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On the (de)stabilization role of protectionism: Theory and evidence

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On the (de)stabilization role of protectionism: Theory and evidence*

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Abstract

To what extent protectionism affects growth and (de)stabilizes the economies? Since 2018, some countries have resorted to protectionist measures as the United States. Although the impacts of protectionism on growth have been widely explored without reaching a consensus, few has been said on its impacts on macroeconomic stability. The present paper attempts to gauge more precisely its implications using a Barro-type (1990) endogenous growth model with public debt and credit constraint where tariffs are a proxy of protectionism. Our main result is to show that when the debt level is high, and the share of foreign goods in total consumption is large enough, increasing tariffs may have a dramatic destabilizing effect generating some expectation coordination failure between multiple equilibria and the possible existence of large self-fulfilling fluctuations. We also exhibit some trade-off between tariffs and growth as tariffs are beneficial only to the low growth equilibrium which may only appear in the globally indeterminate case. We also propose some numerical illustrations confirming the destabilizing impact of tariffs in the case of the US economy. We finally propose an Event Study analysis to confront our results. While our effects appear short lasting, two quarters, we show that the implementation of protectionism destabilizes the US economy in the short run.

Keywords: *Public debt, tariffs, small open economy, credit constraint, global and local indeterminacy*

JEL classification: C62, E32, F13, F34, F43, O41

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1 Introduction

The aim of this paper is to contribute to the understanding of the consequences of protectionism on macroeconomic stability and on economic growth. Indeed, starting in 2018, protectionist measures have been introduced by some countries, especially the United States. Altogether, the various sets of the tariffs increase represent 13% of their imported goods. At the same time, following the impact of the financial crisis of 2008 and the Covid crisis of 2020, the debt level of most OECD countries has dramatically increased, raising the issue of sustainability and tax reforms to find resources to decrease the structural deficits.

We follow Furceri *et al.* (2019), proxying protectionism with tariffs (i.e. tax on imported goods). In the US, on top of starting a trade war with China, the objective of Trump was clearly to promote consumption of domestic goods and thus to promote a relocalization of productive activities in the country. The expected effect was then to boost growth. In the literature, the impact of protectionism has not clearly been identified yet. As discussed in the literature review provided in the next section, the implications of protectionism are still uncleared. On one hand, its impacts on economic growth has been widely explored, without reaching a consensus. On the other hand, its implications on macroeconomic stability has been mainly disregarded.

The contribution of the paper is to focus on three types of questions. First, we want to give clues on the possible (de)stabilizing role of protectionism, especially when economies are indebted. Second, we aim to clarify the impact of tariffs on economic growth. Finally, we want to the gauge the link between tariffs and public debt and check whether there is an interplay between tariffs and public debt.

We consider a small open economy where endogenous growth is driven by public spending as in Barro (1990).[1] We use the same basic formulation as in Modesto *et al.* (2021) where public debt is financed through taxes and external borrowing. A collateral constraint *à la* Fahri and Tirole (2012) allows usto consider that borrowing on the international markets is limited, the loans provided by the rest of the world being proportional to the capital invested in the home country. Contrary to Modesto *et al.* (2021), we do not consider that the representative agent derives utility from holding domestic bonds. However, we assume that he consumes a basket of goods, composed by domestic and foreign goods. The novelty of our approach is to add an international trade dimension through tax on imported goods.

We will consider two cases depending on the level of public debt: low and high public debt.

[1]See also Morimoto *et al.* (2017).

Contrary to the standard framework of a closed economy[2], we show that when public debt is low, global indeterminacy based on the existence of multiple stationary equilibria (actually two balanced growth rates) cannot be ruled out[3]. Indeed, under a low debt burden, both, a low and a high balanced growth rate can be sustained. However, there is no significative impact of tariffs.[4]

When public debt is high instead, we provide the main conclusion of our paper. We show that there exists a unique stationary equilibrium if and only the tax on imports and/or the share of the foreign good into total utility are low enough. As soon as tariffs and the share are large enough, two balanced growth rates coexist leading to global indeterminacy, the existence of expectations coordination failures and then the possible existence of large self-fulfilling fluctuations. In this case, because the debt burden is high, the low growth equilibrium cannot be sustained anymore if there is no enough additional revenues from tariffs (for the government). But, this is no longer true when the representative household consumes a large share of the foreign good which is significantly taxed. In such a case, the government revenues can sustain the debt reimbursement even if the growth rate remains low. We then show that under a large debt level, if the share of foreign good into consumption is large enough, increasing tariffs may have a dramatic destabilizing effect generating some expectation coordination failure between multiple equilibria and the possible existence of large self-fulfilling fluctuations.

We also provide some comparative statics exercise focusing on the impact of tariffs on the stationary equilibria. We emphasize that the high balanced growth rate is always negatively affected by tariffs while the low balanced growth rate, when it exists, is always positively affected. This difference appears to be explained by the origin of growth. At the high steady state, growth is driven by productive spending from the government which is relatively large compared to capital. At the low steady state, growth is driven by private capital and thus private investment instead. Any increase of tariffs has two opposite effects: first households can dedicate less revenues to productive investment, and second, the government experiences additional resources that are used to increase public spending. Along the high growth equilibrium, the small increase of tariffs relative to the size of government spending has a limited impact on the government spending capacities that weakly increase while the tariffs strongly impacts the households' income. The first effect is then dominant and growth declines. Along the low growth equilibrium on the contrary, the first effect is dominated by the second one since the increase of tariffs generates a relatively large

[2]as studied by Chéron *et al.* (2019), Futagami *et al.* (2008), Maebayashi *et al.* (2017) or Minea and Villieu (2013).

[3]The same result is found in Modesto *et al.* (2021).

[4]Except that too large tariffs may prevent the existence of any equilibria. As they act as an externality, such a result is not surprising.

increase of government spending that compensates the negative impact on households income. Our results therefore suggest the existence of a trade-off between tariffs and growth. Indeed, tariffs may enhance growth for the low equilibrium while they have a negative impact on the high equilibrium.

Focusing on the local stability property, we show that the high BGP is always characterized by local indeterminacy, while the low BGP, when it exists, is always a saddle-point. In the case of a high BGP, assume that along an equilibrium path, the agents expect an increase of the growth rate. Due to the access to international market, they may borrow, consume and invest more (preventing a crowding out effect) and they will expect a higher public spending. Then, for given tariffs, the government's revenue can increase significantly leading to an increase of public spending that generates a higher growth rate. The expectations are therefore self-fulfilling.

When it exists, the low BGP has quite different properties. The economy is now characterized by a low growth rate and thus by a low public spending to capital ratio. The credit constraint is then more tightened since the collateral needed to borrow is relatively low. The inflows of capital thus remain limited. Assume again that along an equilibrium, agents are expecting an increase of the growth rate. Being more constrained, the agents consume less and decrease their investment of productive capital. Unable to rely on tariffs in this case, since growth, consumption and tariffs' income remain low, the government does not invest enough and growth cannot increase. The expectations cannot be self-fulfilling and the equilibrium remains locally determinate.

Building on these theoretical results, we then provide some numerical illustrations and empirical evidence supporting our main conclusions. First, considering a set of six countries characterized by large levels of debt, France, Greece, Japan, Spain, United Kingdom and United States, we notice that between 2018 and 2019, the US are the only country to have significantly increased its tariffs. While all the other countries are characterized by the existence of two BGPs, we show that the US moved from one BGP in 2019 to two BGPs in 2019 after the tariffs increase. We therefore conclude that tariffs may have had a destabilizing effect on the US economy.

In a second step, we aim to find empirical evidence for this conclusion, exploiting the introduction of protectionist measures by the US in 2018. Using an Event Study approach, we document the implications of this trade shock on US volatility of growth. To precisely capture the effects of the shock on the volatility, we propose various specifications based on different definitions of macroeconomic stability. The effects appear short lasting, two quarters, suggesting thus that the implementation of protectionism destabilizes the US economy in the short run.

The rest of this paper proceeds as follows: Section 2 propose a literature review. Section 3 presents the model. In Section 4, We study the existence and possible multiplicity of BGPs, while the Section 5 is dedicated to some comparative statics and policy implications. Section 6 analyses the local stability and Section 7 provides a numerical illustration. In Section 8 we employ an empirical strategy based on an Event Study approach and the Section 9 concludes. All technical detail are relegated to the Appendix.

2 Related literature

Looking at the link between international trade, macroeconomic stability and economic growth, many papers in the literature focus on the role of trade openness. In particular, Nishimura *et al.* (2010) consider a two-country, two-good, two-factor general equilibrium model with sector-specific externalities and show that some country's expectation-driven fluctuations can spread throughout the world once trade opens even if the other country has determinacy under autarky.[5] Globalization and market integration then appear to have destabilizing effects on a country's competitive equilibrium.[6] On the contrary, Doi *et al.* (2007) formulate a two-country endogenous growth model, which explain joint determination of long-run trade patterns and world growth rates and prove the existence and local stability of a continuum of balanced growth paths. The destabilizing effect does not hold here but the continuum of equilibria generates some indeterminacy. Some similar conclusions are found around the Compensation hypothesis (Iversen (2001), Down (2007), Kim (2007), Ehrlich and Hern (2014)). According to this view, higher exposure to trade leads to less domestic macroeconomic stability as soon as trade partners share risks. The latter induces a higher demand for compensation through more transfers. Open countries expand security programs that inflate public expenditures, making countries even more vulnerable to shocks. In the same vain, Krugman (1993) shows that it may be explained by more specialization (i.e. geographical concentration of an industry). Following countries tend to specialize more their production, being then more subject to regional shock. On the empirical side, some evidence suggest that the link between trade openness and macroeconomic stability (i.e. growth volatility) is not straightforward where other factors play a role. Bejan (2004) emphasizes the role of the government size in an Econometric analysis based on 111 countries. Developed and developing countries exhibit different patterns. Trade openness allows to smooth volatility for developed countries whereas developing counties experience more volatility. Jansen (2004), Cavallo and Frankel (2008) shed light on other factors, as export concentration and product diversification for example.

[5]For some references on expectation-driven fluctuations, see Le Van *et al.* ((2007).

[6]See also Ghiglino (2007). Le Riche *et al.* (2022) derive similar results in a one-sector model of differentiated products with productive labor externalities, considering two OLG countries, one with wage rigidity and the other with full employment.

Concerning the impact of trade on growth, again many papers in the literature considers the role of trade openness. Ho (2017) examines the effect of externalities on the consequences of financial market globalization in a two-country growth model augmented with domestic credit market imperfections. He finds that depending on the externalities formation financial market globalization can improve growth at the world level, or in the rich country only, and may in some cases imply that both the rich and the poor countries become locked in a stage with no meaningful growth. For the impact of tariffs on the growth rate the results are most of the time not conclusive. Osang and Pereira (1996) consider a small open economy where growth is endogenously driven by human capital accumulation. They examine the effects of an unanticipated increase in one of the tariff rates under different replacement regimes: a lump-sum transfer (LST) or an investment tax credit (ITC). An increase in the tariff of the consumption good is shown not to affect growth in the LST scenario while it positively affects growth under an ITC as the accumulation of capital is accelerated.[7] Naito (2003) examines how a revenue-neutral tariff reform affects growth in an endogenous growth small open economy model with two final goods. In contrast to the previous paper, he argues that tariff-reforms have ambiguous effects on growth, which depend on the pattern of trade and the elasticities of substitution between the inputs and consumption of final goods. Closer to our framework, Osang and Turnovsky (2000) also analyze the effects of consumption and investment tariffs on growth. However, conversely to the previous work, they develop an endogenous growth model in which the economy faces restricted access to the world capital market. A higher consumption tariff, by reducing the growth rate of consumption, is shown to have a negative impact on the long-run growth rate. On the empirical side, Furceri *et al.* (2020) use a local projection method on a data set composed by 150 countries over the period 1963-2014 and emphasize the detrimental effect of protectionism on economic growth. Following the implementation of protectionist measures, economy experiences a rise in unemployment and inequalities, together with a significant decrease in labor productivity. All in all, long run growth declines.[8]

3 Theoretical framework

Our framework builds on Barro (1990) model where production benefits from externalities due to public spending. We consider a decentralized, continuous time intertemporal model of a small

[7]See also Chaudhry (2011) where the innovation degree of the export sector and the quality of institutions are key.

