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Abstract

The sustainability of resource use and the management of public finances are both long run issues that are linked to each other through savings decisions. In order to study them conjointly, this paper introduces a public debt stabilization constraint in an overlapping generation model in which non-renewable resources constitute a necessary input in the production function and belong to agents. It shows that stabilization of public debt at high level (as share of capital) may prevent the existence of a sustainable development path, *i.e.* a path on which per capita consumption is not decreasing. Public debt thus appears as a threat to sustainable development. It also shows that higher public debt-to-capital ratios (and public expenditures-to-capital ones) are associated with lower growth. Two transmission channels are identified. As usual, public debt crowds out capital accumulation. In addition, public debt tends to increase resource use which reduces the rate of growth. We also provide a numerical analysis of the dynamics that shows that the economy is characterized by saddle path stability. Finally, we show that the public debt-to-capital ratio may be calibrated to implement the social planner optimal allocation according to which the growth rate is increasing in the degree of patience.

Keywords: Non-renewable Resources; Growth; Public Finances; Overlapping Generations
JEL Codes: Q32; Q38; H63

1. Introduction

In the 1970s, "The limits to growth" report alerted the general audience on the (in)feasibility of long-run growth. The report notably points out that economic development relies on natural resources, some of which being finite and non-renewable. The pessimistic Malthusian point of view exposed in the report has since been challenged by neoclassical economists who have highlighted

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the role of increasing returns to scale, technological progress and substitution between natural capital and man-made capital (Dasgupta and Heal, 1974, 1979; Solow, 1974; Stiglitz, 1974). Still, while they have shown that it is possible to experiment an infinite growth in a finite world, this is far from being guaranteed and the question of resource exhaustion is still a question of major interest.

Another major concern that has been developed during the last decades is linked with public indebtedness. Indeed, the last decades have been marked by an increase in public debt-to-GDP ratios. In the Eurozone, the Maastricht treaty imposes a public debt-to-GDP ratio lower than 60% of GDP.[1] This stabilization level has been largely exceeded by many developed economies in the last decades, and the CoViD-19 crisis has generated a huge increase of public debt-to-GDP ratios. Actually, in response to the pandemic, the European Commission activated a derogatory clause to allow governments to perform necessary expenditures to support healthcare, social protection, and economic measures, breaching the usual Maastricht fiscal rules. This clause has been extended due to the ongoing war in Ukraine. The situation in Europe is not unique, as several countries around the world have experienced an increase in their debt levels due to the pandemic. If these increases in public indebtedness are justified in the short run, and are in line with what suggested the major international economic organizations, the impact of public debt on growth has been widely studied in the literature and there are numerous theoretical papers that document a negative relationship between public debt and growth in the long run (Diamond, 1965; Blanchard, 1985; Barro, 1990; Saint-Paul, 1992). The famous paper by Reinhart and Rogoff (2010) documents a threshold effects and argues that a stabilization ratio of public debt larger than 90% of GDP is associated with significant decreases in the rate of growth.[2] To summarize, it is widely accepted that high levels of public indebtedness are associated with lower economic performances in the long run.

From these observations, it appears that the economic literature has identified two potential threats to long run economic development: resource exhaustion and public indebtedness. Despite the large literature on these topics, the two threats have never been studied conjointly. This is quite surprisingly since both issues are linked to agents' saving behaviors. In a nutshell, it is well known that public debt may crowd out investment in physical capital, and that physical capital

[1]Several other countries applies explicit public debt management rules (see Appendix A).

[2]While this paper is subject to controversy due to a shortcoming in the methodology (see Herndon et al., 2014), this type of results strongly influenced IMF policy recommendation in the last decades.

accumulation is one major way to compensate for resource exhaustion.

The interaction between public debt and resource exhaustion has not been widely studied in the literature. This is probably due to the situation experienced by some oil-producing countries (mainly middle-eastern ones) for which the resource rents ease the fiscal constraints, leading to low level of public indebtedness. Nevertheless, one can argue that these experiences are not representative of all resource-rich countries experiences. In Appendix A, we provide some empirical evidence that there exist no clear cut elements on a strong negative relationship between resource wealth and public indebtedness. Instead, we show that experiences regarding public debt and resource wealth are diverse and that there exist resource-rich countries with a significant level of public debt. For example, a country like Algeria combines a positive net debt around 44% of its GDP to relatively abundant resources. In addition, we show that the type of resources under consideration matters. If we find, on average, that large fuel exporters have lower levels of debt, this is not true for mineral exporters. With this elements in mind, we think that the interactions between debt and natural resources should be studied, and this is what we propose in the present paper. At this point, it should be noticed that we are not interested in the role that resource rents may have on public debt,[3] but rather in the impacts of public debt on the speed of resource exhaustion and economic growth, which is a different question that may be consistent even when the level of public indebtedness is rather low.[4]

In the present paper, we propose an OLG model in which firms produce combining labor, capital, non-renewable resources, and public infrastructures. Public infrastructures are provided by the government and could be financed by taxes or debt. We consider however that the government faces public finances stabilization constraints in the spirit of *e.g.* Maastricht Treaty. Such a framework allows us to study the impact of the public debt stabilization ratio on the sustainability of growth in the presence of a necessary finite input, namely non-renewable resources.

The paper shows that high public debt stabilization ratios are incompatible with sustainable growth, because they prevent the existence of a positive balanced growth path. In addition, it shows that the rate of growth achieved by the market economy is negatively linked with the size of

[3]We acknowledge that this is an important question and we refer the interested reader to Manzano and Rigobon (2001).

[4]The results presented in this paper are consistent even for negative public debt levels although bounded away from below.

the stabilization ratio. The underlying mechanisms are quite simple. As usual, public debt crowds out savings from physical capital accumulation. More interestingly, public debt also increases the rate of resource extraction, because it crowds out households investment in the non-renewable resource stock. More resources are thus used in the production by firms in early periods, which threatens future growth feasibility.

The paper also analyzes the centralized economy where a benevolent social planner is free to choose the economic path on the ground of its time preference. Intuitively, the more the social planner cares about future generations, the lower will be the extraction rate and the larger will be the rate of growth. We then show, as a result of the monotonic and negative relationship occurring between the public debt stabilization ratio and the economy rate of growth, that there exists one level of public debt stabilization ratio that allows to decentralize the optimal allocation.

This paper is related to the seminal papers on the feasibility of growth with non-renewable resources written in the 1970s that have highlighted the importance of (exogenous) technical progress, increasing returns to scale and substitution between natural and man-made production factor (Dasgupta and Heal, 1974, 1979; Solow, 1974; Stiglitz, 1974). This literature enjoyed a revival with the development of endogenous growth models. The interested reader can refer to Barbier (1999). One weakness of these papers is their use of the Infinitely Lived Agents (ILA) framework which implicitly assume dynastic altruism between agents, not supported by empirical results (Altonji et al., 1992). Such an assumption makes a sustainable management of resources more probable since each generation takes care of the following as of itself. This observation has lead to the work of Agnani et al. (2005) who have addressed this shortcoming using an OLG framework to analyze the sustainability of growth with a necessary non-renewable resource. They have shown that economic growth then requires that the labor share in production be sufficiently high to allow a level of savings (and then capital accumulation) sufficient to compensate for resource depletion. We improve this literature by considering the existence of public productive infrastructures financed by debt, that could crowd out saving from capital accumulation. Our contribution highlights the impact of public debt on resource use sustainability.

This paper is also related to the literature on public debt. Using an OLG framework, Diamond (1965) shows that debt crowds out capital accumulation and reduces growth, a result confirmed by Blanchard (1985). The negative impact of public debt on growth also appears in the endogenous growth framework (Barro, 1990; Saint-Paul, 1992; Bräuningen, 2005). Futagami et al. (2008) introduce public debt in an endogenous growth model with productive government spending. They show that financing productive public expenditures with debt might be growth reducing (enhancing) in developed (developing) countries. In a close set-up, Minea and Villieu (2013) show that increases in public debt reduce growth. As in our own, these papers introduce public debt stabilization targets in the size of the economy.[5] Futagami and Konishi (2023) use an OLG model with endogenous growth to examine fiscal sustainability under two distinct fiscal rules: a primary balance-to-GDP rule and a deficit-to-GDP rule.[6] The authors demonstrate that achieving fiscal sustainability is possible with a constant deficit-to-GDP rule, provided that the initial levels of both the debt-to-GDP ratio and the public deficit-to-GDP ratio are sufficiently low. However, when employing a primary balance-to-GDP rule, attaining fiscal sustainability becomes challenging if the initial primary balance is either negative or zero. Conversely, when the primary balance is in surplus and the debt is adequately small, fiscal sustainability is ensured. A discussion on the dynamics of public debt under alternative fiscal rules in an OLG model of endogenous growth with productive public goods may also be found in Agénor and Yilmaz (2017). The stabilization of government debt has also been studied in Michel et al. (2010). In the present paper, we adopt a fixed public debt-to-GDP ratio, ensuring fiscal sustainability, and delve into analyzing how this ratio influences the sustainability of resource extraction and economic development.

Several papers exist that incorporate the environment into the discussion surrounding public debt and economic growth. Notably, Fodha and Seegmuller (2012) analyze the implications of an environmental tax under public debt stabilization constraint. This work has been extended in several ways (Fodha and Seegmuller, 2014; Clootens, 2017; Fodha et al., 2018; Davin et al., 2022, 2023).[7] However, these papers analyze the links between debt and environmental quality and don't take into account the resource exhaustion issue in their analysis. The present paper is an attempt to fill this gap. In this aspect, our paper does not focus on typical issues such as pollution

[5]They actually differ in the type of target. In Futagami et al. (2008), the size of the economy is captured with private capital while it is captured with GDP in Minea and Villieu (2013). This slight change explain the differences in their results.

[6]The concept of fiscal sustainability in this context pertains to the convergence of the public debt-to-GDP ratio toward a constant level. For a more comprehensive exploration of the notion of fiscal sustainability, readers are encouraged to consult Debrun et al. (2019).

[7]There also exist some empirical works on the relationship between public debt and the environment. See for example Carratù et al. (2019).

and emissions that are detrimental to the quality of the environment but rather investigates the problems related to the exhaustion of non-renewable resources as fossils and minerals in an economy with debt.[8]

The remainder of the paper is organized as follows. Section 2 presents the model. The competitive equilibrium and the balanced growth path are characterized in Section 3. Section 4 analyses how movements in public debt and public stabilization ratios affect the balanced growth path. Section 5 is devoted to the presentation of the central planner problem and Section 6 presents the decentralization of the optimal allocation. Section 7 concludes.

2. The Model

The model is in essence that of Diamond (1965) in which productive public expenditures in infrastructures and non-renewable resources are introduced. For the sake of simplicity, the size of population has been normalized to one and a no demographical growth assumption is used. Lowercase letters represent per worker variables.

2.1. The Resource

A grandfathering economy is considered here. Thus, the economy is endowed with a finite quantity m_{-1} of a non-renewable resource which is held by the first generation of agents. At each date t, old agents sell their resource endowments m_{t-1} to the new generation of agents and to firms at a price p_t. A quantity x_t is used in the production. Thus the rate of resource use is given by

$$q_t = \frac{x_t}{m_{t-1}} \tag{1}$$

The law of motion of the resource stock is thus

$$m_t = (1 - q_t)m_{t-1} \tag{2}$$

Since the resource is finite and non-renewable, the initial stock imposes a limit on total quantities that can be extracted

$$1 \geq \sum_{t=0}^{+\infty} q_t \prod_{j=1}^{t} (1 - q_{j-1}) \tag{3}$$

[8]This is the reason why we decide to not consider an explicit demand for environmental quality. Nevertheless, we do not deny that emissions are often a byproduct of resource use that may affect the utility of households, but this is beyond the purpose of our analysis. Clootens (2021) analyzes the effects of flow emissions in an OLG economy with non-renewable resources where emissions are a byproduct of resource use and extraction.

