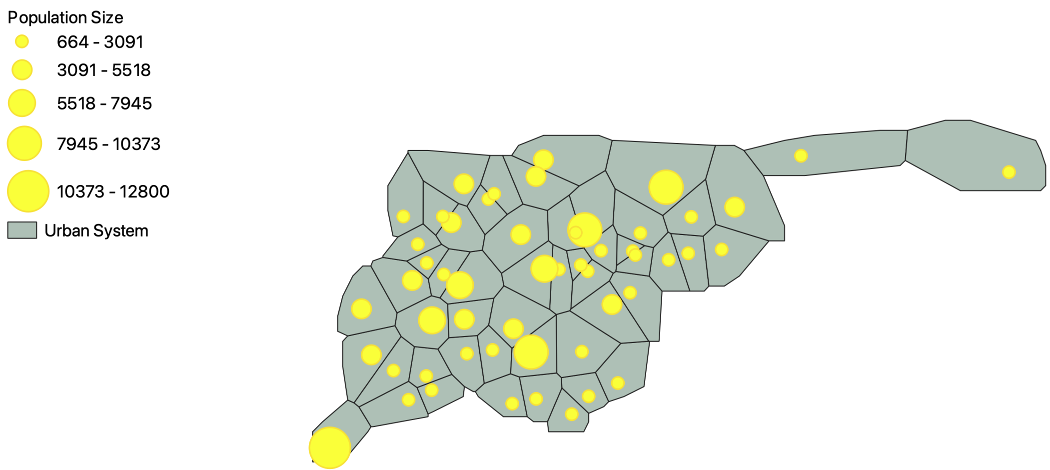
Figure 3: Urban Systems



Notes. The 618 points are grouped into 53 urban systems indicated by a grey regions and constructed using Voronoi polygons. The size of the yellow dots indicate the size of the urban system which includes manufacturing and non-manufacturing sites.

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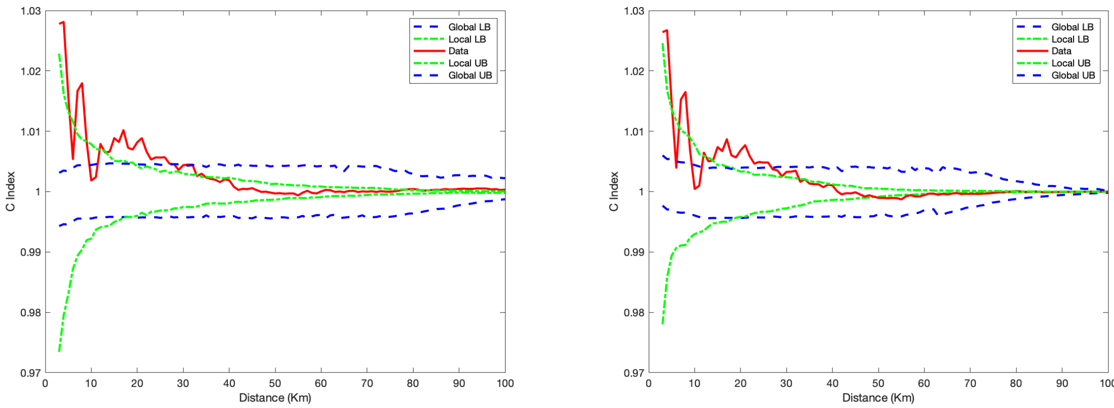
Figure 4: Co-concentration Index



(a) Archaeological Oasis

(b) Restricted Oasis

Notes. The index is calculated based on equation (1). The confidence intervals are created by generating 1000 homogeneous Poisson point processes with intensity equal to 618 for the archaeological oasis and to 611 for the restricted oasis. The local confidence intervals are chosen after dropping of the observations 5% above and below for each distance. Global confidence intervals are created keeping confidence intervals for each level of distance such that we reject only 5% of the total population.

