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Variational rationality. Self regulation success as a succession of worthwhile moves that make sufficient progress.

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Abstract

This paper provides a general and formalized theory of self-regulation success and failures as an application of the recent Variational rationality approach of stay and change human dynamics (Soubeyran, 2009, 2010, 2021.a,b,c,d). For concreteness purposes, it starts with an example in psychology: how to gain or to loose weight ? It ends with a general, conceptual, dynamical and computable formulation of self-regulation and goal pursuit in the context of variational principles and adaptive optimizing algorithms in mathematics.

Key words. Variational rationality, self regulation, variational principles, adaptive optimizing algorithms.

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1 Introduction

This paper provides a conceptual and formalized theory of self-regulation success (failures) that can have, given its generality, a lot of applications in psychology, behavioral sciences, dynamical systems, variational principles and optimizing algorithms (the modern variational analysis). It starts with a novel and broad definition of self regulation: it is what must be done to satisfy enough and speedy enough different changing and recurrent needs without too much frustration and sacrifices. That is, relatively to desired and undesired outcomes, "self-regulation is the dynamic process by which people manage competing demands on their time and resources as they strive to achieve desired outcomes, while simultaneously preventing or avoiding undesired outcomes" (Neal et al., 2017). This paper also starts with a spatial vision of human activities : each period, to satisfy his current needs, an individual must do something, i.e., a bundle of activities. That is, he must move from having done a bundle of activities in the previous period to do a new or the same bundle of activities in the current period. In this way we show that, each period, self regulation success comes with doing a worthwhile and satisficing move. The opposite works with self regulation failure. In a short run perspective, a move is worthwhile if the satisfactions derived from the fulfilment of different needs (through doing this move) are high enough compared with the sacrifices required to do it. In a long run perspective, a move is satisficing if it makes sufficient progress compared to long term aspiration levels.

Hint. **Several figures are added in the text. They are all given at the end of the paper. See Appendix 1: FIGURES, after the conclusion.** They are very important for understanding. They show that, with six simple figures where three straight lines intersect a parabola, the main ideas of our mathematical approach can be fully understood. They include Figure 1 for a satisficing move, Figure 2 for a worthwhile move, Figure 3 for traps, Figure 4 for self regulation failures, and Figures 5, 6 for self regulation success.

Variational rationality: a mathematical approach to psychology. Because the two concepts of satisficing move and worthwhile move represent the heart of the recent (VR) variational rationality approach of stay and change human dynamics (Soubeyran, 2009, 2010, 2021.a,b,c,d), the present behavioral paper is one of its final outputs. With work still in progress, the behavioral side of the VR approach has not yet been published in psychology. In contrast, parts of the mathematical side have been published[1]. As a major advance our VR approach paves the way to show that variational analysis and optimizing algorithms in mathematics and famous behavioral principles in psychology are the two faces of the same coin. In such a perspective the VR approach of stay (continue doing) and change (stop and start doing) human dynamics realizes the dream of Kurt Lewin to build a spatial and mathematical theory of psychology, i.e., his famous principles of topological psychology (Lewin, 1936).

As a very short reminder of the VR perspective we list in section 2 some of the difficulties that the VR approach of self regulation success and failures must overcome on both the conceptual (behavioral) and mathematical levels:

- how to model human life as an entanglement of stop, continue and start dynamics;
- how to model the need satisfaction problem as a traveler problem;
- how to define self regulation in a context where all things are changing;
- how to model valence expectancies as expected satisfaction levels derived from doing a move and self efficacy expectancies as expected costs of moving;
- how to model and to enrich the famous TOTE model of self regulation as doing, each period, a move that is both, satisficing and worthwhile.

Two core concepts help to model self regulation success: satisficing and worthwhile moves. The (conceptual and formal) construction of worthwhile and satisficing moves requires a succession of stages that helps to model a self regulation process as a way to become able to satisfy enough and speedy enough different needs without too much frustrations and sacrifices. Then, a complete understanding of their multidisciplinary construction is not easy as long as we do not consider different examples in different contexts. See

[1] see https://sites.google.com/view/antoine-soubeyran for at least 50 publications in mathematical journals.

Soubeyran (2021.a,b,c,d). Thus, to save space, a reminder of the general construction of satisficing moves and worthwhile moves is given in verbal terms in the appendix.

On one side, in the short run, a move is worthwhile if motivation to undertake this move is high enough relative to resistance to do this move. As we will see, this is equivalent to say that the satisfactions derived from the fulfilment of different needs are higher than the related sacrifices. This provides, at the conceptual and mathematical levels, a new look at the famous force field/topological psychology approach (Lewin, 1938, 1951), given that a move is worthwhile if driving forces (= motivation to move) surpass enough resisting forces (= resistance to move).

On the other side, in the long run, a move is satisficing if it makes sufficient progress (Carver & Scheier, 1990), i.e., if it satisfies enough one or several current needs, compared to long term aspirations levels. This is a way to revisit and to give a mathematical formulation of the TOTE model (Carver & Scheier, 1990). This poses the the fundamental problem of monitoring sufficient progress toward goals (Liberman & Dar, 2009).

Thus, a satisficing worthwhile move, i.e., a worthwhile enough move, is not myopic. Better, it is strongly adaptive, as it makes the proximal and distal views of a situation consistent one with the other (i.e., linking proximal goals with distal goals).

To economize space and to promote intuition, let us start with a daily life example.

Starting with a concrete example: lose or gain weight. Because motivation and self regulation theories are so vast and highly multidisciplinary (see Kruglanski et al, 2015), and because of a lot of difficulties, before dealing with a general case in the last section of this paper, sections 2, 3 and 4 provide an example in the field of psychology: how to gain or to lose weight. In this example, in the context of dynamical systems, the state is the present weight of an individual. His objectives refer to desired satisfaction levels coming from a better satisfaction of different needs including becoming healthier, more beautiful, more admired, less mocked, feeling better about his/her body, having a better self esteem..... Controls represent actions like eating more or less, do more or less physical exercises.... Then, each period, to self regulate his weight an individual must succeed to,

1) stop doing things that he wants to do right now (watching TV instead of jogging), ii) continue doing things he still wants to do (working on his thesis) and, iii) start doing things that he wants to do in a near future in order to,

2) satisfy enough his different needs (being healthier, ...), without too much sacrifices (eat more, or less,...);

3) satisfy quickly enough these needs to escape frustration, when having to wait too long for their satisfaction.

Point 1 poses the problem of overcoming resistance to move, i.e., becoming able to stop, continue and start doing things. Point 2 defines worthwhile moves while point 3 defines satisficing moves that make sufficient progress.

The novelties of our VR perspective: an adaptive and algorithmic

approach of self regulation. Using the VR approach, the last section of our paper provides an algorithmic approach of self regulation success and failures in the context of gradient (= linearized proximal) algorithms. In the concrete example and in the appendix, we have devoted much space to define and model self regulation success and failure through a VR structure that defines what must be done to satisfy enough current needs in a worthwhile way. However, most of the time this VR structure is not well known to an individual in each given situation. That is, what he does not know well includes the main expectancies that help him to define aspiration levels and aspiration gaps, levels of frustration feelings, valences and self efficacy beliefs, advantages and inconveniences to move, motivation and resistance to move and finally worthwhile balances. Then, the next problem is to build, in each situation, approximate models of a VR structure, ending with good enough approximate models of satisficing and worthwhile moves. Finally, we show how the consideration of approximate satisficing and worthwhile moves lead to good enough self regulation success. We sketch the way to proceed in the following sequence. We start with a black box algorithm and build a dictionary that translates behavioral (VR) concepts into algorithmic concepts. This means that the algorithm (an individual) does not know the objective function (valence) $g(.)$ to be maximized. Information about the objective function can only be accessed by querying an oracle. If a maximum $x^* \in X$ exists, it models a desire (desired action). The supremum of $g(.)$ is the ideal level of satisfaction. If it is not well known to the individual it models an aspiration level. At any given step (period) the optimizer (an individual) queries the oracle with a guess x (doing some action x) and the oracle responds with information about the function (valence) around x (eg., value, gradient, Hessian,....). The complexity of the algorithm is the number of queries that can be made to the oracle until the individual can find an $\epsilon-$ approximate maximum x_* for the function (a satisficing action). A first order algorithm considers the value of a function $g(.)$ and its gradient. A famous example is a gradient algorithm, i.e., a linearized proximal algorithm. The status quo is, each period, the couple (what you have done, the satisfaction derived from having done it).....etc....At this stage, the definition of an aspiration level is the big problem. At x, it comes with a (local) quadratic approximate model function of the objective. Then, the definition of an (adaptive) aspiration level that changes with the guess x is the following: it is the maximum level of the quadratic approximate model function. This approximate model function can be an upper bound or a lower bound. The optimizer can use otherwise two approximate model functions, an upper bound and a lower bound (sandwiching). Then, the definition of an aspiration gap is the difference between the approximate aspiration level and the status quo level of satisfaction. The definition of a satisficing move follows with the definition of a sufficient rate of progress....In this way we are able to provide a new and algorithmic construction of the TOTE model. The definition of a worthwhile move goes together with the consideration of a linearized proximal algorithm, via the introduction of a perturbation function (costs or inconveniences to move) and the definition of a proximal payoff (payoff to move).....In this context a worthwhile move models a sufficient descent condition and a trap appears as a perturbed equilibrium, i.e., the end of a succession of worthwhile moves....In the same vein the definition of self regulation success requires : i) to satisfy needs sufficiently, i.e., to solve an inexact algorithm, ii) without too many sacrifices, i.e., using perturbation functions and, iii) quickly enough, i.e., finding when linear convergence occurs. That is, when the speed of convergence of the algorithm is high enough. This strongly depends of the shape of the objective function (sharp or flat).

This is enough to point a strong connexion between (VR) behavioral concepts and algorithmic concepts.

Applications in different disciplines. Given that the development of the VR approach is driven by two main tools, a satisficing move and a worthwhile move, the present paper opens the door to a unifying perspective in the fields of psychology, behavioral sciences and dynamical systems in mathematics. For example,

A) In psychology, this paper initiates,

a) a conceptual and mathematical reformulation of the theory of self-regulation that starts the unification of its two main aspects. That is, the strong interactions between, i) motivation to move aspects and, ii) resistance to move aspects including self efficacy expectancies (Bandura, 1997) and ego depletion effects (Baumeister et all., 1998).

b) a mathematical reformulation of, i) the celebrated Lewin 's force field theory of tension production-tension reduction systems (Lewin, 1951) and of, ii) the famous Bandura's discrepancy production-discrepancy reduction dual processes (Bandura, 1989). This happens because a worthwhile and satisficing move models, within a period, a discrepancy production (= goal setting) - discrepancy reduction (goal striving + goal realization) process.

c) a grand, flexible, algorithmic and computable approach of motivation, emotion and self regulation, where resistance to move and the pursuit of moving goals play a major role.

B) In economics and psychology, this paper begins a dynamic and generalized version of Simon 's bounded rationality theory of satisficing (Simon, 1955), where an individual satisfices (improves enough) with respect to multiple reference points: both the status quo and ideal points.

C) In mathematics, this paper initiates the use in psychology of many famous computable, flexible and inexact optimizing algorithms and variational principles in generalized and asymmetric metric spaces. More precisely, the last section of this paper starts showing that to end in self regulation success, gradient-like algorithms and a lot of variants can be seen as a succession of different formulations of worthwhile and satisficing moves.

D) In cybernetics, this paper will help to show rather easily how iterative gradient algorithms and their variants can be seen as feedback control algorithms with integral quadratic constraints.

In this multidisciplinary context the VR approach helps to mark the distinction between two polar cases:

i) Weak resistance to move in psychology. This situation includes, in mathematics sufficient descent algorithms, like gradient algorithms (see Drusvyatskiy, 2020), proximal algorithms (Martinet, 1970, see Parikh & Boyd, 2013) and proximal gradient algorithms (Aravkin et al., 2017), with error bounds or not, and,

b) Strong resistance to move in psychology. This case refers to famous variational principles in mathematics (Ekeland, 1974, Caristi, 1976, Takahashi, 1989).

Summary. The introduction defines the concept of self regulation and poses the need satisfaction problem that self regulation helps to solve. Section 2 highlights the difficulty to define self regulation concepts. Section 3 and section 4 define satisficing moves (= moves that make sufficient progress) and worthwhile moves in a space of needs and in a space of actions, in the context of the weight gain and loss example. Section 5 models and starts to solve self regulation failures and success in the general context of gradient (= linearized proximal) algorithms.

2 Facing a lot of difficulties

2.1 Difficulty. 1: how to model human life

First of all, to model self regulation at a moment of our life or across the lifespan (Geldhof et al., 2010) requires to model human life.

How can we describe human life in few words?. To be able to give a mathematical formulation of human life the VR approach describes it as a myriad of stop and go dynamics. That is, following Lewin (1935, 1936, 1938, 1951) " life is a constant interplay between completing old situations and opening up new ones" (see Victor Daniels' Website in The Psychology Department at Sonoma State University: Kurt Lewin Notes). More precisely, human dynamics interlace a succession of stays = "continue doing desirable things" and changes = "stop and start doing undesirable and desirable things". These stays and changes entwine a succession of disengagements, reengagements and engagements in different situated activities when individuals leave, get started, stay on track and approach or avoid..... " (Wrosch et al., 2003.a, 2003.b, 2007). Moreover human life interlaces doing and being activities where, i) "doing" means completing tasks towards a predetermined goal and, ii) "being" refers to allowing oneself to experience the present moment. A core example of stop and go dynamics is breaking, continue and forming habits (Jager, 2003, Duhigg, 2014).

Why people do what they do, how and when ?. That is, why such a succession of stays (= continue doing old things) and changes (= stop doing old things and start doing new things) ?. This question is the very topic of a grand and dynamical theory of motivation when resistance to move plays a major role. The main answer of the VR approach has been: if people constantly stop, continue and start doing the same or different things, this is because they must solve the main problem of human life : they must adapt their behaviors to be able to satisfy enough recurrent and changing needs in stable and changing situations, stopping to fulfill some of them, continuing to fulfill others and starting to fulfill new ones.

2.2 Difficulty. 2: how to model the need satisfaction-frustration problem

The need satisfaction-frustration problem. The main goal of the VR approach is to understand how to satisfy enough a set of different needs when the latter can be ill perceived, recurrent or changing with the changing internal and external environments of each individual. Thus, the main task of the VR approach is to provide a conceptual and mathematical answer to the need satisfaction-frustration problem. It is probably one of the most (if not the most) basic question in behavioral sciences. Think at any human problem you can encounter, given that any dissatisfied need is a problem to solve now or later. As Einstein (1949, pp 24- 28) puts it " everything that men do or think concerns the satisfaction of the needs they feel or the escape from pain".

How to adapt your behavior in a world where all things are changing. We face a world where all things are constantly changing, not only in an exogenous way (shocks), but mainly in an endogenous way. It happens like this because as soon as an individual does something to fulfill a need, he changes his internal environment in an endogenous way: this need being satisfied, the set of his dissatisfied needs have changed. Hence his goals change. He also have changed his external environment, moving from having done different things in a given environment to do different or the same things in a new environment. The concept of "situated action" that couples an activity with an environment describes well these fundamental endogenous aspects. Thus, dealing with repeated discrepancies, self regulation theory appears, intuitively, to be the most suited tool that can help to solve the need satisfaction problem in an adaptive way in a changing world.

The need satisfaction-frustration problem as a traveler problem. To be able to build a general mathematical model of human dynamics, the trick is to see human as travelers in a space of bundles of activities. That is, we portray each individual and his different roles (consumer, producer, father, athlete, musician, ...) as a traveler in a space of actions (consuming, producing, ...). Actually, doing something is like moving from what you have done before to what you want to do now. Moving means to stop, continue and start doing things. For example, given limited resources, to be able to start doing something requires to stop doing another thing. Thus, even if this is not so clear at first sight, doing something is a traveler problem because it requires to jump over several obstacles when moving from the beginning to the end. To conclude, in this spatial context, one of the main lessons that the VR approach highlights is that any human dynamic is like travelling in a space filled with resources and means, capabilities to use them, actions, levels of fulfilment of different needs, and derived satisfaction levels.

2.3 Difficulty. 3 : how to define self regulation.

Starting with a broad definition of self regulation. In the context of the VR approach self regulation is what must be done to satisfy enough and speedy enough some needs, without too much frustration of not meeting enough and speedy enough other needs. This definition is very large. It encompasses the existing ones because "what must be done" includes a lot of different things (see the appendix). Its merits is that it greatly helps to provide a clear modelization of self regulation, because it gives a purpose to it. That is, self regulation is at the service of the resolution of need satisfaction-frustration problems, needs being recurrent or changing.

How to unify the myriad of definitions of self regulation ?. Following the literature self regulation "means change, especially change to bring behavior (or other states) into line with some standard such as an ideal or goal" (Baumeister, & Vohs, 2007). That is, "self-regulation is the self 's capacity for altering its behaviors", based on standards. Self-regulation is also defined as the mental processes we use to control our mind's functions, states, and inner processes. Or, self-regulation may be defined as control over oneself. It may involve control over our thoughts, emotions, impulses, appetites, or task performance. Self-regulation is often thought to be the same thing as self-control (Baumeister et al., 2007). That is, self-regulation can be defined as acquiring the capacity to control and then control one's behaviors and emotions when challenged in order to pursue a goal. In this context, the VR approach defines control as the ability to stop, continue and start doing things. This involves inhibition, perseverance and activation processes (Simons et al., 2009) to be able to stop (inhibit), continue and start doing (initiate) an action (Kreemers, 2014). The neuroscience of self and self-regulation emphasized this stop and go aspect of self regulation, driven by inhibition and activation processes (Heatherton, 2011, Rothman et al., 2004). For example "inhibition is a core feature of self-regulation, which refers to the process by which people initiate, adjust, interrupt, stop, or otherwise change thoughts, feelings, or actions in order to effect realization of personal goals or plans or to maintain current standards. ...." (Shah, 2005). Activation is also an "important features of self-regulation, such as initiating self regulatory efforts in order to achieve personal goals" (Shah, 2005). More generally (Higgins, 1997), self-regulation is the dynamic process by which people manage competing demands on their time and resources as they strive, i.e., spend regulatory efforts, to achieve desired outcomes (start doing things), while simultaneously preventing or avoiding undesired outcomes (stop doing things). In contrast to older times, the recent literature marks a profound difference between self-regulation and self-control. "Self-control is about inhibiting strong impulses; self-regulation is about reducing the frequency and intensity of strong impulses by managing stress-load and recovery. In fact, self-regulation is what makes self-control possible, or, in many cases, unnecessary. The reason lies deep inside the brain". See BLOG Self-regulation vs. self-control (Psychokogy today, Stuart Shanker D.Phil., 2016).

Ending in a more precise definition of self regulation. In total, starting from a broad definition of self regulation, we choose a more specific definition: self-regulation is " the ability to flexibly activate, monitor, inhibit, persevere and/or adapt one's behavior" (McClelland et al., 2018). Then, in changing internal and external environments, the goal of self regulation of thoughts, actions and emotions is,

a) to become able to choose, and then choose, when and how to stop, continue and start doing old and new things (goal setting);

b) to become able to stop, continue and start doing these things (goal striving), and then;

c) do these things (goal realization).

This definition supposes that, i) individuals move in a context where motivation to move matters much to overcome resistance to move, ii) during all their life, the implicit or explicit goal of each individual is to satisfy enough his recurrent and changing needs, iii) to satisfy recurrent and changing needs requires to stop, continue and start doing old and new things.

2.4 Difficulty. 4: how to model changing expectancies

The self regulation of human activities and thoughts requires to set expectancies about the consequences of what we consider doing.

How to model expectancies ?. Several famous concepts of expectancies exist in psychology. They are not easy to model. To succeed in this task, the VR approach connects each of them, directly, to the fulfilment of needs or, indirectly, to do something that can help to fulfill these needs :

- a) expected level of satisfaction (satisfaction level expectancy): the fulfilment of a need is expected to provide some satisfaction level;

- b) outcome level expectancy: doing something is expected to fulfill a high enough portion of a need;

- c) valence expectancy: doing something is expected to provide some satisfaction level through the fulfilment of a need. It is worth noting that this fundamental psychological concept is much more useful than the economic concept of utility which has no clear links with the changing state of needs (being basically a classification device).

- d) self efficacy expectancy: becoming able to do something is expected to help to do this thing.

- e) the definition of ideal and aspiration levels follow. They are the known or the approximate highest satisfaction level expectancy that an individual can hope to reach.

As Lewin (1935) puts it, "the valence of an object usually derives from the fact that the object is a mean to the satisfaction of a need, or has indirectly something to do with the satisfaction of a need". Regarding Bandura (1986), self efficacy expectancies represent an individual's belief in his capacity to do something. Then, a valence represents the expected desirability aspect of doing something and self efficacy expectancies define the expected feasibility aspect of doing this thing (Bandura, 1986). According to Bandura's social cognitive theory (Bandura, 1986), the two key determinants of behavior are outcome expectancies and self-efficacy expectancies. According to the VR approach, the key determinants of behavior are the four concepts a), b), c), d), e).

How expectancies change endogenously with the internal and external environments ?.

The (VR) mathematical formulation of expectancies shows that,

1) The valence of doing a thing is the expected level of satisfaction of doing it, given that doing something helps to fulfill different needs. That is, the valence of doing something is itself the composition of two more basic concepts: a satisfaction level expectancy and an outcome level expectancy. This finding is very important because it shows clearly that the valence of doing something depends of the state of different needs.