[8]see also Bairoch (1972) and Eichengreen (1981) where protectionism affects negatively economic growth but only on the short run.

open economy composed by three agents: a large number of identical competitive firms, a constant population of identical infinitely lived households and a government. Firms and households operate in competitive markets, they are price takers. The government levies import tariffs on all imported goods, taxes on the global output of the country and issues public debt. Tariffs, tax revenues and debt are used to produce a public good affecting the aggregated production function to maintain its budget balanced at each period of time. The country imports a consumption good. We assume that the country can borrow on the international market subject to a borrowing constraint based on its domestic capital as a collateral. Since we consider small open economy, the price of imported good is taken as given. All the prices are expressed in units of the domestically produced good, the numeraire good.

3.1 Production

We consider a perfectly competitive economy where the final output y is produced using capital k and labor supplied in one unit. As in Barro (1990), the production benefits from an externality due to public spending G, and is given by $y = k^s G^{1-s}$. Public spending is thus the driver of endogenous growth.[9] The rental rate of capital $r(t)$ and the wage rate $w(t)$ satisfy:

$$r(t) = sx(t)^{1-s} \tag{1}$$

$$w(t) = (1-s)x(t)^{1-s}k(t), \tag{2}$$

where $x \equiv G/k$.

3.2 Households

The size of population is normalized to one. The infinitely-lived households derive utility from consumption, c_t. Each consumer is initially endowed with one unit of labor and an initial stock of private physical capital which depreciates at a constant rate $\delta \in [0,1]$. Agents supply inelastically one unit of labor. Households can save through capital $k(t)$, buy/sell the international asset $d(t)$ from/to foreign, and hold domestic public debt. In our set up, contrary to Modesto *et al.* (2021), domestic residents do not gain utility from holding domestic debt, $B_h(t)$. The two financial assets are freely traded on international markets, whereas capital used in production is not mobile.

[9]See also Boucekkine and Ruiz-Tamarit (2008) and Brito and Venditti (2010) for Lucas-type endogenous growth models based on human capital.

The intertemporal maximization program of a representative agent is given by:

$$\max_{c(t),B_h(t),k(t),d(t)} \int_{t=0}^{+\infty} e^{-\rho t} \ln c(t) dt$$

$$\begin{aligned} \text{s.t. } P_c(t)c(t) + \dot{k}(t) + \dot{d}(t) + \dot{B_h}(t) &= (1-\tau)(r(t)k(t) + w(t)) + rd(t) + rB_h(t) && (3) \\ rd(t) &\geq -\theta(1-\tau)r(t)k(t) && (4) \\ B_h(t) &\geq 0. && (5) \end{aligned}$$

where $\rho > 0$ corresponds to the discount rate. We normalize the price of Home good to unity and we denote P^* the price of imported good.[10] Considering the per capita net foreign asset (NFA) expressed in terms of foreign goods, we derive that $d = P^*\tilde{d}$ representing the NFA expressed in domestic good. τ is the tax rate on income which is assumed to be constant and such that $\tau \in [0, 1)$. The international interest rate is denoted by r, expressed in domestic goods and we assume that it is constant and strictly positive. We follow the same type of formulation as Farhi and Tirole (2012) for the borrowing constraint, where $\theta \in [0, 1]$ is a parameter that captures credit market imperfection.[11]

In our framework, the portfolio decisions of the households are based of three assets: domestic public debt, physical capital and foreign assets. These assets are imperfect substitutes, letting the residents not indifferent between holding international asset and domestic public debt.[12] While, foreigners are indifferent between both assets since they offer the same return, r.

Besides this portfolio choice, households consume to gain utility. They consume a single consumption good that is a composite of the domestic and the foreign consumption goods, denoted $c_h(t)$ and $c_f(t)$ respectively. As Osang and Pereira (1996), the consumption good $c(t)$ is expressed as follow:

$$c(t) = c_f(t)^\alpha c_h(t)^{1-\alpha}. \tag{6}$$

with $\alpha \in [0, 1]$, the share of imported foreign goods in total consumption. The consumption bundle is a combination of foreign and domestic goods, where the two goods are imperfect substitutes. In our model, we introduce tariffs τ_c that are imposed on the foreign composite. We express the total consumption spending of the household as:

$$P_c(t)c(t) = c_h(t) + (1 + \tau_c)P^*c_f(t), \tag{7}$$

[10] The price of the imported good is exogenously determined and supposed to be constant

[11] See also Boucekkine *et al.* (2015) and (2017) for additional references with similar formulations.

[12] since borrowing on the international markets requires a collateral: the capital

Maximizing (6) subject to (7), leads to

$$c_h(t) = (1-\alpha)P_c(t)c(t) \text{ and } c_f(t) = \frac{\alpha P_c(t)c(t)}{(1+\tau_c)P^*}. \tag{8}$$

which implies a constant price P_c such that

$$P_c(t) = \left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}(1+\tau_c)^{\alpha}P^{*\alpha}. \tag{9}$$

Our model is build on a unique dynamical equation corresponding to the budget constraint of the representative household. Therefore, it is more convenient to solve the model to use the standard method of calculus of variations based on the consideration of the Euler equation.

Let us then introduce the following Lagrangian:

$$\begin{aligned}\mathcal{L} = \quad & e^{-\rho t}\left\{\frac{\ln\left((1-\tau)(r(t)k(t)+w(t))+rd(t)+rB_h(t)-\dot{k}(t)-\dot{d}(t)-\dot{B_h}(t)\right)}{P_c(t)}\right\}\\ & +\lambda(t)[rd(t)+\theta(1-\tau)r(t)k(t)]+\mu(t)B_h(t).\end{aligned}$$

$\lambda(t)$ corresponds to the Lagrange multiplier associated to the borrowing constraint while $\mu(t)$ is the Lagrange multiplier of the domestic debt.

The first order conditions are derived from the Euler equation $\frac{\partial \mathcal{L}}{\partial \omega} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\omega}}$, with $\omega = \{k, d, B_h\}$:

$$\frac{e^{-\rho t}(1-\tau)r(t)}{P_c c(t)} + \lambda(t)(1-\tau)\theta r(t) = \frac{e^{-\rho t}}{P_c c(t)}\left(\rho + \frac{\dot{c}(t)}{c(t)}\right) \tag{10}$$

$$\left(\frac{e^{-\rho t}}{P_c c(t)} + \lambda(t)\right) r = \frac{e^{-\rho t}}{P_c c(t)}\left(\rho + \frac{\dot{c}(t)}{c(t)}\right) \tag{11}$$

$$e^{-\rho t}\left(\frac{r}{P_c c(t)}\right) + \mu(t) = \frac{e^{-\rho t}}{P_c c(t)}\left(\rho + \frac{\dot{c}(t)}{c(t)}\right) \tag{12}$$

Any solution needs also to satisfy the transversality conditions:

$$\lim_{t\to+\infty} e^{-\rho t}\omega(t)/c(t) = 0, \qquad \text{with } \omega(t) = \{k(t), d(t), B_h(t)\} \tag{13}$$

Using (10) and (11), the Lagrange multiplier associated to the borrowing constraint is given by:

$$\lambda(t) = \frac{e^{-\rho t}}{c(t)}\left(\frac{(1-\tau)r(t)-r}{r-\theta(1-\tau)r(t)}\right), \tag{14}$$

and we easily get a condition that ensures $\lambda(t) > 0$:

$$(1-\tau)r(t) > r > \theta(1-\tau)r(t). \tag{15}$$

Glancing out the second Lagrange multiplier $\mu(t)$, we easily point out that it is strictly positive. Using indeed Kuhn Tucker conditions, λ(t), (11) and (12), we find that $\lambda(t)r = \mu(t)$, but from (15) and the return of international asset, we obtain that μ is strictly positive. Hence, we prove from Kuhn Tucker conditions that domestic agents do not hold any domestic bonds (i.e. $B_h(t) = 0$), and thus all public debt is held only by foreigners, i.e. $B(t) = B_f(t)$. This result is explained by the fact that, since the borrowing constraint is binding, $(1-\tau)r_k(t) > r$, the domestic public asset is strictly dominated by capital.

We focus on configurations where the credit constraint expressed in (4) is binding (i.e. condition (15) is satisfied). Substituting (14) in (10), we obtain the consumption growth rate:

$$\tfrac{\dot{c}(t)}{c(t)} = \tfrac{(1-\tau)r(t)(1-\theta)}{1-\theta(1-\tau)r_k(t)/r} - \rho. \tag{16}$$

In (16), we can remark that the assets are not perfect substitutes in this case. The growth rate of consumption is not constant, whereas if the assets would have been perfect substitutes, the consumption growth rate will be constant and equal to $r - \rho$.

As mentioned previously, every borrowing on the international market is subject to a collateral. Using (16), we express the expected return of capital as:

$$\frac{(1-\tau)r(t)(1-\theta)}{1-\theta(1-\tau)r(t)/r}. \tag{17}$$

The marginal benefit of investing one unit of capital is equal to $(1-\tau)r(t)(1-\theta)$ while the expected cost of investing one unit of capital is given by $1-\theta(1-\tau)r(t)/r$. One key feature of our model is the imperfect substituability between two assets. It is noteworthy that capital used in production has different return than the international asset.

3.3 Government

The government levies tax on production and on imported goods and issues debt to finance public spending G. The government budget constraint is then given by:

$$\dot{B}(t) = G(t) + rB(t) - \tau(w(t) + r(t)k(t)) - \tau_c P^* c_f(t), \tag{18}$$

where $B(t)$ corresponds to the newly-issued government bonds, $G(t)$ the amount of public investments in time t and $rB(t)$ the debt repayment.

Since the last financial crisis, debt stability became a major concern for governments. To reduce the debt-to-GDP ratio, some reforms have been led as the Stability and Growth Pact (SGP) for example.[13] We replicate these types of reforms assuming that the government adjusts its debt-to-GDP ratio according to a specific rule. As in Minea and Villieu (2011), we denote $b \equiv B/y$ as the ratio of debt over GDP, where the public debt must converge to a certain level b^*, using the following rule:

$$\frac{\dot{b}(t)}{b(t)} = -\phi\left(1 - \frac{\bar{b}}{b(t)}\right), \text{with } \phi. > 0 \tag{19}$$

In (19), two policy parameters allow the government to design its policy: $\bar{b}$ and ϕ. Firstly, $\bar{b}$ represents the target to be reached such that any difference with this threshold requires a debt adjustment. The adjustment of public debt is calibrated with the second policy parameter: ϕ.

3.4 Intertemporal equilibrium

We describe now the intertemporal equilibrium. Let us denote $v = c/y$ the consumption as a proportion of GDP.