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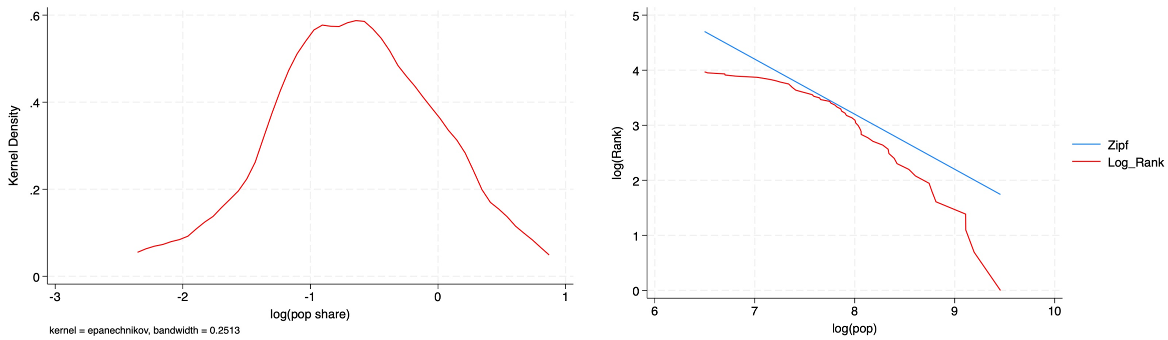
Figure 5: Distribution of Urban Systems in 9th Century



(a) Kernel Density Plot

(b) Rank Size Relation

Notes. Panel (a) plots the K-density of log of population shares of the 53 urban systems. Panel (b) plots the log rank and log size relationship along with a linear slope of -1 (blue line).

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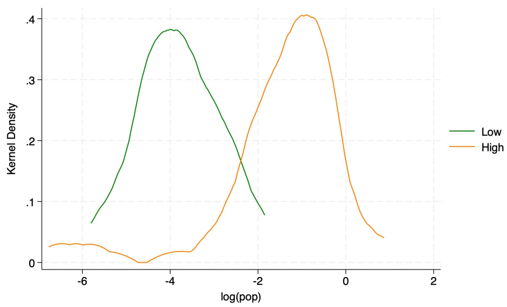
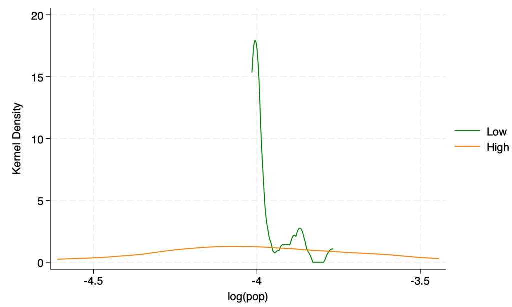
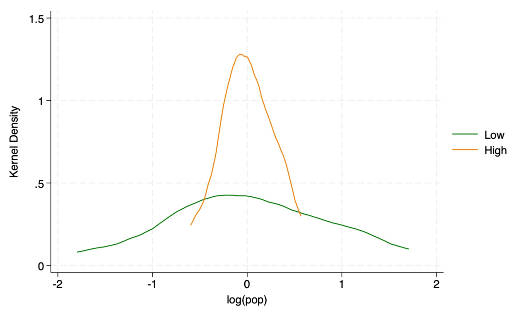
Figure 6: Size Distribution Sensitivity to Parameter Values



(a) μ_C



(b) μ_Z



(c) μ_L

Notes. The figure shows the model predicted distribution for log of population for low and high μ_c (panel a), μ_Z (panel b), and μ_L (panel b). In panel a, the values of μ_c are set as 2 for high and 0.1 for low while μ_Z and μ_L set to 3 and 1. In panel b, the values of μ_z are set as 3 for high and 1 for low while μ_c and μ_L set to 0 and 3. In panel c, the values of μ_l are set as 3 for high and 1 for low while μ_c and μ_L set to 0 and 3.