2) Self efficacy expectancies represent the expected costs of becoming able to do (and then do) something. They are also the composition of different capability costs: a long list of expected costs to be able to stop, continue and start doing things. Our modelisation is an important step which helps to reason in a generalized metric space of activities and moves where a generalized asymmetric distance represents the expected cost to do a move. In this way we provide a new formulation of self efficacy expectancies. It is the expected costs of becoming able to fulfill (and then fulfill) a need or different needs.

3) Expectancies depend of the internal and external environments of an individual.

2.5 Difficulty 5: how to better formulate the TOTE model

2.5.1 The TOTE self regulation model

The TOTE cybernetic model (Miller et al., 1960) represents in psychology the last building block of a self regulation process (Carver, & Scheier, 1982, 1996). It considers a trial and error discrepancy reduction process. The two first blocks being goal setting and preparation for action (Carver, Scheier, 1998). TOTE describes a feedback loop that replaces the stimulus-response as the basic unit of behavior. The aim of this loop is to reduce a discrepancy. It is a succession of four stages, Test, Operate, Test again, Exit. This model is very important because it provides a cognitive and dynamic approach of motivation which shows clearly how plans (mental representations of moves in the VR approach) motivate behavior (Miller et al.,1960). In particular, Carver and Scheier's interpretation of the TOTE model has shown that " emotions reflect progress toward a goal: positive affect is often a cue that a person is moving toward the goal and negative affect signals movement away from the goal " (Vohs, & Baumeister, 2004). In this way TOTE links motivation, cognition and emotion. This model (Miller et al, 1960, see Reeves, 2018, pp 181-183, for a nice presentation) is a succession of TEST-OPERATE rounds until some possible EXIT final round. The definition of each term is the following:

1) In the current period,

- TEST = given the choice of a present ideal state or goal state, become aware of the present state, then, compare the mental representation of the present state against the mental representation of the ideal state.

- OPERATE = if there is a discrepancy between the mental representation of the present state and the ideal state, operate on the environment via the execution of a plan of actions.

- EXIT = if the mental representation of the present state and the ideal state are the same, exit the plan of actions.

2) Next period: repeat the process if the mental representation of the present state and the ideal state are not the same.

TOTE as the basic structure of human trial and error learning. To sum up, at the price of a repetition, (Vohs, & Baumeister, 2004), "people enter the TOTE loop when they establish a goal. The first action, test, refers to the comparison between the current and the desired state (goal). Assuming there is a mismatch between current and desired state, the person engages the operate function of the model in which responses are enacted to close the gap. The person tests again and, depending on whether he or she deems the goal to be met or whether more work is needed, the person either returns to the operate phase or exits the loop". Then, given the choice of a possibly not realistic ideal state or of a more realistic goal state, the TOTE process performs a succession of trials (actions) to, if possible, approach and reach this ideal. In this way, this model provides a simple description of human trial and error learning.

Moving ideals. Notice that this presentation does not exclude the fact that an action plan done in the current period can change, non only the next state, but also the choice of the next ideal state.

Cybernetic aspects. Carver, & Scheier (1982) interpret the TOTE model as follow: "The input function is the sensing of a present condition. That perception is then compared against a point of reference via a mechanism called a comparator. If a discrepancy is perceived between the present state and the reference value, a behavior is performed (output function), the goal of which is to reduce the discrepancy. The behavior does not counter the discrepancy directly but by having an impact on the system's environment (i.e., anything external to the system). Such an impact creates a change in the present condition, leading to a different perception, which in turn is compared anew with the reference value. This arrangement thus constitutes a closed loop of control, the overall purpose of which is to minimize deviations from the standard of comparison".

2.5.2 Critics of the TOTE model

Outside all its great merits, leading in particular to a powerful theory of emotions, the main critic we address to TOTE is that it misses half of the reality if, in psychology, one of the goal is to solve self regulation failure and success problems. TOTE models satisficing moves, but fails to models worthwhile moves. In contrast, the VR approach models self regulation success as a succession of moves that are both worthwhile (short term view) and satisficing (long term view), while self regulation failures represent moves that are either not worthwhile or not satisficing. In this dual way, we articulate the TOTE model (satisficing aspects of a move) with the other side of self regulation (worthwhile aspects of the same move) to better highlight the role of resistance to move: Bandura's self efficacy theory (1978,1997) and Baumeister's ego depletion theory (Baumeister et. al., 1998, 2007.a,b, Baumeister, 2002).

An other important critic is the following. TOTE represents a crude model of satisficing moves. In particular, because, in general, it is described in verbal terms, it does not show, at the mathematical level, how to build (with the help of approximative upper and lower bounds models of the reality) aspiration levels and target levels (goal setting) and more generally, expectancies. It appears too much as a direct translation in psychology of the Miller et. al.(1960) model in cybernetics (see above). Then, its connexions with a general formulation of need satisfaction-frustration problems and motivational dynamics in the context of resistance to move (Lewin, 1935,1936, 1938) is too weak. Mathematically oriented control formulations seem to be absent in psychology. In particular, no links has been done with a long list of famous adaptive (flexible) and computable optimizing algorithms in mathematics.

3 Satisficing and worthwhile moves in a space of needs

The human problem that self regulation wants to solve is the following: how to satisfy enough recurrent and changing needs when resistance to move matters. To simplify the presentation, this section starts with the satisfaction of one need. In this context, it provides a very simple example of a self regulation process. The general case with several needs will be examined elsewhere because it requires to use high level mathematical tools. In the apparently simple context of one need, the problem is to approach an ideal weight (Stotland et al., 2006, Testa et al., 2015). This is an upstream way to satisfy more fundamental downstream needs relative to health, self esteem, affective and relational problems. Our aim is to show that a self regulation process will try to find how to do and do, each period, a move that will be at the same time,

i) satisficing; that is, making sufficient progress relative to the complete satisfaction of this need, i.e., desirable enough and,

ii) worthwhile, i.e., feasible and rewarding enough; that is, such that motivation to move is higher than resistance to move.

In this simple example a move refers to gain or to loose some weight.

3.1 A satisficing move.

3.1.1 An expected need satisfaction function: TEST to discover it.

A simple expected need satisfaction function. Let $w \in R_+$ be the current weight of an individual. The level of satisfaction that w provides, via the fulfilment of downstream health and social needs, is $g = \mathfrak{g}(w) = nw - (1/2)(w)^2 \in R$, where $n = w^* \in R_+$ is the ideal weight for this individual. This means that n is, by construction, the maximum of the need satisfaction function $\mathfrak{g}(.)$:

$w \in R_+ \longmapsto w = \mathfrak{g}(w) \in R$. That is, $\mathfrak{g}(n) \geq \mathfrak{g}(w)$ for all $w \in R_+$. This is the case because $\mathfrak{g}'(w) = n - w = 0$, and $\mathfrak{g}(.)$ is concave. Then, $\mathfrak{g}^* =$ sup $\{\mathfrak{g}(w), w \in R_+\} = \mathfrak{g}(n) < +\infty$ defines an aspiration level, i.e., the ideal level of satisfaction.

TEST. This first phase of TOTE compares the present state, i.e., the weight $v =$ where you are right now, with the ideal state, i.e., the ideal weight $n = w^*$ = where you want to be at the end of this period, or later. Then, there are two cases. Either the comparison (test) reveals that the initial weight $v > 0$ of an individual is too high, i.e., $v > n$, or too low, i.e., $v < n$.

Moves in the space of needs: losses, no loss and no gain, and gains. To satisfy a new need or to better satisfy a recurrent need an individual must do a move. He must go from having the weight v in the previous period to have the weight w in the current period. In this context, within the current period, a move $\mathfrak{m} = (v, \delta, w) \in R_+.R_+$ couples a starting position, i.e., an initial weight v, a translocation or progress $\delta = |w - v|$, and a final position, i.e., a final weight w. A move $\mathfrak{m} = (v, \delta, w)$ is a change if $\delta \neq 0$. It is a stay $\sigma = (v, \delta, v)$ if $\delta = 0$. The translocation stage $\delta > 0$ models how much weight an individual will lose or gain, i.e., $\delta = |w - v| > 0$. In the example, losing (gaining) weight will increase the need satisfaction level from $\mathfrak{g}(v)$ to $\mathfrak{g}(w)$, i.e., $\mathfrak{g}(w) > \mathfrak{g}(v)$ only if the individual loses (gains) some weight, but not too much. That is, iff $\mathfrak{g}(w) > \mathfrak{g}(v)$ i.e., $[w - v]\,[n - (1/2)(w + v)] > 0$. Then, if, for example, the individual has a weight too high and wants to decrease it, i.e., if $v > n$ and $w < v$, this requires $n - (1/2)(w + v) < 0$, i.e., $2n < v + w$, i.e., $2n - v < w < v$. Thus, in the space of needs, the translocation δ of a move models losses, no loss - no gain, and gains, i.e., to lose, to maintain, or to gain weight.

3.1.2 Looking at the satisfaction of doing progress : the impact of emotions

A main finding of the VR approach is that a need satisfaction function $\mathfrak{g}(.)$: $\mathfrak{m} \in \mathfrak{M} \longmapsto \mathfrak{g}(\mathfrak{m}) \in R$ depends of a move $\mathfrak{m} = (v, \delta, w)$, instead of, only, the end w of the movement. In the weight example where a move is $\mathfrak{m} = (v, \delta = |w - v|, w)$, a need satisfaction function $\mathfrak{g}(.)$ depends, both, of the new weight w that an individual wants to reach, and of the weight progress $\delta = |w - v|$. In a separable case, $\mathfrak{g}(\mathfrak{m}) = \mathfrak{g}^a(w - v) + \mathfrak{g}^b(w) \in R$ defines the need satisfaction of doing a move $\mathfrak{m}$. It has two parts,

i) in the translocation phase, the satisfaction $\mathfrak{g}^a(\delta)$ of progressing towards the ideal weight $n > 0$, making the progress $\delta = w - v \geq 0$ ($\delta = v - w \geq 0$), when moving from the too low (too high) initial weight $v < n$ ($v > n$) to a higher (lower) weight w.

ii) at the end of the move, the satisfaction $\mathfrak{g}^b(w)$ of having a higher weight w.

This general formulation defines a reference dependent need satisfaction function. In this way, it introduces some dynamical and emotional aspects. For example, if an individual has a too low weight and wants to gain weight (but not too much), i.e., if $v < w \leq n = w^*$, the satisfaction of doing progress

$\mathfrak{g}^a(w-v) = \pi(w-v) - \eta(n-w)$ can be the difference of two terms: the contentment $\pi(w-v) \in R_+$ of doing progress and the frustration $\eta(n-w) \in R_+$ of not doing enough progress towards the ideal weight n.

We will see later that this point of view is at the heart of acceleration process in the most recent and famous optimizing algorithms (Nesterov 2013), making a surprising link with the gradient hypothesis in psychology. For a similar reference dependent value (utility) function in a static context, without reference to a move, see Köszegi & Rabin (2006). For simplification, in the previous example, the level of satisfaction $\mathfrak{g}^a(w-v)$ of doing progress is not considered.

3.1.3 A satisficing move: making a high enough rate of progress

Most of the time, because of a lack of resources, a lot of needs cannot be satisfied immediately. That is, their full satisfaction (if any) requires several periods. Then, the main question is: how, each period, to satisfy enough and quickly enough some recurrent and changing needs and to wait for the satisfaction of others. For example, how much weight to lose or to gain, each period ? That is, how to choose a high enough rate of progress towards achieving an ideal weight.

Aspiration failure problems. The usual presentation of the TOTE process offers no guaranty to reduce fast enough the discrepancy between the present state and the ideal state. Then, in an important paper relative to TOTE, Carver & Scheier (1990, p 22) asked the question: how rapidly discrepancies can be reduced ? They suggest that " in this case the reference value is an acceptable or desired rate of behavioral discrepancy reduction". They make the distinction between three cases: progress towards the goal is lower, equal or higher than a standard rate of discrepancy reduction. The important point being that the sign of the discrepancy between the present rate of discrepancy reduction and the standard rate of discrepancy reduction causes negative, null or positive emotions. These suggestions show clearly that the usual presentation of TOTE is too simple. This is clear in the context of poor peoples who suffer from aspiration failures when individuals have no idea about their ideal state, or when, in the present case, the aspiration gap between their present state and their ideal (but not realistic) state is too large compared to their current resources (Appadurai 2002, Ray, 2003). This pushes them to give up reaching a better position. In the weight example, this occurs, for example, when an individual is too big.

Escaping to aspiration traps : satisfice (divide the difficulty). What is lacking is the definition of an intermediate concept between the present level and ideal level of need fulfilment. In this context, the VR approach defines a satisficing level as a high enough level of need fulfilment. That is, instead of giving up, an individual can try moving from the present state to a more realistic state of need satisfaction. In other words, this individual can try to move, more modestly, from desirable fantasies, hopes and wishes to lower goal levels, both desirable and feasible enough (Oettingen et al., 2001). A satisficing level of need satisfaction is a driving goal level. It differs from an action goal. The concept of satisficing comes from Simon (1955). It means "to improve enough", instead of optimizing. Simon provides a static presentation of this concept. The VR approach provides a dynamic presentation of a satisficing move in terms of a sufficient rate of progress (Carver & Scheier, 1990).

To be able to set a satisficing level, it is essential to know the present satisfaction level $\mathfrak{g}(v)$ and the ideal level g^*.

The frustration gap. Consider, as the TOTE model suggested,

i) the initial level of satisfaction $g = \mathfrak{g}(v)$ related to the initial weight $v > 0$;

ii) its ideal level of satisfaction $g^* = \sup\{\mathfrak{g}(u), u \in R_+\} = \mathfrak{g}(n) < +\infty$ and the related ideal weight $n = v^* \in R_+$ such that $g^* = \mathfrak{g}(n)$;

iii) the frustration gap $\mathfrak{f}(v) = g^* - \mathfrak{g}(v) \in R_+ \geq 0$. This is the level of dissatisfaction resulting from the discrepancy between the present weight v and the ideal weight n.

If the need satisfaction function is quadratic, for example, if $\mathfrak{g}(.) : w \in R_+ \longmapsto \mathfrak{g}(w) = nw - (1/2)(w)^2 \in R$, the marginal satisfaction level of having the weight w is $\nabla\mathfrak{g}(w) = \mathfrak{g}'(w) = n - w$, the ideal weight $n \in R_{++}$ satisfies $\nabla\mathfrak{g}(n) = \mathfrak{g}'(n) = n - n = 0$ and the ideal level of satisfaction is $g^* = n^2/2$. Furthermore, the level of dissatisfaction at v is the discrepancy $\mathfrak{f}(v) = (1/2)\left[\nabla\mathfrak{g}(v)\right]^2$. It depends of the gradient (= the derivative in this scalar case) of the frustration gap at v.

We will show later (in the mathematical part of the paper) that this striking formula is quite general and plays a major role in the resolution of gradient algorithms. It emphasizes that, when using a concave quadratic approximation of a need satisfaction function $\mathfrak{g}(.)$, the lower the gradient $\nabla\mathfrak{g}(v)$ of the need satisfaction function $\mathfrak{g}(.)$ is, the lower is the level of dissatisfaction at v.

A threshold level of satisfaction: a satisficing standard. This level $g^{\triangleright}$ lies between the present satisfaction level $\mathfrak{g}(v)$ and the ideal level g^*. That is, $g^{\triangleright} = \mathfrak{g}(v) + \theta\left[g^* - \mathfrak{g}(v)\right], 0 < \theta \leq 1$. A satisficing level $g^{\triangleright}$ is a moving goal in the space of satisfaction levels. It models a standard, or a realistic enough aspiration level. Notice that a satisficing weight w exists because g^* is a supremum. The satisficing standard $g^{\triangleright} = g^{\triangleright}(v, g^*, \theta)$ depends on the initial level of satisfaction $\mathfrak{g}(v)$, the rate of improvement $0 < \theta \leq 1$ and the initial discrepancy $\mathfrak{f}(v) = g^* - \mathfrak{g}(v) \geq 0$, where $\theta\left[g^* - \mathfrak{g}(v)\right]$ is a share of this gap.

A satisficing move. A new weight w satisfices if $\mathfrak{g}(w) \geq g^{\triangleright}$. That is, if $\mathfrak{g}(w) - \mathfrak{g}(v) \geq \theta\left[g^* - \mathfrak{g}(v)\right]$. This means that the rate of discrepancy reduction $\left[\mathfrak{g}(w) - \mathfrak{g}(v)\right] / \left[g^* - \mathfrak{g}(v)\right]$ must be higher than the standard rate θ. Thus, our condition models what can be a sufficient rate of progress (Carver & Scheier, 1990). Our condition gives also a sufficient rate of decrease of the frustration gap $\mathfrak{f}(w)$.

That is, $\mathfrak{g}(w) - \mathfrak{g}(v) = \mathfrak{f}(v) - \mathfrak{f}(w) \geq \theta\mathfrak{f}(v)$ iff $0 \leq \mathfrak{f}(w) \leq (1-\theta)\mathfrak{f}(v)$. (*)

The size of a satisficing move. Let us show that a weight gain or loss must be not too low and not too large. On the example, the equality $f(v) = (1/2)\left[n - v\right]^2$ shows that condition (*) is equivalent to $0 \leq \left[n - w\right]^2 \leq (1-\theta)\left[n - v\right]^2$, i.e., $0 \leq |n - w| \leq (1-\theta)^{1/2}|n - v|$. That is,

i) if $w < n$, $n - w \leq (1-\theta)^{1/2}|n - v|$ gives $w \geq v_{\diagdown} = n - (1-\theta)^{1/2}|n - v|$;

ii) if $w > n$, $w - n \leq (1-\theta)^{1/2}|n - v|$ gives $w \leq v_{+} = n + (1-\theta)^{1/2}|n - v|$.

Then, the set of ends of satisficing moves starting from the initial weight v is $\$_\theta(v) = \{w \in R_+, 0 < v_\swarrow \leq w \leq v_+\}$. Thus, a satisficing weight gain or loss $w - v$ must be not to low and not too high: $v_\swarrow - v \leq w - v \leq v_+ - v$, i.e.,

$(n-v) - (1-\theta)^{1/2} |n-v| \leq w - v \leq (n-v) + (1-\theta)^{1/2} |n-v|$, i.e.,

$(v-n) + (1-\theta)^{1/2} |n-v| \geq v - w \geq (v-n) - (1-\theta)^{1/2} |n-v|$.

Let us note $0 \leq \underline{\eta}(\theta) = 1 - (1-\theta)^{1/2} \leq 1$ and $\overline{\eta}(\theta) = 1 + (1-\theta)^{1/2} \geq 1$.

Then,

i) if the initial weight is too low, $v < n$, a satisficing weight gain $\delta = w - v \geq 0$ is such that $\underline{\eta}(\theta)(n-v) \leq w - v \leq \overline{\eta}(\theta)(n-v)$;

ii) if the initial weight is too high, $v > n$, a satisficing weight loss $\delta = v - w \geq 0$ is such that $\underline{\eta}(\theta)(v-n) \leq v - w \leq \overline{\eta}(\theta)(v-n)$.

Then, a satisficing weight gain or loss $\delta = |w - v|$ must be not too low and not too large, i.e., low enough and large enough relative to the initial discrepancy $|n - v|$.

A satisficing move provides a **high enough advantage to move**. In the example the advantage to do the move $\mathfrak{m} = (v, \delta, w)$ rather than to do the stay $\sigma = (v, 0, v)$ is $A(\mathfrak{m}/\sigma) = \mathfrak{g}(w) - \mathfrak{g}(v) \geq 0$ while $A(\mathfrak{m}^*/\sigma) = \mathfrak{g}(n) - \mathfrak{g}(v) \geq 0$ models the ideal advantage to do a move starting from the initial weight v. In this context, a satisficing move is such that the advantage to move $A(\mathfrak{m}/\sigma)$ is high enough with respect to the ideal advantage to move $A(m^*/\sigma)$. That is, $A(\mathfrak{m}/\sigma) \geq \theta A(\mathfrak{m}^*/\sigma), 0 < \theta \leq 1$. On the example $A(\mathfrak{m}^*/\sigma) = g^* - \mathfrak{g}(v) = \mathfrak{f}(v) = (1/2)[n-v]^2$.

Emotional aspects linked to satisficing standards. To satisfice balances four emotional aspects relative to where you are and to where you want to be:

- before satisficing: an initial frustration feeling to do not have the ideal weight $\mathfrak{f}(v) = g^* - \mathfrak{g}(v) \geq 0$;

- before (and after) satisficing: an ex ante (and an ex post) satisfaction feeling to imagine to improve enough (ex ante) and to improve enough (ex post) the weight $\mathfrak{g}(w) - \mathfrak{g}(v)$;

- after satisficing: a frustration feeling to do have reached, yet, the ideal weight, given the residual gap $g^* - \mathfrak{g}(w) \geq 0$.

3.2 A worthwhile move

3.2.1 Expected costs to do a move.

Try to do a move and do it = Operate. To satisfy a need an individual must do a move. That is, he must do something (the end of the move). This can be a different thing than before if this individual wants to satisfy a new need or to better satisfy a recurrent need. This can be the same thing if the recurrent need is the same. For example, an individual who wants to lose weight must eat less than before and must increase physical exercises,.... Then, i) in an initial goal setting stage, he must choose to do some move (choosing how much weight to lose and what must be done to loose it) and this choice being done, ii) in a goal striving stage, he must try to become able to do this move (what must be done) and finally, iii) in a goal realization stage, he must do (implement) these things (eat less, doing more exercises).