This condition establishes that we cannot extract more resources than the quantity available at the beginning of the time horizon. This implies that the sequence of the extraction rates is subject to some restrictions.

2.2. The Consumers

In each period of time, the economy is composed of two generations of finite lived agents. An agent born in period t maximizes the following inter-temporal log-utility function

$$u(c_t; d_{t+1}) = \ln(c_t) + \frac{1}{1+\rho}\ln(d_{t+1}) \tag{4}$$

where c represents the consumption while young, d the consumption while old, and ρ is the individual rate of time preferences. In his/her first period of life, the agent works and earns a wage w. This wage is used to consume c, to save s in capital or in public bonds yielding a real interest rate r, to buy property rights on the resource stock m at a price p, and to pay lump-sum taxes τ. In his/her second period of life, the agent uses his/her savings (both in capital, public bonds and in resources) to consume. Thus, normalizing to one the price of the output, his/her budget constraints when young and old are respectively

$$w_t - \tau_t = c_t + s_t + p_t m_t \tag{5}$$

$$d_{t+1} = (1 + r_{t+1})s_t + p_{t+1}m_t \tag{6}$$

The intertemporal budget constraint is then

$$w_t = c_t + \frac{d_{t+1}}{1+r_{t+1}} - \frac{p_{t+1}m_t}{1+r_{t+1}} + p_t m_t + \tau_t \tag{7}$$

Maximization of (4) subject to (7) leads to the following first order conditions

$$\frac{d_{t+1}}{c_t} = \frac{1+r_{t+1}}{1+\rho} \tag{8}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1} \tag{9}$$

(8) is the standard Euler equation establishing that the marginal rate of substitution between the two consumptions has to be equal to their relative price. (9) represents the Hotelling rule. It is a non-arbitrary condition between the two types of savings which sets that the resource price has to increase at the interest rate. It means that our agents are indifferent between the three types of assets: resources, public bonds, and capital.

2.3. The Government

The government has the responsibility to provide indivisible and non excludable public infrastructures H using debt B or taxes $\mathcal{T}$. In addition, we assume that there are no congestion effects. The flow of new public infrastructures is G and public infrastructures depreciate at a rate $\delta_H \in [0; 1]$. The stock of public infrastructures in t writes

$$H_t = G_t + (1 - \delta_H)H_{t-1} \tag{10}$$

The government's budgetary constraint writes (in per worker variable)

$$b_{t+1} = (1 + r_t)b_t - \tau_t + g_t \tag{11}$$

We assume that the government faces public finances stabilization rules in the spirit of the Maastricht Treaty which imposes to Eurozone countries a public debt no larger than 60% of GDP.[9] We thus suppose that the public debt-to-capital ratio is a constant $\hat{B}$.[10] In the same spirit, we assume that public expenditures are kept constant as a share of national capital. We denote this constant $\hat{G}$. The choice of focusing simultaneously on two fiscal instruments implies that taxes are endogenously given in a way that balances the government budgetary constraint.

2.4. The Firms

The production sector of this economy is composed by a large number of perfectly competitive firms. This means that each firm is price taker and does not take into account the effect of its own production decisions on the other firms. Each firm produces the consumption and investment good Y by combining capital K, resources X, labor L and using public infrastructures H. Capital and public infrastructures depreciate respectively at rates $\delta_K \in [0, 1]$ and $\delta_H \in [0, 1]$ over a period. A represents the technical level and grows at an exogenous rate a. The production function of the representative firm is

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} X_t^{\nu} H_t^{\theta}$$

[9]Beside the countries concerned by the Maastrich Treaty, there exist many countries that have implemented explicit debt management rules (see Appendix A). In addition, our modeling framework also applies to many countries that, although not subject to explicit debt rules, do not allow their debt to grow indefinitely.

[10]It will be demonstrated later (Proposition 2) that GDP and the capital stock increase at the same rate along the BGP. Thus, along the BGP, the public debt-to-GDP ratio is also constant.

We assume that technology displays constant returns to scale at the private level, *i.e.* $\alpha+\beta+\nu = 1$. However, at the social level, returns to scale are increasing in view of the contribution to production of public infrastructures. Such a contribution is measured by the parameter $\theta > 0$. Nevertheless, we should introduce an upper bound on θ, namely $\alpha + \theta < 1$. If such an equality would not hold, the problem of the rate of resource extraction would no more be so dramatically relevant for the purpose of growth. In fact, the contribution of private capital and infrastructure to production would be large enough to display constant or increasing returns and therefore the maximization problem would not be well defined. Of course, such an inequality implies $\theta < \beta+\nu$.[11]

It follows that real aggregate profits Π are given by

$$\Pi_t = A_t K_t^\alpha L_t^\beta X_t^\nu H_t^\theta - (r_t + \delta_K)K_t - w_t L_t - p_t X_t \tag{12}$$

Production in per worker terms writes

$$y_t = A_t k_t^\alpha x_t^\nu H_t^\theta \tag{13}$$

where lower case letters denote per capita variables.

$$r_t = \alpha A_t k_t^{\alpha-1} x_t^\nu H_t^\theta - \delta_K \tag{14}$$

$$p_t = \nu A_t k_t^\alpha x_t^{\nu-1} H_t^\theta \tag{15}$$

$$w_t = \beta A_t k_t^\alpha x_t^\nu H_t^\theta \tag{16}$$

Notice that conditions (14)-(16) simply claim that production factor are remunerated at their marginal productivity. In addition, it is immediate to verify that public infrastructures increase all the marginal productivities.

3. Intertemporal Equilibrium and Balanced Growth Path

Using the following market clearing condition

$$s_t = k_{t+1} + b_{t+1} \tag{17}$$

[11] Notice that our production function differs from Barro (1990) one where A_t is constant, $\nu = 0$ and $\theta = 1 - \alpha$. While in Barro (1990) the main growth engine rest upon constant returns in aggregate capital at the social level, in our case, by contrast, perpetual growth will be mainly the fruit of the exogenous technological progress and resource preservation.

and equations (1), (2), (5), (6), (8), (9),(10), (11), (13), (14), (15), and (16), an intertemporal equilibrium may be found. We denote $\mu_{\ell,t+1} = \frac{\ell_{t+1}}{\ell_t}$ the growth factor between t and $t+1$ of any variable ℓ, and we define $\hat{H}_t = H_t/K_t$, which represents the size of public infrastructure relative to the economy. The next proposition characterizes the intertemporal equilibrium of the decentralized economy. Such an equilibrium involves the evolution across time of μ_k, q, and $\hat{H}$. μ_k and $\hat{H}$ appears lagged once and q lagged twice.

Proposition 1. An intertemporal equilibrium is defined by the following equations:

$$\mu_{k,t+1}\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right] = \left[\frac{\beta q_t - \nu(1-q_t)(2+\rho) - \alpha\hat{B}q_t}{\alpha(2+\rho)q_t}\right] \times \\ \left[(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1}\left[\hat{G}\hat{H}_{t-1}^{-1}\mu_{k,t}+(1-\delta_H)\right]^{\theta} - 1 + \delta_K\right] - \frac{\hat{G}+(1-\delta_K)\hat{B}}{2+\rho} \tag{18}$$

$$\frac{(1+a)\mu_{k,t+1}^{\alpha}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-1}\left[\hat{G}\hat{H}_t^{-1}\mu_{k,t+1}+(1-\delta_H)\right]^{\theta} - 1 + \delta_K}{(1+a)\mu_{k,}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1}\left[\hat{G}\hat{H}_{t-1}^{-1}\mu_{k,t}+(1-\delta_H)\right]^{\theta} - 1 + \delta_K} = \\ (1+a)\mu_{k,t+1}^{\alpha-1}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu}\left[\hat{G}\hat{H}_t^{-1}\mu_{k,t+1}+(1-\delta_H)\right]^{\theta} \tag{19}$$

$$\hat{H}_t = \hat{G} + (1-\delta_H)\hat{H}_{t-1}\mu_{k,t}^{-1} \tag{20}$$

together with the initial conditions $\mu_{k,0}$, q_0, q_{-1} and $\hat{H}_{-1}$ and the usual transversality condition. Let us emphasize that the unique not inherited condition is q_0.

Proof. Proof is reported in Appendix B. □

(18) represents the dynamics of the accumulation of wealth by households. More precisely, it explains how households allocate their savings between resources, public bonds, and capital. (19) represents the dynamics of assets prices in the economy. (20) represents the dynamics of infrastructures size in the economy.

The present paper focuses on the balanced growth path because it constitutes the only case where a non-declining consumption path may be sustained in the presence of necessary non-renewable resources, as pointed out by Agnani et al. (2005). Due to the presence of public debt and public expenditures, we will use the following assumption that ensures the existence of a balanced growth path.

Assumption 1. The debt-to-capital ratio satisfies $\max\left\{-\frac{(2+\rho)}{(1+\rho)}, -\frac{\hat{G}}{1-\delta_K}\right\} < \hat{B} < \beta/\alpha$.

This assumption is not restrictive. Indeed, standard parameter calibration implies that the debt-to-capital ratio should be lower than 2, which is a threshold rarely achieved. In addition, our paper is interested in positive levels of debt stabilization as it is the case for most developed countries. However, our results are fully consistent even for negative public debt stabilization ratios although bounded away from below.[12]

In the present paper, we focus on the balanced growth path, *i.e.* paths characterized by constant growth factors. We can thus introduce the following proposition and show the existence of a unique balanced growth path.

Proposition 2. Under Assumption 1, the balanced growth path exists, is unique, and is defined by the following equations

$$\mu = \mu_y = \mu_k = \mu_c = \mu_d = \mu_s = \mu_b = \mu_g = \mu_w = \mu_\tau = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}$$

$$\mu_x = \mu_m = 1-q$$

$$\mu_p = \frac{\mu_y}{\mu_x} = 1+r = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}-1}$$

$$\mu_r = 1$$

$$\mu_A = 1+a$$

and q solving the following non-linear equation

$$LHS(q) \equiv \frac{\alpha(1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}[(2+\rho)+(1+\rho)\hat{B}]q + \alpha\hat{G}q + (1-\delta_K)\hat{B}\alpha q}{\beta q - (2+\rho)\nu(1-q) - \alpha\hat{B}q}$$
$$= (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{v}{1-\alpha-\theta}-1} - (1-\delta_K) \equiv RHS(q) \quad (21)$$

Proof. Proof is reported in Appendix C. □

The study of the local stability around the balanced growth path is reported in Appendix F. It shows numerically that the standard configuration of the steady state corresponds to the saddle path one and is determinate.

The balanced growth path could represent either a growing or a decreasing economy. In the present paper, we are interested in the sustainability of positive growth defined as follows:[13]

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[12]Notice that the lower bound is a sufficient condition (not necessary) that ensures that all results given in the paper hold. It could be also noticed that in an OLG framework, in which one period represents about 30 years, the depreciation rate of capital approaches unity so that $-\hat{G}/(1-\delta_K)$ is very low. It implies that the lower bound is probably low enough to be empirically plausible for almost all countries.

[13]It can be noticed that we use here a concept of weak sustainability which follows naturally from the use of the Cobb-Douglas production function that we consider.

Definition 1. A balanced growth path is defined as sustainable if it is compatible with non-declining per capita consumption, *i.e.* if $\mu \geq 1$.