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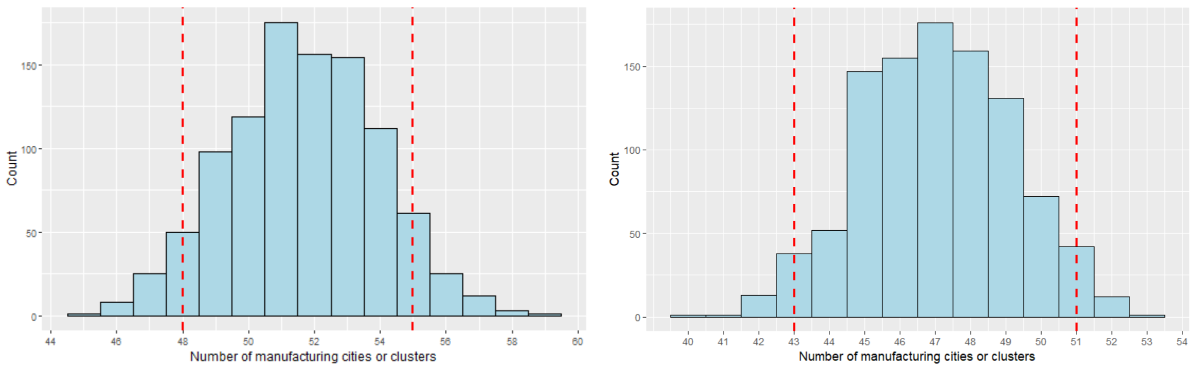
Figure 7: Simulated number of manufacturing cities



(a) Archaeological Oasis (b) Restricted Oasis

Notes. Panel (a) show the distribution of simulated number urban clusters that are formed by randomly dropping 618 points in the oasis. The first cluster in the simulated data is defined such that it has the maximum number of points within the 7 kilometersx radius. We count it as one cluster, and then move to the next set of points using the same criteria. This we repeat till we have exhausted all the points. Every simulation gives us a different number of clusters, the frequency of which is then reported in the figure above. Panel (b) does the same analysis as in Panel a but in restricted oasis which excludes the border sites on the north east and the south west frontier.

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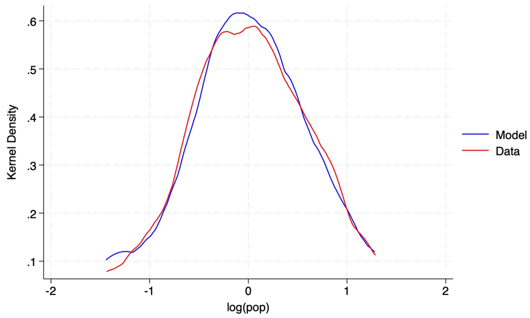
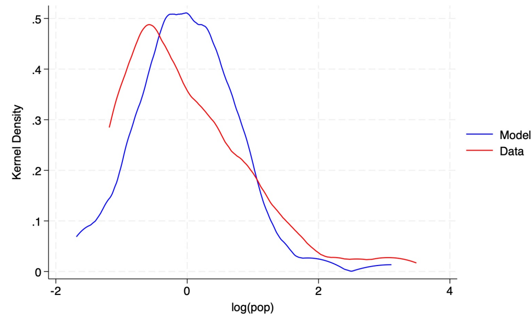
Figure 8: Distribution of Predicted versus Actual Shares



(a) $9^{th} Century$



(b) $21^{st} Century$

Notes. The red line indicates the distribution of population shares (in logs) across 53 urban systems as observed in the data. The blue line is the same distribution of shares (in logs) as predicted by the baseline model. Panel a is for the 9^{th} century while panel b is for the 21^{st} century.