Lemma 1 *The intertemporal equilibrium is determined by the following three dynamical equations:*

$$\begin{aligned}
\frac{\dot{x}(t)}{x(t)} &= \frac{\left(\frac{r-\theta(1-\tau)sx(t)^{1-s}}{r}\right)\left[\frac{x(t)^s-\tau}{b(t)}+r-\frac{\alpha\tau_c}{(1-\alpha)(1+\tau_c)}\frac{v(t)}{b(t)}+\phi\left(1-\frac{b^*}{b(t)}\right)\right]}{1-s} - \frac{x(t)^{1-s}\left[(1-\tau)(1-\theta s)-\frac{v(t)}{1-\alpha}\right]}{1-s}, \\
\frac{\dot{v}(t)}{v(t)} &= \frac{r(1-\tau)sx(t)^{1-s}(1-\theta)}{r-\theta(1-\tau)sx(t)^{1-s}} - \rho - \frac{x(t)^s-\tau}{b(t)} - r + \frac{\alpha\tau_c}{(1-\alpha)(1+\tau_c)}\frac{v(t)}{b(t)} - \phi\left(1-\frac{b^*}{b(t)}\right), \\
&\equiv V(b(t),x(t),v(t)) \\
\frac{\dot{b}(t)}{b(t)} &= -\phi\left(1-\frac{b^*}{b(t)}\right), \textit{with } \phi > 0.
\end{aligned} \tag{20}$$

Proof. See appendix 10.1

We have a three-dimensional dynamic system which involves three variables: $b(t)$, $x(t)$ and $v(t)$. Among these three variables, one is pre-determined, $b(t)$ and the two remaining, $x(t)$ and $v(t)$, are forward. Any intertemporal equilibrium path also needs to satisfy the transversality conditions (described in Section 3.2).

[13] In SGP, the member states for which the current debt-to-GDP is above a threshold (60% fixed by the Maastricht treaty) must reduce the distance between their ratio and the threshold by an average rate of one-twentieth per year. If the debt-to-GDP ratio is beyond this threshold, government must reduce it at a steady peace level.

4 The balanced growth path (BGP): uniqueness versus multiplicity

In this section, we analyse the conditions for the existence of a unique or multiple BGPs. We show that the results are driven by three elements. In the first place, the share of foreign goods in total consumption and tariffs are central in the existence of Balanced Growth path. Depending on their values, they promote multiplicity. We then shed light on the importance of the interest rate that have a strong impact on the characterization of the BGP.

A Balanced Growth path is a steady state of the dynamical system (20), i.e. is a stationary solution (b,v,x) solving $\dot{b}(t) = \dot{v}(t) = \dot{x}(t) = 0$. Along the BGP, the following equality is satisfied: It follows that at the equilibrium, the growth rate should be propertional to the total deficit

$$\Gamma(x) = H(x), \tag{21}$$

where:

$$\Gamma(x) \equiv \frac{(1-\tau)sx^{1-s}(1-\theta)}{1-\theta(1-\tau)sx^{1-s}/r} - \rho, \tag{22}$$

$$H(x) \equiv \frac{x^s - \tau}{b^*} + r - \frac{\alpha\tau_c\left[(1-\tau)(1-s)x^{1-s} + \rho\frac{r-\theta(1-\tau)sx^{1-s}}{r}\right]}{(1+\tau_c)b^*x^{1-s}}. \tag{23}$$

Using (1), (2) and the binding (4) we necessarily have $\underline{x} < x < \bar{x}$ with:

$$\bar{x} \equiv \left(\frac{r}{(1-\tau)s\theta}\right)^{\frac{1}{1-s}} \text{ and } \underline{x} \equiv \left(\frac{r}{(1-\tau)s}\right)^{\frac{1}{1-s}}. \tag{24}$$

We assume

Assumption 1 *$r > \rho$ and $\theta > s/(2-s)$.*

The first part of the assumption ensures that the growth rate (22) is positive whatever the value of θ, while the second part allows to simplify the analysis, ensuring that $\Gamma(x)$ is a convex function.[14] Replacing x by the upper bound $\bar{x}$ and $\underline{x}$, yields respectively to:

$$\bar{r} \equiv \tau^{\frac{1-s}{s}}(1-\tau)s \text{ and } \underline{r} \equiv \tau^{\frac{1-s}{s}}(1-\tau)\theta s = \bar{r}\theta. \tag{25}$$

[14] All our results on existence, uniqueness and multiplicity of the steady states could be obtained even under $\theta < (2-s)$, but at the cost of cumbersome technical details.

We will consider in the rest of the paper that $r \in (\underline{r}, \bar{r})$.

Beside the analysis of existence and uniqueness of the steady state, we need also to determine whether the steady state is characterized by a primary surplus or rather by a primary deficit as this property will be partially related to the number of equilibria. A primary surplus (deficit) is obtained if and only if $\tau y + \tau_c P^* c_f - G > (<) 0$. It is therefore immediate to derive that a stationary solution x features a primary surplus if $x^s - \tau - \alpha\tau_c v/[(1-\alpha)(1+\tau_c)] < 0$, and a primary deficit if $x^s - \tau - \alpha\tau_c v/[(1-\alpha)(1+\tau_c)] > 0$.

4.1 The case of large debt: a fundamental role of tariffs

In the following Proposition, we first consider the case of a high enough debt-output ratio. We show that there is a unique BGP, as illustrated in Figure 1, if the share α of imported foreign goods in total consumption is low enough, or if the tariff τ_c on imported good is low enough. On the contrary, for large values of both α and τ_c, two BGPs may occur. Moreover, depending on the value of the interest rate r, the steady states can be characterized by a primary deficit or surplus:

Proposition 4.1 *Under Assumption 1, let $r > \rho$ and $r \in (\underline{r}, \bar{r})$. Consider the critical values:*

$$\hat{b} \equiv \frac{\tau - \left[\frac{r}{(1-\tau)s}\right]^{\frac{s}{1-s}}}{\rho} < \bar{b} = \hat{b} + \frac{(1-\tau)(1-s)}{\rho} \tag{26}$$

and assume $b^* \in (\hat{b}, \bar{b})$. Then, there exist $1 > \bar{\theta} > \underline{\theta} > 0$ such that when $\theta \in (\underline{\theta}, \bar{\theta})$, the following cases hold:

1. *There is a unique stationary solution $x \in (\underline{x}, \bar{x})$ of (22) in the following cases:*

   (a) for any $\tau_c \in (0,1)$ if $\alpha \leq \bar{\alpha}(\theta)$ with:

   $$\bar{\alpha}(\theta) \equiv \frac{r\rho[\bar{b}-\hat{b}]}{(1-\tau)[(1-s)r+s\rho(1-\theta)]}$$

   (b) if $\alpha > \bar{\alpha}(\theta)$ and $\tau_c \leq \bar{\tau}_c(\theta)$ with:

   $$\bar{\tau}_c(\theta) \equiv \frac{\rho r[\bar{b}-\hat{b}]}{(1-\tau)[(1-s)r+s\rho(1-\theta)][\alpha-\bar{\alpha}(\theta)]}$$

   Moreover, there exists $r_0 \in (\underline{r}, \bar{r})$ such that the steady state is characterized by a primary surplus (deficit) for $r < (>) r_0$.

2. *There exists $\epsilon > 0$, such that there are two stationary solutions $x_1, x_2 \in (\underline{x}, \bar{x})$ of (22), with $x_1 < x_2$, if $\alpha > \bar{\alpha}(\theta)$ and $\tau_c \in (\bar{\tau}_c(\theta), \bar{\tau}_c(\theta) + \epsilon)$. Moreover, x_1 is always characterized by*

a primary surplus, while x_2 is characterized by a primary surplus when $r \in (\underline{r}, r_0(\epsilon))$ and a primary deficit when $r \in (r_0(\epsilon), \bar{r})$ with $r_0(\epsilon)$ close to r_0.

Proof. See appendix 10.2.

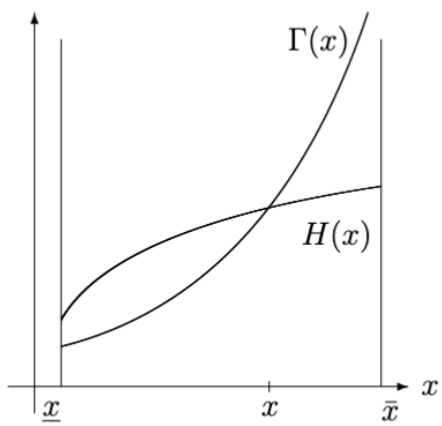

Figure 1: Uniqueness versus Multiplicity of BGP

In a closed economy with perfectly substitutable assets (public debt and capital), it has been shown by Minea and Villieu (2013) that multiplicity of BGPs is ruled out under a log linear utility function in consumption. Indeed, if agents expect an increase of public expenditures, this will induce a higher future income. To finance this increase in public spending, a larger debt emission is required, which crowds out private investment having a negative impact on future income. Then, expectations may not be self-fulfilling and uniqueness is obtained.

In contrast, in our framework, multiplicity is driven first by the coexistence of two key mechanisms, (i) the inflow of international assets and (ii) the existence of an investment multiplier due to the credit constraint with collateral, and second by the existence of tariffs. As a small open economy can import international funds, such a crowding out effect is no longer relevant. Therefore, a higher public spending may now be compatible with an increase of productive investment. The resulting effect on growth is magnified by the collateral role of capital which generates an investment multiplier. In this case, an expected increase of public spending can be self-fulfilling because of higher future income and growth, which sustain a long run equilibrium with larger public spending. However, in Modesto *et al.* (2021), it is shown that this mechanism strongly depends on the size of public debt. Indeed, in the case of high debt, it is shown that no matter what is the value of the preference parameter for domestic debt, uniqueness of the BGP holds.

As Proposition 4.1 makes clear, the international trade dimension is central in our framework.

Indeed, when public debt is high enough, the share of foreign goods and tariffs drive the existence of two BGPs. We note that when domestic households do not consume an important share of goods, or when tariffs are low enough, a unique BGP exists. This configuration is in a sense similar to the framework of Modesto *et al.* (2021).

To understand why uniqueness occurs under a large debt-output ratio as soon as the tariffs income is limited,[15] we rewrite the government budget constraint (20) as $\Gamma(x)b^* = G/y - \tau + rb^* - \frac{\alpha\tau_c}{(1-\alpha)(1+\tau_c)}\upsilon$. When τ_c and/or α is low, for a too low x, growth is not sufficient to allow the repayment of a high level of debt. Hence, a large debt-output ratio is not compatible with the government budget constraint and cannot be sustained, so that a too low steady state cannot exist. We then recover the result of Modesto *et al.* (2021).

On the other hand, when tariffs and the share of foreign goods into total consumption are high enough, a second BGP exists, which is characterized by a lower growth. Henceforth, when two BGPs coexist, the economy may be located at either the low (x_1) or high growth (x_2) steady state. In such a configuration, households consume an important share of goods that are highly taxed. The government therefore earns some extra revenues that allow to sustain a low equilibrium characterized by a low growth rate and a large debt at the same time. In such a configuration there is a potential of expectations coordination problem. Indeed, the crowding out effect on private investment generated by the large debt can be more than compensated by government expenditures allowed by tariffs income. It follows that even if agents expect a low growth, the related equilibrium can be self-fulfilled as they expect the government will be able to sustain the large debt burden from the tariffs income. We then conclude that when the debt level is high, and the share of foreign goods in total consumption is large enough, increasing tariffs may have a dramatic destabilizing effect generating some expectation coordination failure between multiple equilibria and the possible existence of large self-fulfilling fluctuations.