Proposition 2 has important implications in terms of sustainability. Indeed, a balanced growth path is the only path compatible with a non-declining consumption (Agnani et al., 2005). High public debt-to-capital stabilization ratios, preventing the existence of a balanced growth path, thus prevent any sustainable growth possibility.

According to Definition 1 and taking into account Proposition 2, a sustainable balanced growth path requires

$$\frac{(2+\rho)\nu}{\beta - \alpha\hat{B} + (2+\rho)\nu} < q^* \leq 1 - (1+a)^{\frac{-1}{\nu}}$$

Thus, the economy is contracting if $1 - (1+a)^{\frac{-1}{\nu}} < \frac{(2+\rho)\nu}{\beta - \alpha\hat{B} + (2+\rho)\nu}$ which is less likely to hold for low level of public debt-to-capital stabilization ratios. Thus, a high level of public debt is associated with an unsustainable use of resources.

Since our economy can display long-run positive as well as negative balanced growth, some restrictions on the structural as well as policy parameters are necessary in order to guarantee a sustainable balanced growth path. The following proposition explains such a point.

Proposition 3. If $\frac{\beta}{\alpha} > \hat{B} > \frac{1}{\alpha}\left[\beta + (2+\rho)\nu - \frac{(2+\rho)\nu}{1-(1+a)^{-1/\nu}}\right]$, the economy is characterized by an unsustainable balanced growth path.

Proof. $1 - (1+a)^{\frac{-1}{\nu}} < \frac{(2+\rho)\nu}{\beta - \alpha\hat{B} + (2+\rho)\nu} \Leftrightarrow \hat{B} > \frac{1}{\alpha}\left[\beta + (2+\rho)\nu - \frac{(2+\rho)\nu}{1-(1+a)^{-1/\nu}}\right]$ □

Proposition 3 shows that there exists an intermediary level of debt-to-capital which is compatible with a balanced growth path, but which is (environmentally) unsustainable, in the sense that the corresponding rhythm of resource extraction is to high to ensure perpetual growth. Public debt thus appears as a threat to sustainable development.

Figure 1 summarizes the previous findings. Depending on the level of the public debt-to-capital ratio, the economy could experience no balanced growth, negative balanced growth, or positive balanced growth. Only the last case could be defined as sustainable.

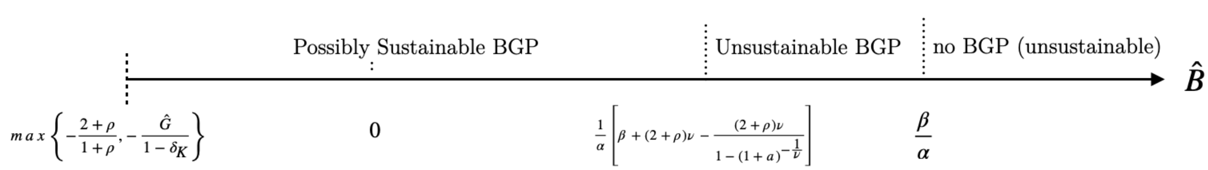

Figure 1: The growth experience is debt-to-capital dependent

For low levels of public debt-to-capital, the economy experiences sustainable growth. Above a certain threshold, the growth becomes negative, but is still balanced. Then, if the debt-to-capital becomes very large, the existence of balanced growth is no longer possible, which disable the sustainability of development. Interestingly, no such conditions are found in the model once the resource dimension is removed. In the latter case, the rate of growth is simply $(1+a)^{\frac{1}{1-\alpha-\theta}}$. While this observation could be attributed to our modeling framework, it nevertheless proves the importance of the natural resource dimension in the debt-growth nexus. In Table 1, we propose to calibrate the model to obtain values for both thresholds. Two calibrations are thus performed. Calibration 1 captures standard developed economies features, while calibration 2 is more consistent for emerging or developing countries. For high income economies, the two thresholds are very close, so the likelihood to be trapped in the unsustainable BGP is very low. Interestingly, the spread between the two thresholds increases when the resource share increases, which is consistent for emerging and developing economies. It makes the occurrence of unsustainable balanced growth more likely.[14] This little exercise illustrate the importance of considering the resource dimension in studies that deal with public debt sustainability. It paves the way for future research analyzing the sustainability of a given country's debt.

	Calibration 1	Calibration 2
α	0.3	0.2
β	0.65	0.5
ν	0.05	0.3
β/α	2.16667	2.5
$\frac{1}{\alpha}\left[\beta+(2+\rho)\nu-\frac{(2+\rho)\nu}{1-(1+a)^{-\frac{1}{\nu}}}\right]$	2.16664	1.55875

$\rho = 0.016$ and $a = 0.028$ (annual rates)

Table 1: Threshold Values

4. The Impact of Public Finances Stabilization Ratios on Growth

This section analyses how movements of public debt-to-capital and public expenditures-to-capital ratio affects the rate of growth of the economy using comparative statics. In the following

[14]Anecdotally, depending on the labor share importance relative to the capital share, both thresholds may increase or decrease.

propositions, we prove that an increase in the public expenditures stabilization ratio or in the public debt stabilization ratio is growth detrimental.

Proposition 4. An increase in the public expenditures-to-capital stabilization ratio increases the extraction rate and decreases the rate of growth.

Proof. Proof is reported in Appendix D. □

This effect is quite intuitive. When the weight of public expenditures in the economy increases, it implies an increase in taxes and reduces the disposable income of households. They consume less but they also save less in both capital and resources. The rate of resource extraction increases which in turn affects negatively the rate of growth. More capital is needed in the long run to compensate for resource depletion but in the same time less capital is offered by households.

Proposition 5. An increase of the public debt-to-capital stabilization ratio increases the extraction rate and decreases the rate of growth.

Proof. Proof is reported in Appendix E. □

In the long run, an increase in $\hat{B}$ imposes a higher level of taxes which depresses growth as explained above. In addition, an increase in the level of debt-to-capital stabilization ratio implies a larger crowding out effect of public debt on other assets (capital and resources) since a larger share of household savings is devoted to public bonds. Capital accumulation is reduced and resource use increases.

5. The Central Planner's Problem

This section is devoted to the study of the social planner program. We assume that the social planner faces the same expenditure stabilization rule than the one we imposed previously to the market economy. Formally, it means that the public expenditures-to capital ratio should be stabilized at a level $\hat{G}$. This assumption has two advantages. i) It allows to see how the optimal rate of growth is affected by a change in the ratio of public expenditures-to-capital, i.e. in change in budgetary treaties; ii) It will also allow us to find an instrument (the public debt-to-capital stabilization ratio) that is able to decentralize the optimal equilibrium for each level of public expenditures stabilization ratio. Let's assume that the social planner discounts time at a rate ψ. As a consequence, it solves the following Ramsey problem:

$$\max_{\{c_t,d_t,k_t,m_t,H_t\}_{t=0}^{+\infty}} \frac{1}{1+\rho}\ln(d_0)+\sum_{t=0}^{+\infty}\frac{1}{(1+\psi)^{t+1}}\left[\ln(c_t)+\frac{1}{1+\rho}\ln(d_{t+1})\right] \tag{22}$$

subject to:

$$c_t+d_t+k_{t+1}+\left(\hat{G}-(1-\delta_k)\right)k_t = A_t k_t^{\alpha} x_t^{\nu} H_t^{\theta} \tag{23}$$

$$A_{t+1}=(1+a)A_t \tag{24}$$

$$m_t = m_{t-1}-x_t \tag{25}$$

$$m_{-1}=\sum_{t=0}^{+\infty} q_t m_{t-1} \tag{26}$$

$$H_t=\hat{G}k_t+(1-\delta_H)H_{t-1} \tag{27}$$

$$k_0>0, A_0>0, m_{-1}>0, H_{-1}>0 \text{ given} \tag{28}$$

where (23) is the resource constraint of the economy, (24) is the law of technical progress, (25) is the law of motion of the resource stock, (26) is a total exhaustibility condition for the resources, (27) is the law of accumulation of public infrastructure while (28) represents initial endowments. The first order condition of the planner's program may be reduced to the following system:

$$\frac{1+\psi}{1+\rho}=\frac{d_t}{c_t} \tag{29}$$

$$\frac{d_{t+1}}{c_t}(1+\rho)=\frac{A_{t+1}k_{t+1}^{\alpha}H_{t+1}^{\theta}\nu x_{t+1}^{\nu-1}}{A_t k_t^{\alpha}H_t^{\theta}\nu x_t^{\nu-1}} \tag{30}$$

$$\begin{aligned}\frac{A_t H_t^{\theta} k_t^{\alpha}\nu x_t^{\nu-1}}{A_{t-1}H_{t-1}^{\theta}k_{t-1}^{\alpha}\nu x_{t-1}^{\nu-1}} &= \hat{G}A_t\theta H_t^{\theta-1}k_t^{\alpha}x_t^{\nu}+A_t H_t^{\theta}\alpha k_t^{\alpha-1}x_t^{\nu}+(1-\delta_k)-\hat{G}\\ &+(1-\delta_H)\left[1-\left[\frac{A_t H_t^{\theta}k_t^{\alpha}\nu x_t^{\nu-1}}{A_{t+1}H_{t+1}^{\theta}k_{t+1}^{\alpha}\nu x_{t+1}^{\nu-1}}\right]\left[A_{t+1}H_{t+1}^{\theta}\alpha k_{t+1}^{\alpha-1}x_{t+1}^{\nu}+(1-\delta_k)-\hat{G}\right]\right]\end{aligned} \tag{31}$$

$$\lim_{t\to+\infty}\left(\frac{1}{1+\psi}\right)^t\frac{k_{t+1}}{c_t}=0 \tag{32}$$

where (29) and (30) are, respectively, the intergenerational and intragenerational optimality conditions, (31) characterizes the optimal intertemporal resources allocation and (32) is the transversality condition.