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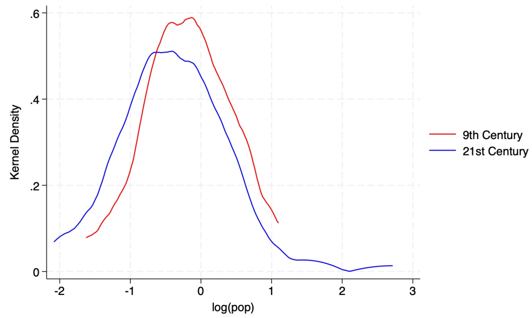
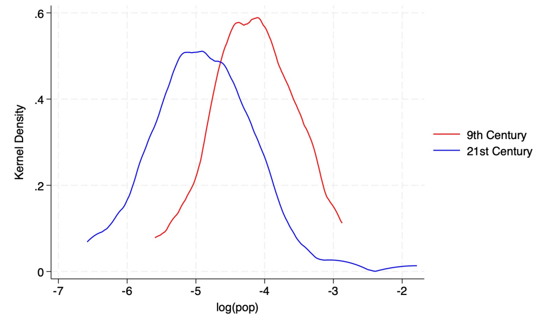
Figure 9: Empirical City Size Distributions



(a) Demeaned



(b) Share

Notes.
The red line indicates the log distribution of population size across 53 urban systems as observed in the 9th century cities. The blue line is the same distribution for the 21st century cities. Panel a demeans the population size by the average city size during that period. Panel b is the distribution of population shares in each period.

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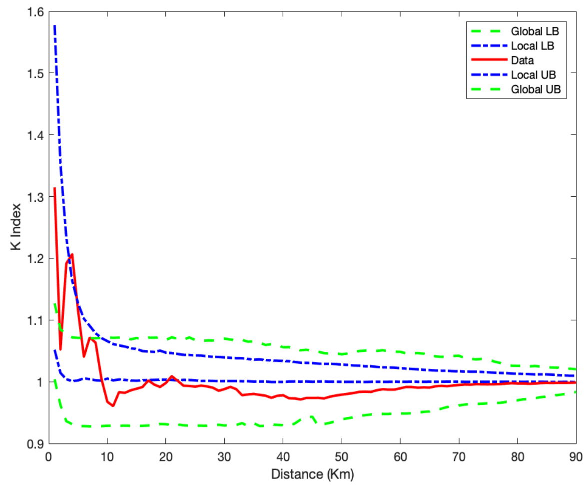
Figure 10: 21^{st} century cities' co-concentration around 9^{th} century cities



Notes. The index is calculated based on equation (1) but in this case it measures the co-concentration of manufacturing cities of the 9^{th} and the 90 large cities of the 21^{st} century. The confidence intervals are created by generating 1000 homogeneous Poisson point processes with intensity equal to 53 for the historical cities and 90 for the modern cities. The local confidence intervals are chosen after dropping of the observations 5% above and below for each distance. Global confidence intervals are created keeping confidence intervals for each level of distance such that we reject only 5% of the total simulated samples.

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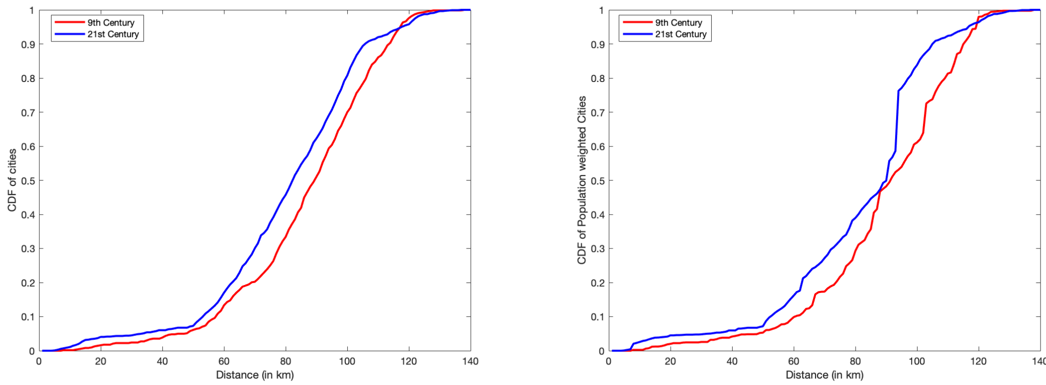
Figure 11: Cumulative shares of cities near to the North-East frontier



(a) Unweighted (b) Weighted

Notes.
The red line is the share of total cities in the 9th century that are within x kilometers of distance to the North East frontier of the oasis of Bukhara. The coordinate of the North-East frontier is 40.17^o N, 65.42^o E. The blue line are the same shares for the 21st century cities. Panel b takes into account the population size of the cities in computing the shares while panel a uses a simple count of cities.