We now discuss the properties of the steady state in terms of primary deficit/surplus. In the case of a unique steady state we find a primary deficit/surplus when r is high/low. A sufficiently high interest rate $r > r_0$ pushes down growth since $\partial\Gamma(x)/\partial r < 0$. To sustain the reimbursement of the high level of public debt, a sufficient high level of growth is however needed, which means that public spending should be high enough. This explains that there is a primary deficit. Of course, when $r < r_0$, we have exactly the opposite situation.

When two BGPs exist, we argue here that the high steady state x_2 has the same properties

[15] As suggested by Figure 1, uniqueness corresponds to a configuration where a unique "large" steady state x occurs while a lower one is outside of the admissible set $(\underline{x}, \bar{x})$ as it is unsustainable.

as the one in the case of uniqueness. On the other hand, x_1 is always characterized by a primary surplus. This steady state is characterized by a lower growth but at the same time by low public spending. Since we are in a case of large tariffs, the total income of the government is large enough to guarantee that the low steady state is characterized by a primary surplus no matter what is the value of the interest rate.

4.2 The case of low debt: tariffs do not really matter

Let us focus now on the case where public debt is low enough, i.e. $b^* < \hat{b}$. Unlike the previous case, we show that two BGPs always exist.

Proposition 4.2 *Under Assumption 1, let* $\tilde{r} \equiv \tau^{\frac{1-s}{s}}(1-\tau)s\tilde{\theta}$ *and*

$$\tilde{b} \equiv \frac{\left[\left[\frac{r}{(1-\tau)s\tilde{\theta}}\right]^{\frac{s}{1-s}} - \tau\right]}{r\frac{1-\tilde{\theta}}{\tilde{\theta}-\theta} - \rho}$$

For given (b^*, θ, r)*, there exist* $\rho_0 > 0$, $\widetilde{\theta} \in (\theta, 1)$ *and* $\hat{r} \in [\tilde{r}, \bar{r}]$ *such that if* $\rho < \rho_0$, $r \in (\widetilde{r}, \bar{r})$ *and* $b^* < \min\{\tilde{b}, \hat{b}\}$*, there are two stationary solutions* $x_1, x_2 \in (\underline{x}, \bar{x})$ *of (22), with* $x_1 < x_2$*, in the following cases:*

1. *for any* $\tau_c \in [0,1]$*, if* $\alpha \leq \tilde{\alpha}(\theta)$ *with:*

$$\tilde{\alpha}(\theta) \equiv \frac{[\rho_0 - \rho](\tilde{b} - b^*)}{(1-\tau)\left[1 - s + \rho s \frac{\theta - \tilde{\theta}}{r}\right]}$$

2. *if* $\alpha > \tilde{\alpha}(\theta)$ *and* $\tau_c \leq \bar{\tau}_c(\tilde{\theta})$*, with:*

$$\bar{\tau}_c(\tilde{\theta}) \equiv \frac{[\rho_0 - \rho](\tilde{b} - b^*)}{(1-\tau)\left[1 - s + \rho s \frac{\theta - \tilde{\theta}}{r}\right](\alpha - \tilde{\alpha}(\theta))}$$

Moreover, x_2 *is always characterized by a primary deficit, and there exists* $r_0 \in [\underline{r}, \bar{r}]$ *such that* x_1 *is characterized by a primary deficit when* $r \in [\underline{r}, r_0]$ *and a primary surplus when* $r \in [r_0, \bar{r}]$.

Proof. See appendix 10.3

Interestingly, when public debt is low enough, two BGPs exist. To understand this result consider again the government budget constraint (20) rewritten as $\Gamma(x)b^* = G/y - \tau + rb^* - \frac{\alpha\tau_c}{(1-\alpha)(1+\tau_c)}v$. If the debt-output ratio is low enough, the government budget constraint is sustainable even with a low growth rate, which explains the existence of the low steady state (x_1). In contrast, at a high steady state (x_2), the growth rate is high enough to sustain the government budget whatever the level of debt. Given a sufficiently low level of public debt, the multiplicity of

BGPs is essentially explained by the coexistence of the same two key mechanisms: (i) the inflow of international assets and (ii) the existence of an investment multiplier due to the credit constraint with collateral. Compared to the case with a high debt, the role of tariffs income is here not crucial as it just appears as a complement of resources allowing to support the two steady states.[16] We clearly observe indeed that even if $\alpha = 0$, the two steady states still exist as initially shown by Modesto *et al.* (2021). However, as we will see in the next section, tariffs have a crucial role on the value of the long-run growth rate.

5 Comparative statics: a trade-off between tariffs and growth

We now provide comparative statics. We focus on the behavior of the equilibrium when tariffs are modified. Consider the two equations characterizing the intertemporal equilibrium, namely $\Gamma(x)$ and $H(x)$. As we have already noticed before, $\Gamma(x)$ corresponds to the growth rate of the economy. Let us now turn to the economic interpretation of $H(x)$. Multiplying $H(x)$ by b^*, gives:

$$H(x)b^* = x^s - \tau - \frac{\alpha\tau_c}{(1-\alpha)(1+\tau_c)}v + rb^*. \tag{27}$$

In (27), we easily recognize the primary balance and the debt burden, rb^*. We finally argue that (27) expresses the total public expenditures net of taxes proceedings. It follows therefore that at the equilibrium, the growth rate should be proportional to the total deficit of the government. Notice that only $H(x)$ depends on tariffs and its impact appears to be negative. We formulate the following proposition where we consider the case of two BGPs since in the case of uniqueness the same result as the high steady state x_2 is obtained:

Proposition 5.1 *Under Assumption 1, Propositions 4.1 and 4.2, for a given* $\tau_c \in [0, 1]$:

$$\frac{\partial x_1}{\partial \tau_c} > 0 \text{ and } \frac{\partial x_2}{\partial \tau_c} < 0$$

Proof. See Appendix (10.4)

In Figure 2, we depict an increase of tariffs.

The increase of tariffs allows to decrease the deficit of the government. However it has different impacts on growth depending on the amount of public spending relative to capital. On one hand at

[16] It is worth noticing however that too large tariffs may prevent the existence of any equilibria. As they act as an externality, such a result is not surprising.

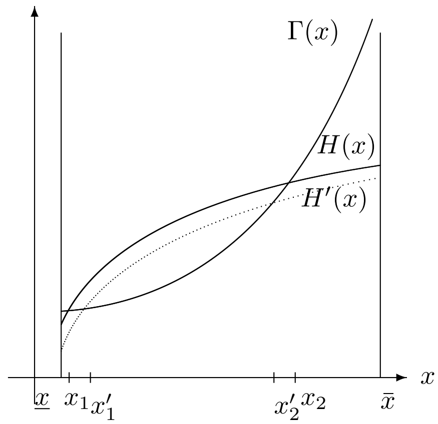

Figure 2: The impact of an increase of τ_c

Note: In this figure we display the effects of increasing tariffs. Dotted curve represents H(x) following an increase of tariffs, while the solid H(x) curve represents H(x) before the increase of tariffs. We easily remark that steady states move in opposite directions following tariffs' increase.

the low steady state x_1 with a relatively low public spending over capital, any increase of tariffs is pro-growth. On the other hand at the high steady state x_2, with a relatively large public spending over capital, this increase is detrimental for growth. We argue that this difference comes from the origin of growth. At x_2, growth is driven by productive spending from the government which is relatively large compared to capital. On the contrary, at x_1, growth is driven by private capital and thus private investment. Following an increase of tariffs two effects counteract. Firstly, households can dedicate less revenues to productive investment and this has a negative impact on growth. At the same time, government experiences additional resources that are used to increase public spending, boosting growth. The source of growth now matters. At x_2, growth comes mainly from productive spending. But the small increase of tariffs relative to the size of government spending has a limited impact on the government spending capacities that weakly increase while the tariffs strongly impacts the households' income. Therefore, the first effect dominates the second one and growth declines. At x_1 on the contrary, the first effect is dominated by the second one. Indeed, even though growth is driven by private investment, the increase of tariffs generates a relatively large increase of government spending that compensates the negative impact on households income. Our results therefore suggest the existence of a trade-off between tariffs and growth. Indeed, tariffs may enhance growth for the low equilibrium while they have a negative impact on the high equilibrium.

6 Local stability analysis

We now investigate the local stability properties of the BGPs. Let us consider the three-dimensional dynamic system as given by equations (20) in Lemma 1 and that can be written as follows

$$\begin{aligned} \dot{b}(t) &= -\phi(b(t) - b^*) \\ \dot{v}(t) &= V(b(t), x(t))v(t) \\ \dot{x}(t) &= X(b(t), v(t), x(t))x(t). \end{aligned} \tag{28}$$

For the local stability analysis, we linearize this three-dimensional dynamic system given around the steady state. From the eigenvalues of the Jacobian matrix, we determine the properties of the steady states. It is worth noting that $-\phi$ is a negative eigenvalue.[17]. The two others variables $x(t)$ and $v(t)$ are forward, thereby if the two remaining eigenvalues have a positive real part, then the steady state $(\bar{b}, v^*, x^*)$ is a *saddle-point stable*. While, if λ_2 or λ_3 has negative real part, then the steady state is *locally indeterminate*. Following the conditions presented in the Propositions 4.1 and 4.2, we formulate the following proposition:

Proposition 6.1 *Under Assumption 1, let $r > \rho$, $r \in (\underline{r}, \bar{r})$, and consider Propositions 4.1 and 4.2. Then the following results hold:*

i) When $b^* \in (\hat{b}, \bar{b})$ and $\theta \in (\underline{\theta}, \bar{\theta})$, if $\alpha \leq \bar{\alpha}(\theta)$, or $\alpha > \bar{\alpha}(\theta)$ and $\tau_c \leq \bar{\tau}_c(\theta)$, then the unique steady state x is locally indeterminate.

ii) When $b^* \in (\hat{b}, \bar{b})$ and $\theta \in (\underline{\theta}, \bar{\theta})$, if $\alpha > \bar{\alpha}(\theta)$ and $\tau_c \in (\bar{\tau}_c(\theta), \bar{\tau}_c(\theta) + \epsilon)$, then the high steady state x_2 is locally indeterminate while the low one x_1 is saddle-point stable.

iii) When $\rho < \rho_0$, $r \in (\widetilde{r}, \bar{r})$ and $b^* < \min\{\tilde{b}, \hat{b}\}$, if $\alpha \leq \tilde{\alpha}(\theta)$, or $\alpha > \tilde{\alpha}(\theta)$ and $\tau_c \leq \bar{\tau}_c(\tilde{\theta})$, then the high steady state x_2 is locally indeterminate and the low one x_1 is saddle-point stable.

Proof. See Appendix (10.6)

When the balance growth path is unique, the equilibrium is locally indeterminate. The steady state is characterized by sunspot fluctuations around it. If two BGPs coexist, instead, the lowest steady state, x_1 is a saddle-point stable while x_2 is locally indeterminate. Global indeterminacy then occurs under multiplicity.

We can resume the local stability analysis in the following table:

[17] Since the equation which is driving public debt is linear

BGP existence	***Local stability properties***
unique BGP	Local indeterminacy
two BGPs	Global indeterminacy

Table 1: Local stability analysis

The main result of this paper is then to show that when debt is large, the fact that domestic households consume an important share of foreign goods under large enough tariffs is the key ingredient to explain global indeterminacy, i.e. the existence of expectations coordination failures that may lead to the occurrence of large sunspot fluctuations.