Combining equations (23)-(31), the dynamics of the economy is defined by the following system:

$$\frac{(1+a)\,\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-1}\left[(1+a)\,\mu_{H,t}^{\theta}\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-1}-(1-\delta_k)+\hat{G}-(1-\delta_H)\left[(1-\delta_k)-\hat{G}\right]\right]+(1-\delta_H)\left[(1-\delta_k)-\hat{G}\right]}{(1+a)\,\mu_{H,t}^{\theta}\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-1}\left[(1+a)\,\mu_{H,t-1}^{\theta}\mu_{k,t-1}^{\alpha}\mu_{x,t-1}^{\nu-1}-(1-\delta_k)+\hat{G}-(1-\delta_H)\left[(1-\delta_k)-\hat{G}\right]\right]+(1-\delta_H)\left[(1-\delta_k)-\hat{G}\right]}=$$
$$(1+a)\,\mu_{H,t}^{\theta}\mu_{k,t}^{\alpha-1}\mu_{x,t}^{\nu}\frac{\left[\hat{G}\theta\hat{H}_t^{-1}+\alpha\right](1+a)\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-1}+(1-\delta_H)\alpha(1+a)\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha-1}\mu_{x,t+1}^{\nu}\left[(1+a)\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-1}-1\right]}{\left[\hat{G}\theta\hat{H}_{t-1}^{-1}+\alpha\right](1+a)\mu_{H,t+1}^{\theta}\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-1}+(1-\delta_H)\alpha(1+a)\mu_{H,t}^{\theta}\mu_{k,t}^{\alpha-1}\mu_{x,t}^{\nu}\left[(1+a)\mu_{H,t}^{\theta}\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-1}-1\right]} \tag{33}$$

$$(1+a)\,\mu_{k,t+1}^{\alpha}\mu_{H,t+1}^{\theta}\mu_{x,t+1}^{\nu-1}=(1+\psi)\,\mu_{k,t+1}\frac{\gamma_{c,t+1}}{\gamma_{c,t}} \tag{34}$$

$$\gamma_{c,t}\left[1+\frac{1+\psi}{1+\rho}\right]+\mu_{k,t+1}+\hat{G}-(1-\delta_k)=$$
$$\frac{(1+a)\,\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-1}\left[(1+a)\,\mu_{H,t}^{\theta}\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-1}-(1-\delta_k)+\hat{G}-(1-\delta_H)\left[(1-\delta_k)-\hat{G}\right]\right]+(1-\delta_H)\left[(1-\delta_k)-\hat{G}\right]}{\left[\hat{G}\theta\hat{H}_t^{-1}+\alpha\right](1+a)\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-1}+(1-\delta_H)\alpha(1+a)\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha-1}\mu_{x,t+1}^{\nu}\left[(1+a)\mu_{H,t+1}^{\theta}\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-1}-1\right]} \tag{35}$$

$$\hat{H}_t=\hat{G}+(1-\delta_H)\,\hat{H}_{t-1}\mu_{k,t}^{-1} \tag{36}$$

where $\gamma_{c,t} \equiv c_t/k_t$ stands for the consumption to capital ratio in period t and $\mu_{k,t} \equiv k_t/k_{t-1}$, $\mu_{H,t} \equiv H_t/H_{t-1}$ $\mu_{x,t} \equiv x_t/x_{t-1}$, $\hat{H}_t = H_t/k_t$, as it was the case in the decentralized economy. Evaluating the system at the BGP and using the definition of the extraction rate $q_t = x_t/m_{t-1}$, one can define the balanced growth path of this Ramsey economy.

Proposition 6. The optimal balanced growth path is defined by the following growth rates:

$$\tilde{\mu}_y = \tilde{\mu}_k = \tilde{\mu}_H = \tilde{\mu}_c = \tilde{\mu}_d = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}}$$
$$\tilde{\mu}_x = \tilde{\mu}_m = 1-\tilde{q}$$
$$\tilde{\mu}_A = 1+a$$
$$\text{where } \tilde{q} = \frac{\psi}{1+\psi}$$

Proof. Proof is reported in Appendix G □

Notice the important feature according to which the optimal extraction rate is completely determined by the social rate ψ of time preference. We can see here that neither the stabilization rule for public expenditures nor the depreciation rate of private and public capital don't affect optimal extraction and growth, which are solely determined by the exogenous rate of technological progress, the social rate of time preference and factor elasticities in the production function. The Ramsey economy is thus a kind of cake-eating problem where the speed of resource exhaustion depends on a trade-off between different generations' welfare. It thus appears that the optimal rate of growth depends also on such a trade-off. When ψ is low, the rate of resource use is low so that the resource stock is preserved for future generations and long-run growth is enhanced. In opposition, when ψ is large, the rate of resource use is large so that the resource stock exhaustion goes faster depressing future growth.

6. Decentralization of the Optimal Allocation

Comparing the growth rates that appear in Propositions 2 and 6, it is immediate that the decentralization of the optimal allocation requires to put the market equilibrium extraction rate at the optimal level.

Proposition 7. The optimal allocation may be decentralized with a public debt-to-capital ratio $\tilde{\hat{B}}$.

Proof. The debt-to-capital ratio affects the rate of resource extraction as highlighted in Proposition 5. It is thus possible to find the level of public debt-to-capital ratio such that $q = \tilde{q}$. Using equation (21), it can be inferred that the optimal level of debt-to-capital ratio $\tilde{\hat{B}}$ should satisfy

$$\frac{\alpha(1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}}[(2+\rho)+(1+\rho)\tilde{\hat{B}}]\tilde{q}+\alpha\hat{G}\tilde{q}+(1-\delta_k)\alpha\tilde{q}\tilde{\hat{B}}}{\beta\tilde{q}-(2+\rho)\nu(1-\tilde{q})-\alpha\tilde{\hat{B}}\tilde{q}} = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{v}{1-\alpha-\theta}-1}-(1-\delta_k)$$

where $\tilde{q} = \psi/(1+\psi)$. It is straightforward to demonstrate that this condition imposes

$$\tilde{\hat{B}} = \frac{\left[(1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}-1}-(1-\delta_k)\right][\beta\tilde{q}-(2+\rho)\nu(1-\tilde{q})]-\alpha\tilde{q}\left[(2+\rho)(1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}}+\hat{G}\right]}{\alpha\tilde{q}(1+a)^{\frac{1}{1-\alpha-\theta}}(1-\tilde{q})^{\frac{\nu}{1-\alpha-\theta}-1}[1+(1+\rho)(1-\tilde{q})]} \tag{37}$$

□

From equation (37), it immediately appears that a higher level of public expenditures-to-capital ratio imposes a lower level of debt. The reason is quite simple. The optimal extraction rate is exclusively defined by the social rate of time preference. However, the market equilibrium extraction rate is endogenously determined and $\hat{G}$ and $\hat{B}$ are key parameters in its determination as highlighted by Propositions 4 and 5. Since increases in each of these two parameters increase the extraction rate, it is not surprising that an increase of the (exogenous) level of public expenditures-to-capital ratio reduces the optimal level of public debt-to-capital ratio.

7. Conclusion

Environmental issues are ones of the main cores of economic agenda. The faith in a lasting capital accumulation process and in a never-ending increase in GDP, consumption and investment is more and more questioned on the ground of an environment characterized by its limited amount of resources not all of them renewable. This seems apparently to put an upper bound on the increase of what Adam Smith referred as to the "wealth of the nations". Another highly questioned issue in recent literature is the impact of public debt on growth. Indeed, it is often thought that savings devoted to finance public debt is crowded out from productive capital accumulation. However, public expenditures allowed by the emission of public debt may be growth enhancing since they can endow the economy with productive infrastructures (Barro, 1990). In addition, public debt can be seen as a speculative bubble able to restore dynamic efficiency in economies characterized by capital over-accumulation (Tirole, 1985).

In this paper we have focused on such issues by studying an OLG model with stationary population, log-linear preferences and a Cobb-Douglas technology of production in which individuals accumulate physical capital, a non-renewable resource, and government liabilities. The government fiscal policy consists in targeting the public debt-to-capital ratio and the public expenditures-to-capital ratio, where public spending is used to finance public infrastructure that contribute to production. As a consequence, in order to respect public budget constraint, taxation is endogenously adjusted in response to the evolution of aggregate variables. Within such a framework, we have studied the impact of the fiscal rules on the balanced growth path. More in details, we have proved that a higher public debt stabilization ratio and/or a larger public spending stabilization ratio is growth detrimental, since it compels agents to reallocate their savings from physical capital and the stock of natural resources to debt and this in turn accelerates the extraction rate at the cost of reducing the long run sustainability of the system.

In a further section of the paper we have carried out an analysis of the centralized economy where a benevolent social planner is free to choose the economic path on the ground of its time preference and subject uniquely on the public expenditures stabilization target. The main result we obtain is that the stationary growth rate increases as soon as the social planner cares more and more about future generations and therefore tries to avoid a too much fast exploitation of the non-renewable resource which may be detrimental for the future economic growth. In addition, we show that the optimal balanced growth path can be opportunely decentralized by calibrating the fiscal instruments, as the public debt ratio; this is the immediate consequence of the monotonic and negative relationship occurring between public debt stabilization ratio and the economy rate of growth.

Since the rate of growth chosen by the social planner is increasing in his degree of patience, we find that the optimal public debt stabilization ratio will be lower in economies run by less short-sighted rulers. Therefore, in some circumstances, larger public debts could be welfare-improving. Of course, such results hold within our economy characterized by short-lived agents, log-linear preferences, Cobb-Douglas technology and in the absence of increasing returns to scale sufficient to generate unbounded growth (with the necessary requirement of exogenous technological progress). By removing each of these hypotheses, one could improve the analysis in terms of the role of the fiscal policy on growth, of the equilibrium (in)determinacy and of the extraction rate of non-renewable resources. In addition, the polluting features of non-renewable resources could also be considered by introducing environmental quality in the utility function. We leave such purposes for future research.

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Appendix A. Public Debt and Resource Wealth : Some Descriptive Statistics

In this Appendix, we propose to give the reader some descriptive statistics on public debt and resource wealth. Notably, Table A.2 reports both gross debt and net debt, resource rents, fuel exports and mineral exports shares of total exports, and informs the reader on the existence of an explicit debt rule. It appears that some resource rich countries have large levels of debt to GDP. The information given in Table A.2 is summarized in Figure A.2. What appears is that there exist very different experiences. Among resource rich countries, some have a high level of debt to GDP, while other have low (or even negative) debt to GDP ratios. In addition, it appears that the type of non-renewable under consideration matters. While we can find (some) negative association between fuel exports share and public debt to GDP, we find some positive association between ores and mineral exports and public debt. It should be noticed that the coefficient associated with the slope of the linear adjustment line is never statistically significant (excepted the one concerning net debt to GDP and fuel exports share), which tends to confirm that these two variables have an ambiguous relationship.

Table A.2: Debt to GDP and Resource Abundance

Country	Gross Public Debt (%GDP)	Net Public Debt (%GDP)	Resource Rent (%GDP)	Fuel Exports (% total exports)	Mineral Exports (% total exports)	Explicit Debt Rule
Afghanistan	7.4		.65			.
Albania	75.93	69.9	1.25	.91	4.09	.
Algeria	52.27	44.05	11.93			.
Andorra	46.33					yes
Angola	136.54		25.52	93.11	5.32	.
Antigua and Barbuda	101.48		0			yes
Argentina	102.79		1.83	2.68	.22	no
Armenia	63.48		2.48	3	36.82	yes
Aruba	110.14			.05	4.16	.
Australia	57.24	34.55	5.92	13.91	40	yes
Austria	83.31	59.6	.09	1.94	2.99	yes
Azerbaijan	21.33		18.8	87.25	1.29	no
Bahrain	129.73		8.68	29.82	31.16	.
Bangladesh	34.18		.32			.
Barbados	147.02	145.73	.22	4.78	2.78	.
Belarus	47.49		1.7	12.61	1.26	.
Belgium	112.82	98.13	.02	5.02	4.37	yes
Belize	104.49		.83	1.34	.56	.
Benin	46.14		2.32	.01	.07	yes
Bhutan	130.89		2.88			.
Bolivia	77.97	68.05	2.99	29.47	26.99	.
Bosnia and Herzegovina	36.53	24.95	.77	6.74	5.51	.
Botswana	19.03	15.09	.68	.39	1.52	yes
Brazil	98.68	62.54	3.99	11.89	16.15	no
Brunei Darussalam	2.86		17.01	81.51	.08	.