Costs to do a move: disengagement, reengagement and engagement costs. They include costs to choose to do something and costs to try to do and do this thing. Costs to choose to do something are spend before doing a thing. They become sunk costs when an individual tries to do this thing and does it. Costs to try to do and do something refer to capability costs and execution costs. That is, costs to become able to stop, continue and start doing something and costs to effectively stop, continue and do this thing. These costs to try to do and do something are disengagement, reengagement and engagement costs. They model a long list of costs, like switching costs and transaction costs, with physiological, physical, material, financial, cognitive, emotional and social aspects. See Soubeyran (2021.a,b,c,d) for a lot of examples. For disengagement, reengagement and engagement processes, see among others, Wrosch et al. (2003.a,b, 2007) and Carver & Scheier (2005).

In this example, try to do something and do it means OPERATE. That is, try to loose or to gain weight and do it, i.e., do the move $\mathfrak{m} = (v, \delta, w)$. This means, first, become able to perform the translocation $\delta = |w - v| > 0$. For example, if eating less (more) is the only way to lose (gain) weight, this requires to become able to eat less (more), and then, to eat less (more), to finally get the weight $w \lesseqgtr v$.

Then, the cost $\mathfrak{C}(m) = \mathfrak{C}(v, \delta, w) \in R_+$ to do the move $\mathfrak{m}$ is

$$\mathfrak{C}(\mathfrak{m}) = \begin{bmatrix} c_= v + c_+(w - v), \text{ if } w - v \geq 0 \\ c_= w + c_\swarrow(v - w) = c_= v + (c_\swarrow - c_=)(v - w), \text{ if } w - v < 0 \end{bmatrix},$$

while $\mathfrak{C}(\sigma) = c_= v$ is the cost to do the stay σ,

where $c_= = c^a_= + c^b_=$, $c_+ = c^a_+ + c^b_+$ and $c_\swarrow = c^a_\swarrow + c^b_\swarrow \in R_+$ define the costs to be able to "maintain, gain and lose", and then "maintain, gain and lose" one unit of weight. Thus, the cost to stay (= maintain his weight) is $\mathfrak{C}(\sigma) = c_= v$. Costs to move (change or stay) $\mathfrak{C}(\mathfrak{m}) = p(v, w)$ define a partial quasi distance if $c_\swarrow - c_= > 0$. See Soubeyran (2021.c) and Hai et al.(2022) for an application.

Then, $\mathfrak{C}(\mathfrak{m}) = \mathfrak{C}(\sigma) + \mathfrak{C}(\delta)$,

where $\mathfrak{C}(m) = \begin{bmatrix} \rho_= v + \rho_+(w - v), \text{ if } w - v \geq 0 \\ \rho_= v + +\rho_\swarrow(v - w), \text{ if } w - v < 0 \end{bmatrix}$, and

$\mathfrak{C}(\sigma) = \rho_= v$, with $\rho_= = c_= > 0, \rho_+ = c_+ > 0$ and $\rho_\swarrow = c_\swarrow - c_= > 0$.

3.2.2 Definition of a worthwhile move.

This definition comes at the end of a short list of concepts, i.e., a variational rationality structure. Given, as it has been done previously, the definitions of a move $\mathfrak{m} = (v, \delta, w)$, a need satisfaction function $\mathfrak{g}(.) : w \in \mathcal{V} = R_+ \longmapsto \mathfrak{g}(w) \in Z = R$, and costs to move $\mathfrak{C}(\mathfrak{m})$, and using the "lose or gain weight" example, we define successively:

Advantages to move: $A(\mathfrak{m}/\sigma) = \mathfrak{g}(\mathfrak{m}) - \mathfrak{g}(\sigma) \in Z = R^l$. They represent the difference between the satisfaction level to do a move (a change or a stay) and the satisfaction to do a stay. In the "lose or gain weight" example,

$A(\mathfrak{m}/\sigma) = \mathfrak{g}(w) - \mathfrak{g}(v) = n(w - v) - (1/2)(w^2 - v^2)$, i.e.,

$A(\mathfrak{m}/\sigma) = (w - v)\left[(n - v) - (1/2)(w - v)\right].$

Extension. If, on the weight example, the satisfaction function

$\mathfrak{g}(\mathfrak{m}) = \mathfrak{g}^a(\delta) + \mathfrak{g}^b(w)$ depends not only on the end w of the move $\mathfrak{m} = (v, \delta, w)$ but also on the translocation δ, with $\mathfrak{g}^b(w) = nw - (1/2)w^2$ and $\mathfrak{g}^a(\delta) = \pi(w - v) - \eta(n - w)$, where $\pi, \eta \in R_+$, then, $\mathfrak{g}^a(\delta) = (\pi + \eta)\delta - \eta(n - v)$ and $\mathfrak{g}^a(0) = -\eta(n - v)$ give a similar formula $A(\mathfrak{m}/\sigma) = (n + \pi + \eta)(w - v) - (1/2)\left[(w^2 - v^2\right].$

Then, if, in the weight example, we want to consider the satisfaction of making progress as a part of advantages to change, we must change n in $n + \pi + \eta$ in the formulation of a worthwhile balance.

Inconveniences to move: $I(m/\sigma) = \mathfrak{C}(m) - \mathfrak{C}(\sigma) \in Z = R^l_+$. They are the difference between the costs to do a move (a change or a stay) and the costs to do a stay. In the example,

$I(\mathfrak{m}/\sigma) = \mathfrak{C}(\mathfrak{m}) - \mathfrak{C}(\sigma) = \begin{bmatrix} \rho_+(w - v), \text{ if } w - v \geq 0 \\ \rho_\swarrow(v - w), \text{ if } w - v < 0 \end{bmatrix}.$

with $\rho_= = c_=, \rho_+ = c_+$ and $\rho_\swarrow = c_\swarrow - c_= > 0$.

Inconveniences to move are non negative if $c_\swarrow > c_=$, i.e., if the unit cost to lose weight is higher than the unit cost to maintain it. This seems to be true.

Motivation to move: $M(\mathfrak{m}/\sigma) = U\left[A(\mathfrak{m}/\sigma)\right]$ where,

$U[.] : A \in \mathcal{A} = R^l \longmapsto M = U[A] \in \mathcal{U} = R$. It defines the utility $U[A]$ of advantages to move A. In the example, $M(\mathfrak{m}/\sigma) = U\left[A(\mathfrak{m}/\sigma)\right] = a\left[A(\mathfrak{m}/\sigma)\right]^\alpha$, with $\alpha, a \in R_{++}$. The scalar $a > 0$ models the importance given to motivation to move. Given this formula, motivation to move means "wanting to change or wanting to stay". This anticipates, generalizes and formalizes (Soubeyran, 2009, 2010, 2021.a,b,c,d), the recent definition of motivation = "wanting to change" suggested by Baumeister (2016). See also Reeves (2016), and Kruglanski et al.(2016).

Resistance to move: $R(\mathfrak{m}/\sigma) = D\left[I(\mathfrak{m}/\sigma)\right]$ where,

$D[.] : I \in \mathcal{I} = R^l_+ \longmapsto R = D[I] \in \mathcal{D} = R$. It defines the disutility $D[I]$ of inconveniences to move I. In the example, $R(\mathfrak{m}/\sigma) = D\left[I(\mathfrak{m}/\sigma)\right] = b\left[I(\mathfrak{m}/\sigma)\right]^\beta$, with $\beta, b \in R_{++}$. The scalar $b > 0$ models the importance given to inconveniences to move. Our formulation generalizes and formalises the famous Lewin's concept of resistance (Lewin, 1935, 1936, 1938, 1951).

Payoff of a move: if $\alpha = \beta = 1$, it is $P(\mathfrak{m}) = \mathfrak{g}(\mathfrak{m}) - \xi\mathfrak{C}(\mathfrak{m})$, where $\xi > 0$ is the weight given to the costs to move.

Worthwhile balance: $B(\mathfrak{m}/\sigma) = M(\mathfrak{m}/\sigma) - \xi R(\mathfrak{m}/\sigma)$. It generalizes and models the concept of "force of motivation" (Lewin, 1938). It is the weighted difference between motivation and resistance to move. The weight $\xi \in R_+$ models the relative importance given to resistance to move. In the example, if for simplification $\alpha = 1 = \beta > 0$,

$B(\mathfrak{m}/\sigma) = A(\mathfrak{m}/\sigma) - \xi I(\mathfrak{m}/\sigma)^\beta$, i.e.,

$B(\mathfrak{m}/\sigma) = (w - v)\left[(n - v) - (1/2)(w - v)\right] - \xi\rho_+^\beta(w - v)^\beta$ if $w - v \geq 0$;

$B(\mathfrak{m}/\sigma) = (w - v)\left[(n - v) - (1/2)(w - v)\right] - \xi\rho_\swarrow^\beta(v - w)^\beta$ if $w - v \leq 0$, that is,

$B(\mathfrak{m}/\sigma) = (v - w)\left[(v - n) - (1/2)(v - w) - \xi\rho_\swarrow^\beta(v - w)^{\beta - 1}\right]$ if $w - v \leq 0$.

. The two benchmark cases are $\alpha = 1, \beta = 1$ (strong resistance do move) and $\alpha = 1, \beta = 2$ (weak resistance do move). This distinction is essential because the shape of a worthwhile dynamic depends a lot on the strength of resistance to move, relative to the strength of motivation to move.

In the linear-linear case, $\alpha = \beta = 1$, a worthwhile balance is the difference $B(\mathfrak{m}/\sigma) = \mathfrak{g}(m) - \mathfrak{g}(\sigma) - \xi\,[C(m) - C(\sigma)] = P(\mathfrak{m}) - P(\sigma)$ between the payoff to change $P(\mathfrak{m})$ and the payoff to stay $P(\sigma)$. Then, a change is worthwhile if the payoff to change is greater than the payoff to stay.

3.2.3 A worthwhile move: trying not to endure too many sacrifices to satisfy some needs.

The indecision (= ambivalence) problem. The most basic question that human dynamics can pose is the following: should I stay, or should I change ?. This is an indecision problem where an individual hesitates between stay or change. In this setting, coexist, within an individual, ambivalent (positive and negative) feelings toward the same thing (person, object, or action), simultaneously drawing him or her in opposite directions. Here the thing is a move (stay or change) and the conflicting feelings are motivation and resistance to move. The VR concept of worthwhile move provides a unifying and simple answer to such an indecision problem.

Definition. A change $m = (v, \delta, w), \delta \neq 0$ is worthwhile, relative to the stay $\sigma = (v, 0, v)$ when motivation to move is high enough (= not too low) relative to resistance to move. That is, when its worthwhile balance is non negative: $B(m/\sigma) = M(\mathfrak{m}/\sigma) - \xi R(\mathfrak{m}/\sigma) \in R_+, \xi > 0$.

This means that when a move is worthwhile, it cannot be rejected without a careful examination, unless you find a better one, with a largest worthwhile balance. In other words, an individual will consider changing when driving forces are high enough relative to resisting forces (Lewin, 1935,1936, 1938, 1951), i.e., when a goal (= the desirable end of the move) is desirable enough and feasible enough (Oettingen & Gollwitzer, 2015), that is, when an individual wants enough to move and expects that he can move easily enough.

The strength of an intention to move. A non negative worthwhile balance means also that the ratio between motivation and resistance to move is high enough: $M/R = M(\mathfrak{m}/\sigma)/R(\mathfrak{m}/\sigma) \geq \xi > 0$. The weight $\xi = 1 + h > 0$, $h \lesseqgtr 0$, is a threshold level. Then, if $M/R \geq \xi$, the larger is ξ, the larger is M relative to R. That is, the higher is ξ, the less the situation is ambivalent, the less the individual hesitates between change or stay, the more an individual is determined to do a change m rather than to do the stay σ, i.e., the stronger is his intention to change rather than to stay. Thus, when an individual faces an ambivalent change situation (Soubeyran, 2021.d), if $M/R \geq \xi$, the threshold level ξ represents the strength of his intention to move. See Mladinic (1998) for different indexes of ambivalence.

How much sacrifices you can accept to improve your current situation ? When the initial situation of an individual is bad, his main problem is, within a period, to try to improve it, moving from the satisfaction level $\mathfrak{g}(v)$ to a higher satisfaction level $\mathfrak{g}(w) > \mathfrak{g}(v)$, after moving from having the weight v to have a weight w closer to the ideal weight n. A way to overcome the presence of too high sacrifices is to devaluate them, giving less importance on resistance to move R. In this setting, an individual can accept to make more sacrifices to improve his current situation. This refers to an implicit or explicit devaluation process in psychology, when activating a need devalues unrelated objects (Brendl et al., 2003). Formally, a way to devaluate the sacrifices is to lower the weight $\xi > 0$,until $0 \leq \xi \leq 1 \Longleftrightarrow h \leq 0$. This means that it is "as if" motivation to move is lower than resistance to move, but not too much: $0 \leq \xi R \leq M \leq R$. The size of his short term sacrifices being $(1-\xi)R \geq 0$.

Making a change worthwhile in the long run when it is not worthwhile in the short run. When, within the current period, an individual is weakly determined to change rather than to stay, his intention to change rather than to stay is weak. In this situation, even if changing is weakly worthwhile in the short run, changing can become strongly worthwhile in the long run. Because changing helps to improve the current situation. Thus, choosing to stay for a while in a new and better situation over several periods can be much better than to stay at the status quo, because of the benefits of cumulated advantages. This is the essence of the famous exploration-exploitation trade off (March, 1991). Then, even if motivation to change does not balance resistance to change in the short run, it can do it in the long run, after a sufficient number of stays in the new and better situation.

3.2.4 Sets of ends of worthwhile moves (with strong resistance to move)

Consider first the case of strong resistance to move, i.e., $\alpha = \beta = 1$. The opposite case of weak resistance to move being examined later, at the end of this section when $\alpha = 1, \beta = 2$.

As seen before, when resistance to move is strong, worthwhile balances are, in the weight gain or loss example,

$B^{+}(\mathfrak{m}/\sigma) = (w-v)\,[n-v-\xi\rho_{+}) - (1/2)(w-v)]$ if $w-v \geq 0$;

$B^{\searrow}(\mathfrak{m}/\sigma) = (v-w)\,[(v-n-\xi\rho_{\searrow}) - (1/2)(v-w)]$ if $w-v \leq 0$.

Then, in this example, we can define two sets of ends of worthwhile moves.

Sets of ends of worthwhile weight gains. Suppose $0 < v < n$. Then, the set of ends $w \in X = R_{+}$ of worthwhile weight gains $\mathfrak{m} = (v, w-v \geq 0, w) \in \mathcal{M}$ starting from the initial weight v is,

$\mathcal{W}^{+}(v) = \{w \in R_{+}, w \geq v \text{ and } B^{+}(\mathfrak{m}/\sigma) \geq 0\}$. That is,

$\mathcal{W}^{+}(v) = \{w \in R_{+},\ v \leq w \leq \overline{v}\}$ where $\overline{v} = v + 2(n-v-\xi\rho_{+})$.

To stay at the initial weight is worthwhile: $v \in \mathcal{W}^{+}(v)$. A worthwhile weight gain $\delta = w-v > 0$ exists iff $n-v-\xi\rho_{+} > 0$. This requires,

i) $n-v > \xi\rho_{+} > 0$. That is, a large enough discrepancy $n-v > 0$;

i) $0 < v < n-\xi\rho_{+}$, i.e., a low enough initial weight compared to the ideal weight.

ii) $0 \leq w-v \leq \overline{v}-v = 2(n-v-\xi\rho_{+})$, i.e., a not too large weight gain $\delta = w-v \geq 0$.

Sets of ends of worthwhile weight losses. The set of ends $w \in X = R_+$ of worthwhile weight losses $m = (v, v - w \geq 0, w) \in \mathcal{M}$ starting from the initial weight $v > 0$ is $\mathcal{W}^{\searrow}(v) = \{w \in R_+, w \leq v \text{ and } B^+(\mathfrak{m}/\sigma) \geq 0\}$. That is,

$\mathcal{W}^{\searrow}(v) = \{w \in R_+,\ \underline{v} \leq w \leq v\}$ where $\underline{v} = v - 2(v - n - \xi\rho_{\searrow})$.

To stay at the initial weight is worthwhile: $v \in \mathcal{W}^{\searrow}(v)$. A worthwhile weight loss $\delta = v - w > 0$ exists iff $v - n - \xi\rho_{\searrow} > 0$. This requires,

i) $v - n > \xi\rho_+ > 0$. That is, a large enough discrepancy $v - n > 0$, i.e.,

ii) $v > n + \xi\rho_{\searrow}$, i.e., an initial weight v large enough compared to the ideal weight;

iii) $0 \leq v - w \leq v - \underline{v} = 2(v - n - \xi\rho_{\searrow})$, i.e., a not too large weight loss $\delta = v - w \geq 0$.

A worthwhile move must be small enough. That is, to be worthwhile a move $m = (v, \delta = |w - v|, w)$, i.e., a worthwhile weight gain $\delta = w - v \geq 0$ or a weight loss $\delta = v - w \geq 0$ must be small enough, not larger than $2(n - v - \xi\rho_+)$ and $2(v - n - \xi\rho_{\searrow})$.

3.3 A new characterization of self regulation failures and success

The question that pose self regulation failures is the following: what is, each period, relative to the satisfaction of recurrent needs and new needs, the highest rate of progress that does not require too many current sacrifices.

Self regulation failures: when a worthwhile (satisficing) move is not satisficing (worthwhile). "Self regulation is required when people face resistance or conflict to attaining their desired future" (Gollwitzer, Oettingen, 2011, James, 1890). Self regulation is also required when people lack motivation to move. Self regulation is a process. It occurs, i) in a changing internal environment including needs, aspirations, goals, thoughts and feelings and, ii) in a changing external environment plenty of objects, persons and landscapes that can help (as means) or forbidd (as obstacles) to satisfy needs.

In our context, this desired future is the satisfaction of a list of proximal and distal needs. Then, a self regulation process lists what must be done, to become able to, i) stop fulfilling some intermittent needs, ii) continue to fulfill (more or less) some recurrent needs and, iii) start fulfilling news needs. Given that, to fulfill needs requires to move from having done some things (a bundle of activities) in the previous period to do the same or other things in the current period, what must be done is, first, to become able to disengage, reengage and engage in different activities that help or forbid the satisfaction of recurrent and new needs. Then, it is to effectively do that. Self regulation failures (success) occur when, ex ante, what must be done cannot (can) be done or, ex post, has not (has) been done.

Then, to self regulate his activities, an individual must navigate between two benchmark cases:

i) To do a satisficing move that makes enough progress relative to the full satisfaction of his short term and long term needs. This satisficing move satisfices, i.e., satisfies enough, in a speedy enough way, his recurrent and new needs. This means that doing this move must be motivating enough.

ii) To make a worthwhile move. Such a move must improve enough the satisfaction of his needs without requiring too high sacrifices. That is, this move must overcome enough resistance to move.

Then, one of the major finding of the VR approach is to show that the goal of self regulation is, each period, to find and, then, to implement a move that can be, both, satisficing and worthwhile. If not, a move that is satisficing, but not worthwhile, can require too high sacrifices. And a move that is worthwhile, but not satisficing, can present a too low rate of progress.

However, there are several difficulties to find and to perform a satisficing (worthwhile) move that is also a worthwhile (satisficing) move. Self regulation failures lists these difficulties. Thus, consider the weight gain- loss example and examine when the sets of ends of satisficing and worthwhile moves intersect (or not).

3.4 Self regulation failures

Consider self regulation failures to gain or to lose enough weight.

3.4.1 Fail to gain enough weight.

To be concrete, consider first the weight gain example where the initial weight $v > 0$ is too low with respect to the ideal weight, i.e., $0 < v < n$. Then, starting from v, the problem is to know when a weight gain $\delta = w - v \geq 0$ can be both a satisficing and a worthwhile weight gain move $m = (v, \delta, w)$. The set of ends w of all worthwhile and satisficing weight gains m starting from the weight v is the intersection set $\mathcal{W}_\xi^+(v) \cap \$_\theta^+(v)$, where $\mathcal{W}_\xi^+(v) = \{w \in R_+, v \leq w \leq \overline{v}\}$ and $\$_\theta^+(v) = \{w \in R_+, v_\swarrow \leq w \leq v_+\}$, with $0 < v < n$, $\overline{v} = v + 2(n - v - \xi\rho_+) \geq v$,

$v_\swarrow = n - (1-\theta)^{1/2} |n - v| = n - (1-\theta)^{1/2}(n - v)$,

$v_+ = n + (1-\theta)^{1/2} |n - v| = n + (1-\theta)^{1/2}(n - v)$.

Notice that $v \leq v_\swarrow$ because $v_\swarrow - v = (n-v)\left[1 - (1-\theta)^{1/2}\right] > 0$ if $0 \leq \theta \leq 1$ and $0 < v < n$.

Let $s_+ = (n-v)/\rho_+ > 0$ and $\pi(\theta) = (1/2)\left[1 + (1-\theta)^{1/2}\right] \in [1/2, 1]\,, \pi(0) = 1, \pi(1) = 1/2$ with $\pi(\theta') < \pi(\theta')$ if $\theta' > \theta$.

Result.1. Self regulation failures to gain enough weight occur when there exists no worthwhile move that provides a satisficing rate of progress (higher than a desired rate of progress θ). That is, when the intersection $\mathcal{W}_\xi^+(v) \cap \$_\theta^+(v)$ is empty, i.e., iff $\xi > s_+\pi(\theta)$.