Let us present the mechanisms at stake in the two cases: uniqueness and multiplicity. We first discuss the existence of self-fulfilling expectations when BGP is *unique*. We consider an equilibrium along which agents suddenly formulate expectations about a possible higher growth rate. Since assets are imperfect substitutes and due to the access to international market, agents may borrow, consume and invest more (preventing a crowding out effect). The investment multiplier allows to achieve a higher growth rate. Moreover, since the consumption of the foreign good increases, tariffs income are quite large, improving the government's revenue dedicated to increase its spending. All these mechanisms highlight the possibility for self-fulfilling expectations leading to multiple transitional paths and expectation-driven fluctuations when the BGP is unique.

When *two BGPs* coexist, a low steady state x_1 coexists with a high one, x_2. Nevertheless, x_2 keeps the same local indeterminacy property as in the uniqueness case as all the mechanisms previously mentioned remain exactly the same. We explain the mechanisms are stake for the lowest steady state. The economy is now characterized by a low growth rate and thus by a low government spending to capital ratio. The credit constraint is then more tightened since the collateral needed to borrow is relatively lower. Assume again that along an equilibrium, agents are expecting an increase of the growth rate. Being able to get less inflows of capital, they cannot increase significantly their consumption and their investment in productive capital. At the same time, the government becomes more constrained, being unable to rely on sufficient revenues, since growth, consumption and tariffs' income remain low. As a result, the government does not invest more and growth cannot increase. Therefore, the expectations cannot be self-fulfilling. However, due to expectations coordination failures already mentioned earlier, the existence of global indeterminacy with two steady states leads to possible large fluctuations around them.

7 Numerical illustration

This section is devoted to a numerical illustration of Properties presented in Section 4. The data used have been collected on OECD, IMF and the World Bank websites for the year 2019 (and 2018 for the tariffs rate) and for six countries: France, Greece, Japan, Spain, United Kingdom and United States.[18] We select 2019 data to model the impacts of US protectionism measures as we can notice that contrary to all the other countries, the US have strongly increased their tariffs between 2018 and 2019. This year then appears as the most suitable one to explore the impacts of tariffs increases. For the calibration step, we follow Modesto et al. (2021), where $\theta = 0.06$ and $\rho = 0.01$. For each separate country, we use the specific data of Table 2 and we check the conditions provided in Propositions 4.1 and 4.2.

	s (%GDP)	τ (%GDP)	b^* (%GDP)	r (%)	$\tau_c(2018)$ (%)	$\tau_c(2019)$ (%)	α (%GDP)
France	30.7	44.89	97.62	0.13	3.10	3.08	32.55
Greece	29.30	39.48	184.91	2.59	3.10	3.08	41.86
Japan	36.45	31.41	235.45	-0.11	3	3.23	17.4
Spain	34.16	34.68	95.54	0.66	3.08	3.10	32.02
UK	29.30	32.72	85.24	0.94	3	2.65	31.92
US	35.8	24.97	108.46	2.14	3.2	13.8	14.54

Table 2: Data for countries sample

Note: Data for 6 developed countries in 2019, where: s corresponds to 1 - labour share, labour share has been collected on OECD website, τ the tax pressure on OECD, b^* on IMF, r represents the long-term interest rate on OECD, τ_c is the tariff rate on World Bank and α the share of import goods in total consumption on World Bank website.

We conclude that with the 2019 data all the countries are characterized by the existence of two equilibria. We find the same result for all the countries except the US using the tariffs of 2019. It is worth noting in Table 2 that only US experienced a significant change in tariffs rate between 2018 and 2019 as it has been multiplied by more than 4. Using 2018 rate shows that US are characterized by a unique equilibrium. According to our framework, the increase of tariffs by the US in 2018 triggered instability generating multiplicity of equilibrium and therefore large fluctuations associated to global indeterminacy.

8 Empirical strategy

Our numerical exercise suggests that because of the strong increase of their tariffs rate between 2018 and 2019, the US may have been subject to a greater volatility of GDP in 2019. We need

[18]The selection of the sample is motivated by their heterogeneity on: public debt level, tariffs and tax pressure. Countries like Japan, Greece and United States are characterized by high public debt; France by strong tax pressure, United States have the highest tariffs. Spain and United Kingdom dedicate one third of their consumption to the consumption of foreign goods.

however to check whether such a conclusions could be observed in the data. We then employing an Event study approach ton confirm our claim. Event study methodology appears to be suitable to understand if and how the increase of tariffs by the US have generated macroeconomic (in)stability.

8.1 Data and methodology

In this section, we describe the source of data and the construction of the growth volatility (i.e. macroeconomic stability).

8.1.1 Data

In this section, we focus on the United States. The data used in this exercise are quarterly data from 2000Q1 to 2020Q4, collected on IMF, OECD and World Bank websites. We collected: the growth rate, tariff rate, fiscal deficit, public debt, interest rates, inflation rate and exchange rate volatility (nominal). In this paper, we document the impacts of the increase of tariffs in 2019 by the US on macroeconomic stability. We assume that volatility of growth can be a proxy for macroeconomic stability (Ramey and Ramey, 1995; Sangnier, 2013). We first estimate growth volatility using GARCH (Generalized Auto Regressive Conditional Heterosckedasticity)[19].

8.1.2 Modeling macroeconomic stability with GARCH

We start from a simple GDP growth equation:

$$Y_t = \beta X_t + \varepsilon_t \tag{29}$$

$$\varepsilon_t \sim \mathcal{N}(0, \sigma_t^2)$$

We can model the volatility of growth (conditional variance of ε_t) as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_j \sigma_{t-j}^2 \tag{30}$$

where $\varepsilon_t^2 = \omega + \alpha(\mathcal{L})\varepsilon_t^2 + V_t$.

In (30), α represents how persistent the time series is, ε_{t-1}^2 corresponds to the contribution of disturbances of the previous quarter to the actual volatility, β represents how persistent the

[19]GARCH model are often used to model volatility in financial markets. This type of models allows to deal with heteroskedasticity that could alter the conclusions drawn since other models tend to create observations clusters

conditional variance is, $\mathcal{L}$ a lag operator and finally σ^2_{t-j} corresponds to the contribution of past volatility. Moreover α_i and β_j are constant and ≥ 0. If,

$$\textstyle\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j \leq 1.$$

We have a positive and stable conditional variance of ε_t, we also affirm that ε_t asymptotically stationary to the second order. We can define V_t, the variance as :

$$V_t \equiv \varepsilon_t^2 - \sigma_t^2,$$

we impose in (29) that disturbances are distributed as a Gaussian law. In fact, the disturbances do not follow necessarily a Normal distribution, particularly when time series has fat tail, Student-t error distribution appears more suitable. For this reason, we choose to simulate GARCH model with four types of error distributions: Normal, skew-Normal, Student-t and skew-Student. To choose the appropriate distributions we implement tests[20] and use the Akaike information criterion. After investigations [21], we choose the Skewed-Student distribution for the error terms.

8.2 The Event Study approach

Now, we start the Event study to understand the impacts of the increase of tariffs on US growth volatility. But, let us briefly the US context before the presentation of the Event Study.

The US context – In January 2018, the United States implemented five sets of measures representing 12.6% of the total goods imported by the US. The main goods targeted by those measures are semi conductors, steel, aluminium, etc,. from China, European Union, South Korea and Mexico. The tax on semi conductors from China for example have been increased by 30% for example. In tables 3 [22] and 4 [23] are reported the tariffs rate on primary products and on manufactured products from 2014 to 2020. We exploit this trade shock using an Event study to analyze whether it generated more volatility. Let us now describe the central specification.

The baseline specification – As mentioned in Section 8.1.1, we follow the BMS approach where fiscal and monetary factors are added. Our specification can be written as:

$$R_t = \alpha_t K_t + \beta_t X_t + e_t, \tag{31}$$

[20]Shapiro-Wilk test for Normality of the series, Ljung-Box test for auto correlation testing and the Jarque Bera test to see if the error terms follow a Normal distribution.

[21]See details Appendix

[22]See Appendix

[23]See Appendix

where R_t is the growth volatility for the time period t relative to the event; K_t the expected (predicted) growth volatility and e_t the component of growth volatility that can be considered as abnormal, unexpected, X_t is the vector of control variables composed by the inflation rate, public debt, fiscal deficit, interest rates, the nominal exchange rate volatility. In (31), we easily obtain a measure of the change of growth volatility associated to the event, i.e. the *abnormal growth volatility*, e_t from it. We then define the event window, we set it at nine quarters, including four quarters before and four quarters after [24] the tariffs increase.

8.3 Results

This section is devoted to the presentation of the main results, firstly from the central specification (BMS approach) and then from the various robustness checks.

8.3.1 Main results

Figure 3 plots the results from our experiment[25].

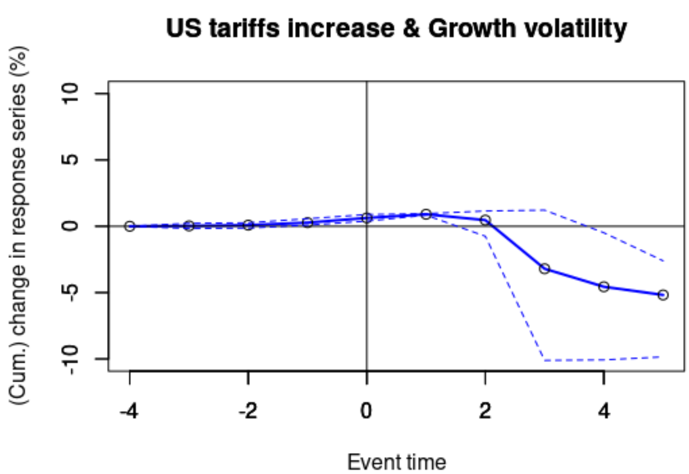

Figure 3: Event study on US tariffs increase

Note: The x-axis represents the time horizon, quarterly. On the y-axis are displayed the growth volatility. The dotted lines represent the confidence intervals. It is worth noting that the increase of tariffs starting 2018 significantly generated more volatility for at least two quarters. After three quarters, the effects do not seem significant anymore.

Following the increase of tariffs in January 2018, the US economy experiences more volatility

[24]In Section 8.3.2, other time windows are used

[25]The coefficients associated to the impact of tariffs increase on growth volatility are reported in Table 4 available in Appendix. The coefficients are positive and statistically different from zero, suggesting that the increase of tariffs by US in 2018 has generated more volatility in the US. The coefficients and our interpretation from are in line with the Figure 3.

of growth. Nonetheless, we remark that the effects are short-lasting, two quarters. These results could be associated to different behaviors, from the production sectors and or households like a reallocation of production factors or some modifications of the consumption bundle.

8.3.2 Robustness Checks

We perform various robustness checks to assess the stability and the validity of our results. To do so, we first add control variables and then we change the time window.

Control variables– In our central specification, we control for some factors according to the BMS approach. In this subsection, we add new control variables in the specification from others as the Narrow Monetary Stability approach or an approach closely related to the five Maastricht convergence criteria. The Narrow Monetary Stability (NMS) focuses solely on the value and stability of inflation (as discussed by Fischer, 1992). According to this view, macroeconomic stability is dependent on maintaining inflation at a stable and desirable level. In the latter, macroeconomic stability is promoted only if the criteria are satisfied[26] (see among others Obstfeld *et al.* (1997)) We obtain similar results: the increase of tariffs generates more growth volatility for two quarters, whatever the approach used.

Time window– As second robustness check, we also vary the time window to assess the robustness of our results. We first increase to 11 and 13 quarters the time window. Then, we decrease it to 7 quarters. We do not observe significant differences compared to the main specification with 9 quarters.