Bulgaria	23.3	13.41	.59	4.64	14.75	yes
Burkina Faso	46.37		8.96	.29	2.69	yes
Burundi	65.97		12.41	1.86	5.56	yes
Cabo Verde	145.13	132.24	11.47		0	yes
Cambodia	35.16		1.02	0	.56	yes
Cameroon	44.86	43.03	4.68			yes
Canada	117.75	33.64	1.5	19.33	8.34	no
Central African Republic	43.42		9.32	.04	16.71	yes
Chad	54.22		15.76			yes
Chile	32.6	13.4	2.97	.68	57.37	no
China	68.06		1.09	1.21	1.12	.
Colombia	65.66	54.61	3.78	41.56	1.3	no
Comoros	24.02		1.5	0	.74	.
Costa Rica	67.17		.92	.02	1.42	no
Croatia	87.34		.52	9.04	4.39	yes
Cyprus	114.96	53.93	.01	20.04	4.87	yes
Czech Republic	37.65	23.58	.44	1.2	1.22	yes
CÙte d'Ivoire	47.58		2.02	10.28	1.43	yes
Democratic Republic of the Congo	16.49		14.88	0	74.15	.
Denmark	42.2	14.71	.22	2.08	1.37	yes
Djibouti	43.99	42.76	.3			.
Dominica	114.53		.05			yes
Dominican Republic	71.49	57.45	1.37	.32	1.53	.
Ecuador	60.89		4.76	26.06	4.12	yes
Egypt	85.31	79.73	2.69	17.62	3.82	.
El Salvador	89.4		.76	3.13	.92	.
Equatorial Guinea	48.38	37.61	23.35			yes
Eritrea	179.66					.
Estonia	18.56	2.98	1.09	10.68	1.97	yes
Eswatini	41.4	34.92	3.92	1.13	.17	.
Ethiopia	53.7	50.08	5.09	0	.32	.
Fiji	63.09	62.45	1.51	.01	1.28	.
Finland	68.99	33.3	.37	7.01	6.87	yes
France	114.65	102.28	.03	1.88	2.04	yes
Gabon	78.28		17.69			yes
Georgia	60.19		.81	.38	36.9	yes
Germany	67.95	45.77	.09	1.82	2.83	yes
Ghana	79.06	74.95	9.47			.
Greece	212.45		.06	21.87	8.69	yes
Grenada	71.41		0	.03	.66	yes
Guatemala	31.52		1.34	2.36	.97	.
Guinea	47.49		4.13			yes
Guinea-Bissau	76.51		10.53			.
Guyana	51.08	48.22	19.19	43.52	3.29	.
Haiti	21.35		.57			.
Honduras	52.42		1.21	.01	3.16	.
Hong Kong SAR	1		0	.09	1.47	no
Hungary	79.56	72.61	.26	2.19	1.23	yes
Iceland	77.16	60.56	0	.72	36.11	yes
India	89.18		1.86	10.03	4.7	yes
Indonesia	39.76	36.14	2.76	15.63	5.57	yes
Iraq	84.23		32.42			.
Ireland	58.44	52.36	.02	.43	.61	yes
Islamic Republic of Iran	44.08	36.14	22.34			yes
Israel	70.65	67.6	.11		1.33	no
Italy	155.31	141.82	.08	2.14	2.37	yes
Jamaica	108.07		.21	18.76	45.45	yes
Japan	259.43	162.65	.1	1.19	3.13	no
Jordan	87.98	87.88	.03	1.33	6.53	.
Kazakhstan	26.36	-8.62	15.54	58.21	18.5	yes
Kenya	67.95	63.01	1.22	6.75	5.13	yes
Kiribati	19.02		.05	3.4	.45	.
Korea	48.7	18.25	.1	4.95	2.61	.
Kuwait	11.71		32.01	92.88	.19	.

Kyrgyz Republic	67.65		11.21	4.38	10.24	.
Lao P.D.R.	82.75		3.14	16.87	15.74	.
Latvia	43.28	33.42	1.26	3.57	1.75	yes
Lebanon	150.58	147.94	0	.34	6.98	.
Lesotho	54.19	18.7	5.1	.02	.23	.
Liberia	58.66	53.15	15.66			yes
Lithuania	46.58	41.12	.31	7.04	2.18	yes
Luxembourg	24.75	-10.48	.01	.08	4.41	yes
Macao SAR	0		0	0	17.92	.
Madagascar	50.82		5.34	.97	17.44	.
Malawi	54.79		3.97	.15	.53	.
Malaysia	67.72		5.23	11.39	3.5	yes
Maldives	154.39		0			yes
Mali	47.34	40.72	9.37			yes
Malta	53.38	42.93	0	3.26	.38	yes
Marshall Islands	21.62		0			.
Mauritania	55.82	54.81	2.48	.34	42.4	.
Mauritius	99.18		0	.29	2.13	yes
Mexico	60.15	51.62	2.09	3.84	2.85	no
Micronesia	18.28		.02			.
Moldova	36.63		.24	.07	1.75	.
Mongolia	97.37		14.78	30.11	40.09	yes
Montenegro	107.35		.67	16.65	29.1	yes
Morocco	72.25	71.61	.32	.52	5.49	.
Mozambique	119.96		11.7	36.38	38.81	.
Myanmar	39.28		4.41	20.52	6.49	.
Namibia	66.61	64.13	2.01	.75	29.76	yes
Nauru	61.38		0			.
Nepal	42.44		.51		.9	.
Netherlands	54.59	44.7	.15	7.46	2.21	yes
New Zealand	43.16	10.24	1.45	.78	2.42	no
Nicaragua	48.05		1.63	.44	.77	.
Niger	44.99	41.04	5.57	14.7	23.01	yes
Nigeria	34.49	34.05	6.23	88.7	.3	no
North Macedonia	51.88	51.11	.56	1.41	4.46	.
Norway	46.8	-80.16	6.06	49.32	8.2	no
Oman	69.68	28.45	20.97	63.63	6.18	.
Pakistan	79.56	72.91	.89	.87	3.33	yes
Panama	65.56	43.16	.12	.38	32.3	yes
Papua New Guinea	47.06		10.78			.
Paraguay	36.9	32.25	1.63	20.4	.63	no
Peru	34.98	20.31	2.32	3.78	43.51	yes
Philippines	51.64		.74	1.09	6.6	.
Poland	57.14	45.08	.62	1.58	3.22	yes
Portugal	135.18	123.24	.2	4.6	2.17	yes
Puerto Rico	50.19		0			.
Qatar	72.61		14.98	81.81	2.6	.
Republic of Congo	113.98		37.39	75.11	.83	yes
Romania	49.64	40.2	.63	2.42	2.3	yes
Russia	19.2		10.16	42.1	8.64	no
Rwanda	65.57		3.9	.04	8.24	yes
Samoa	43.19		.3			.
San Marino	71.65					.
Saudi Arabia	32.4	15.85	18.21	67.62	1.84	.
Senegal	69.17		3.21	15.99	6.75	yes
Serbia	58.71	54.94	1.01			yes
Seychelles	84.84	76.67	.16	11.41	.34	.
Sierra Leone	76.33		7.81			.
Singapore	151.95		0	8.09	.72	no
Slovak Republic	59.74	49.64	.22	2.4	1.83	yes
Slovenia	79.59	49.73	.21	3.06	3.64	yes
Solomon Islands	13.65	3.38	19.03			.
South Africa	69	62.2	3.91	8.11	31.44	.
South Sudan	36.35	36.26				yes

Spain	119.95	103	.04	4.05	3.2	yes
Sri Lanka	95.69		.09	2.74	.55	yes
St. Kitts and Nevis	56.85		0			yes
St. Lucia	96.9		.02	3.4	5.54	yes
St. Vincent and the Grenadines	79.25		.03			yes
Sudan	263.37		12.4			.
Suriname	146.1		8.1	0	.24	.
Sweden	39.22	8.57	.41	4.54	4.99	yes
Switzerland	43.33	20.47	.01	.61	2	no
Sao Tome and Principe	81.37		1.92		.14	.
Tajikistan	50.43		5.73	7.75	47.56	.
Tanzania	40.53		3.86	.82	7.17	yes
Thailand	49.47		1.3	2.67	1.86	yes
The Bahamas	74.96		.02	28.39	5.82	yes
The Gambia	85.89		2.82	0	7.46	.
Timor-Leste	11.47		7.19			yes
Togo	60.28		4.31	2.36	12.45	yes
Tonga	43.62		.04			.
Trinidad and Tobago	59.27	9.03	5.77	29.23	2.57	.
Tunisia	82.85		1.38	5.67	1.83	.
Turkmenistan	13.12					no
Tuvalu	7.29		0			.
Turkiye	39.65	30.14	.4	2.69	4.03	.
Uganda	46.32		7.27	2.38	.25	yes
Ukraine	60.56		1.46	1.13	11.09	.
United Arab Emirates	39.67		11.94	71.43	3.41	.
United Kingdom	102.61	90.19	.39	7.06	6.6	yes
United States	134.54	99.08	.42	12.72	3.21	no
Uruguay	68.31	57.52	2.27	1.24	.39	no
Uzbekistan	37.62		11.57	5.96	8.34	.
Vanuatu	47.53		.62			.
Venezuela	319.09					.
Vietnam	41.67		2.62	.95	1.05	yes
Yemen	83.96	83.34				.
Zambia	140.21	138.09	11.81	1.61	78.78	.
Zimbabwe	102.49		6.8	1.1	44.76	.

Data source : The information on the existence of an explicit debt rule is taken from the IMF fiscal rule database (Davoodi et al., 2022). 2021 is the reference period. Resource rents, fuel and mineral exports are taken from the World Development Indicator database for year 2020. Public debt information is taken from the IMF World Economic Outlook Database for year 2021.

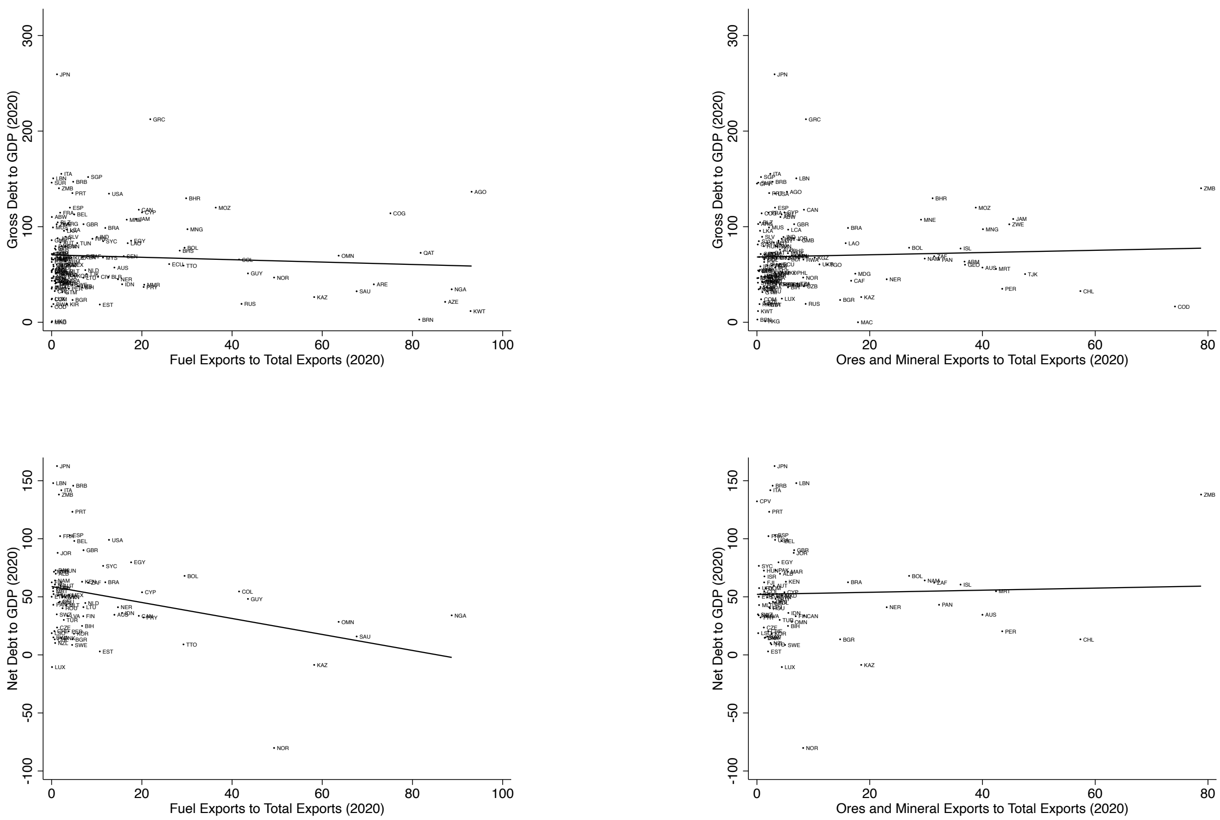

Figure A.2: Resource Exports Shares and Public Debt

Appendix B. Proof of Proposition 1

- Dividing (10) by K_t, we get

$$\frac{H_t}{K_t} = \frac{G_t}{K_t} + (1-\delta_H)\frac{H_{t-1}}{K_t}\frac{K_{t-1}}{K_{t-1}}$$

  Since $\frac{G_t}{K_t} = \hat{G}$ and $\frac{H_t}{K_t} = \hat{H}_t$, we can write

$$\hat{H}_t = \hat{G} + (1-\delta_H)\hat{H}_{t-1}\mu_{k,t}^{-1}$$

  which is equation (20).