Proof: $\mathcal{W}_\xi^+(v)$ and $\$_\theta^+(v)$ are not empty. Then, $\mathcal{W}_\xi^+(v) \cap \$_\theta^+(v) = \phi$ iff $\overline{v} < v_\swarrow$. That is, $\overline{v} < v_\swarrow$ iff $\overline{v} = v + 2(n - v - \xi\rho_+) < v_\swarrow = n - (1-\theta)^{1/2}\left[n - v\right]$, i.e.,

$2(n - v) - 2\xi\rho_+ < (n - v) - (n - v)(1-\theta)^{1/2}$, i.e.,

$(n - v) - 2\xi\rho_+ + (n - v)(1-\theta)^{1/2} < 0$, i.e.,

$2\xi\rho_+ > (n - v)\left[1 + (1-\theta)^{1/2}\right]$, i.e., $\xi > s_+\pi(\theta)$.

Remark. If $\theta = 0$, $\pi(0) = 1$ requires $\xi > s_+$. If $\theta = 1$, $\pi(1) = 1/2$ requires $\xi > s_+/2$

3.4.2 Fail to lose enough weight.

In this second case the initial weight is too high relative to the ideal weight, i.e., $v > n$. Then, $\$^{\searrow}_{\theta}(v) = \{w \in R_+, 0 < \upsilon_{\searrow} \leq w \leq \upsilon_+\}$ and

$\mathcal{W}^{\searrow}_{\xi}(v) = \{w \in R_+,\ \underline{v} \leq w \leq v\}$,

$\upsilon_{\searrow} = n - (1-\theta)^{1/2} |v - n| = n - (1-\theta)^{1/2}(v - n)$,

$\upsilon_+ = n + (1-\theta)^{1/2} |v - n| = n + (1-\theta)^{1/2}(v - n)$, and

$\underline{v} = v - 2(v - n - \xi\rho_{\searrow})$.

Let $s_{\searrow} = (v - n)/\rho_{\searrow} > 0$.

Result.2. Self regulation failures to lose enough weight occur when the intersection $\mathcal{W}^{\searrow}_{\xi}(v) \cap \$^{\searrow}_{\theta}(v)$ is empty, i.e., iff $\xi > s_{\searrow}\pi(\theta)$. That is, when,

Proof: $\mathcal{W}^{\searrow}_{\xi}(v) \cap \$^{\searrow}_{\theta}(v) = \phi$ iff $\upsilon_+ < \underline{v}$. Then,

$\upsilon_+ < \underline{v}$ iff $n + (1-\theta)^{1/2}(v - n) < \underline{v} = v - 2(v - n - \xi\rho_{\searrow})$, i.e.,

$(1-\theta)^{1/2}(v - n) < (v - n) - 2(v - n - \xi\rho_{\searrow}) = -(v - n) + 2\xi\rho_{\searrow}$, i.e.,

$(v - n)\left[1 + (1-\theta)^{1/2}\right] < 2\xi\rho_{\searrow}$, i.e., $\xi > s_{\searrow}\pi(\theta)$.

3.5 Two main examples of self regulation failures.

3.5.1 Being stuck in a stationary trap

The problem of being trapped in a too low or a too high weight $v \lesseqgtr n$ can occur when, at the beginning of a need satisfaction story, an individual is not able to do a worthwhile move. That is, when trying to approach the ideal weight requires too high sacrifices relative to the advantages. For example, suppose that,

i) an individual is not where he wants to be, i.e., at the ideal weight n (a current and recurrent need);

ii) he wants to approach where he wants to be, to better meet their needs. That is, he would like to fill, at least, a portion of the discrepancy $|n - v|$;

iii) unfortunately, changing his initial weight from v to w is not worthwhile, whatever the change $v \curvearrowright w$ he can do. Therefore, despite the fact that he will remain frustrated to stay there, he will still prefer to stay at his initial weight v, because the satisfaction to approach more the ideal requires too much sacrifices.

Stationary traps. The VR approach (Soubeyran, 2009, 2010, 2021.a,b,c,d) defines a stationary trap as a not worthwhile to leave position. In the weight gain-loss example,

i) if $v < n$, v is a stationary trap iff $B^{+}_{\xi}(\mathfrak{m}/\sigma) < 0$ for all weight gains $m = (v, w - v > 0, w) \in \mathcal{M}$. That is, iff $\mathcal{W}^{+}_{\xi}(v) = \{v\}$, i.e., iff $v \geq n - \xi\rho_+$, i.e., $\xi \geq s_+ = (n - v)/\rho_+$.

ii) if $v > n$, v is a stationary trap iff $B^{\searrow}_{\xi}(\mathfrak{m}/\sigma) < 0$ for all weight losses $m = (v, v - w > 0, w) \in \mathcal{M}$. That is, iff $\mathcal{W}^{\searrow}_{\xi}(v) = \{v\}$, i.e., iff $v \leq n + \xi\rho_{\searrow}$, i.e., $\xi \geq s_{\searrow} = (v - n)/\rho_{\searrow}$.

Proof: $\mathcal{W}^{+}_{\xi}(v) = \{v\} \Longleftrightarrow \overline{v} = v + 2(n - v - \xi\rho_+) \leq v$, i.e., iff $n - v - \xi\rho_+ \leq 0$ and

$\mathcal{W}_{\xi}^{\frown}(v) = \{v\} \Longleftrightarrow v \leq \underline{v} = v - 2(v - n - \xi\rho_{\frown})$, i.e., iff $v - n - \xi\rho_{\frown} \leq 0$.

Then, the set of stationary traps is $T_{\xi} = \{v \in R_{+}, n - \xi\rho_{+} \leq v \leq n + \xi\rho_{\frown}\}$. This shows that if an individual has a weight close enough to the ideal weight n, it is a stationary trap. Being there, he prefers to stay with his initial weight than to change his weight. That is, he prefers to remain frustrated because it is not worthwhile (too costly relative to his motivation) to change his weight.

Stationary traps as self regulation failures. If the initial weight $v \neq n$ is not the ideal weight n, being stuck in a stationary trap $v \in T$ forbids to make worthwhile and satisficing changes $v \curvearrowright w \neq v$. This is the case if $\xi \geq s_{+} = (n - v)/\rho_{+}$ $(v \in T, v < n)$ or if $\xi \geq s_{\frown} = (v - n)/\rho_{\frown}$ $(v \in T, v > n)$. Then, an individual is trapped in an initial weight $v \neq n$ when,

i) the importance $\xi > 0$ given to inconveniences to move is too high. This situation occurs when an individual has a strong aversion for losses (Kahneman & Tversky, 1979);

ii) the aspiration gap $|n - v|$ is too low. This represents one aspect of an aspiration failure. The other aspect is when the aspiration gap is too large (Ray, 2003);

iii) the unit cost to change ρ_{+} or $\rho_{\frown}$ is too high. This occurs when an individual is ego depleted (Baumeister, 2002), has not enough resources or does not have enough skills.

Optimal worthwhile moves (weight gains or losses). An optimal worthwhile weight gain starting from the initial weight $v < n$ verifies $\partial B^{+}/\partial w = (n - v - \xi\rho_{+}) - (w - v) = 0$, given that $\partial^{2}B/\partial w^{2} = -1 < 0$. Then, the optimal weight starting from the initial weight $v < n$ is $w_{*} = n - \xi\rho_{+} \geq 0$. Similarly, an optimal worthwhile weight starting from the initial weight $v > n$ is $w_{*} = n + \xi\rho_{\frown}$.

Variational trap: failure at the end of a move. The VR approach defines variational traps as worthwhile to reach but not worthwhile to leave. In the weight gain-loss example, if the initial weight v is not a stationary trap, the optimal weight $w_{*} = n - \xi\rho_{+}$ or $w_{*} = n + \xi\rho_{\frown}$ can be reached in a worthwhile way. Then, $w_{*} = n - \xi\rho_{+}$ or $w_{*} = n + \xi\rho_{\frown}$, are variational traps, worthwhile to reach from $v < n$ or from $v > n$, i.e., $w_{*} \in \mathcal{W}_{\xi}^{+}(v)$ or $w_{*} \in \mathcal{W}_{\xi}^{\frown}(v)$, and not worthwhile to leave, i.e., $\mathcal{W}_{\xi}^{+}(w_{*}) = \{w_{*}\}$ or $\mathcal{W}_{\xi}^{\frown}(w_{*}) = \{w_{*}\}$.

3.5.2 Self regulation failures: the inability to solve goal conflicts

Proximal and distal goal conflicts. Each period, an individual who wants to satisfy some needs has solve goal conflicts. In the present paper we consider, for simplification, only one need (i.e., to meet some ideal weight). As seen before, self regulation failures occur when, within a period, starting from the status quo, each worthwhile move does not provide a sufficient rate of progress. That is, when worthwhile moves are not satisficing or inversely. Then, within the current period, and relative to the satisfaction of one need, the main conflict is between,

i) a long term and promotion goal: to choose to do a satisficing move that guaranties a sufficient rate of progress (θ high enough);

ii) a short term and prevention goal: to choose to do a worthwhile move that limits the size of the sacrifices (ξ high enough).

The conflict comes from the following fact: a too high rate of progress θ can require to make a large enough satisficing change, while a low enough rate of sacrifices (ξ high enough) can require to make a small enough worthwhile change. Then, self regulation failures occur when changes are too large (worthwhile enough) or too small (high enough rate of progress).

3.5.3 Aspiration failures: failures to meet in a worthwhile way the weight you aspire to

Let us comment Result.1(see above). This will illustrate how, starting from a too low weight $0 < v < n$, i) all satisficing weight gains $\delta = w - v > 0$ must be too large if the required rate of progress θ_* is too high and, ii) all worthwhile weight gains must be too small if the importance ξ_* given to resistance to move is too high.

Consider the sets of ends of satisficing and worthwhile moves relative to the given θ_* and ξ_* : $\$^+_{\theta_*}(v) = \{w \in R_+, v_\swarrow \leq w \leq v_+\}$, with $v_\swarrow = n - (1 - \theta_*)^{1/2}(n - v)$ and $v_+ = n + (1 - \theta_*)^{1/2}(n - v)$;

$\mathcal{W}^+_{\xi_*}(v) = \{w \in R_+, v \leq w \leq \overline{v}\}$ with $\overline{v} = v + 2(n - v - \xi_*\rho_+)$,and $n - v - \xi_*\rho_+ > 0$.

Then, the lowest satisficing weight gain $\delta = w - v \geq 0$ is $\underline{\delta} = v_\swarrow - v = (n - v)\left[1 - (1 - \theta_*)^{1/2}\right] \geq 0$, while the largest worthwhile weight gain $\delta = w - v \geq 0$ is $\overline{\delta} = \overline{v} - v = 2(n - v - \xi_*\rho_+) \geq 0$ if $\xi_* \leq s_+ = (n - v)2\rho_+$.

When all satisficing weight gains are too large. This occurs when $\underline{\delta} = (n - v)\left[1 - (1 - \theta_*)^{1/2}\right]$ is large. This happens when, i) the desired rate of progress θ_* is high and, ii) the aspiration gap $n - v > 0$ is large. This situation is linked to several facts: perfectionism, too big an ambition, a high self esteem, impatience, short termism, not being ready to delay the full satisfaction of a need, not wanting to suffer from frustration a too long time, wanting to improve the speed of moving,....

When all worthwhile weight gains are too small. This happens when $\overline{\delta} = 2(n - v - \xi_*\rho_+)$ is low. That is, when,

i) v is a stationary trap, i.e., when $n - v - \xi_*\rho_+ \leq 0 \iff \xi_* \geq s_+$, ($\xi_*$ is large enough, no sacrifices are accepted in the translocation), or when,

ii) v is not a stationary trap, i.e., $\xi_* < s_+$ (ξ_* is low enough, some sacrifices are accepted in the translocation), but the largest worthwhile weight gain is too small compared to the smallest satisficing weight gain. That is, iff $\overline{v} - v < v_\swarrow - v = w_* - v$, i.e., (see Result .1): $\overline{v} - v = 2(n - v - \xi_*\rho_+) < (n - v)\left[1 - (1 - \theta_*)^{1/2}\right] = v_\swarrow - v$

$\iff \xi_* > s_+\pi(\theta_*)$ if $\pi(\theta_*) = (1/2)\left[1 + (1 - \theta_*)^{1/2}\right]$.

Because $s_+ = (n-v)2\rho_+$,this inequality allows the aspiration gap $n-v > 0$ to be large when the unit cost of moving ρ_+ is large enough (high enough resistance to move). This case models aspiration "failures of the poor" (Appadurai, 2004, Ray 2003). This situation is linked to several facts: too high costs of moving

ρ_+ coming from a lack of different resource, poor abilities, and ego depletion, a too high importance ξ_* given to resistance to move, i.e., refusal to make too many sacrifices.

When an individual is unable to meet the ideal weitght n in one worthwhile step within a period. This occurs iff $\xi_* > s_+\pi(\theta_* = 1) = s_+/2$. In this case the satisficing set is reduced to the ideal weight, i.e., $\$^+_{\theta=1}(v) = \{n\}$ and it is not worthwhile to meet the ideal weight in one step within the period, i.e., $n \notin \mathcal{W}^+_{\xi^*}(v)$.

Conclusion. If all worthwhile moves are small enough, and all satisficing moves are too large (setting a too ambitious rate of progress) a worthwhile move cannot be a satisficing move. This illustrates, in the short term (one period), the impossibility, when motivation is too low and resistance is too high, to reach desires quickly enough, making sufficient progress to divide the difficulty, without having to undergo too many sacrifices. This shows the importance of trying progressively to reach the ideal weight.

3.5.4 Self regulation failures, feelings and emotions

Emotions play a major role in self regulation (Wagner & Heatherton, 2015). Let us give three examples among many others.

A. **To have low morale undermines self regulation**. In this setting, an individual puts a too little weight on the translocation phase δ (doing progress) and on the end of the move, i.e., the final weight w.

B. **Short-termism (myopia)**. That is, giving not enough importance to frustration feelings $f(w) = g^* - \mathfrak{g}(w) > 0$ in the next period compared with the contentment feeling $\mathfrak{g}(w)$ of having a better weight w in the current period. To clearly see how myopia works, let us define the satisfaction feelings of the individual as the weighted difference $\mathfrak{g}^e(w) = \mathfrak{g}(w) - \lambda f(w) = \mathfrak{g}(w) - \lambda\left[g^* - \mathfrak{g}(w)\right]$ between $\mathfrak{g}(w)$ and $f(w)$.Notice that $\lambda \geq 0$ is the importance given to frustration feelings in the next period. In this setting, advantages to move are $A^e(\mathfrak{m}/\sigma) = \mathfrak{g}^e(w) - \mathfrak{g}^e(v) = (1+\lambda)\left[\mathfrak{g}(w) - \mathfrak{g}(v)\right] = (1+\lambda)A(\mathfrak{m}/\sigma)$.

Therefore a worthwhile move is such that $A^e(\mathfrak{m}/\sigma) \geq \xi\ I(\mathfrak{m}/\sigma)$. That is, it is "as if" $A(\mathfrak{m}/\sigma) \geq [\xi/(1+\lambda)]\ I(\mathfrak{m}/\sigma)$. This shows that if an individual fails to take care of frustration feelings ($\lambda = 0$), this puts less importance on inconveniences to move, $\xi > \xi/(1+\lambda)$. This pushes him not to resist the temptation to eat more. Hence, leading to a self regulation failure.

C. **Giving too much importance to inconveniences to change**. This will lower the maximal size of a worthwhile move. Thus, it will be more difficult for this move to be satisficing. This gives rise to a new self regulation failure. More precisely, let $M = aA$ and $R = bI$, $a, b > 0$ be linear formulations of motivation and resistance to move. Then, a move is worthwhile iff $M \geq \xi R \iff A \geq [(\xi b)/a]\, I$. This shows that devaluating resistance to move ($0 < \xi < 1$) lower the importance $\xi' = (\xi b)/a < \xi$ given to inconveniences to move.

D. **Aspiration failures and the poors**. As an application of our findings, the concept of aspiration failures (Appadurai, 2004, Ray, 2002, 2003) will be examined elsewhere, in the context of the psychology of the poors when they try to raise their future standard of living to approach the standard they aspire to (Turner & Lehning, 2007, Haushofer & Fehr, 2014).

3.6 Self regulation success

Successful self regulation must consider, each period, two reference points. Each period, i) a worthwhile move considers the status quo (where you are) as the reference point. It compares the advantage and the inconvenience to move with respect to the status quo and, ii) a satisficing move considers the ideal end (where you want to be later) as the reference point. It compares the advantage to move with the ideal advantage to move (aspiration gap).

Self regulation success. In the present context a self regulation success occurs when, within a period, an individual is able to solve a choice conflict between the two following goals: i) a short term goal: to improve his position with respect to the initial position without too many sacrifices (a worthwhile move) and, ii) a long term goal: a sufficient rate of progress (= high enough speed) to approach enough the ideal weight (a satisficing move). Then, the choice of a long term promotion goal (choosing the rate of progress θ) must be coupled with the choice of a short term prevention goal (choosing the rate of sacrifices ξ). A difficult problem is to choose them simultaneously. That is, each period, a translocation is successful when it makes sufficient progress towards an ideal end without having to suffer too much pain. Thus, an individual will accept and will want to change rather than to stay because, within the current period, i) if he changes, he will enjoy to better satisfy his needs, and, ii) if he stays, he will suffer from the frustration of not being able to better satisfy his current needs. Given that, most of the time, an individual cannot reach his goals immediately, he must go progressively over several periods. That is, he must monitor his rate of progress θ to divide the difficulty over several periods.

Success: choosing a good size for a move. If, for example an individual has a too low weight, $v < n$, our findings show that to succeed to reach the ideal weight n, an individual must make a not too small and a not too large move $w \in [v_{\swarrow}, \overline{v}]$ where,

$v_{\swarrow} = n - (1-\theta)^{1/2}(n-v) \leq \overline{v} = v + 2(n - v - \xi\rho_+)$. Then, the window of self regulation success is $\overline{v} - v_{\swarrow} = (n-v)\left[1 + (1-\theta)^{1/2}\right] - 2\xi\rho_+ \geq 0$.

This window of success is large when the size of the window $\overline{v} - v_{\swarrow}$ is large. That is, when the aspiration gap $n - v$ is large enough, the rate of progress θ is low enough, the importance given to sacrifices ξ is low enough, and the unit cost of gaining weight ρ_+ is low enough. Hence, the choice of the step size $\delta = |w - v|$ matters much to escape to self regulation failures. Line search algorithms and modified Armijo rules will model this finding. See Shi & Shen (2005). The intersection condition $v_{\swarrow} \leq \overline{v}$ is equivalent to $\xi \leq s_+\pi(\theta)$, where $s_+ = (n-v)/\rho_+ > 0$ and $\pi(\theta) = (1/2)\left[1 + (1-\theta)^{1/2}\right]$. Failure happens in the opposite case, as it is expected. The success condition $\xi \leq s_+\pi(\theta)$ is equivalent to the condition $0 \leq \theta \leq \psi_{s_+}(\xi) = 4\left[(\xi/s_+) - (\xi/s_+)^2\right]$.

The threshold map $\xi \in [0, s_+] \longmapsto \psi_{s_+}(\xi) \in [0,1]$ shows that the success condition $0 \leq \theta \leq \psi_{s_+}(\xi)$ requires a low rate of progress when the weight ξ

given to the inconvenience to move is either close to zero or close to s_+.

Using slow-rapid dynamics: decoupling worthwhile moves from satisficing moves. As seen before a way to succeed in self-regulation is, each period, to do a medium size move that is, both, satisficing and worthwhile. An other possibility is to decouple these two kinds of moves, doing in the current period a worthwhile (shorter) move and, then, in the next period, a satisficing (larger) move. We will examine this very important point elsewhere, making the link with famous Nesterov's acceleration processes in first order optimization algorithms (Nesterov, 2012, 2013). As a leading example we will consider a champion who agrees to make sacrifices in the short run to better achieve his goals in the future. In this context, we will examine the importance of proximal goals that divide the difficulty and help to reach distal goals (Bandura & Schunk, 1981). An opposite example is a firm that exploits more than it explores over several periods. After each improving enough (satisficing) change, it prefers to stay for a while, to make this satisficing change worthwhile. This is a way to compensate for the previous sacrifices that must be done to satisfy enough (March, 1991).

3.7 Overcoming weak (instead of strong) resistance to move

As seen before, resistance to move can be strong or weak (in the small). Then, the next question is : how self regulation success depends much of the strength of resistance to move. When resistance to move is weak ($\beta = 2$), a worthwhile balance is (see before the weight example),

$B^{+}(\mathfrak{m}/\sigma) = (n-v)(w-v) - \left[1/2 + \xi\rho_+^2\right](w-v)^2$ if $w > v$;

$B^{\searrow}(\mathfrak{m}/\sigma) = (n-v)(w-v) - \left[1/2 + \xi\rho_{\searrow}^2\right](w-v)^2$ if $w < v$.

Then,

If $w > v$, $B(\mathfrak{m}/\sigma) = (w-v)\left[(n-v) - (1/2 + \xi\rho_+^2)(w-v)\right] \geq 0$ requires $v < n$ and $0 < w - v \leq \chi_+(n-v)$, where $\chi_+ = 1/\left[1/2 + \xi\rho_+^2\right] > 0$.

If $w < v$, $B(\mathfrak{m}/\sigma) = (v-w)\left[(v-n) - (1/2 + \xi\rho_{\searrow}^2)(v-w)\right] \geq 0$ requires $v > n$ and $0 < v - w \leq \chi_{\searrow}(v-n)$, where $\chi_{\searrow} = 1/\left[1/2 + \xi\rho_{\searrow}^2\right] > 0$.