9 Conclusion

In this paper, we have considered a small open economy with endogenous growth driven by public spending where agents consume some foreign good which is subject to tariffs taxation and are subject to some borrowing constraint. The government finances its expenditures through taxes on income, imported goods and by the issue of debt. Tariffs are here considered as proxy of protectionism.

We have first proved that when debt is sufficiently high, the trade dimension is central. Indeed, when agents consume a sufficiently large share of the foreign good and tariffs are high enough,

[26]low and stable inflation, long-run interest rates, public sector deficit to GDP, public debt to GDP and a stable currency fluctuations

two BGPs coexist while uniqueness holds without foreign good consumption. The high BGP is always negatively affected by some tariffs increase and is always locally indeterminate. On the contrary, the low BGP is always positively affected by some increase of tariffs and is always locally determinate. We then shown that tariffs may have a dramatic destabilizing effect generating some expectation coordination failure between multiple equilibria and the possible existence of large self-fulfilling fluctuations. We also exhibit some trade-off between tariffs and growth as tariffs are beneficial only to the low growth equilibrium which may only appear in the globally indeterminate case.

Building on these theoretical results, we provide some numerical illustration and empirical evidence supporting our main conclusions. Exploiting that the US are the only country to have significantly increased its tariffs in 2019, we show numerically that its economy is characterized by more volatility after this change. Moreover, using an Event Study approach, we find empirical evidence for the existence of more growth volatility for the US economy from 2019, although these effects are short-lasting.

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10 Appendice

10.1 Proof of Lemma 1

Using the equilibrium prices (1) and (2), the constraint on public debt (19) and $G/y = x^s$, we derive from equation (18)

$$x(t) = \frac{\dot{B}(t)}{k(t)} + \tau x(t)^{1-s} + \frac{\tau_c P^* c_f(t)}{k(t)} - \frac{rB(t)}{k(t)} \tag{32}$$

Let us introduce the variable $v \equiv c/y$. From (8) we get

$$\frac{P^* c_f(t)}{k(t)} = \frac{\alpha}{(1+\tau_c)} \frac{P_c c(t)}{k(t)} = \frac{\alpha}{(1+\tau_c)(1-\alpha)} v(t) x(t)^{1-s}$$

Substituting this expression into (32) and using the fact that $\dot{b}/b = \dot{B}/B - \dot{y}/y$ give the growth rate of production:

$$\frac{\dot{y}(t)}{y(t)} = \frac{x(t)^s - \tau}{b(t)} + r - \frac{\alpha\tau_c}{(1+\tau_c)(1-\alpha)} \frac{v(t)}{b(t)} + \phi\left(1 - \frac{b^*}{b(t)}\right) \tag{33}$$

and the growth rate of consumption (16) can be rewritten as:

$$\Gamma(x(t)) \equiv \frac{\dot{c}(t)}{c(t)} = \frac{\dot{c}_h(t)}{c_h(t)} = \frac{(1-\tau)sx(t)^{1-s}(1-\theta)}{1-\theta(1-\tau)sx(t)^{1-s}/r} - \rho. \tag{34}$$

From (8) we also get

$$\frac{P_c c(t)}{k(t)} = \frac{1}{1-\alpha} \frac{c_h(t)}{k(t)} = \frac{1}{1-\alpha} v(t) x(t)^{1-s}.$$

Moreover, a binding credit constraint (4) means that $d = -\theta(1-\tau)sy/r$ and also implies that $\dot{d}/d = \dot{y}/y$. Using these results and (3), we then derive the growth rate of capital:

$$\begin{aligned}
\frac{\dot{k}(t)}{k(t)} &= x(t)^{1-s}\left[(1-\tau)(1-\theta s) - \frac{1}{1-\alpha}v(t)\right] + \frac{\theta(1-\tau)s}{r} x(t)^{1-s} \frac{\dot{y}(t)}{y(t)} \\
&= x(t)^{1-s}\left[(1-\tau)(1-\theta s) - \frac{1}{1-\alpha}v(t)\right] \\
&+ \frac{\theta(1-\tau)s}{r} x(t)^{1-s}\left[\frac{x(t)^s - \tau}{b(t)} + r - \frac{\alpha\tau_c}{(1+\tau_c)(1-\alpha)} \frac{v(t)}{b(t)} + \phi\left(1 - \frac{b^*}{b(t)}\right)\right]
\end{aligned} \tag{35}$$

Note that $x = G/k = (y/k)^{\frac{1}{1-s}}$. Using (33) and (34), we easily get:

$$\begin{aligned}
\frac{\dot{x}(t)}{x(t)} &= \frac{\left(\frac{r-\theta(1-\tau)sx(t)^{1-s}}{r}\right)\left[\frac{x(t)^s-\tau}{b(t)} + r - \frac{\alpha\tau_c}{(1+\tau_c)(1-\alpha)}\frac{v(t)}{b(t)} + \phi\left(1-\frac{b^*}{b(t)}\right)\right]}{1-s} \\
&- \frac{x(t)^{1-s}\left[(1-\tau)(1-\theta s) - \frac{v(t)}{1-\alpha}\right]}{1-s} \\
&\equiv X(b(t), v(t), x(t)).
\end{aligned} \tag{36}$$

Recalling that $v \equiv c/y$, we finally get:

$$\begin{aligned}
\frac{\dot{v}(t)}{v(t)} &= \frac{r(1-\tau)sx(t)^{1-s}(1-\theta)}{r-\theta(1-\tau)sx(t)^{1-s}} - \rho - \frac{x(t)^s-\tau}{b(t)} - r + \frac{\alpha\tau_c}{(1+\tau_c)(1-\alpha)}\frac{v(t)}{b(t)} - \phi\left(1 - \frac{b^*}{b(t)}\right) \\
&\equiv V(b(t), v(t), x(t)).
\end{aligned} \tag{37}$$

10.2 Proof of Proposition 4.1

From (22)-(23) we derive:

$$\begin{aligned} \Gamma'(x) &= \frac{r^2(1-\tau)s(1-s)(1-\theta)x^{-s}}{[r-\theta(1-\tau)sx^{1-s}]^2} \quad (38) \\ H'(x) &= s\frac{x^{s-1}}{b^*} + \frac{\rho\alpha\tau_c(1-s)}{(1+\tau_c)b^*x^{2-s}} \quad (39) \end{aligned}$$

We have $\Gamma'(x) > 0$ and $H'(x) > 0$. We also easily see that $H''(x) < 0$, while $\Gamma''(x)$ has the same sign than $\theta(1-\tau)(2-s)x^{1-s} - r$. Since $x > \underline{x}$, $\Gamma''(x) > 0$ is ensured by Assumption 1. In addition, we have:

$$\Gamma(\bar{x}) = +\infty \text{ and } \Gamma(\underline{x}) = r - \rho > 0$$

$$H(\bar{x}) = \frac{\left(\frac{r}{(1-\tau)s\theta}\right)^{\frac{s}{1-s}} - \tau}{b^*} + r - \frac{\alpha\tau_c(1-\tau)}{(1+\tau_c)b^*r}\left[(1-s)r + \rho s(1-\theta)\right]$$

$$H(\underline{x}) = \frac{\left(\frac{r}{(1-\tau)s}\right)^{\frac{s}{1-s}} - \tau}{b^*} + r - \frac{\alpha\tau_c(1-\tau)(1-s)}{(1+\tau_c)b^*}$$

Considering that $\Gamma(x)$ is convex and $H(x)$ is concave with $\Gamma(\bar{x}) > H(\bar{x})$, there is a unique solution $x \in (\underline{x}, \bar{x})$ if $\Gamma(\underline{x}) < H(\underline{x})$. When $\alpha = 0$ or $\tau_c = 0$, $\Gamma(\underline{x}) < H(\underline{x})$ if and only if $b^* > \widehat{b}$, with $\widehat{b}$ as given by (26). Consider now $\alpha > 0$ and $\tau_c > 0$.

1. (a) We easily derive that $\Gamma(\underline{x}) < H(\underline{x})$ for any $\tau_c \in (0, 1)$ if $\alpha < \bar{\alpha}(\theta)$ with

$$\bar{\alpha}(\theta) \equiv \frac{r\rho(b^* - \widehat{b})}{(1-\tau)[(1-s)r + \rho s(1-\theta)]}$$

Moreover, straightforward computations show that for any $\theta \in (0, 1)$, $\bar{\alpha}(\theta) < 1$ if $b^* \in (\widehat{b}, \bar{b})$ with $\bar{b}$ as given in (26).

(b) On the contrary if $\alpha > \bar{\alpha}(\theta)$, we easily derive that $\Gamma(\underline{x}) < H(\underline{x})$ for any $\tau_c < \bar{\tau}_c(\theta)$ with

$$\bar{\tau}_c(\theta) \equiv \frac{r\rho(b^* - \widehat{b})}{(1-\tau)[(1-s)r + \rho s(1-\theta)](\alpha - \bar{\alpha}(\theta))}$$

2. Assume now that $\alpha > \bar{\alpha}(\theta)$ and $\tau_c > \bar{\tau}_c(\theta)$. By continuity, there exists $\epsilon > 0$ such that if $\tau_c \in (\bar{\tau}_c(\theta), \bar{\tau}_c(\theta) + \epsilon)$, there exist two stationary solutions $x_1, x_2 \in (\underline{x}, \bar{x})$ of (21) with $x_1 < x_2$.

Steady state characterization. We characterize the steady state of the economy. Considering (33) and (34) and solving for v yields:

$$v = \frac{1-\alpha}{x^{1-s}}\left[(1-\tau)(1-s)x^{1-s} + \rho\frac{r - \theta(1-\tau)sx^{1-s}}{r}\right] \quad (40)$$

We can express the primary deficit as:

$$DP(x) = x^s - \tau - \frac{\alpha\tau_c}{(1+\tau_c)}\left[(1-\tau)(1-s)x^{1-s} + \rho\frac{r - \theta(1-\tau)sx^{1-s}}{r}\right] \quad (41)$$

We easily get $DP'(x) > 0$. Consider the bounds $\underline{x}$ and $\bar{x}$ as given by (24). We get:

$$DP(\underline{x}) = \left(\frac{r}{(1-\tau)s}\right)^{\frac{s}{1-s}} - \tau - \frac{\alpha\tau_c(1-\tau)}{(1+\tau_c)r}[r(1-s) + \rho s(1-\theta)] < 0 \quad (42)$$

since $r < \bar{r}$. Moreover, since $r > \underline{r}$, we have

$$DP(\bar{x}) = \left(\frac{r}{(1-\tau)s\theta}\right)^{\frac{s}{1-s}} - \tau - \frac{\alpha\tau_c(1-\tau)(1-s)}{(1+\tau_c)} \gtreqqless 0 \tag{43}$$

if and only if

$$\alpha \lesseqqgtr \frac{1+\tau_c}{\tau_c(1-\tau)(1-s)}\left[\left(\frac{r}{(1-\tau)s\theta}\right)^{\frac{s}{1-s}} - \tau\right] \equiv \tilde{\alpha}(\theta) \tag{44}$$

We can easily derive that under $b^* \in (\widehat{b}, \bar{b})$ we have $\bar{\alpha}(\theta) > \tilde{\alpha}(\theta)$ and that if $\theta > \underline{\theta}$ with

$$\underline{\theta} = \max\left\{\frac{s}{2-s}; \left(\frac{\tau}{1-s+\tau s}\right)^{\frac{1-s}{s}}\right\} \tag{45}$$

then $\bar{\alpha}(\theta) < 1$ if

$$\tau_c > \frac{\left(\frac{r}{(1-\tau)s\theta}\right)^{\frac{s}{1-s}} - \tau}{1-s+\tau s - \left(\frac{r}{(1-\tau)s\theta}\right)^{\frac{s}{1-s}}} \equiv \tilde{\tau}_c(\theta) \tag{46}$$