- Combining (9), (15) and (14), we get

$$\frac{\nu A_{t+1}k_{t+1}^{\alpha}x_{t+1}^{\nu-1}H_{t+1}^{\theta}}{\nu A_t k_t^{\alpha}x_t^{\nu-1}H_t^{\theta}} = 1 + \alpha A_{t+1}k_{t+1}^{\alpha-1}x_{t+1}^{\nu}H_{t+1}^{\theta} - \delta_K$$

  Taking the ratio of this equation evaluated in $t+1$ and in t, we obtain

$$\frac{(1+a)\mu_{k,t+1}^{\alpha}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-1}\left[\hat{G}\hat{H}_t^{-1}\mu_{k,t+1}+(1-\delta_H)\right]^{\theta}-1+\delta_K}{(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1}\left[\hat{G}\hat{H}_{t-1}^{-1}\mu_{k,t}+(1-\delta_H)\right]^{\theta}-1+\delta_K} = (1=a)\mu_{k,t+1}^{\alpha-1}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu}\left[\hat{G}\hat{H}_t^{-1}\mu_{k,t+1}+(1-\delta_H)\right]^{\theta}$$

which is equation (19).

- Combining equations (5) and (6) together with the market clearing condition for capital and bonds (17) and the Euler equation (8), it is immediate that

$$(2+\rho)(k_{t+1}+b_{t+1}) = w_t - \tau_t - (2+\rho)p_t m_t$$

Using the government budget constraint (11), it gives

$$(2+\rho)k_{t+1} + (1+\rho)b_{t+1} = w_t - (1+r_t)b_t - g_t - (2+\rho)p_t m_t$$

Since the government stabilizes its debt-to-capital at $\hat{B}$ and its expenditures to GDP ratio at $\hat{G}$ and substituting w_t, p_t and r_t by their expressions, it follows that we obtain

$$(2+\rho)k_{t+1} + (1+\rho)b_{t+1} = \left[\beta - \frac{\nu(2+\rho)(1-q_t)}{q_t} - \alpha\hat{B}\right] A_t k_t^{\alpha} x_t^{\nu} H_t^{\theta} - (1-\delta_K)b_t - g_t$$

Dividing both sides by k_t, and noticing than $A_t k_t^{\alpha-1} x_t^{\nu} H_t^{\theta} = \frac{p_t/p_{t-1}-1+\delta_K}{\alpha}$, we obtain that

$$(2+\rho)\mu_{k,t+1} + (1+\rho)\hat{B}\mu_{k,t+1} = \left[\beta - \frac{\nu(2+\rho)(1-q_t)}{q_t} - \alpha\hat{B}\right]\left[\frac{p_t/p_{t-1}-1+\delta_K}{\alpha}\right] - (1-\delta_K)\hat{B} - \hat{G}$$

Since $\frac{p_t}{p_{t-1}} = (1+a)\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-1}\mu_{H,t}^{\theta}$ and taking into account that $\mu_{H,t} = \hat{G}\mu_{k,t}\hat{H}_{t-1}^{-1} + (1-\delta_H)$, we obtain

$$\mu_{k,t+1}\left[1 + \frac{1+\rho}{2+\rho}\hat{B}\right] = \left[\frac{\beta q_t - \nu(1-q_t)(2+\rho) - \alpha\hat{B}q_t}{\alpha(2+\rho)q_t}\right] \times \left[(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-1}\left[\hat{G}\hat{H}_{t-1}^{-1}\mu_{k,t} + (1-\delta_H)\right]^{\theta} - 1 + \delta_K\right] - \frac{\hat{G}+(1-\delta_K)\hat{B}}{2+\rho}$$

which is precisely equation (18).

Appendix C. Proof of Proposition 2

- Equation (20) evaluated on the BGP gives

$$\hat{H} = \frac{\hat{G}}{1-(1-\delta_H)\mu_k^{-1}}$$

which corresponds to the BGP level of the ratio H_t/K_t.

- Equation (19) evaluated at the BGP gives

$$1 = (1+a)\mu_k^{\alpha-1}(1-q)^{\nu}\left[\hat{G}\hat{H}^{-1}\mu_k + (1-\delta_H)\right]^{\theta}$$

Taking into account the BGP level of $\hat{H}$, it is immediate that

$$\mu_k = (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}$$

- Equation (18) evaluated at the BGP gives

$$\mu_k\left[\frac{(2+\rho)+(1+\rho)\hat{B}}{2+\rho}\right] = \left[\frac{\beta q - \nu(1-q)(2+\rho) - \alpha\hat{B}q}{\alpha(2+\rho)q}\right]\times$$
$$\left[(1+a)\mu_k^{\alpha}(1-q)^{\nu-1}\left[\hat{G}\hat{H}^{-1}\mu_k + (1-\delta_H)\right]^{\theta} - 1 + \delta_K\right] - \frac{\hat{G}+(1-\delta_K)\hat{B}}{2+\rho}$$

using the BGP values of $\hat{H}$ and μ_k, we can write as we have already seen

$$LHS(q) \equiv \frac{\alpha(1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{\nu}{1-\alpha-\theta}}[(2+\rho)+(1+\rho)\hat{B}]q + \alpha\hat{G}q + (1-\delta_K)\hat{B}\alpha q}{\beta q - (2+\rho)\nu(1-q) - \alpha\hat{B}q}$$
$$= (1+a)^{\frac{1}{1-\alpha-\theta}}(1-q)^{\frac{v}{1-\alpha-\theta}-1} - (1-\delta_K) \equiv RHS(q)$$

$RHS(q)$ is a positive, increasing and convex function defined on $[0;1[$ admitting a vertical asymptote for $q=1$. Depending on the size of the public debt stabilization ratio, $LHS(q)$ may be positive or negative.

Since the numerator of $LHS(q)$ is always positive under assumption 1, the sign of the denominator determines the sign of the function.

If q satisfies $(2+\rho)\nu < (\beta-\alpha\hat{B}+(2+\rho)\nu)q$, $LHS(q)$ is positive and admits a vertical asymptote in $q = \hat{q} = \frac{(2+\rho)\nu}{\beta-\alpha\hat{B}+(2+\rho)\nu}$. Otherwise, $LHS(q)$ will be negative and will never intersect with $RHS(q)$. Since $\lim_{q\to\hat{q}^+} LHS(q) = +\infty$ and $\lim_{q\to 1} LHS(q) = \frac{\alpha\hat{G}+\alpha(1-\delta_K)\hat{B}}{\beta-\alpha\hat{B}} > 0$, it exists a unique $q^* > \hat{q}$ such that equation (21) is satisfied. Thus, under assumption 1, there exists a unique balanced growth path. Moreover, it can be shown that, for $q > \hat{q}$

$$\frac{\partial LHS}{\partial q} = -\frac{1}{\left[\beta q - \alpha\hat{B}q - \nu\left(2+\rho\right)\left(1-q\right)\right]^2}\times$$
$$\left\{\left[\alpha\left(1+a\right)^{\frac{1}{1-\alpha-\theta}}\left(1-q\right)^{\frac{\nu}{1-\alpha-\theta}}\left[(2+\rho)+(1+\rho)\hat{B}\right]\left(\hat{G}\alpha+(1-\delta_k)\hat{B}\alpha\right)\right]\nu\left(2+\rho\right)+\right.$$
$$\left.\left[\frac{q}{1-q}\frac{\nu}{1-\alpha-\theta}\alpha\left(1+a\right)^{\frac{1}{1-\alpha-\theta}}\left(1-q\right)^{\frac{\nu}{1-\alpha-\theta}}\left[(2+\rho)+(1+\rho)\hat{B}\right]\right]\left[\beta q - \alpha\hat{B}q - \nu(2+\rho)(1-q)\right]\right\} < 0$$

This is represented on Figure C.3

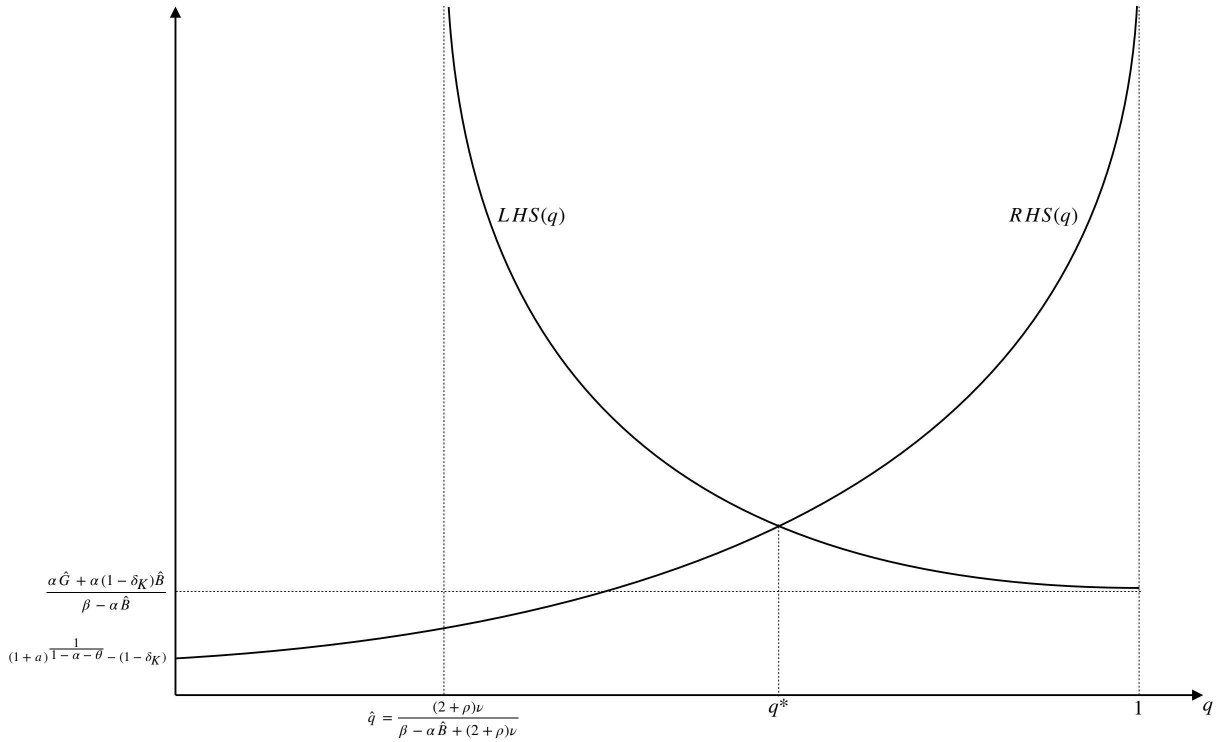

Figure C.3: The market equilibrium extraction rate

Finally, all the BGP growth factors of other variables may be expressed as follows :