Thus, if, for example, the initial weight is too low, $v < n$, the set of ends of worthwhile moves starting from the initial weight v is

$\mathcal{W}^{+}(v) = \{w \in R_+, w \geq v \text{ and } B^{+}(\mathfrak{m}/\sigma) \geq 0\} = \{w \in R_+,\ v \leq w \leq \overline{v}'\}$

where $\overline{v}' = v + \chi_+(n-v)$ when resistance to move is weak instead of $\overline{v} = v + 2(n - v - \xi\rho_+)$ when resistance to move is strong (see before). Thus, the largest worthwhile move is $\delta' = \overline{v}' - v = \chi_+(n-v)$ when resistance to move is weak and $\delta = \overline{v} - v = 2(n - v - \xi\rho_+)$ when resistance to move is strong.

RESULTS: To save space, let us consider the case $v < n$. Then,

A) stationary traps do not exist when resistance to move is weak ($\beta = 2$), because $\overline{v}' = v + \chi_+(n-v) > v$;

B) stationary traps exist when resistance to move is strong ($\beta = 1$) if the importance given to resistance to move is strong enough, i.e., if $\xi \geq (n-v)/\rho_+$;

C) if the importance given to resistance to move is low enough, i.e., if $0 \leq \xi < (n-v)/\rho_+$, stationary traps do not exist in both cases. However, the largest worthwhile move is lower when resistance to move is strong than when it is weak if $(n-v)/\rho_+ - 1/2\rho_+^2 \leq \xi < (n-v)/\rho_+$. If the left hand side of these inequalities is ≤ 0, i.e., $(n-v)/\rho_+ - 1/2\rho_+^2 \leq 0$, this condition is $0 < n - v \leq 1/(2\rho_+)$.

Proof: let us show point C). That is, let us examine the inequality $\overline{v} < \overline{v}'$, i.e., $0 \leq 2(n-v-\xi\rho_+) \leq \chi_+(n-v)$, i.e., $(n-v)\,[2-\chi_+] \leq 2\,\xi\rho_+$ (*). Given that $2-\chi_+ = 4\xi\rho_+^2/(1+2\xi\rho_+^2)$, condition (*) is equivalent to $(n-v)4\xi\rho_+^2/(1+2\xi\rho_+^2) \leq 2\,\xi\rho_+$, i.e., $(n-v)/\rho_+ - 1/2\rho_+^2 \leq \xi$.

Self regulation success. These results show clearly that self regulation is less successful when resistance to move is strong instead of weak. Because the largest worthwhile move being smaller, it will be more difficult for it to be a given satisficing move. In particular, it will require several moves to reach the ideal weight $n > v$. The condition $n \in \mathcal{W}_\xi(v)$ being more difficult to satisfy.

Remark. The case $v > n$ is similar.

4 Satisficing and worthwhile moves in a space of actions.

4.1 Moves in the space of actions = operate.

Moves. As seen before, in a space of needs, a move models losses, neither losses nor gains, and gains. In a related upstream space of activities (to eat, to make exercise), moves model the operate stage of TOTE, including disengagements = stop doing things, reengagements = continue doing things and engagements = start doing new things. For example,

- stop and start doing things, like eating less and doing more exercises to lose weight,

- continue doing the same things, like eating the same things and doing the same excercises to maintain the same weight,

- start and stop doing things, like eating more and doing less exercises to gain weight.

Intensive and extensive formulations of a move. Let X be a space of actions (situated activities). Let $x, y \subset X$, be two bundles of actions. Then a move $m = (\underline{x}, \omega, y)$ in the space of actions X starts with $\underline{x}$ = having done the bundle of actions x in the previous period, follows with the translocation $\omega \in \Omega(x, y)$ and ends with y = doing the bundle of actions y in the current period. The translocation ω is a succession of disengagements $\omega_{\diagdown} = x \diagdown y$, reengagements $\omega_= = x \cap y$ and engagements $\omega_+ = y \diagdown x$.

Example of an intensive move $m_j = (x_j, \omega_j, y_j)$. For example eating less or more pieces of the same chocolate $y_j \lesseqgtr x_j$, $x_j, y_j \in R_+$. The translocation $\omega_j = |y_j - x_j|$ models the size (= how much units) of disengagements $x_j - y_j > 0$, reengagments $y_j - x_j = 0$ and engagements $y_j - x_j > 0$. That is, doing less, the same or more of the same thing. In this context $x_j \in R_+$ is identified with the interval $x_j = [0, x_j] \subset R_+$.

Example of an extensive move : doing less, the same or more of different things. For example, doing two different activities instead of one, and more or less of each.

Example of intensive-extensive move: $m = (m_1, ..., m_j, ...m_l)$ with $m_j = (x_j, \omega_j, y_j), \omega_j = |y_j - x_j|, j = 1, 2, ..., l$. For example, if $l = 2$, eating more or less chocolate, making more or less exercises. In this setting, $x = (x_1, x_2)$ represents the box $[0, x_1] . [0, x_2] \subset R_+^2 = X$.

4.2 Different kinds of expectancies

In psychology, Bandura (1978) and others (see Maddux, et al., 1986 for a survey) defined three kinds of expectancies: outcome expectancy, value expectancy and self efficacy expectancy. Outcome-expectancy refers to the result ones anticipates from having performed a task, value expectancy is the importance given to the outcome, i.e., the satisfaction level derived from taking advantage of an outcome. A self-efficacy expectancy defines one's beliefs in the ability to perform a task. A last expectancy, rarely cited in this context, is a valence expectancy (Lewin, 1935). It represents the satisfaction level derived from doing some activities, through the explicit fulfilment of a need. For simplification and to save space we continue our presentation with the help of the weight example. We will start to consider a much more general case in the last section and in other papers.

4.2.1 Moves in a space of actions

1) Remind that a move $\mathfrak{m} = (v, \delta, w)$ in a space of weights $V = R_+$ (see before) models an individual who passes from having the weight $v \in V = R_+$ in the previous period to having the weight $w \in V = R_+$ in the current period. The term $\delta = \delta(u, v) = |w - v|$ means gaining weight if $\delta = w - v \geq 0$ or losing weight if $\delta = v - w \geq 0$.

2) A move $m = (x, \omega, y)$ in a space of different levels of an activity $X = R_+$ represents an individual who passes from eating $x \in R_+$ units of a good in the previous period to eating $y \in R_+$ units of this good in the current period. The term $\omega = \omega(x, y) = |y - x|$ means consuming more of the good if $\omega = y - x \geq 0$ or consuming less if $\omega = x - y \geq 0$.

4.2.2 Outcome, value and valence expectancies

3) An outcome expectancy $w = \varphi(y) \in R$ models the expected weight w an individual fears to have if he eats $y \in R_+$ units of a good. In this simple example, for simplification, $\varphi(y) = \mu y, \mu > 0$. That is, the more of the good he eats, the higher his weight will be. Furthermore, the higher is $\mu > 0$, the more he thinks that eating a given amount of the good will lead to a high weight. Then, an outcome expectancy function $\varphi(.) : y \in X \longmapsto w = \varphi(y) \in R_+$ couples each bundle of actions $y \subset X$ with the outcome level w that y helps to reach.

4) A value expectancy $\mathfrak{g}(w)$ represents the satisfaction level (= value) derived from having the weight w, given that the ideal weight is $n > 0$. In the example

$\mathfrak{g}(w) = nw - (1/2)w^2, w \in R_+$. Then, a value expectancy function is $\mathfrak{g}[.] : w \in R_+ \longmapsto g = \mathfrak{g}[w] \in R$.

5) A valence expectancy defines the satisfaction level derived from of eating y units of a good, i.e., $g(y) = \mathfrak{g}[\varphi(y)] \in R$. In the simple example where $\mathfrak{g}(w) = nw - (1/2)w^2$, $g(y) = n\varphi(y) - (1/2)\varphi(y)^2$. Then, if $\varphi(y) = \mu y, \mu > 0$,

$g(y) = n\mu y - (\mu^2/2)y^2 = \mu^2 \left[x^* y - (1/2)y^2\right]$,

where $x^* = n/\mu > 0$ is the optimal level of food that helps to reach the ideal weight n. This valence defines the level of satisfaction $g(y)$ with eating y units of a good, because of its indirect utility. That is, not for its own taste, but because eating some quantity of it helps to gain or lose weight (a nutritious food if you want to gain weight, and a diet food if you want to loose weight). Then, a valence function is the composition of an outcome expectancy function $\varphi(.)$ with a value expectancy function $\mathfrak{g}[.] : w \in R_+ \longmapsto g = \mathfrak{g}[w] \in R$. That is, $g(.) : y \in X \longmapsto g = \mathfrak{g}[\varphi(y)] = g(y) \in R$.

Comments. Lewin (1935, p.78) defined the fundamental concept of valence as follow: "the valence of an object usually derives from the fact that the object is a means to the satisfaction of a need, or has indirectly something to do with the satisfaction of a need". The VR approach provides, and this seems very new, the first mathematical formulation of this fundamental concept. Thus, the valence of doing something (for example, to eat chocolate) depends not only of this thing, but also of the state of needs (to be hungry or not). This means that it changes if the size of the need changes. Notice, following Lewin, that the valence of an object y is the valence of $y =$ using this object. i.e., the valence of doing something, y. Then, a valence expectancy is also a kind of outcome expectancy. It refers to beliefs about the consequences (in term of satisfaction level) of performing a behavior.

4.2.3 Aspiration and frustration levels

6) The aspiration level of satisfaction is $g^* = \sup\{g(y), y \in X = R_+\} < +\infty$. In the simple weight example where $\varphi(y) = \mu y, \mu > 0$, $g(y) = \mu^2 \left[x^* y - (1/2)y^2\right]$, the aspiration level $g^* = g(x^*) = n^2/2$ is reached when an individual eats the ideal amount $y = x^* = n/\mu$ of the diet good. Then, the gradient $\nabla g(y) = \mu^2 [x^* - y]$ of the satisfaction function is proportional to the discrepancy between what an individual presently eats, y, and what he would like to eat, x^*.

7) The frustration level of eating not enough or too much, for example $y \neq x^*$ units of the good instead of x^*, is, $f(y) = g^* - g(y) = (\mu^2/2)[x^* - y]^2$. Furthermore $f(y) = (1/2\mu^2)\nabla g(y)^2$. This simple formula links the level of frustration $f(y)$ at y with the square of the gradient $\nabla g(y)$ at y of the satisfaction function. This fact will appear to be very important in the last section, in the general context of gradient algorithms.

8) More complex formulations can be $\varphi(y) = \mu\sqrt{y}$ or $\mu y^\tau, \tau > 0$. For example, if $\varphi(y) = \mu\sqrt{y}$, $g(y) = n\mu\sqrt{y} - (\mu^2/2)y$.

4.2.4 Self-efficacy expectancies.

They refer to the degree of conviction that one can successfully execute the behavior(s) required to produce an outcome. As we have seen before in the weight example (section 3), the VR approach models these self efficacy beliefs as the expected costs $\mathfrak{C}(v,\delta,w) \in R_+$ to do a move $\mathfrak{m} = (v,\delta,w)$ in a space of needs. This move goes from v = having fulfilled the level v of a given need (= having the weight v) in the previous period to w = fulfilling the level w of this need (to have the weight w) in the current period, where the translocation is the modulus of the change in weight $\delta = \delta(u,v) = |w-u|$.

Let $v = \varphi(x)$ and $w = \varphi(y) \in R_+$ be the expected weights (outcome expectancies) derived from eating the quantities of a good $x, y \in R_+$. Then, in a space of actions X self efficacy expectancies refer to the expected costs $C(x,\omega(x,y),y) = \mathfrak{C}(v,\delta,w) = \mathfrak{C}\left[\varphi(x), \delta\left[\varphi(x),\varphi(y)\right], \varphi(y)\right] \in R_+$ to do a move $m = (x, \omega(x,y) = |y-x|, y)$, where an individual passes from eating $x \in R_+$ units of a good in the previous period to eating $y \in R_+$ units of this good in the current period. In this setting $\delta\left[\varphi(x),\varphi(y)\right] = |\varphi(y)-\varphi(x)|$.

For simplification we will write $C(x,y) = C(x,\omega(x,y),y)$.

Example. Let

$$\mathfrak{C}(\mathfrak{m}) = \mathfrak{C}(v,\delta,w) = \left[\begin{array}{c} \rho_{=}v + \rho_{+}(w-v), \text{ if } w-v \geq 0 \\ \rho_{=}v + +\rho_{\searrow}(v-w), \text{ if } w-v < 0 \end{array}\right].$$

Then, if $w = \varphi(y)$,

$$\mathfrak{C}(m) = C(x,y) = \left[\begin{array}{c} \rho_{=}\varphi(x) + \rho_{+}\left[\varphi(y)-\varphi(x)\right], \text{ if } y-x \geq 0 \\ \rho_{=}\varphi(x) + +\rho_{\searrow}\left[\varphi(x)-\varphi(y)\right], \text{ if } y-x < 0 \end{array}\right].$$

If, for example, $w = \varphi(y) = \mu y$,

$$\mathfrak{C}(m) = C(x,y) = \mu\left[\begin{array}{c} \rho_{=}x + \rho_{+}(y-x), \text{ if } y-x \geq 0 \\ \rho_{=}x + +\rho_{\searrow}(x-y), \text{ if } y-x < 0 \end{array}\right].$$

These costs to move refer to costs to be able to eat then, eat, more or less chocolate.

The formal resolution of a long debate. Our VR formalization solves an important debate between Bandura and a lot of researchers working on outcome and self efficacy beliefs defined as perceived capability to perform a behavior (see Williams, 2010 for a long list of references). Specifically, Bandura has argued that, (a) self-efficacy causally influences expected outcomes of behavior; (b) but not vice versa. However, research has shown that expected outcomes causally influence self-efficacy judgments, and some authors have argued that this relationship invalidates self-efficacy theory. Bandura has rebutted those arguments saying that self-efficacy judgments are not invalidated when influenced by expected outcomes. Our formalization shows very clearly that, if, to be concrete, the outcome is the weight level, $w = \varphi(y) = \mu y$, outcome expectancy influences self efficacy via the appearence of μ in expected costs $C(x,y)$. So Bandura's claim (b) seems wrong. Moreover, in our setting, influence (a) is indirect. This last point will be examined elsewhere.

4.2.5 Expected worthwhile and satisficing balances.

In the simple weight gain-loss example where $\varphi(z) = \mu z, \mu > 0$, and $g(y) = \mu^2 \left[x^* y - (1/2)y^2\right]$, expected advantages and inconveniences to move are,

8) $A(m/\sigma) = g(m) - g(\sigma) = g(y) - g(x) = \mu^2 (y-x) \left[(x^* - x) - (1/2)(y-x)\right]$

9) $I(m/\sigma) = C(m) - C(\sigma) = C(x,y) - C(x,x) = \mu \begin{array}{l} \rho_+(y-x), \text{ if } y-x \geq 0 \\ \rho_\diagdown(x-y), \text{ if } y-x < 0 \end{array}.$

Suppose, for simplification, that $U[A] = A^\alpha, D[I] = I^\beta, \alpha, \beta > 0$. Then, motivation and resistance to move are,

10) $M(m/\sigma) = U\left[A(m/\sigma)\right] = A(m/\sigma)^\alpha$ and,

11) $R(m/\sigma) = D\left[I(m/\sigma)\right] = I(m/\sigma)^\beta$;

Then, a worthwhile balanc**e** is,

12) $B(m/\sigma) = U\left[A(m/\sigma)\right] - \xi D\left[I(m/\sigma)\right] = A(m/\sigma)^\alpha - \xi I(m/\sigma)^\beta$.

If $\alpha = 1, \beta > 0$, the simple example gives,

$B(m/\sigma) = \mu^2 (y-x) \left[(x^* - x) - (1/2)(y-x)\right] - \xi \mu^\beta \left[\begin{array}{l} \rho_+(y-x), \text{ if } y-x \geq 0 \\ \rho_\diagdown(x-y), \text{ if } y-x < 0 \end{array}\right]^\beta.$

As seen before, the two benchmark cases are $\alpha = 1, \beta = 1$ (strong resistance do move) and $\alpha = 1, \beta = 2$ (weak resistance do move).

Remark. As seen in section 3 if, in the weight example, we want to consider the satisfaction of making progress as a part of advantages to change, we must change n in $n + \pi + \eta$ in the formulation of a worthwhile balance.

4.2.6 Satisficing and worthwhile moves in a space of actions

In the weight example, an individual wants to gain (lose) weight because his initial weight $v < n$ ($v > n$) is lower (larger) than the ideal weight n. Then, he must try to eat more (less) of a food that makes you fat. The ideal level of diet food is $x^* = n/\mu > 0$. Thus, in the initial position x he will try to eat more (less) if $x < x^*$ ($x > x^*$).

Satisficing moves. A satisficing move $m = (x, \omega = |y-x|, y)$ verifies $g(y) - g(x) = f(x) - f(y) \geq \theta\left[g^* - g(x)\right] = \theta f(x)$, $0 < \theta \leq 1$. That is, $0 \leq f(y) \leq (1-\theta) f(x)$. Given that $f(y) = (\mu^2/2)\left[x^* - y\right]^2$, a satisficing move going from x to y provides a sufficient decrease $|x^* - y| \leq (1-\theta)^{1/2} |x^* - x|$. That is, $x_\diagdown \leq y \leq x_+$:

if $y \leq x^*$, then, $y \geq x_\diagdown = x^* - (1-\theta)^{1/2} |x^* - x|$.

if $y \geq x^*$, then, $y \leq x_+ = x^* + (1-\theta)^{1/2} |x^* - x|$.

This shows that the set of ends of satisficing moves starting from x is $\$_\theta(x) = \{y \in X, x_\diagdown \leq y \leq x_+\}$.

Worthwhile moves. Take, for example, $\beta = 1$ (strong resistance to move). Then, in a space of actions,

i) $B(m/\sigma)/\mu^2 = (y-x)\left[(x^* - x - \xi\rho_+/\mu) - (1/2)(y-x)\right]$ if $y - x \geq 0$;

ii) $B(m/\sigma)/\mu^2 = (x-y)\left[(x - x^* - \xi\rho_\diagdown/\mu) - (1/2)(x-y)\right]$ if $y - x \leq 0$.

Thus,

i) $B(m/\sigma) \geq 0$ iff $x \leq y \leq \overline{x}$, if $y - x \geq 0$ and $\overline{x} = x + 2(x^* - x - \xi\rho_+/\mu)$. Thus, $\mathcal{W}_\xi^+(x) = \{y \in X,\ x \leq y \leq \overline{x}\}$ and $y \in \mathcal{W}_\xi^+(x) \diagdown \{x\}$ requires $x^* - x > \xi\rho_+/\mu$.

ii) $B(m/\sigma) \geq 0$ iff $\underline{x} \leq y \leq x$, if $y - x \leq 0$ and $\underline{x} = x - 2(x - x^* - \xi\rho_{\searrow}/\mu)$. Thus, $\mathcal{W}_{\xi}^{\searrow}(x) = \{y \in X,\ \underline{x} \leq y \leq x\}$ and $y \in \mathcal{W}_{\xi}^{\searrow}(x) \setminus \{x\}$ requires $x - x^* > \xi\rho_{\searrow}/\mu$.

In both cases, the existence of a worthwhile change requires a high enough aspiration gap $x - x^* > \xi\rho_{\searrow}/\mu$ or $x - x^* > \xi\rho_{\searrow}/\mu$.

4.3 Self regulation failures /success in a space of actions

Let us show how self regulation failures come from a lack of motivation and a too high resistance to move**.**

Results. Self regulation failure occurs when accepting to make sacrifices to gain (or to lose) weight is low. That is, when the importance ξ given to inconveniences to change weight is too high, i.e., when,

A) $\xi > s_+\pi(\theta)$ with $s_+ = (n - v)/\rho_+ = (x^* - x)(\mu/\rho_+)$ if $v = \mu < n = \mu x^*$ or,

B) $\xi > s_{\searrow}\pi(\theta)$ with $s_{\searrow} = (v - n)/\rho_{\searrow} = (x - x^*)(\mu/\rho_{\searrow})$ if $v > n$.

We must interpret these results in a new way, given that we move in the space of actions, taking care about what we must eat to gain or lose weight. That is, the aspiration gap is not $|n - v|$, but $|x^* - x|$. Thus, we start with a given discrepancy between what we presently eat, x, and what we will want to eat, x^*.

First aspect: given this discrepancy $|x^* - x|$, self regulation failures occur when, A') $0 < x^* - x < (\xi\rho_+)/(\pi(\theta)\mu)$, or, B') $0 < x - x^* < (\xi\rho_{\searrow})/(\pi(\theta)\mu)$. That is, when:

i) the desired importance ξ given to sacrifices (inconveniences to move) is too high, i.e., the individual is not ready to make high enough sacrifices, i.e., he is not able to resist to temptations or to decrease their saliences;

ii) the desired rate of progress θ is too high, i.e., the individual is impatient and does not want to stay frustrated a too long time with the same too low or too high weight;

iii) the cost ρ_+ of being able to gain or to lose weight and effectively gain or lose weight, is too large (low self efficacy expectancy, negative mood, ego depletion, not having enough time to learn and train) and,

iv) outcome expectancy μ is low enough, i.e., someone does not expect to lose enough weight if he eats less (pessimism, negative mood).

Second aspect: given the situation ξ, θ, ρ_+ or $\rho_{\searrow}$, self regulation failures occur when the aspiration failure $|x^* - x|$ is too small (lack of motivation).