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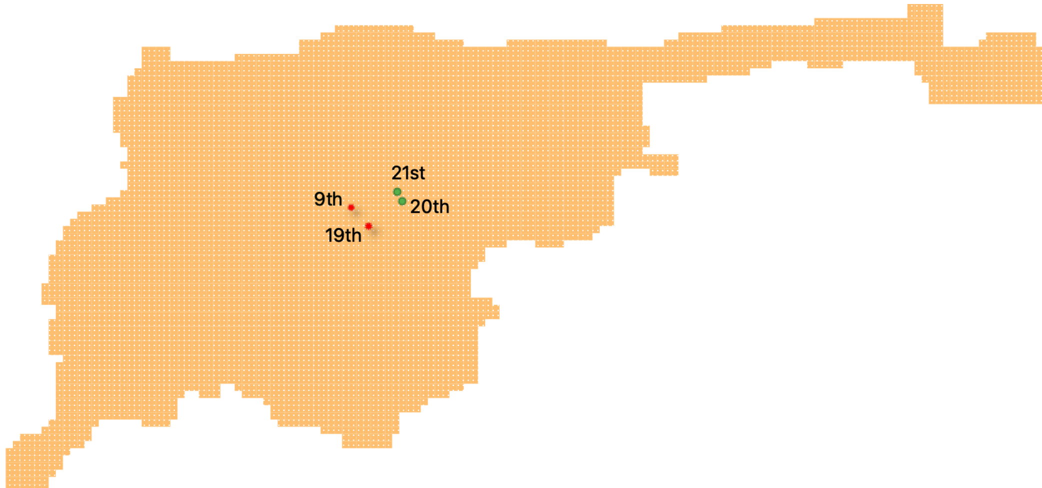
Figure 12: Centroids of the oasis since the 9^{th} century



Notes. The red dots correspond to the centroid of the cities in the 9th and the 1893. This is pre-industrial revolution times. The points in green correspond to centroid of cities post-industrial revolution in 1970 and 2018. The centroid of city locations moved northeast by 7.19 kilometers between the 9^{th} and the 21^{st} century. In the intermediate period, the centroid of 19^{th} century points was south-east of the 9^{th} cities by 3.8 kilometers. This is pre-industrialization period. Post that, the cities moved north-east by 6.8 kilometers on average. This North-East shift shows that cities started moving closer to the industrial and political centers (Samarkand and Tashkent) after the industrialization. Between the 20^{th} and the 21^{st} century, the shift was only around 1.6 kilometers.

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Table 1: Summary statistics

Urban Systems

Size	Obs.	Mean	Std. Dev.	Min	Max
Residential Area (ha)	53	19.29	13.77	3.42	64
	51	18.59	12.46	3.42	56.7
Total Area (ha)	53	21.88	15.14	4.52	70
	51	21.10	13.78	4.52	60
Population	53	3249	2513	663	12800
	51	3090	2163	663	9840

Population is rounded up to the nearest integer.

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Table 2: Population

	(1) Pop. Share	(2) Pop. Share	(3) Pop. Share	(4) Pop. Share
Centrality	0.899***	0.754***	0.727***	0.666***
	(0.000)	(0.000)	(0.000)	(0.000)
Period I		0.0111*		0.00705
		(0.013)		(0.111)
Stopover			0.0155**	0.0126*
			(0.002)	(0.013)
N	53	53	53	53
R^2	0.623	0.667	0.690	0.706

p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes. The dependent variable is the population share of urban systems. Period I is a dummy for urban systems which contain a city that existed already in Period I. Stopover is the dummy for urban systems which contain a stopover city on the silk road.