Moreover, there exists $\bar{\theta} \in (\underline{\theta}, 1)$ such that when $\theta \in (\underline{\theta}, \bar{\theta})$ we have $\tilde{\tau}_c(\theta) > \bar{\tau}_c(\theta)$. Obviously we have $\tilde{\tau}_c(\theta) > \bar{\tau}_c(\theta) + \epsilon$

1. We immediately derive that in the case of a unique BGP, i.e. if $\alpha < \bar{\alpha}(\theta)$, or $\alpha > \bar{\alpha}(\theta)$ and $\tau_c < \bar{\tau}_c(\theta)$, then $DP(\bar{x}) > 0$ and there exists $x_0 \in (\underline{x}, \bar{x})$ such that $DP(x_0) = 0$. It follows therefore that $H(x_0) = r$ and thus the unique BGP x^* is such that $x^* > x_0$ if and only if $\Gamma(x_0) < H(x_0) = r$, i.e.

$$F(r) \equiv r^2 + r\left[\rho - s(1-\tau)x_0^{1-s}\right] - \theta\rho s(1-\tau)x_0^{1-s} > 0$$

We conclude that $F(r) \gtreqqless 0$ if and only if r_0 with

$$r_0 = \frac{s(1-\tau)x_0^{1-s} - \rho + \sqrt{\left[\rho - s(1-\tau)x_0^{1-s}\right]^2 + 4\theta\rho s(1-\tau)x_0^{1-s}}}{2} \in (\underline{r}, \bar{r}) \tag{47}$$

Notice that if $r = r_0$, the BGP is by definition equal to x_0. We have then proved that the unique BGP is characterized by a primary surplus (deficit) when $r < (>) r_0$.

2. In the case of two BGPs such that $x_1 < x_2$, i.e. when $\alpha > \bar{\alpha}(\theta)$ and $\tau_c \in (\bar{\tau}_c(\theta), \bar{\tau}_c(\theta) + \epsilon)$, notice first that the equality $\Gamma(x) = H(x)$ can be equivalently written $[\Gamma(x) - r]b^* = DP(x)$. Since $b^* > \widehat{b}$, we easily derive that when $r = r_0$ we have $x_2 = x_0$. We conclude by continuity using the same argument as in 1. above that x_1 is always characterized by a primary surplus, while there exists $r_0(\epsilon)$ close to r_0 such that x_2 is characterized by a primary surplus (deficit) when $r < (>) r_0$.

10.3 Proof of Proposition 4.2

If $b^* < \widehat{b}$ we get $\Gamma(\underline{x}) > H(\underline{x})$. As $\Gamma(\bar{x}) > H(\bar{x})$, it follows that there are two steady states if there exists a $\widetilde{x} \in (\underline{x}, \bar{x})$ such that $\Gamma(\widetilde{x}) < H(\widetilde{x})$. Let us define

$$\widetilde{x} \equiv \left(\frac{r}{(1-\tau)s\widetilde{\theta}}\right)^{\frac{1}{1-s}}$$

where $\widetilde{\theta} \in (\theta, 1)$ ensures that $\widetilde{x}$ can take any value between $\underline{x}$ and $\bar{x}$. $\Gamma(\widetilde{x}) < H(\widetilde{x})$ is equivalent to:

$$\left(\frac{r}{(1-\tau)s\widetilde{\theta}}\right)^{\frac{s}{1-s}} - \tau - b^* \left[r\frac{1-\widetilde{\theta}}{\widetilde{\theta}-\theta} - \rho\right] > \frac{\alpha(1-\tau)\tau_c\left[1-s+\frac{\rho s(\widetilde{\theta}-\theta)}{r}\right]}{1+\tau_c} \tag{48}$$

Assume from now on that $b^* < \widetilde{b}$ and $\rho < \rho_0$ with

$$\widetilde{b} \equiv \frac{\left(\frac{r}{(1-\tau)s\widetilde{\theta}}\right)^{\frac{s}{1-s}} - \tau}{\rho_0 - \rho} \text{ and } \rho_0 \equiv r\frac{1-\widetilde{\theta}}{\widetilde{\theta}-\theta}$$

so that the right-hand side (RHS) is positive. Note that $\widetilde{b} > 0$ for $r > \widetilde{r}$ with

$$\widetilde{r} = \tau^{\frac{1-s}{s}}(1-\tau)s\widetilde{\theta} \in (\underline{r}, \bar{r}) \tag{49}$$

Assume that for a given $\theta \in (0,1)$, $\tau_c < \tilde{\tau}_c(\theta)$ and consider the value $x_0 \in (\underline{x}, \bar{x})$ such that $DP(x_0) = 0$. Let us then choose $\widetilde{\theta}$ such that $\widetilde{x} < x_0$. It follows therefore that $\widetilde{r} < r_0$.

Then, it is easy to check that inequality (48) is equivalent to

$$(\rho_0 - \rho)(\widetilde{b} - b^*) > \tau_c \left[\alpha(1-\tau)\left[1 - s + \frac{\rho s(\widetilde{\theta}-\theta)}{r}\right] - (\rho_0 - \rho)(\widetilde{b} - b^*)\right] \tag{50}$$

This inequality holds for any $\tau_c \geq 0$ if and only if $\alpha \leq \widetilde{\alpha}$ with

$$\widetilde{\alpha} \equiv \frac{(\rho_0-\rho)(\widetilde{b}-b^*)}{(1-\tau)\left[1-s+\frac{\rho s(\widetilde{\theta}-\theta)}{r}\right]}$$

On the contrary, when $\alpha > \widetilde{\alpha}$, inequality (50) holds if and only if $\tau_c < \widetilde{\tau}_c$ with

$$\widetilde{\tau}_c \equiv \frac{(\rho_0-\rho)(\widetilde{b}-b^*)}{(1-\tau)\left[1-s+\frac{\rho s(\widetilde{\theta}-\theta)}{r}\right](\alpha-\widetilde{\alpha})}$$

Assuming $r \in (\widetilde{r}, \bar{r})$ and $\bar{b} < \min\{\widetilde{b}, \widehat{b}\}$, we then conclude that if $\alpha \leq \widetilde{\alpha}$, or $\alpha > \widetilde{\alpha}$ and $\tau_c < \widetilde{\tau}_c$, there are two stationary solutions $x_1, x_2 \in (\underline{x}, \bar{x})$, with $x_1 < x_2$.

Let us study now whether the steady states are characterized by a primary deficit or a primary surplus. Recall that the equality $\Gamma(x) = H(x)$ can be equivalently written $[\Gamma(x) - r]b^* = DP(x)$. Since now $b^* < \widehat{b}$, we easily derive that when $r = r_0$ as given by (47) we have $x_1 = x_0$. We conclude by continuity using the same argument as in 1. above that x_2 is always characterized by a primary deficit, while x_1 is characterized by a primary surplus (deficit) when $r > (<) r_0$.

10.4 Proof of Proposition 5.1

Since $H(x) = \Gamma(x)$ at a steady state and tariffs appear only in $H(x)$, we have:

$$\frac{dx}{d\tau_c} = \frac{\partial H/\partial \tau_c}{\Gamma'(x) - H'(x)} \tag{51}$$

$$= -\frac{\alpha\left[(1-\tau)(1-s)x^{1-s} + \rho\frac{r-\theta(1-\tau)sx^{1-s}}{r}\right]}{(1+\tau_c)^2 b^* x^{1-s}[\Gamma'(x) - H'(x)]} \tag{52}$$

In then case of a unique BGP x^*, we know that $H'(x) < \Gamma'(x)$ and thus $dx^*/d\tau_c < 0$. In the case of two steady states, the impact of the tariffs depends on the difference $H'(x) - \Gamma'(x)$. At x_1, since $H'(x_1) > \Gamma'(x_1)$, we get $dx_1/d\tau_c > 0$, while at x_2, since $H'(x_2) < \Gamma'(x_2)$, we get $dx_2/d\tau_c < 0$.

10.5 Proof of Proposition 5.2

Since $H(x) = \Gamma(x)$ at a steady state and b^* only enters $H(x)$, we have: :

$$\frac{dx}{db^*} = \frac{\partial H/\partial b^*}{\Gamma'(x) - H'(x)} \tag{53}$$

$$= -\frac{DP(x)}{b^{*2}[\Gamma'(x) - H'(x)]} \tag{54}$$

with $DP(x)$ the primary deficit as given by (41). We already know that x_1 is such that $\Gamma'(x_1) - H'(x_1) < 0$ while x_2 is such that $\Gamma'(x_2) - H'(x_2) > 0$. The case of a unique steady state corresponds to the same properties as x_2. We then easily deduce that the sign of the primary deficit will drive the effects of increasing public debt.

In the case of a unique steady state x^*, using Proposition 4.1, since $\Gamma'(x^*) - H'(x^*) > 0$, we easily derive that $dx^*/db^* < 0$ when $r > r_0$ while $dx^*/db^* > 0$ when $r < r_0$.

In the case of two steady states we have to distinguish the configurations $b^* > \widehat{b}$ and $b^* < \widehat{b}$.

- If $b^* > \widehat{b}$ we know that the low steady state is such that $\Gamma'(x_1) - H'(x_1) < 0$ and $DP(x_1) < 0$. We then get $dx_1/db^* < 0$. For the high steady state x_2, since $\Gamma'(x_2) - H'(x_2) > 0$, we conclude from Proposition 4.1 that $dx_2/db^* < 0$ when $r > r_0$ while $dx_2/db^* > 0$ when $r < r_0$.

- If $b^* < \widehat{b}$ we know that the high steady state is such that $\Gamma'(x_2) - H'(x_2) > 0$ and $DP(x_2) > 0$ which imply $dx_2/db^* < 0$. For the low steady state x_1, since $\Gamma'(x_1) - H'(x_1) < 0$, we conclude from Proposition 4.2 that $dx_1/db^* > 0$ when $r < r_0$ while $dx_1/db^* < 0$ when $r > r_0$.