Then, our formula summarizes in a very compact way almost all the reasons given in the literature to self regulation failures. Several other important reasons must be examined elsewhere in great details:

1) When minor lapses in self-control snowball into self-regulatory collapse (Wagner & Heatherton, 2015).

2) The impact of ego depletion must also be examined more carefully in at least a two periods model and cumulative efforts that lead to fatigue (two consecutive tasks, Baumeister, 2002).

3) The difficulties to buid good habits (falling in a good trap) and to break bad habits (escaping from a bad trap). In our example (see section 3), traps

occur with the inability to make high enough sacrifices: ξ too high means, $\xi \geq s_+ = (n-v)/\rho_+ = (x^*-x)(\mu/\rho_+)$ or $\xi \geq s_\searrow = (n-v)/\rho_\searrow = (x-x^*)(\mu/\rho_\searrow)$.

Lack of self control: having difficulties to sacrifice immediate gratifications. Our VR formulation is well adapted to model and solve lack of self control in the following way. In consumer research, self control is often conceptualized as the abstinence from hedonic consumption. Rejecting that notion, Vosgerau et. al.(2020) argue that self-control failures are choices in violation of superordinate long-term goals accompanied by anticipated regret, rather than choices of hedonic over utilitarian consumption. Self-control describes the sacrifice of immediate, short-term gratification in service of more important, long-term benefits. " The 'now' self prefers consuming a tempting good now, but the 'future' self would regret having consumed the tempting good in the past. Thus, self-control is distinct from impatience and self-regulation. According to this conceptualization, self-control conflicts are characterized by three criteria " (Vosgerau et. al., 2020):

i) time-inconsistent preferences: preferences change over time,

ii) a hierarchy of preferences: the importance of the self that demands immediate gratification fades quickly as time passes, giving way to the self that serves long-term goals", and,

iii) anticipated regret: one expects to regret resolving a self-control conflict in favor of immediate gratification.

Then, following this conceptualization, a lack of consumer self control does not anticipate these three criteria. Our VR approach shows that very clearly. Much more can be said relative to this point.

5 Gradient algorithms. Using good enough models of the reality is required for self regulation success.

5.1 What is the information left on the table : what is known and what is unknown

5.1.1 Expected ex ante and revealed ex post (= real) aspects of a thing.

We have shown before how self regulation failures and self regulation successes occur when expectancies are supposed to match the reality. That is, when, ex ante, what you expect to come is realized ex post. For convenience, if some aspect of a thing is unknown ex ante (before gathering some information), we can make, ex ante, an expectation about this unknown aspect. When this aspect of a thing is revealed ex post (after gathering this information) we will say that it is its real aspect. Then, we must contrast an expected-ex ante aspect of a thing with a revealed-ex post aspect of a thing.

For example, if ex ante, before having done a given bundle of activities $y \in X$, we do not know the level of satisfaction derived from doing y, the ex ante-expected level of satisfaction derived from doing y is an expected valence (valence expectancy). In contrast, ex post (after having done y), the revealed level of satisfaction derived from having done y is the real valence of the bundle of activities y. Then, we can contrast the ex ante-expected valence of doing something with the revealed-ex post valence of doing this thing. That is, we can contrast a valence expectancy with a real valence.

For a better understanding and to economize notations, we will start with revealed-ex post functions. That is, in the weight example, we start to consider,

i) a real value $\mathfrak{g}(w) \in R$. It represents the revealed-ex post level of satisfaction derived from having the weight w;

ii) a real outcome. It models the revealed-ex post weight $w = \varphi(y) \in R_+$ derived from having eaten the quantity $y \in R_+$ of a good;

iii) a real valence. It is the revealed-ex post level of satisfaction $g(y) = \mathfrak{g}\left[\varphi(y)\right] \in R$ derived from having the weight w, after having eaten y units of a good;

iv) a real self efficacy $\mathfrak{C}(v, w)$ or $C(x, y) \in R_+$. It links a move (u, w) or (x, y) with the revealed -ex post cost $\mathfrak{C}(v, w)$ or $C(x, y)$ of having done this move.

Then, the list of revealed- ex post functions,

i) $\mathfrak{g}(.) : w \in R_+ \longmapsto \mathfrak{g}(w) \in R$, say a real value function;

ii) $\varphi(.) : y \in R_+ \longmapsto w = \varphi(y) \in R_+$, say a real outcome function;

iii) $g(.) : y \in R_+ \longmapsto g(y) \in R$, i.e. a real valence function; and,

iv) $\mathfrak{C}(.,.) : (u, w) \in R_+^2 \longmapsto \mathfrak{C}(v, w) \in R_+$ or $C(.,.) : (x, y) \in R_+^2 \longmapsto C(x, y) \in R_+$, i.e., real self efficacy functions.

These real functions being unknown ex ante, we will consider expectancies functions as functions that approximate real functions. For example, if, to save space, we start with an unknown real valence function $g(.)$, a valence expectancy function $\widetilde{g}_x(.) : y \in X \longmapsto \widetilde{g}_x(y) \in R$ will represent a function that approximates, locally, around x, the real valence function $g(.)$. See below approximation functions (lower and upper bounds).

5.1.2 Building good enough expectancies.

Most of the time, expectancies can be wrong, or at least, not good enough, because of a lack of information and knowledge. For example, expectancies can be too optimistic or too pessimistic. This will push an individual to be too ambitious or too conservative, ending in overshooting or undershooting moves. Then, if these expectancies are too wrong, making too wrong satisficing and worthwhile moves will not end in high enough levels of fulfilment of the different needs. In this context, we must show now how individuals can built not too bad expectancies that are close enough to the reality, depending of what they know and what they do not know, ex ante, in the current period. More technically, how to build good enough models of satisficing and worthwhile moves ? This will be the case if we can build a good enough (approximate) model of a worthwhile balance.

We must start to show explicitly how self regulation success can be modeled and solved through all the machinery of the most famous and up to date variational principles and optimizing algorithms in mathematics. Due to lack of space, this last section will only start to do that task in the context of gradient algorithms, leaving aside a lot of famous variational principles. See,

a) Soubeyran (2022.a). Variational rationality: the formulation of need satisfaction - need frustration dynamical systems. Forthcoming.

b) Soubeyran (2022.b). Variational rationality: the resolution of need satisfaction - need frustration dynamical systems. Forthcoming.

5.2 The enclosing principle: lower and upper bounds of a VR structure

Ex ante expectancy functions as approximations of ex post revealed functions. For an individual, a cheap way to better know his value, outcome, valence and self expectancy functions is to use an enclosing principle. That is, to try to know what they cannot be (negative information), rather that trying to know what they can be (positive information). This helps to better learn from errors and partial success (Martinez Legaz & Soubeyran, 2016). In artificial intelligence, the No-Yes game is an extreme example where the problem is to find what an individual has in mind. Questions are : is what you have in mind this or that ? Answers can only be : yes or not. Then, a yes answer ends the game (positive information), while a no question gives an information about what this thing cannot be (negative information). The accumulation of enough negative informations leading to determine a small subset of things that this thing can be.

Examples. In mathematics a lot of different enclosing principles exist. For example, i) the famous Hahn–Banach separation theorem (Zălinescu, 2002) tells us when two convex sets can be separated by an hyperplan, ii) a cutting plane method uses an hyperplane to separate the current point from the optimal solution of a linear program, iii) a differentiable concave function can be majorized by each of its linear approximation (tangent line), iv) M.M algorithms (Giles, 2015) use a majorization (minorization) function as an upper (lower) bound of a given function to be minimized (maximized). Finally sandwiching algorithms (Drusvyatskiy et al., 2018, Drusvyatskiy, 2020, Drusvyatskiy et al., 2021) use simultaneously or sequentially an upper bound and a lower bound to enclose a function to be minimized. In this paper we will enclose, locally, each period, real value, outcome, valence and self efficacy functions between a changing lower bound model and a changing upper bound model. In this way, we build a VR enclosing principle that will help, each period, to build good enough models of satisficing and worthwhile moves ending in a successful self regulation.

5.2.1 Lower bound and upper bound of a valence function

Local approximation models. Given the initial bundle of activities $x \in X$, an ex ante local approximation function around x of an ex post-revealed function

$g(.) : y \in X \longmapsto g(y) \in R$ is a function $\widetilde{g}_x(.) : y \in X \longmapsto \widetilde{g}_x(y) \in R$ such that, i) $\widetilde{g}_x(x) = g(x)$ and, ii) the difference $|\widetilde{g}_x(y) - g(x)|$ goes to zero as y approachs x. This approximation model is a global upper bound $\overline{g}_x(.)$ of $g(.)$ at x if $\overline{g}_x(y) \geq g(y)$ for all $y \in X$. It is a global lower bound $\underline{g}_x(.)$ of $g(.)$ if $\underline{g}_x(y) \leq g(y)$ for all $y \in X$. A local upper bound or a local lower bound requires $y \in \mathcal{V}(x) \subset X$.

A sandwiching model is, for a function $g(.)$, a couple $(\underline{g}_x(.), \overline{g}_x(.))$ including a global lower bound and a global upper bound, with i) $\underline{g}_x(x) = g(x) = \overline{g}_x(x)$ and $\underline{g}_x(y) \leq g(y) \leq \overline{g}_x(y)$ for all $x, y \in X$.

Linear lower and upper bound of a function $g(.)$**.** For example**,** if the function $g(.)$ is convex (concave) and differentiable, with gradient $\nabla g(x) = g'(x)$ at x, a linear lower bound (upper bound) function around x is $\widetilde{g}_x(.) : y \in X \longmapsto \widetilde{g}_x(y) = g(x) + < g'(x), y - x >$, where $\nabla g(x) = g'(x) = [\partial g/\partial x_1(x), ..., \partial g/\partial x_l(x)]$ if $X = R^l$. The same is true with a subgradient $g'(x) \in \partial g(x)$.

Example. In a previous example $\mathfrak{g}(w) = nw - (1/2)w^2$, $\widetilde{\mathfrak{g}}_v(w) = \mathfrak{g}(v) + (n - v)(w - v)$ with $\mathfrak{g}'(v) = n - v$.

Quadratic lower and upper bound of a function $g(.)$. By definition a function $g(.) : y \in X \longmapsto g(y) \in R$ (Drusvyatskiy et al., 2018) is,

i) Lipschitz smooth if it has a quadratic lower bound $\underline{g}_x(.)$:

$g(y) \geq \underline{g}_x(y) = g(x) + < g'(x), y - x > - (\overline{r}/2) \|y - x\|^2$, $\overline{r} > 0$, for all $x, y \in X$;

ii) strongly concave if it has a quadratic upper bound $\overline{g}_x(y)$:

$g(y) \leq \overline{g}_x(y) = g(x) + < g'(x), y - x > - (\underline{r}/2) \|y - x\|^2$, $\underline{r} > 0$, for all $x, y \in X$, with $0 < \underline{r} \leq \overline{r} < +\infty$.

Then, a strongly concave function is not too flat, while a Lipschitz smooth function is not too curvy. These two local lower and upper bound models of a satisfaction function or of a valence function depend of the move $\delta = y - x$, as emphasized by the VR approach.

5.2.2 Lower bound and upper bound of a self efficacy function

Suppose that $X = R^l$ and consider the cost to move bi-function $C(., .) : (x, y) \in X.X \longmapsto C(x, y) \in R_+$. Then, the difference $I(y/x) = C(x, y) - C(x, x)$ models inconveniences to move if $I(y/x) \in R_+$ for all $x, y \in X$. Given the huge diversity of possible costs to move in applications, we will give two main examples of costs to move with lower and upper bounds of inconveniences to move.

Inconveniences to move as non smooth asymmetric norms. Given the vector space X, let $p(.) : x \in X \longmapsto p(x) \in R_+$ be an asymmetric norm, that is, a function satisfying: i) for all $x, y \in X$, $p(x + y) \leq p(x) + p(y)$, ii) for all $x \in X$, $x = 0 \Longleftrightarrow p(x) = 0$ and, iii) for all $x \in X$, for all $\lambda \geq 0$, $p(\lambda x) = \lambda p(x)$.

If we replace the second condition ii) by ii)' $x = 0 \Longleftrightarrow p(x) = p(-x) = 0$, then, we say that $p(.)$ is an asymmetric hemi-norm on X. See Danilidis et al. (2020). As a consequence, the function $q(., .) : x, y \in X \longmapsto q(x, y) = p(y - x) \in R_+$ is an extended quasi- (hemi-) distance on X. That is, i) for all $x, y, z \in X$, $q(x, z) \leq q(x, y) + q(y, z)$ and, ii) for all $x, y \in X$, $x = y \Longleftrightarrow q(x, y) = 0$ or $x = y \Longleftrightarrow q(x, y) = q(y, x) = 0$.

In a real vector space, a fundamental way to construct a plenty of asymmetric norms is to start with a Minkowski functional of a convex subset $K \subset X$ that contains the origin,

$p_K(.) : x \in X \longmapsto p_K(x) = \inf \{r \in R, r > 0, x \in rK\} \in R_+$. This functional is an asymmetric norm if K is an absorbing set. This functional is an asymmetric seminorm (where $p(x)$ can be zero even if $x \neq 0$) if K is an absorbing set and $p_K(x)$ is finite for all $x \in X$ (Cobzas, 2013).

In this context of an asymmetric normed space where a change is $\delta = y - x$, inconveniences to do this move $I(y/x) = C(x, y) - C(x, x) = p(y - x)$ can be identified with the asymmetric norm $p(y - x)$. Then, costs to move are $C(x, y) = C(x, x) + p(y - x)$.

Canonical example. Start with $x, y \in X = R_+$. As seen before (section 3),

$I(y/x) = C(x,y) - C(x,x) = \begin{bmatrix} \rho_+ |y - x|, \text{ if } y - x \geq 0 \\ \rho_\diagdown |y - x|, \text{ if } y - x \leq 0 \end{bmatrix} = p(y-x)$ defines an asymmetric norm on R_+ where $\rho_+ = c_+ > 0$ and $\rho_\diagdown = c_\diagdown - c_= > 0$.

In the more general case where $X = R^l_+$, $C(x, y) = \Sigma^l_{j=1} C^j(x^j, y^j)$ and $I(y/x) = \Sigma_{j \in J_+(x,y)} \rho^j_+ \left|y^j - x^j\right| + \Sigma_{j \in J_\diagdown(x,y)} \rho^j_\diagdown \left|y^j - x^j\right|$, where $J_+(x, y) = \left\{j, y^j - x^j \geq 0\right\}$ and $J_\diagdown(x, y) = \left\{j, y^j - x^j \leq 0\right\}$.

Enclosing inconveniences. Let $\overline{\rho}^j = \max \left\{\rho^j_+, \rho^j_\diagdown\right\}$, $\underline{\rho}^j = \min \left\{\rho^j_+, \rho^j_\diagdown\right\}$, $\overline{\rho} = \max \left\{\overline{\rho}^j, j \in J\right\}$ and $\underline{\rho} = \min \left\{\underline{\rho}^j, j \in J\right\}$. Then, if $0 < \underline{\rho} \leq \overline{\rho} < +\infty$, $\underline{\rho} \left\|y - x\right\|_1 \leq I(y/x) \leq \overline{\rho} \left\|y - x\right\|_1$ where $\left\|y - x\right\|_1 = \Sigma_{j \in J} \left|y^j - x^j\right|$ is the l_1 norm. Given that in $X = R^l$ all norms are equivalent, we have $\left\|y - x\right\|_2 \leq \left\|y - x\right\|_1 \leq \sqrt{l} \left\|y - x\right\|_2$.

Then, the final enclosing lower and upper bounds for inconveniences to move are $\underline{I}(y/x) = \underline{h} \left\|y - x\right\|_2 \leq I(y/x) \leq \overline{I}(y/x) = \overline{h} \left\|y - x\right\|_2$ where $\underline{h} = \underline{\rho}$ and $\overline{h} = \sqrt{l}\overline{\rho}$.

Remark. If $p(.)$ is an asymmetric norm on X, the function $p^{-1}(.)$: $X \to [0, \infty)$ defined by $p^{-1}(.) = p(-x)$ is also an asymmetric norm on X. Then $p^s : X \to [0, \infty)$ defined by $p^s(x) = max\{p(x), p(-x)\}$ is a norm on X. A simple but important special case is the following: if $X = R, p(x) = \max \{x, 0\}$, then $p^{-1}(x) = \max \{-x, 0\}$ and $p(x) = |x|$. Then, if $X = R^l$, all norms being equivalent, an asymmetric norm $p(.)$ is majorized by the symmetric norm $p^s(.)$, which is equivalent to the euclidian norm. That is $p(y - x) \leq p^s(y - x) \leq \lambda \left\|y - x\right\|_2, \lambda > 0$.

Inconveniences to move as a smooth Bregman distance. Inconveniences to move $I(y/x) = \chi(x, y) \left\|y - x\right\|_2$ can be modeled as the product of the unit inconvenience to move, $\chi(x, y) \geq 0, y \neq x$, time the Euclidian length of the move $\left\|y - x\right\|_2$. A canonical example largely used in optimization (Bregman, 1967) is the famous Bregman distance $L(y/x)$ relative a differentiable and strongly convex kernel $k(.) : x \in X \longmapsto k(x) \in R$, where $L(y/x) = k(y) - k(x) - < \nabla k(x), y - x > = \chi(x, y) \left\|y - x\right\|_2 \geq 0$, with $\gamma(x, y) = [k(y) - k(x)] / \left\|y - x\right\| - < \nabla k(x), (y - x)/ \left\|y - x\right\| \geq 0$ if $y \neq x$.

In this setting costs to change and costs to stay are $C(x, y) = k(y) - < \nabla k(x), y - x > \geq 0$, and $C(x, x) = k(x) \geq 0$ for all $x, y \in X = R^l$. Then, inconveniences to move are $I(y/x) = L(y/x) \geq 0$ for all $x, y \in X = R^l$. If $k(x) = (1/2) \|x\|^2 = C(x,x)$, costs to move are $C(x,y) = (1/2) \|x\|^2 + (1/2) \|y-x\|^2$. This interpretation of a Bregman distance in term of costs to move is entirely new, both in mathematics and in behavioral sciences.

To save space, we will examine this important case elsewhere.

Inconveniences to move as a strongly convex function. Given a vector space X, inconveniences to move $I(y/x) = C(x,y) - C(x,x) = p(y-x) \geq 0$ can be a convex function of the size of the change $\delta = y - x$, i.e., $p(.) : \delta \in X \longmapsto p(\delta) \in R_+$ with $p(0) = 0$. They represent travelling costs $p(y-x)$. These travel costs are not symmetric, i.e., $p(-\delta)$ can be different from $p(\delta)$ for $\delta \neq 0$. Notice that, in general, a convex function is not subadditive, nor positively homogeneous. In contrast a positively homogeneous function defined on a convex cone K in $X = R^l$ is convex if and only if f is subadditive (Niculescu, 2005, p 28). Travel functions can have lower and upper bounds if they are smooth and strongly convex. These bounds are linear quadratic functions $\widetilde{p}(y-x) = < \nabla p(0), y-x > +(\eta/2) \|y-x\|_2^2$, given that $p(0) = 0$, with $\eta \in \left\{\underline{\eta}, \overline{\eta}\right\}, 0 < \underline{\eta} < \overline{\eta} < +\infty$.

Remark. When costs to move are differentiable and lipschitz continuous with respect to the second variable y, an other possibility would be to use quadratic lower and upper bounds models of costs to move,

$C(x,y) = C(x,x) + < \nabla_2 C(x,x), y-x > +(\nu/2) \|y-x\|^2, \nu \in \{\underline{\nu} < \overline{\nu}\}, 0 < \underline{\nu} < \overline{\nu} < +\infty$.

5.2.3 Lower and upper bounds of a worthwhile balance

Consider the case of weak resistance to move, i.e., for example $\beta = 2$. Then, enclosing a worthwhile balance works as follows for all $x, y \in X$:

$\underline{A}(y/x) = \underline{g}_x(y) - \underline{g}_x(x) \leq A(y/x) \leq \overline{A}(y/x) = \overline{g}_x(y) - \overline{g}_x(x)$ where $g(x) = \underline{g}_x(x) = \overline{g}_x(x)$;

$\underline{I}(y/x) = \underline{C}(x,y) - \underline{C}(x,x) \leq I(y/x) \leq \overline{I}(y/x) = \overline{C}(x,y) - \overline{C}(x,x)$,

i.e., $\underline{h} \|y-x\| \leq I(y/x) \leq \overline{h} \|y-x\|, 0 < \underline{h} \leq \overline{h} < +\infty$ and,

$\underline{B}(y/x) = \underline{A}(y/x) - (\xi/2)\overline{I}(y/x)^2 \leq B(y/x) \leq \overline{B}(y/x) = \overline{A}(y/x) - (\xi/2)\underline{I}(y/x)^2$.

In the quadratic case,

$\underline{B}(y/x) = \underline{g}_x(y) - \underline{g}_x(x) - (\xi/2)\overline{h}^2 \|y-x\|^2$, i.e.,

$\underline{B}(y/x) = \left[< \nabla g(x), y-x > -(\overline{r}/2) \|y-x\|^2\right] - (\xi/2)\overline{h}^2 \|y-x\|^2$;

$\overline{B}(y/x) = \overline{g}_x(y) - \overline{g}_x(x) - \xi \underline{h}^2 \|y-x\|^2$, i.e.,

$\overline{B}(y/x) = \left[< \nabla g(x), y-x > -(\underline{r}/2) \|y-x\|^2\right] - (\xi/2)\underline{h}^2 \|y-x\|^2$.