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Table 3: Residential Area

	(1) R.A. Shares	(2) R.A. Shares	(3) R.A. Shares	(4) R.A. Shares
Centrality	0.899***	0.774***	0.763***	0.707***
	(0.000)	(0.000)	(0.000)	(0.000)
Period I		0.00955*		0.00650
		(0.024)		(0.135)
Stopover			0.0122**	0.00961
			(0.010)	(0.052)
N	53	53	53	53
R^2	0.647	0.680	0.690	0.704

p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes. R.A. stands for Residential Area. The dependent variable is the R.A. share of an urban system in the oasis of Bukhara. Period I is a dummy for urban systems which contain a city that existed already in Period I. Stopover is the dummy for urban systems which contain a stopover city on the silk road.

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Table 4: Total Area

	(1)	(2)	(3)	(4)
	T.A. share	T.A. share	T.A. share	T.A. share
Centrality	0.905***	0.759***	0.732***	0.670***
	(0.000)	(0.000)	(0.000)	(0.000)
Period I		0.0112**		0.00717
		(0.005)		(0.068)
Stopover			0.0156***	0.0127**
			(0.000)	(0.005)
N	53	53	53	53
R^2	0.668	0.715	0.740	0.757

p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes. T.A. stands for Total Area. The dependent variable is the share of the T.A. of an urban system. Period I is a dummy for urban systems which contain a city that existed already in Period I. Stopover is the dummy for urban systems which contain a stopover city on the silk road.

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Table 5: Moment matching (9$^{\text{th}}$ century data): Population

Archaeological Oasis	Targeted Moments		
	variance	range	skewness
Data	0.40	2.72	-0.15
Eq. 14 as is	0.40	2.72	-0.15
Eq. 14 w/o Silk Road Effect ($s_i = 1\ \forall i$)	0.47	2.69	-0.15
Restricted Oasis			
Data	0.43	2.69	0.04
Eq. 14 as is	0.43	2.69	0.04
Eq. 14 w/o Silk Road Effect ($s_i = 1\ \forall i$)	0.39	2.70	0.04

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Table 6: Moment matching (9$^{\text{th}}$ century data): Total Area

Archaeological Oasis			
	Targeted Moments		
	variance	range	skewness
Data	0.44	2.74	-0.007
Eq. 14 as is	0.44	2.74	-0.007
Eq. 14 w/o Silk Road Effect ($s_i = 1\ \forall i$)	0.47	2.73	-0.005
Restricted Oasis			
Data	0.42	2.58	-0.09
Eq. 14 as is	0.37	2.61	-0.11
Eq. 14 w/o Silk Road Effect ($s_i = 1\ \forall i$)	0.37	2.60	-0.10

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Table 7: Moment matching (9th century data): Residential Area

Archaeological Oasis	Targeted Moments		
	variance	range	skewness
Data	0.48	2.92	-0.04
Eq. 14 as is	0.48	2.92	-0.04
Eq. 14 w/o Silk Road Effect ($s_i = 1\ \forall i$)	0.53	2.90	-0.03
Restricted Oasis			
Data	0.45	2.80	-0.14
Eq. 14 as is	0.45	2.80	-0.14
Eq. 14 w/o Silk Road Effect ($s_i = 1\ \forall i$)	0.44	2.81	-0.14

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Table 8: Moment matching (21$^{\text{st}}$ century data)

	Targeted Moments			
	variance	range	skewness	kurtosis
Data	0.62	4.79	0.57	4.85
Eq. (14) as is	0.92	4.67	1.26	4.71

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Table 9: Mechanisms' Relative Strength

	Market size (μ_C)	Location (μ_L)	Travel (μ_Z)	Model Fit (γ)
9$^{\text{th}}$ Century				
Population	0.18	1.74	3.30	1.10
Residential Area	0.15	1.47	3.11	1.11
Total Area	0.08	1.40	3.02	1.04
21$^{\text{st}}$ Century	2.78	0.93	4.42	-2.19

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