10.6 Proof of Proposition 6

Linearizing the dynamical system (20) around a steady state (b^*, v^*, x^*) gives the following Jacobian matrix

$$\mathcal{J} = \begin{pmatrix} -\phi & 0 & 0 \\ V_1(b^*, v^*, x^*)v^* & V_2(b^*, v^*, x^*)v^* & V_3(b^*, v^*, x^*)v^* \\ X_1(b^*, v^*, x^*)x^* & X_2(b^*, v^*, x^*)x^* & X_3(b^*, v^*, x^*)x^* \end{pmatrix}$$

We easily derive

$$
\begin{aligned}
\mathcal{D} &= -\phi\left[V_2(b^*,v^*,x^*)X_3(b^*,v^*,x^*) - V_3(b^*,v^*,x^*)X_2(b^*,v^*,x^*)\right]x^*v^* \\
\mathcal{T} &= -\phi + V_2(b^*,v^*,x^*)v^* + X_3(b^*,v^*,x^*)x^* \\
\mathcal{S} &= -\phi\left[V_2(b^*,v^*,x^*)v^* + X_3(b^*,v^*,x^*)x^*\right] \\
&+ \left[V_2(b^*,v^*,x^*)X_3(b^*,v^*,x^*) - V_3(b^*,v^*,x^*)X_2(b^*,v^*,x^*)\right]x^*v^*
\end{aligned}
$$

It follows that the eigenvalues of $\mathcal{J}$ are solution of the following polynomial

$$
\begin{aligned}
\mathcal{P}(\lambda) &= \lambda^3 - \mathcal{T}\lambda^2 + \mathcal{S}\lambda - \mathcal{D} \\
&= (\lambda+\phi)\left[\lambda^2 - \lambda(\mathcal{T}+\phi) - \tfrac{\mathcal{D}}{\phi}\right]
\end{aligned}
$$

We then get three eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$ such that

$$
\begin{aligned}
\lambda_1 &= -\phi \\
\lambda_2 + \lambda_3 &= \mathcal{T} + \phi = V_2(b^*,v^*,x^*)v^* + X_3(b^*,v^*,x^*)x^* \\
\lambda_2\lambda_3 &= -\tfrac{\mathcal{D}}{\phi} = \left[V_2(b^*,v^*,x^*)X_3(b^*,v^*,x^*) - V_3(b^*,v^*,x^*)X_2(b^*,v^*,x^*)\right]x^*v^*
\end{aligned}
$$

Note first that

$$
X(b,v,x) = \frac{\left(\frac{r-\theta(1-\tau)sx^{1-s}}{r}\right)[\Gamma(x) - V(b,v,x)] - x^{1-s}\left[(1-\tau)(1-\theta s) - \frac{v}{1-\alpha}\right]}{1-s}
$$

Straightforward computations give

$$
\begin{aligned}
V_2(b^*,v^*,x^*) &= \tfrac{\alpha\tau_c}{(1-\alpha)(1+\tau_c)b^*} \\
V_3(b^*,v^*,x^*) &= \Gamma'(x^*) - \tfrac{sx^{*s-1}}{b^*} \\
X_2(b^*,v^*,x^*) &= \frac{-\left(\frac{r-\theta(1-\tau)sx^{1-s}}{r}\right)V_2(b^*,v^*,x^*) + \frac{x^{*1-s}}{1-\alpha}}{1-s} \\
X_3(b^*,v^*,x^*) &= -\frac{\theta(1-\tau)sx^{-s}}{r}\Gamma(x^*) + \frac{\left(\frac{r-\theta(1-\tau)sx^{1-s}}{r}\right)\left[\Gamma'(x^*) - V_3(b^*,v^*,x^*)\right]}{1-s} \\
&- x^{*-s}\left[(1-\tau)(1-\theta s) - \tfrac{v^*}{1-\alpha}\right]
\end{aligned}
$$

with

$$
\Gamma(x^*) = \tfrac{r(1-\theta)(1-\tau)sx^{*1-s}}{r-\theta(1-\tau)sx^{*1-s}} - \rho
$$

Note that at the steady state we get

$$
x^{*-s}\left[(1-\tau)(1-\theta s) - \tfrac{v^*}{1-\alpha}\right] = \left(\tfrac{r-\theta(1-\tau)sx^{*1-s}}{r}\right)\Gamma(x^*) \tag{55}
$$

We then get after simplifications

$$
X_3(b^*,v^*,x^*)x^* = \tfrac{s}{1-s}\left(\tfrac{r-\theta(1-\tau)sx^{1-s}}{r}\right)\tfrac{x^{*s}}{b^*} - \Gamma(x^*)
$$

Straightforward computations then yield

$$
-\tfrac{\mathcal{D}}{\phi} = \tfrac{x^{*2-s}v^*}{(1-s)(1+\alpha)}\left[H'(x^*) - \Gamma'(x^*)\right]
$$

In the cases where there exists a unique steady state, it must be such that $H'(x^*) - \Gamma'(x^*) < 0$. It follows that $\lambda_2 > 0$ and $\lambda_3 < 0$. Since $\lambda_1 = -\phi < 0$, we conclude that two eigenvalues are negative and one is positive implying that the steady state is locally indeterminate.

Let us finally consider the cases where two steady states $x_1 < x_2$ exist and are necessarily such that $\Gamma'(x_1) - H'(x_1) < 0$ and $\Gamma'(x_2) - H'(x_2) > 0$. It follows that the highest steady state x_2 satisfies $-\mathcal{D}/\phi < 0$, and we conclude again that two eigenvalues are negative and one is positive implying local indeterminacy.

Let us consider now the lowest steady state x_1 which is such that $-\mathcal{D}/\phi > 0$ implying $\lambda_2\lambda_3 > 0$. We need then to study the sign of $\lambda_2 + \lambda_3$ and thus to compute $V_2(b^*, v^*, x^*)v^* + X_3(b^*, v^*, x^*)x^*$. Let us write $X(b^*, v^*, x^*)$ as follows

$$X(b^*, v^*, x^*) \quad = \quad \tfrac{\Phi(x^*)}{1-s}$$

with $\Phi(x) = \mathcal{B}(x)\left[\mathcal{A}(x) - \frac{\alpha\tau_c v^*}{(1-\alpha)(1+\tau_c)b^*}\right] - x^{1-s}\left[(1-\tau)(1-\theta s) - \frac{v^*}{1-\alpha}\right]$, and

$$\begin{aligned}
\mathcal{A}(x) &= \tfrac{x^s - \tau}{b^*} + r + \phi\left(1 - \tfrac{b^*}{b}\right) \\
\mathcal{B}(x) &= \tfrac{r - \theta s(1-\tau)x^{1-s}}{r}
\end{aligned}$$

Recall that at the steady state $b^* = b$. We obviously get

$$X_3(b^*, v^*, x^*) \quad = \quad \tfrac{\Phi'(x^*)}{1-s}$$

From this we can compute

$$\begin{aligned}
\Phi'(x^*) &= \mathcal{B}'(x^*)\left[\mathcal{A}(x^*) - \tfrac{\alpha\tau_c v^*}{(1-\alpha)(1+\tau_c)b^*}\right] + \mathcal{A}'(x^*)\mathcal{B}(x^*) \\
&- (1-s)x^{*-s}\left[(1-\tau)(1-\theta s) - \tfrac{v^*}{1-\alpha}\right]
\end{aligned}$$

We easily derive

$$\begin{aligned}
\mathcal{A}'(x^*) &= s\tfrac{x^{*s-1}}{b^*} > 0 \\
\mathcal{B}'(x^*) &= -\tfrac{\theta(1-s)s(1-\tau)x^{*-s}}{r} < 0
\end{aligned} \tag{56}$$

At the steady state we get $\Phi(x^*) = 0$ which implies

$$\tfrac{v^* x^{*-s}}{1-\alpha} = x^{*-s}(1-\tau)(1-\theta s) - \tfrac{\mathcal{B}(x^*)}{x^*}\left[\mathcal{A}(x^*) - \tfrac{\alpha\tau_c v^*}{(1-\alpha)(1+\tau_c)b^*}\right]$$

Substituting all this into the expression of $\Phi'(x^*)$ yields

$$\Phi'(x^*) = \quad \mathcal{A}'(x^*)\mathcal{B}(x^*) + \left[\mathcal{A}(x^*) - \tfrac{\alpha\tau_c v^*}{(1-\alpha)(1+\tau_c)b^*}\right]\left[\mathcal{B}'(x^*) - \tfrac{(1-s)}{x^*}\mathcal{B}(x^*)\right]$$

Considering that

$$\mathcal{B}'(x^*) - \tfrac{(1-s)}{x^*}\mathcal{B}(x^*) \quad = \quad -\tfrac{\theta(1-s)s(1-\tau)x^{*-s}}{r} - \tfrac{(1-s)}{x^*}\tfrac{r-\theta s(1-\tau)x^{*1-s}}{r} = -\tfrac{(1-s)}{x^*}$$

we get

$$\Phi'(x^*) \quad = \quad \mathcal{A}'(x^*)\mathcal{B}(x^*) - \tfrac{(1-s)}{x^*}\left[\mathcal{A}(x^*) - \tfrac{\alpha\tau_c v^*}{(1-\alpha)(1+\tau_c)b^*}\right]$$

Recalling that for any steady state we have $\Gamma(x^*) = \mathcal{A}(x^*)$ and

$$\begin{array}{l} \mathcal{A}'(x^*) = H'(x^*) - \frac{\rho\alpha\tau_c(1-s)}{(1+\tau_c)b^* x^{*2-s}} \\ \Gamma'(x^*) = \frac{(1-s)r}{x^*[r-\theta(1-\tau)s x^{*1-s}]}\left[\Gamma(x^*)+\rho\right] \end{array} \tag{57}$$

we derive

$$\Phi'(x^*) \quad = \quad \frac{r-\theta s(1-\tau)x^{*1-s}}{r}\left[H'(x^*) - \Gamma'(x^*) - \frac{\rho\alpha\tau_c(1-s)}{(1+\tau_c)b^* x^{*2-s}}\right] + \frac{\rho(1-s)}{x^*}$$

and thus

$$X_3(b^*, v^*, x^*)x^* \quad = \quad \rho + \frac{r-\theta s(1-\tau)x^{*1-s}}{r(1-s)}\left[H'(x^*)x^* - \Gamma'(x^*)x^* - \frac{\rho\alpha\tau_c(1-s)}{(1+\tau_c)b^* x^{*1-s}}\right]$$

We then derive

$$\begin{array}{rcl} \mathcal{T} + \phi & = & V_2(b^*, v^*, x^*)v^* + X_3(b^*, v^*, x^*)x^* \\ & = & \rho + \frac{\alpha\tau_c v^*}{(1-\alpha)(1+\tau_c)b^*} + \frac{r-\theta s(1-\tau)x^{*1-s}}{r(1-s)}\left[H'(x^*)x^* - \Gamma'(x^*)x^* - \frac{\rho\alpha\tau_c(1-s)}{(1+\tau_c)b^* x^{*1-s}}\right] \end{array}$$

Using the expression of v^* as given by (40), we finally get

$$\mathcal{T} + \phi \quad = \quad \rho + \frac{r-\theta s(1-\tau)x^{*1-s}}{r(1-s)}\left[H'(x^*) - \Gamma'(x^*)\right]x^* + \frac{\alpha\tau_c(1-\tau)(1-s)}{(1+\tau_c)b^*}$$

Since the lowest steady state x_1 si such that $\Gamma'(x_1) - H'(x_1) < 0$, we derive $\mathcal{T} + \phi > 0$ and we conclude that one eigenvalue is negative and two eigenvalues have a positive real part. It follows that x_1 is saddle-point stable.

10.7 Figures and Tables

	tariffs rate primary products
2014	2.5
2015	2.4
2016	2.4
2017	6.6
2018	6
2019	39.5
2020	2.7

Table 3: US tariffs rate on primary products

Note. Data for US from 2014 to 2020 of tariffs rate on primary products.

	tariffs rate manufactured products
2014	3
2015	2.9
2016	2.8
2017	2.8
2018	2.8
2019	3.1
2020	2.9

Table 4: US tariffs rate on manufactured products

Note. Data for US from 2014 to 2020 of tariffs rate on manufactured products.

Dependent variable	*Growth volatility*	
	Broad Monetary Stability	Narrow Monetary Stability
Tariffs	0.18917*	0.17349*
	(0.091)	(0.092)

Table 5: Event Study

Notes: *p<0.10, **p<0.05, ***p<0.01 . Robust standard errors in parentheses. This table summarizes the coefficients associated to the tariffs. In parentheses, the standard deviations. The two approaches that we consider for the notion of stability are used there. In Column (1) the control variables selected correspond to the Broad Monetary Stability approach. In Column (2), these are related to the Narrow Monetary Stability approach. The results suggest that the increase of tariffs have generated more growth volatility. We do not find any statistically difference between both approaches.