Then,

i) $\underline{B}(y/x) = < \nabla g(x), y-x > -(\overline{\eta}/2) \|y-x\|^2 \leq B(y/x) = g(y) - g(x)$;

ii) $\overline{B}(y/x) = < \nabla g(x), y-x > -(\underline{\eta}/2) \|y-x\|^2 \geq B(y/x)$,

with $0 < \underline{\eta} = \underline{r} + \xi\underline{h}^2 < \overline{\eta} = \overline{r} + \xi\overline{h}^2 < +\infty$.

5.3 How adaptive gradient algorithms can lead to quick self regulation success.

In this last section we define a move $m = (x, \delta = y - x, y)$ in a space of bundles of activities $x, y \in X = R^l$. The translocation δ is the shift $\delta = y - x$ instead of the norm $\|y - x\|$. See the weight example where $X = R_+$.

5.3.1 When a gradient step of an approximative model provides a move that, both, satisfices (= makes enough progress) and is worthwhile.

Question: what about self regulation success in the context of gradient algorithms. This question is: when a gradient move can be, both, worthwhile and satisficing. The previous weight gain or loss example helps very much to consider, in a vectorial space of actions (bundles of activities) $x \in X = R^l_+$, a much more general formulation of self regulation success in the context of a gradient algorithm, when,

i) resistance to move is weak ($\beta = 2$), i.e., a worthwhile balance is

$B_\xi(y/x) = A(y/x) - \xi I(y/x)^\beta = g(y) - g(x) - \xi \left[C(x, y) - C(x, x)\right]^2$ where $x, y \in X = R^l_+$;

ii) an optimal bundle of activities $x^* \in X$ exists;

iii) the expected valence function $g(.)$ and the expected cost to move function $C(., .)$ are unknown, except at the current point $x = x_k$, where only $g(x), \nabla g(x), C(x, x)$ and $\nabla_2 C(x, x)$ are known. In particular the optimal bundle of activities $x^* \in X$ and the optimal expected valence $g^* = g(x^*)$ are unknown.

Then, each period, in a much more difficult context than before, given the oracles $x = x_k$, $g(x), \nabla g(x), C(x, x)$ and $\nabla_2 C(x, x)$, a given worthwhile ratio $\xi > 0$ and a given satisficing ratio $0 < \theta < 1$, self regulation success poses the question: when, each period, an individual can find some move $m = (x, \delta = y - x, y)$, with a shift $\delta = y - x$, which is, at the same time,

A) worthwhile, i.e., $g(y) - g(x) \geq (\xi/2) I(y/x)^2$ and,

B) satisficing (leading to sufficient progress), i.e., $g(y) - g(x) \geq \theta \left[g^* - g(x)\right]$.

5.3.2 Main result.

Using lower and upper bounds of valence expectancies and self efficacy beliefs let us show how self regulation success can be achieved if an individual can do a succession of not too large and not too small gradient moves. In this way we start to build a dynamic flexible and adaptive mathematical model for a general theory of motivation emotions and self regulation.

Building lower and upper aspiration levels/aspiration gaps. Let us build, each period, an approximate aspiration level, using an approximate lower or upper expected valence function $\widetilde{g}_x(.) : x \in X \longmapsto \widetilde{g}_x(y) \in R$, with $\widetilde{g}_x(.) \in \left\{\underline{g}_x(.), \overline{g}_x(.)\right\}$, where relative to the unknown valence function $g(.)$,

i) the approximate valence of $g(.)$ is

$\widetilde{g}_x(y) = g(x) + < \nabla g(x), y - x > - (\widetilde{r}/2) \|y - x\|^2 \in R$ and

ii) the approximate advantage to move of $A(./.)$ is
$\widetilde{A}_x(x,y) = \widetilde{g}_x(y) - \widetilde{g}_x(x) = <\nabla g(x), y-x> -(\widetilde{r}/2)\left\|y-x\right\|^2$,

where $\widetilde{r} = \underline{r}$ is small (upper bounded model) and $\widetilde{r} = \overline{r}$ is large (lower bound model), with $0 < \underline{r} < \overline{r}$, given that $\underline{g}_x(.) \leq \overline{g}_x(.)$.

Then, the approximate aspiration level at x is $\widetilde{g}_x^* = \sup\{\widetilde{g}_x(y), y \in X\} < +\infty$. That is, $\widetilde{g}_x^* = g(x) + (1/2\widetilde{r})\left\|\nabla g(x)\right\|^2$ and the related approximate aspiration gap is $\widetilde{A}_x^* = \widetilde{g}_x^* - g(x) = (1/2\widetilde{r})\left\|\nabla g(x)\right\|^2$ for $\widetilde{r} \in \{\underline{r}, \overline{r}\}$.

Proof: starting from x, the derivation of $\widetilde{g}_x(y)$ with respect to y gives the first order condition $\nabla g(x) = \widetilde{r}(y-x)$ where $y = \widetilde{x}^*$. Then, plug $y = \widetilde{x}^*$ in $\widetilde{g}_x(y)$, where $\widetilde{x}^* - x = (1/\widetilde{r})\nabla g(x)$ defines the optimal gradient move of the approximative model.

Make a satisficing move (= make sufficient progress) within the approximate model. Given what has been defined before**,** starting from the status quo x and working on the approximative model at x, approximate advantages to move are $\widetilde{A}_x(x,y) = \widetilde{g}_x(y) - \widetilde{g}_x(x)$ and the aspiration gap is $\widetilde{A}_x^* = \widetilde{g}_x^* - g(x) = (1/2\widetilde{r})\left\|\nabla g(x)\right\|^2$. Then, to satisfice approximatively means: find a shift $\delta = y - x$ such that approximate advantages to move fill a sufficient portion $0 < \widetilde{\theta} < 1$ of the aspiration gap. That is, find an action $y = x_{\widetilde{\theta}}$ such that $\widetilde{A}_x(x, x_{\widetilde{\theta}}) \geq \widetilde{\theta}\widetilde{A}_x^*$, i.e., $\widetilde{g}_x(y) - \widetilde{g}_x(x) \geq \widetilde{\theta}\left[\widetilde{g}_x^* - g(x)\right]$, i.e., $<\nabla g(x), y-x>$ $-(\widetilde{r}/2)\left\|y-x\right\|^2 \geq (\widetilde{\theta}/2\widetilde{r})\left\|\nabla g(x)\right\|^2$.

A way to do a satisficing move within the approximative model: do a well appropriate gradient step. Such a well chosen gradient step $y - x = \alpha\nabla g(x) \iff y = x_\alpha = x + \alpha\nabla g(x)$, with $\alpha > 0$, must be such that $\widetilde{A}_x(x_\alpha/x) \geq \widetilde{\theta}\widetilde{A}_x^* = (\widetilde{\theta}/2\widetilde{r})\left\|\nabla g(x)\right\|^2$. That is, α must satisfy $\alpha - (\widetilde{r}/2)\alpha^2 \geq \widetilde{\theta}/2\widetilde{r}$.

Remark. Given that $1/2\widetilde{r} = \max\left\{\alpha - (\widetilde{r}/2)\alpha^2, \alpha \geq 0\right\}$, the previous condition $\alpha - (\widetilde{r}/2)\alpha^2 \geq \widetilde{\theta}/2\widetilde{r}$ requires $0 < \widetilde{\theta} \leq 1$, as it must be. It is satisfied if $\alpha \in \left[\alpha^{\searrow}(\widetilde{\theta},\widetilde{r}), \alpha^{+}(\widetilde{\theta},\widetilde{r})\right]$. More precisely, given $0 < \widetilde{\theta} \leq 1$, there exist two values of α such that $\alpha - (\widetilde{r}/2)\alpha^2 = \widetilde{\theta}/2\widetilde{r} \iff \widetilde{\theta} = 2(\widetilde{r}\alpha) - (\widetilde{r}\alpha)^2$. They are $\alpha(\widetilde{\theta},\widetilde{r}) = (1/\widetilde{r})\left[1 \pm \sqrt{1-\widetilde{\theta}}\right]$.

HINT. This remark shows a very important point: to satisfice relative to the approximative model is equivalent to choose an appropriate gradient step of this approximative model. Then, our strategy will be to show how a gradient step of the approximative model can be a satisficing move of the approximative model.

Choose a pessimistic lower bound model. This can be a quadratic lower bound model where,

i) $g(y) \geq \underline{g}_x(y)$, $\underline{g}_x(x) = g(x)$ for all $x, y \in X = R^l$;

ii) $\underline{g}_x(y) = g(x) + <\nabla g(x), y-x> - (\overline{r}/2)\left\|y-x\right\|^2 \in R$, with $0 < \underline{r} < \overline{r} < +\infty$ for all $x, y \in X = R^l$.

Such a quadratic lower bound model exists if $g(.)$ is Lipschitz smooth, i.e., if $g(.)$ is curvy enough. For equivalent definitions see Ang (2021). In this setting, as seen before, $\underline{A}_x(y/x) = \underline{g}_x(y) - \underline{g}_x(x) = <\nabla g(x), y-x> - (\overline{r}/2)\left\|y-x\right\|^2$, for all $x, y \in X = R^l$.

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Find a gradient move that is a satisficing move relative to the true model.

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FIRST RESULT. We get a first main result. Given the rate of progress $0 < \widetilde{\theta} = \overline{\theta} \leq 1$ required when an individual uses a lower bound (pessimistic) model, the gradient move of the pessimistic model $\delta_\alpha = x_\alpha - x = \alpha \nabla g(x)$ is satisficing, i.e., $g(x_\alpha) - g(x) \geq \overline{\theta}\,[g^* - g(x)]$ (S), if condition (A), $\alpha - (\overline{r}/2)\alpha^2 \geq \overline{\theta}/2\overline{r}$, and condition (B), $\overline{\theta}(\underline{r}/\overline{r}) \geq \theta$, are satisfied.

Proof.

Step 1. In this setting true (= ex-post revealed) advantages to move $A(y/x) = g(y) - g(x)$ are higher than the lower bound advantages to move, $A(y/x) \geq \underline{A}_x(y/x) = \underline{g}_x(y) - \underline{g}_x(x) = \, < \nabla g(x), y - x > - \, (\overline{r}/2) \left\| y - x \right\|^2$, for all $x, y \in X = R^l$. This inequality is also true for a gradient step $(x, y = x_\alpha)$ of the pessimistic model, $y - x = x_\alpha - x = \alpha \nabla g(x)$. Thus,

$A(x_\alpha/x) = g(x_\alpha) - g(x) \geq \underline{A}_x(x_\alpha/x) = \left[\alpha - (\overline{r}/2)\alpha^2\right] \left\| \nabla g(x) \right\|^2$, i.e.,

$g(x_\alpha) - g(x) \geq \left[\alpha - (\overline{r}/2)\alpha^2\right] \left\| \nabla g(x) \right\|^2$ (C).

Then, given $\widetilde{\theta} = \overline{\theta}$ and $\underline{A}_x^* = (1/2\overline{r}) \left\| \nabla g(x) \right\|^2$, $g(x_\alpha) - g(x) \geq \overline{\theta} \underline{A}_x^* = (\overline{\theta}/2\overline{r}) \left\| \nabla g(x) \right\|^2$ if $\alpha - (\overline{r}/2)\alpha^2 \geq \overline{\theta}/2\overline{r}$ (A). If this last inequality is satisfied, $g(x_\alpha) - g(x) \geq (\overline{\theta}/2\overline{r}) \left\| \nabla g(x) \right\|^2$.

Step 2. Let us take advantage of the existence of an upper bound model with $\widetilde{r} = \underline{r} > 0$. Then, $g(y) \leq \overline{g}_x(y) = g(x) + < \nabla g(x), y - x > - \, (\underline{r}/2) \left\| y - x \right\|^2$ for all $x, y \in X$.

Taking the maximum of the two sides of this inequality gives $g^* \leq g(x) + (1/2\underline{r}) \left\| \nabla g(x) \right\|^2$, i.e., $\left\| \nabla g(x) \right\|^2 \geq 2\underline{r}\,[g^* - g(x)]$ (D) for all $x \in X$. Thus, (C) and (D) imply $g(x_\alpha) - g(x) \geq (\overline{\theta}/2\overline{r}) \left\| \nabla g(x) \right\|^2 \geq \overline{\theta}(\underline{r}/\overline{r})\,[g^* - g(x)]$ (E).

Then, the two inequalities (E) and (B) imply $g(x_\alpha) - g(x) \geq \overline{\theta}(\underline{r}/\overline{r})\,[g^* - g(x)] \geq \theta\,[g^* - g(x)]$. This gives $g(x_\alpha) - g(x) \geq \theta\,[g^* - g(x)]$ (F), with $0 < \overline{\theta} \leq 1, 0 < \theta \leq 1$ and $0 < \underline{r} < \overline{r} < +\infty$.

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Show when this true satisficing gradient move is also a true worthwhile gradient move.

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SECOND RESULT. A satisficing gradient move $x_\alpha - x = \alpha \nabla g(x)$ is also worthwhile, i.e., $g(x_\alpha) - g(x) \geq \xi I(x_\alpha/x)^2$, if $0 < \alpha \leq 1/\left[\overline{r}/2 + \xi \overline{h}^2\right]$.

Proof. Given the enclosing inequalities $0 \leq \underline{h} \left\| y - x \right\| \leq I(y/x) \leq \overline{h} \left\| y - x \right\|$, setting a gradient move $\nabla g(x) = (1/\alpha)(x_\alpha - x)$ and pluging this move in inequality $g(x_\alpha) - g(x) \geq \left[\alpha - (\overline{r}/2)\alpha^2\right] \left\| \nabla g(x) \right\|^2$ (C) give,

$g(x_\alpha) - g(x) \geq \left[\alpha - (\overline{r}/2)\alpha^2\right] (1/\alpha^2) \left\| x_\alpha - x \right\|^2 = \left[(1/\alpha) - \overline{r}/2\right] \left\| x_\alpha - x \right\|^2 \geq \xi \overline{h}^2 \left\| x_\alpha - x \right\|^2$.

Then, $g(x_\alpha) - g(x) \geq \xi I(x_\alpha/x)^2$ if $(1/\alpha) - \overline{r}/2 \geq \xi \overline{h}^2$. That is, if $1/\alpha \geq \overline{r}/2 + \xi \overline{h}^2$, i.e., if $0 < \alpha \leq 1/\left[\overline{r}/2 + \xi \overline{h}^2\right]$.

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MAIN RESULT: SUFFICIENT CONDITIONS FOR SUCCESS. To conclude this discussion, given the desirable rate of progress $0 < \theta \leq 1$ and the tolerated level of sacrifices $\xi > 0$,

i) the existence of a satisficing gradient move $\alpha - (\overline{r}/2)\alpha^2 \geq \overline{\theta}/2\overline{r}$ such that $\overline{\theta}(\underline{r}/\overline{r}) \geq \theta$ requires a high enough size α of the gradient step, i.e., $\alpha \geq \alpha^{\smallsetminus} = (1/\overline{r})\left[1 - \sqrt{1-\overline{\theta}}\right]$, and a high enough rate of progress $\overline{\theta}$ when using the lower bound model, i.e., $\overline{\theta} \geq (\overline{r}/\underline{r})\theta$;

ii) this satisficing gradient move will be a worthwhile move if the size of this gradient step is small enough, i.e., $0 < \alpha \leq 1/\left[\overline{r}/2 + \xi\overline{h}^2\right]$.

Then, self regulation success requires that ξ and θ satisfy the two conditions, i) $(1/\overline{r})\left[1 - \sqrt{1-\overline{\theta}}\right] \leq 1/\left[\overline{r}/2 + \xi\overline{h}^2\right]$ and, ii) $\overline{\theta} \geq (\overline{r}/\underline{r})\theta$. That is, a low enough ξ, a low enough $\overline{\theta}$, and a low enough θ.

This result confirms the generality of the weight gain-loss example.

5.3.3 Speed of convergence in the context of adaptive gradient algorithms

Consider now, in the same setting as previously, a succession of periods $k \in N$.

Linear convergence Suppose that, each period $k = 0, 1, ...$, an individual makes, as seen before, a satisficing gradient move $x_\alpha - x = \alpha\nabla g(x)$, with $x_k = x$ and $x_{k+1} = x_\alpha$, that makes, at the same time, sufficient progress (= satisficing) and is worthwhile. Then, the sufficient progress condition (S) $g(x_\alpha) - g(x) \geq \theta\left[g^* - g(x)\right]$, with $0 < \theta \leq \overline{\theta}(\underline{r}/\overline{r})$, gives $g(x_{k+1}) - g(x_k) \geq \theta\left[g^* - g(x_k)\right], k \in N$. Let $f(x) = g(x^*) - g(x) \geq 0$ be the level of frustration of doing x instead of having done the ideal action x^*, i.e., $g^* = g(x^*)$. Condition (S) becomes $g(x_{k+1}) - g(x_k) = f(x_k) - f(x_{k+1}) \geq \theta f(x_k)$. That is, $0 \leq f(x_{k+1}) \leq (1-\theta)f(x_k)$.

This means that, each period, frustration feelings decrease at a sufficient rate $0 < 1-\theta \leq 1$. Then, $0 \leq f(x_{k+1}) \leq (1-\theta)f(x_k) \leq (1-\theta)^2 f(x_{k-1}) \leq .... \leq (1-\theta)^{k+1}f(x_0)$.

Thus, the sufficient decrease condition $0 \leq f(x_{k+1}) \leq (1-\theta)^{k+1}f(x_0)$ shows that $f(x_{k+1}) = g^* - g(x_{k+1}) \geq 0$ goes to zero as fast as $(1-\theta)^{k+1}$ if k goes to infinity. This refers to linear convergence in mathematics (Karimi et al., 2016). This leads to self regulation success in psychology.

Finite convergence The shape of the true valence function $g(.)$ near its maximum x^* (when this ideal position exists) have a great important for the speed of convergence. At the two extremes, in the context of optimizing algorithms, this shape can be sharp or flat near x^* (see Apidopoulos et al., 2019 for a nice classification). This pushes us, in psychology, to make the distinction between sharp or flat ideals or aspirations. Let us show very briefly how a self regulation dynamic can converge in finite time to a sharp ideal, i.e., a self regulation success. Intuitively, when an ideal position is sharp (flat), convergence towards the ideal position will be fast (slow).

Definition. A differentiable valence function $g(.) : x \in X = R^l \longmapsto g(x) \in R$ is sharp if $\|\nabla g(x)\| \geq \eta > 0$ for all $x \in X$. For example $g(x) = \|x\|$.

Then, the size $\|x_\alpha - x\|$ of a gradient move $x_\alpha - x = \alpha \nabla g(x)$ with a constant step $\alpha > 0$ is bounded above, i.e., $\|x_\alpha - x\| = \alpha \|\nabla g(x)\| \geq \alpha\eta > 0$ if $\|\nabla g(x)\| \geq \eta > 0$. Let us show that, in this context, a succession of worthwhile gradient moves $x_{k+1} - x_k = \alpha \nabla g(x_k)$, $k \in N$ converges in finite time.

Proof.

Consider a gradient step $(x, y = x_\alpha)$ of the pessimistic model, i.e., $y - x = x_\alpha - x = \alpha \nabla g(x), \alpha > 0$ (see step 1 of the proof of the main result). Thus, advantages to moves $A(x_\alpha/x) = g(x_\alpha) - g(x)$ are higher than the pessimistic advantages to move $\underline{A}_x(x_\alpha/x) = \left[\alpha - (\overline{r}/2)\alpha^2\right] \|\nabla g(x)\|^2$. That is, $g(x_\alpha) - g(x) \geq \left[\alpha - (\overline{r}/2)\alpha^2\right] \|\nabla g(x)\|^2$ (C). Take any size $\alpha > 0$ such that $\psi(\alpha) = \alpha - (\overline{r}/2)\alpha^2 > 0$. For example, $\alpha^* = 1/\overline{r}$. Thus, $\psi(\alpha^*) = 1/2\overline{r}$ and $g(x_\alpha) - g(x) \geq (1/2\overline{r}) \|\nabla g(x)\|^2$. Then, if $\|\nabla g(x)\| \geq \eta > 0$ for all $x \in X$, $g(x_\alpha) - g(x) \geq (1/2\overline{r}) \|\nabla g(x)\|^2 \geq \eta/2\overline{r} > 0$ for all $x \in X$. Consider now a succession of such pessimistic gradient moves. We have, for each $k \in N, g(x_{k+1}) - g(x_k) \geq \eta/2\overline{r} > 0$.

Given that $g(x_{k+1}) - g(x_k) > 0$ for all $x \in X$, the sequence $\{g(x_k), k \in N\}$ converges if $g(.)$ is bounded above. Then, advantages to move $g(x_{k+1}) - g(x_k)$ goes to zero. This is impossible. Then, a pessimistic gradient move dynamic stops in finite time.

5.3.4 No overshooting.

Suppose that $g(.)$ is smooth, i.e., $g(y) \geq g(x) + < \nabla g(x), y - x > -(\overline{r}/2) \|y - x\|^2$ for all $x, y \in X$. Let $x_{\overline{r}} - x = (1/\overline{r})\nabla g(x)$ be a gradient move. Then, using the smoothness condition, $g(x_{\overline{r}}) - g(x) \geq (1/2\overline{r}) \|\nabla g(x)\|^2$ provides, as seen before, a descent method. Moreover there is no overshooting. That is, $x_{\overline{r}}$ is on the same side of x^* as $x : < x^* - x_{\overline{r}}, x^* - x > \geq 0$ for all $x \in X$.