- From equation (1), we obtain $\mu_x = \mu_m$
- $\mu_A = 1 + a$
- We assume constant ratios of public expenditures to capital and public indebtness to capital ratios. Thus $\mu_g = \mu_b = \mu_k$
- Equation (10) implies $\mu_{H,t} = \frac{G_t}{H_{t-1}} + (1 - \delta_H) = \hat{G}\mu_{k,t}\hat{H}_{t-1}^{-1} + (1 - \delta_H)$. On the BGP, we have $\mu_H = \hat{G}\mu_k\hat{H}^{-1} + (1 - \delta_H) = \mu_k$
- The Hotelling rule (9) implies $\mu_p = 1 + r_{t+1}$. In addition, a BGP requires a constant interest rate because the resource price's rate of growth is constant so $\mu_r = 1$
- The production function at the BGP gives $\mu_y = (1 + a)\mu_k^{\alpha}\mu_x^{\nu}\mu_h^{\theta} = \mu_k$
- Equation (16) gives $\mu_w = \mu_y$
- The Euler Equation (8) along the BGP gives $\mu_c = \mu_d$

- The government budget constraint (11) implies $\mu_\tau = \mu_b = \mu_y$
- Evaluating the budgetary constraint at the BGP leads to $\mu_d = \mu_y$
- Evaluating (17) at the BGP gives $\mu_s s_t = \mu_k (k_{t+1} + b_{t+1})$. Since $s_t = k_{t+1} + b_{t+1}$ we have $\mu_s = \mu_k$

Appendix D. Proof of Proposition 4

The rate of growth of the economy is defined as

$$\mu = (1+a)^{\frac{1}{1-\alpha-\theta}} (1-q)^{\frac{\nu}{1-\alpha-\theta}}$$

The rate of technological progress is considered as exogenous. By contrast, the rate of resource extraction is affected by changes in $\hat{G}$. Changes in $\hat{G}$ thus affect the rate of growth only through their effects on the extraction rate. Differentiating equation (21), it can be written that

$$\frac{dq}{d\hat{G}} = \frac{\frac{\partial LHS}{\partial \hat{G}}}{-\frac{\partial LHS}{\partial q} + \frac{\partial RHS}{\partial q}}$$

Under assumption 1 and for $q > \hat{q}$, we have already established that $\frac{\partial LHS}{\partial q} < 0$. In addition, we have

$$\frac{\partial RHS}{\partial q} = -\left(\frac{\nu}{1-\alpha-\theta} - 1\right) (1+a)^{\frac{1}{1-\alpha-\theta}} (1-q)^{\frac{\nu}{1-\alpha-\theta}-2} > 0$$

and

$$\frac{\partial LHS}{\partial \hat{G}} = \frac{\alpha q}{\left[\beta q - \alpha \hat{B} q - \nu (2+\rho)(1-q)\right]} > 0$$

Then we can conclude that $\frac{dq}{d\hat{G}} > 0$.

Graphically, only $LHS(q)$ is affected by changes in $\hat{G}$ and since $\frac{\partial LHS}{\partial \hat{G}} > 0$, we obtain we obtain Figure D.4.

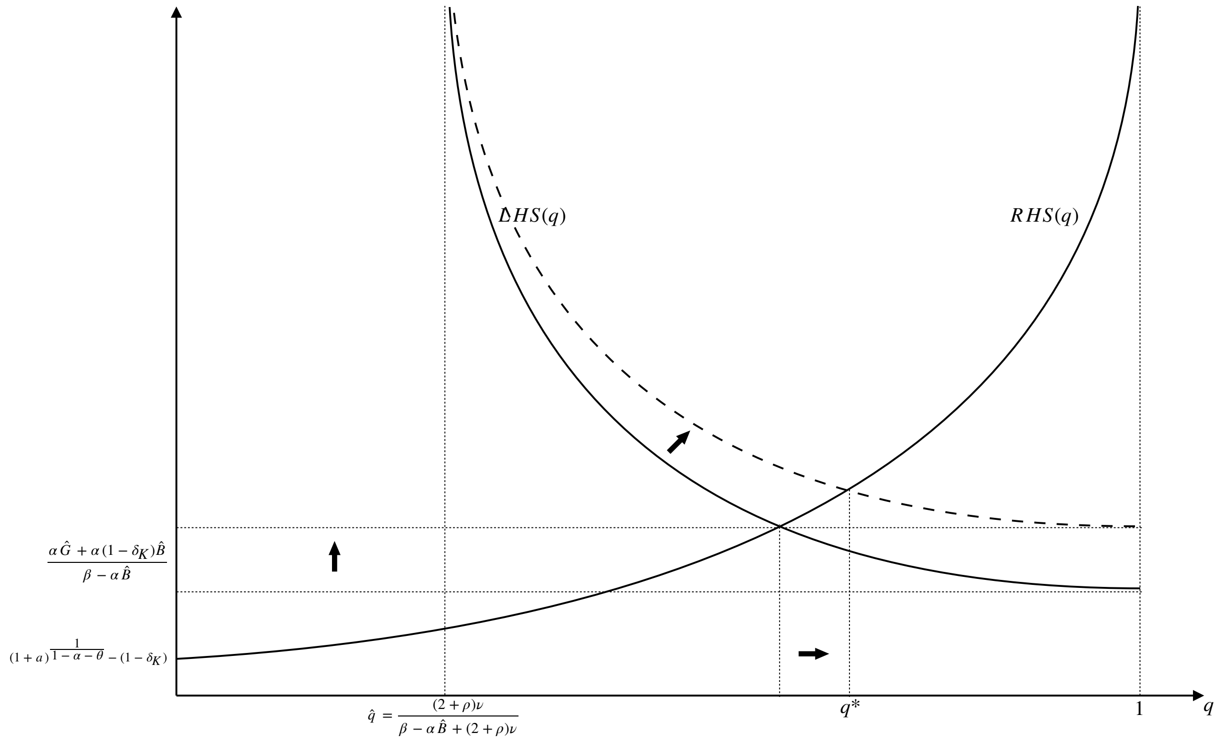

Figure D.4: Effects of an increase in $\hat{G}$ on the extraction rate

Appendix E. Proof of Proposition 5

Proceeding as above, we note that the rate of technological progress is considered as exogenous. By contrast, the rate of resource extraction is affected by changes in $\hat{B}$. Changes in $\hat{B}$ thus affect the rate of growth only through their effects on the extraction rate. Differentiating equation (21), it can be written that

$$\frac{dq}{d\hat{B}} = \frac{\frac{\partial LHS}{\partial \hat{B}}}{-\frac{\partial LHS}{\partial q} + \frac{\partial RHS}{\partial q}}$$

Under assumption 1 and for $q > \hat{q}$, we have already established that $\frac{\partial LHS}{\partial q} < 0$ and $\frac{\partial RHS}{\partial q} > 0$. In addition, we have

$$\frac{\partial LHS}{\partial \hat{B}} = \frac{1}{\left[\beta q - \alpha\hat{B}q - \nu\left(2+\rho\right)\left(1-q\right)\right]^2} \times$$
$$\left\{\left[\alpha\left(1+a\right)^{\frac{1}{1-\alpha-\theta}}\left(1-q\right)^{\frac{\nu}{1-\alpha-\theta}}\left(1+\rho\right)q + \left(1-\delta_k\right)\alpha q\right]\left[\beta q - \alpha\hat{B}q - \nu\left(2+\rho\right)\left(1-q\right)\right] + \right.$$
$$\left.\left[\alpha\left(1+a\right)^{\frac{1}{1-\alpha-\theta}}\left(1-q\right)^{\frac{\nu}{1-\alpha-\theta}}\left[\left(2+\rho\right)+\left(1+\rho\right)\hat{B}\right]q + \hat{G}\alpha q + \left(1-\delta_k\right)\hat{B}\alpha q\right]\alpha q\right\} > 0$$

Then we can conclude that $\frac{dq}{d\hat{B}} > 0$.

Graphically, only $LHS(q)$ is affected by changes in $\hat{B}$ and since $\frac{\partial LHS}{\partial \hat{B}} > 0$, we obtain Figure E.5.

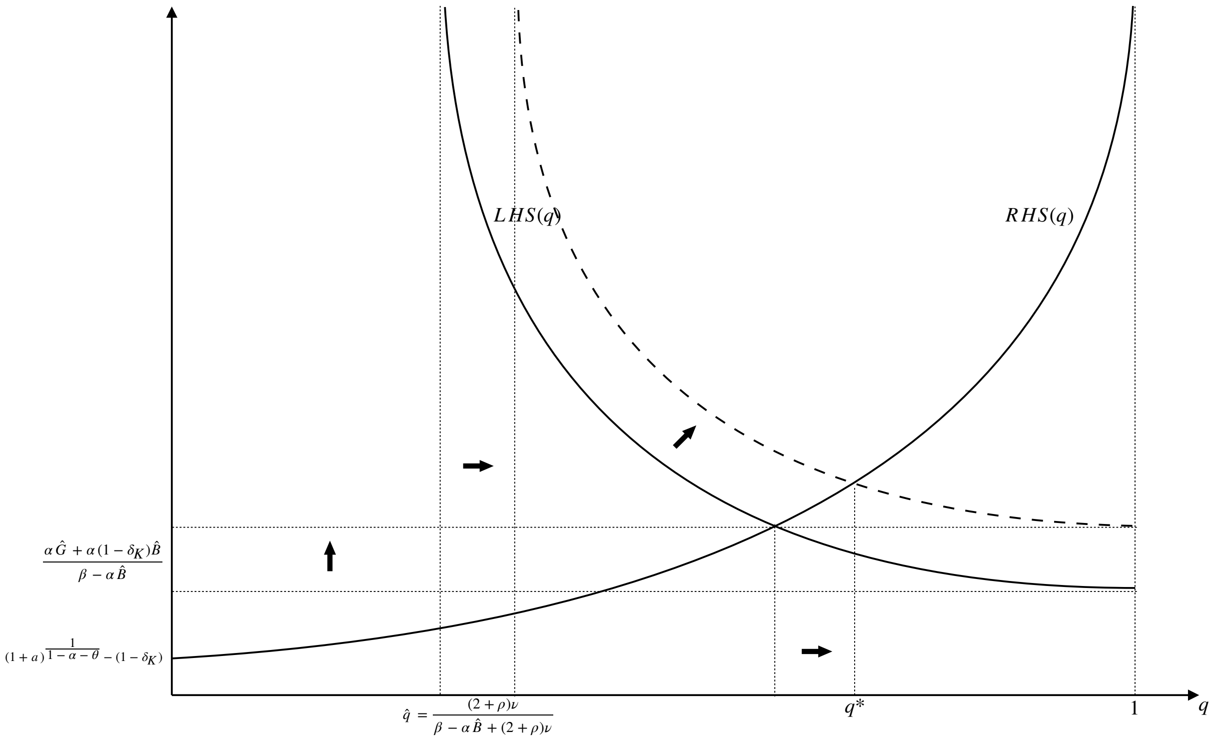

Figure E.5: Effects of an increase in $\hat{B}$ on the extraction rate

Appendix F. Discussion on local stability

This Appendix is devoted to the analysis of local stability of the dynamic system defined by equations (18), (19), and (20). Defining $z_t = q_{t-1}$, the system may be re-written as follows

$$\begin{cases} \mu_{k,t+1}\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right] = \left[\frac{\beta q_t - \nu(1-q_t)(2+\rho) - \alpha\hat{B}q_t}{\alpha(2+\rho)q_t}\right] \times \\ \qquad \left[(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-z_t)}{z_t}\right]^{\nu-1}\left[\hat{G}\hat{H}_{t-1}^{-1}\mu_{k,t} + (1-\delta_H)\right]^{\theta} - 1 + \delta_K\right] - \frac{\hat{G}+(1-\delta_K)\hat{B}}{2+\rho} \\ \frac{(1+a)\mu_{k,t+1}^{\alpha}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-1}\left[\hat{G}\hat{H}_t^{-1}\mu_{k,t+1}+(1-\delta_H)\right]^{\theta}-1+\delta_K}{(1+a)\mu_{k,}^{\alpha}\left[\frac{q_t(1-z_t)}{z_t}\right]^{\nu-1}\left[\hat{G}\hat{H}_{t-1}^{-1}\mu_{k,t}+(1-\delta_H)\right]^{\theta}-1+\delta_K} = \\ \qquad (1+a)\mu_{k,t+1}^{\alpha-1}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu}\left[\hat{G}\hat{H}_t^{-1}\mu_{k,t+1} + (1-\delta_H)\right]^{\theta} \\ z_{t+1} = q_t \\ \hat{H}_t = \hat{G} + (1-\delta_H)\hat{H}_{t-1}\mu_{k,t}^{-1} \end{cases}$$