Proof. We follow Garivier (2019). Since $g(.)$ is smooth,
$\|\nabla g(x)\| = \|\nabla g(x) - \nabla g(x^*)\| \leq \overline{r} \|x - x^*\|$ for all $x \in X$. Then,
$< \nabla g(x), x - x^* > \leq \|\nabla g(x)\| \|x - x^*\| \leq \overline{r} \|x - x^*\|^2$. Thus,
$< x^* - x_{\overline{r}}, x^* - x > = \|x - x^*\|^2 + < (1/\overline{r})\nabla g(x), x^* - x > \geq 0$ for all $x \in X$.

5.3.5 Extensions.

Links between three indexes of self regulation success Let us note, as it is well known (Karimi et al, 2016) that, if a function $g(.)$ is smooth and strongly concave, we have three fundamental inequalities that drive a lot of descent algorithms in optimization. This will suggest that sufficient conditions for self regulation success can be generalized in a lot of different mathematical contexts (work in progress).

Self regulation success can be defined by two different stopping rules:

i) in the satisfaction level space $R: \ 0 \leq f(x) = g^* - g(x) \leq \varepsilon, \varepsilon > 0$;

ii) in the action space X: $\|x^* - x\| \leq \varepsilon', \varepsilon' > 0$.

When $g(.)$ is differentiable, the main way to link these two success conditions is to use the gradient stopping rule $\|\nabla g(x)\| \leq \varepsilon"$ (or $\|\partial g(x)\| \leq \varepsilon"$when a subgradient exists). Three well known conditions help to guaranty self regulation success in more general cases.

Error bound (EB): $\|\nabla g(x)\| \geq (\mu/2)\,\|x^* - x\|$ for all $x \in X$, with $\mu > 0$. This error bound inequality shows that we are close to the ideal position x^* when the gradient at position x is small. Then, $\|\nabla g(x)\| \leq \varepsilon"$ implies $\|x^* - x\| \leq \varepsilon' = 2\varepsilon"/\mu$.

Notice that $g(.)$ smooth implies (EB). See above the no overshooting condition.

Polyak-Lojasiewicz condition (PL): $\|\nabla g(x)\|^2 \geq 2\mu\,[g^* - g(x)]$ for all $x \in X$, with $\mu > 0$, given. This P.L inequality shows that we are close to the aspiration level $g^* = g(x^*)$ when the gradient is small. This means that the lower bound aspiration gap $(1/2\mu)\,\|\nabla g(x)\|^2$ is high enough relative to the aspiration gap $f(x) = g^* - g(x)$. That is, the square of the gradient at x gives an upper bound of the frustration level $f(x)$ at x. Then, $\|\nabla g(x)\| \leq \varepsilon"$ implies $0 \leq g^* - g(x) \leq \varepsilon = (\varepsilon")^2/2\mu$.

Notice that $g(.)$ strongly concave implies P.L. See above inequality (D) in the proof of the first result.

Quadratic growth conditions (GC): $g(x^*) - g(x) \geq (\mu/2)\,\|x^* - x\|^2$ for all $x \in X$, with $\mu > 0$. In the context of the VR approach, this means that, starting from any $x \in X$, the ideal position x^* can be reached through a worthwhile move, i.e., $x^* \in W_\xi(x) = \left\{y \in X,\ g(y) - g(x) \geq (\xi/2)\,\|y - x\|^2\right\}$, with $\xi = \mu$.

It is easy to see that $g(.)$ strongly concave implies (GC): the strongly concave inequality $g(y) \leq g(x) + < \nabla g(x), y - x > -(\underline{r}/2)\,\|y - x\|^2$ for all $x, y \in X$ implies that, if $x = x^*, \nabla g(x^*) = 0$ gives $g(y) - g(x^*) \leq -(\underline{r}/2)\,\|y - x^*\|^2$. That is, if $\mu = \underline{r} > 0$, $g(x^*) - g(y) \geq (\mu/2)\,\|y - x^*\|^2$ for all $y \in X$.

Acceleration process: non monotone self regulation dynamics. Following the famous book of Nesterov (2013), the study of acceleration processes is a hot topic in optimization. In the context of behavioral sciences, the problem is the following: if an individual makes constant gradient steps, he will make big steps when the gradient is large and small steps when it is small. This means that, the more an individual approaches a goal, the lower his speed of convergence (= approach). Then, to accelerate, this individual must put more weight on the motivation side (= advantage to move) of the worthwhile balance when the gradient becomes small. This remark is in line with the famous goal gradient hypothesis in psychology. The more you wait for the satisfaction of a need, the more it is painful. Then, to economize time, an individual will accelerate when he approaches a goal.

Let us show how to give more weight on advantages to do a move. For example, suppose that an individual weights differently his satisfaction $g(x)$ of filling a portion of a need and his residual unsatisfaction $f(x) = g^* - g(x) \geq 0$ of not filling fully this need. This defines a contentment-frustration function $\psi(x) = \mu g(x) - \nu f(x) = \mu g(x) - \nu [g^* - g(x)] = (\mu + \nu)g(x) - \nu g^*$ with $\mu, \nu > 0$. Then, if $\mu + \nu > 1$, this contentment-frustration valence puts more weight on advantages to move $A(y/x) = \psi(y) - \psi(x) = (\mu + \nu) [g(y) - g(x)]$.

These remarks open a vast area of research in psychology in the context of flexible (adaptive) accelerated optimizing algorithms. This will be the topic of our future research.

Cybernetic control processes and the self-regulation of behavior As seen at the beginning of this paper, the interpretation of human self regulation as a cybernetic process is central in psychology (see Carver & Scheier, 2012, for a survey). Following recent papers in mathematics (Lessard et al., 2016, Hu & Lessard 2017), our VR approach will greatly help to offer such a cybernetic control model of human behavior in the context of first-order optimization methods (work in progress). A simple example will introduce this avenue for new research in psychology.

Let 1) $s_{k+1} = As_k + Bu_k$, 2) $y_k = Cs_k + Du_k$, be a linear state space representation of a dynamical system where the state at time k is the vector s_k, the control (input) is the vector u_k and the output is the vector y_k; the dimensions of the matrices A, B, C and D being chosen appropriately. We can connect this linear system in feedback with the non linearity $\phi(.)$ by defining the rule, 3) $u_k = \phi(y_k)$. Then, a gradient step $x_{k+1} - x_k = \alpha \nabla g(x_k) = -\alpha \nabla f(x_k), \alpha > 0$, of the gradient method can be written as

1) $s_{k+1} = s_k + \alpha \nabla g(x_k)$, 2) $y_k = s_k$ and, 3) $u_k = \phi(y_k) = \nabla g(x_k)$.

This shows that the control of a gradient move is the gradient $\nabla g(x_k)$. This formulation is in accordance with the fact that the error bound condition (EB), and the Polyak-Lojasiewicz condition (PL) are driven by the norm of the gradient, $\|\nabla g(x_k)\|$.

6 Conclusion

For simplification and to avoid too much mathematics, the present paper starts to give, at the individual level, necessary and sufficient conditions for self regulation success of a unique need when resistance to move is strong or weak. Future papers will consider more realistic cases including, i) an individual who wants to satisfy several recurrent and changing needs in a changing environment, ii) a group of individuals or an organization that wants to reach different moving goals, and interrelated individuals (games) who want to satisfy several needs; These extensions will require much more (= high level) mathematics.

Their resolution will show that, at the interaction between a lot of different disciplines, two main problems are the two faces of the same coin:

1) in psychology and behavioral sciences a lot of motivational and self regulation problems related to the satisfaction of different recurrent and changing needs in a changing world where resistance to move matters;

2) in mathematics (the modern variational analysis) : a long list of variational and adaptive optimizing problems and theorems that our VR approach helps to classify in two branches:

i) when resistance to move is strong: the main optimization theorems that include the famous Weierstrass theorem, Banach fixed point theorem, Ekeland, Caristi and Takahashi equivalent variational principles, Nash equilibrium and quasi equilibrium theorems with potential functions (game theory);

ii) when resistance to move is weak: the main optimizing algorithms include the famous gradient, line search, descent, proximal, trust region algorithms and the accelerated versions.

In a near future we will consider more carefully the self regulation of multiple goals (Fishbach et al. 2009, Neal et al., 2017) in the explicit context of goal disengagement, reengagement and engagement (Wrosch et al, 2003.a, Wrosch et al, 2003.b, Wrosch et al, 2007). We will also consider the problem of autonomy in the context of interacting individuals (game theory). See Ryan & Deci (2006).

We also want to develop simulations in the context of the relation between real need satisfaction-need frustration problems and modern flexible optimizing algorithms. For two preliminary examples, see Cruz Neto et al.(2020) and Ferreira et al.(2022, submitted).

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Appendix 1: Figures

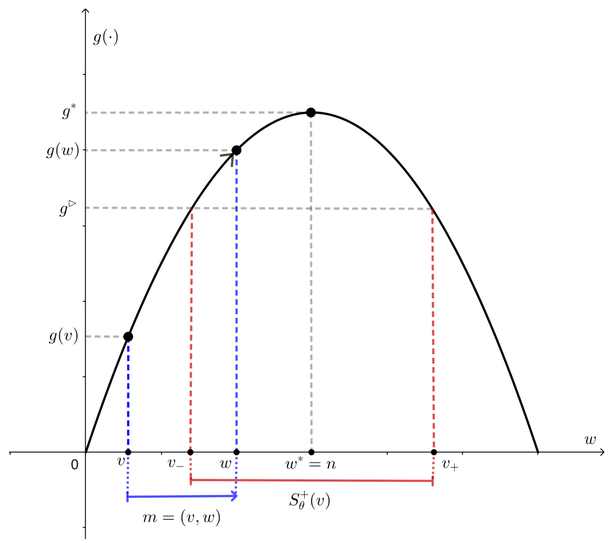

Figure 1: Satisficing move.

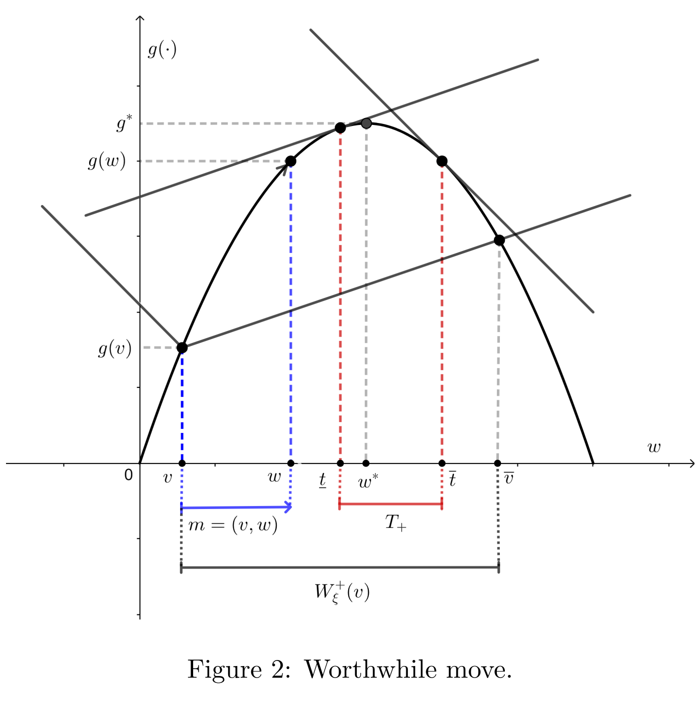

Figure 2: Worthwhile move.

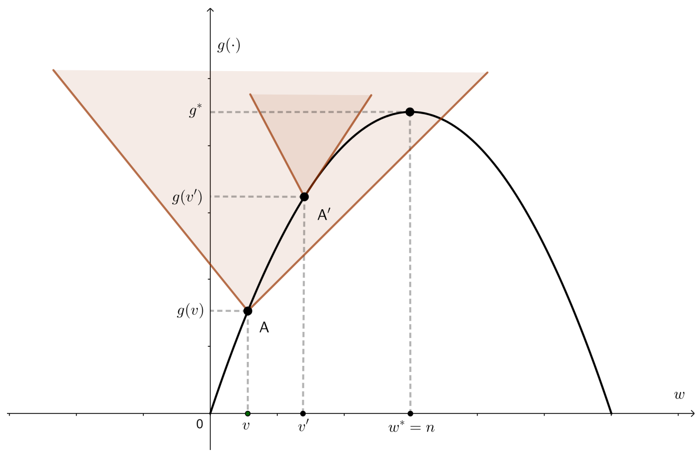

Figure 3: A trap.

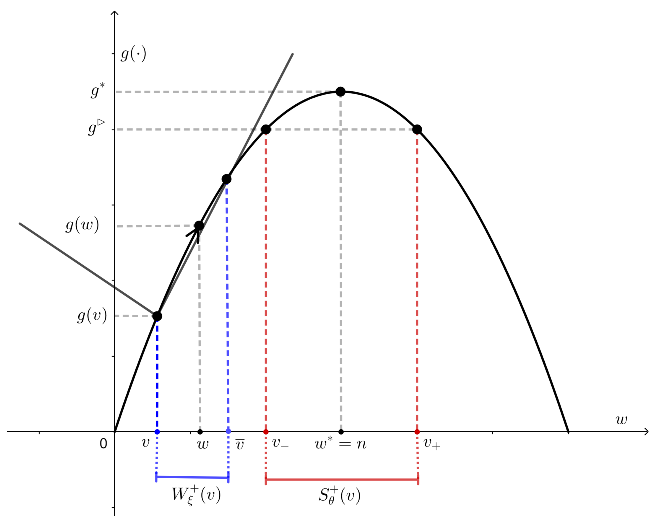

Figure 4: Self regulation failures.

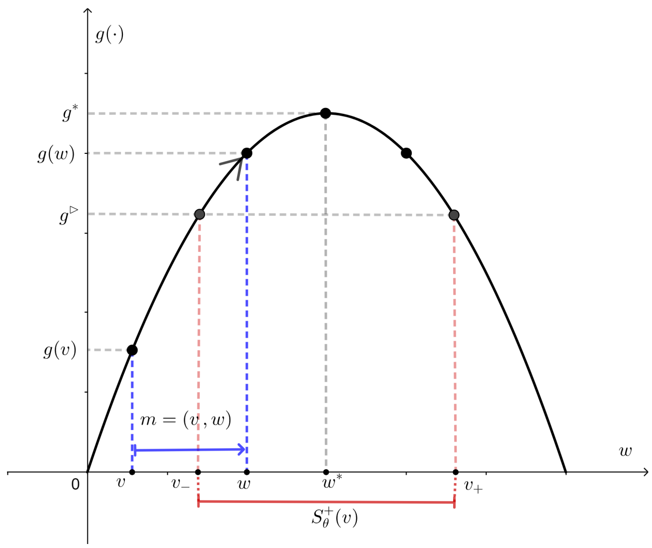

Figure 5: Self regulation success.

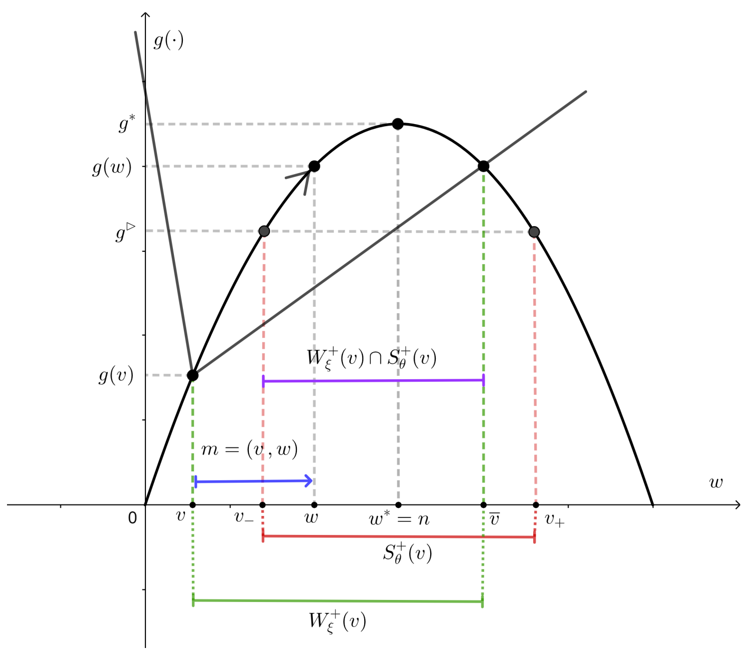

Figure 6: Self regulation success.

Appendix 2. The conceptual construction of satisficing moves and worthwhile moves

The VR approach is driven by two main tools: worthwhile and satisficing moves. Their (conceptual and formal) construction requires a succession of stages that helps to model in a precise way a self regulation process in order, each period, to become able to satisfy enough different needs.

Stage1: define needs and expectancies

A) Define your needs. That is,

1) become aware of your motives (what you like and what you dislike in different domains of your life);

2) become aware of your recurrent and changing needs (unsatisfied motives);

3) define the size of each need;

4) choose which needs you want to fulfill first. That is, determine their relative importance, i.e., define your urgent needs whose fulfilment cannot be delayed and your less important needs that can be fulfilled later.

B) Build expectancies. That is, define successively, for each need,

5) a value expectancy. It is the expected level of satisfaction derived from the degree of fulfilment of this need;

6) an outcome expectancy. It defines how much of this need will be fulfilled after having done a bundle of activities;

7) a valence expectancy. It represents the expected level of satisfaction derived from the degree of fulfilment of this need, through doing a bundle of activities;

8) a self efficacy expectancy. It defines the expected costs of becoming able to do a bundle of activities (capability expectancy) and the expected costs to do it (competence expectancy).

Stage 2: define a satisficing move (long term view)

A satisficing move considers, in the current period, desirability aspects of a move. To model such a move we must define successively, for each recurrent need or new need,

9) the status quo level of satisfaction-frustration. It is derived from the degree of fulfilment of this need through doing some bundle of activities in the previous period. This level is zero for a new need;

10) the ideal level of satisfaction of this need. It is, in the current period, the highest level of satisfaction that an individual can hope to reach when trying to completely fill this recurrent or lasting need. At this ideal level of satisfaction, frustration feelings of not having fully filled this need disappears;

11) the aspiration level of satisfaction. It represents an approximation of the ideal level, when this ideal level is not well known;

12) a satisficing level of satisfaction or target. It is a high enough level of satisfaction derived from the degree of fulfilment of this need through doing some bundle of activities in the current period. This level is between the aspiration level and the status quo level of satisfaction. It must be close enough to the aspiration level. It defines a threshold level between being satisfied enough and not too much dissatisfied (Simon, 1955). At this level, the frustration feelings of not having completely met this need are not to high;

13) the aspiration gap (= discrepancy). It is the difference between the aspiration level and the status quo level of satisfaction;

14) a progress (= improvement). It is the difference between a satisfaction level (higher than the status quo level) and the status quo level;

15) a sufficient rate of progress (Carver & Scheier, 1990). It defines a large enough ratio between some progress and the aspiration gap. It defines a level of satisfaction close enough to the aspiration level and larger enough relative to the status quo level.

Then, a move satisfices when, going from having done something in the previous period to do another thing in the current period, you makes sufficient progress, i.e., you improves enough your satisfaction level. The term " makes sufficient progress" materialises the vague expression " quite close to the aspiration level" used in definition 12). It is more useful than the static Simon's version of satisficing, because it considers, in a dynamic setting, two reference points instead of one, i.e., the status quo level and the aspiration level, instead of only the aspiration level.

Our VR definition of a satisficing move is close to the formulation of the TOTE model. But, being formalized, it is much more precise because it models more easily the past, future (distal) and present (proximal) aspects of a situation.

Stage 3: define a worthwhile move (short term view)

The definition of a worthwhile move requires to define successively,

16) expected advantages to move. They are the difference between two expected valences: the valence of envisioning to do the same thing as before and the valence of envisioning to do another thing;

17) expected inconveniences to move. They represent the difference between two expected costs: the costs of changing from having done something to doing another thing and the costs to do the same thing as before;

18) motivation and resistance to move. They define the value given to variations. That is, motivation and resistance to move are the values (expected utility and expected disutility) given to expected advantages and expected inconveniences to do a move;

19) an expected worthwhile balance. It is the weighted difference between motivation and resistance to do a move (= change rather than stay).

A worthwhile move considers the balance between the desirability and the feasibility aspects of a move. In the current period, a worthwhile move is such that motivation to move is high enough relative to resistance to move. This means that the expected worthwhile balance between motivation and resistance to move is non negative or high enough. This VR concept helps to model a lot of Lewin's concepts in his famous topological psychology and force field theory (Lewin, 1935, 1936, 1938, 1951).

Stage 4: end of goal setting, then goal striving and goal realization

Given the previous concepts that summarize the beginning of goal setting, we will be in a good position to define,

a) the end of goal setting, i.e., choosing an action-goal, that, both, makes a move satisficing and worthwhile.

b) goal striving, i.e., becoming able to do this chosen action (bundle of activities);

c) goal realization, i.e., doing the chosen action, hoping to make a worthwhile and satisficing move.

d) feedbacks, i.e., check that having done the chosen action makes the resulting move worthwhile and satisficing. That is, use feedbacks to see if your needs has been sufficiently met, without having to endure too much sacrifices.

e) If not try an other action, or revise the entire process, ...

In this way, self regulation is a cybernetic way to solve need satisfaction problems in a recursive way (a fixed point approach and its convergence). Then, self-regulation involves "flexible goal setting, tenacious goal striving" and goal attainment (Mischel et al., 1996).

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