Linearizing this system around the BGP, we get

$$\begin{pmatrix} d\mu_{k,t+1} \\ dq_{t+1} \\ dz_{t+1} \\ dH_t \end{pmatrix} = J \begin{pmatrix} d\mu_{k,t} \\ dq_t \\ dz_t \\ dH_{t-1} \end{pmatrix}$$

where

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ 0 & 1 & 0 & 0 \\ -\left(1-\delta_H\right)\hat{H}\mu_k^{-2} & 0 & 0 & \left(1-\delta_H\right)\mu_k^{-1} \end{pmatrix}$$

and

$$J_{11} = \frac{\left[\frac{\beta a-\alpha\hat{B}q-\nu(2+\rho)(1-q)}{\alpha q(2+\rho)}\right]\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta-1}\left(\alpha+\theta\hat{G}\hat{H}^{-1}\right)}{\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right]}$$

$$J_{12} = \frac{\left[\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta}\left[\frac{\nu}{\alpha}\frac{1}{q^2}-\left[\frac{\beta a-\alpha\hat{B}q-\nu(2+\rho)(1-q)}{\alpha q(2+\rho)}\right]\left(1-\nu\right)\frac{1}{q}\right]\right]-\left(1-\delta_k\right)\frac{\nu}{\alpha q^2}}{\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right]}$$

$$J_{13} = \frac{\left[\frac{\beta a-\alpha\hat{B}q-\nu(2+\rho)(1-q)}{\alpha q(2+\rho)}\right]\left(1+a\right)\left(1-\nu\right)\mu_k^{\alpha+\theta}\left(1-q\right)^{\nu-2}\frac{1}{q}}{\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right]}$$

$$J_{14} = -\frac{\left[\frac{\beta a-\alpha\hat{B}q-\nu(2+\rho)(1-q)}{\alpha q(2+\rho)}\right]\left(1+a\right)\left(1-q\right)^{\nu-1}\theta\mu_k^{\alpha+\theta}\hat{G}\hat{H}^{-2}}{\left[1+\frac{1+\rho}{2+\rho}\hat{B}\right]}$$

$$\begin{aligned}
J_{21} = {} & \frac{1}{\frac{(1-q)^{\nu-1}}{q}\left[\frac{(1+a)\mu_k^{\alpha+\theta}(1-\nu)}{(1+a)(1-q)^{\nu-1}\mu_k^{\alpha+\theta}-1+\delta_k}+\left(1+a\right)\mu_k^{\alpha+\theta-1}\nu\left(1-q\right)\right]} \\
& \times\left\{\left[\frac{\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta-1}\left(\alpha+\theta\hat{G}\hat{H}^{-1}\right)}{\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta}-1+\delta_k}-\left(1+a\right)\left(1-q\right)^{\nu}\mu_k^{\alpha+\theta-2}\left(\alpha-1+\theta\hat{G}\hat{H}^{-1}\right)\right]J_{11}\right. \\
& -\left[\frac{\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta-1}\left(\alpha+\theta\hat{G}\hat{H}^{-1}\right)}{\left(1+a\right)\mu_k^{\alpha+\theta}\left(1-q\right)^{\nu-1}-1+\delta_k}\right] \\
& \left.+\left[\frac{\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta}\theta\hat{G}\hat{H}^{-2}}{\left(1+a\right)\left(1-q\right)^{\nu-1}\mu_k^{\alpha+\theta}-1+\delta_k}-\left(1+a\right)\left(1-q\right)^{\nu}\mu_k^{\alpha+\theta-1}\theta\hat{G}\hat{H}^{-2}\right]\left(1-\delta_H\right)\hat{H}\mu_k^{-2}\right\}
\end{aligned}$$

$$J_{22} = \frac{1}{\frac{(1-q)^{\nu-1}}{q}\left[\frac{(1+a)\mu_k^{\alpha+\theta}(1-\nu)}{(1+a)(1-q)^{\nu-1}\mu_k^{\alpha+\theta}-1+\delta_k} + (1+a)\,\mu_k^{\alpha+\theta-1}\nu\,(1-q)\right]}$$

$$\times\left\{\left[\frac{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta-1}\left(\alpha+\theta\hat{G}\hat{H}^{-1}\right)}{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}-1+\delta_k} - (1+a)\,(1-q)^{\nu}\,\mu_k^{\alpha+\theta-2}\left(\alpha-1+\theta\hat{G}\hat{H}^{-1}\right)\right]J_{12}\right.$$

$$\left.+\frac{1}{q}\left[\frac{(1+a)\,\mu_k^{\alpha+\theta}\,(1-\nu)\,(1-q)^{\nu-2}\,(2-q)}{(1+a)\,\mu_k^{\alpha+\theta}\,(1-q)^{\nu-1}-1+\delta_k} + (1+a)\,\mu_k^{\alpha+\theta-1}\nu\,(1-q)^{\nu-1}\right]\right\}$$

$$J_{23} = \frac{1}{\frac{(1-q)^{\nu-1}}{q}\left[\frac{(1+a)\mu_k^{\alpha+\theta}(1-\nu)}{(1+a)(1-q)^{\nu-1}\mu_k^{\alpha+\theta}-1+\delta_k} + (1+a)\,\mu_k^{\alpha+\theta-1}\nu\,(1-q)\right]}$$

$$\times\left\{\left[\frac{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta-1}\left(\alpha+\theta\hat{G}\hat{H}^{-1}\right)}{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}-1+\delta_k} - (1+a)\,(1-q)^{\nu}\,\mu_k^{\alpha+\theta-2}\left(\alpha-1+\theta\hat{G}\hat{H}^{-1}\right)\right]J_{13}\right.$$

$$\left.-\left[\frac{1}{q}\frac{(1+a)\,\mu_k^{\alpha+\theta}\,(1-\nu)\,(1-q)^{\nu-2}}{\left[(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}-1+\delta_k\right]}\right]\right\}$$

$$J_{24} = \frac{1}{\frac{(1-q)^{\nu-1}}{q}\left[\frac{(1+a)\mu_k^{\alpha+\theta}(1-\nu)}{(1+a)(1-q)^{\nu-1}\mu_k^{\alpha+\theta}-1+\delta_k} + (1+a)\,\mu_k^{\alpha+\theta-1}\nu\,(1-q)\right]}$$

$$\times\left\{\left[\frac{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta-1}\left(\alpha+\theta\hat{G}\hat{H}^{-1}\right)}{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}-1+\delta_k} - (1+a)\,(1-q)^{\nu}\,\mu_k^{\alpha+\theta-2}\left(\alpha-1+\theta\hat{G}\hat{H}^{-1}\right)\right]J_{14}\right.$$

$$-\left[\frac{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}\theta\hat{G}\hat{H}^{-2}}{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}-1+\delta_k} - (1+a)\,(1-q)^{\nu}\,\mu_k^{\alpha+\theta-1}\theta\hat{G}\hat{H}^{-2}\right](1-\delta_H)\,\mu_k^{-1}$$

$$\left.+\left[\frac{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}\theta\hat{G}\hat{H}^{-2}}{(1+a)\,(1-q)^{\nu-1}\,\mu_k^{\alpha+\theta}-1+\delta_k}\right]\right\}$$

The stability features of the above Jacobian J depend on the associated eigenvalues. To this end, notice that our dynamic system includes three predetermined variables μ, z and $\hat{H}$, and one forward looking, q. It follows that the equilibrium will be determinate if and only if the number of the stable roots is lower than four. Unfortunately, due to the dimension of our system, we need to rely on numerical simulation to analyze the stability of our system. Table F.3 gives the eigenvalues associated with J for two calibrations corresponding with the two cases analyzed in Table 1. It follows that we can reasonably assume saddle path stability. Indeed, we have performed several simulations and the saddle path stability seems to be very robust to changes in parameters values as long as we take realistic values. The Jacobian eigenvalues calculation notebook can be made available on request.

	Calibration 1	Calibration 2
α	0.3	0.2
β	0.65	0.5
ν	0.05	0.3
θ	0.2	0.2
ρ (annual rate)	0.016	0.016
a (annual rate)	0.028	0.028
δ_k(annual rate)	0.028	0.028
δ_H(annual rate)	0.028	0.028
$\hat{B}$	0.5	0.5
$\hat{G}$	0.1	0.1
q	0.580687	0.756931
μ	3.64678	1.55804
$\hat{H}$	0.100152	0.100356
Eigenvalues	(2.910; 0.497; 0.001; 0)	(10.38; 0.399; 0.002; 0)

Table F.3: Simulated eigenvalues of the Jacobian

Appendix G. Proof of Proposition 6

- From the definition of a, we have $\tilde{\mu}_A = (1 + a)$
- Evaluating (36) on the BGP gives

$$\hat{H} = \frac{\hat{G}}{1 - (1 - \delta_H)\tilde{\mu}_k^{-1}}$$

  which corresponds to the BGP level of the ratio H_t/K_t. Since this ratio is constant on the BGP, it is then immediate that $\tilde{\mu}_k = \tilde{\mu}_H$.

- It follows from the definition of $\tilde{q}$ that $\tilde{\mu}_x = (1 - \tilde{q})$.
- Evaluating (33) on the BGP, we obtain $1 = (1 + a)\,\tilde{\mu}_k^{\alpha+\theta-1}$ which implies that

$$\tilde{\mu_k} = (1 + a)^{\frac{1}{1-\alpha-\theta}}(1 - \tilde{q})^{\frac{\nu}{1-\alpha-\theta}}$$

  .

- Evaluating (35) on the BGP, it is immediate that γ_c is constant. We then conclude that $\tilde{\mu}_c = \tilde{\mu}_k$. Equation (29) implies that $\tilde{\mu}_c = \tilde{\mu}_d$.

- Evaluating (34) on the BGP, we obtain $\tilde{\mu}_x^{-1} = (1+\psi)$. It then follows that $\tilde{q} = \frac{\psi}{1+\psi}$.