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The tragedy of the (anti-)commons: The case of prey-predator fisheries[☆]

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Abstract

We examine the efficiency and environmental consequences of assigning species-specific common-property rights, considering a Lotka-Volterra model in which fisheries are specialized in the harvesting of a single species. We show that the fragmentation of the ecosystem implies the *tragedy of the anticommons* even when fisheries compete for the resource. Indeed, contrasting the private exploitation equilibrium with the socially optimal solution, we demonstrate that the predator stock is too high while the prey stock is too low under private property rights. A puzzling result is that the "abundant" species is actually underused because of insufficient economic incentives; however, the scarce and high-priced species does not necessarily suffer from overexploitation. Biological interactions are consequently the main driver of stock depletion. Finally, we investigate how to simultaneously solve both the tragedy of the commons and that of the anticommons and analyze the economic costs of regulating only the tragedy of the commons.

Keyword: Exclusive property rights, Common-pool resource, Anticommons, Fisheries, Prey-predator relationship, Optimal control.

JEL classification: C61, C73, H23, Q22.

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1. Introduction

Over the past few decades, considerable efforts have been made to ensure sustainable exploitation of fisheries resources. Various management tools have been widely implemented globally; however, rights-based management tools seem to have been the most powerful instruments to overcome overexploitation (see, among others, Costello et *al.* [10], Newell et *al.* [44] and Péreau et *al.* [45]). However, these tools have not systematically provided successful results to ensure sustainable fishing. Many studies have thus examined why regulations may be ineffective in managing fish stock. However, none of the analyses associates the failure of rights-based management with the tragedy of the anticommons. This study fills this gap by examining how a species-based management system may imply issues associated with anticommons resources. We specifically analyze a system that exclusively assigns property rights over a specific species for a limited group of fisheries while species are in a prey-predator relationship.

The literature on fisheries management is essentially driven by comparisons between desirable catch levels and actual harvest rates. One strand of studies contrasts open-access regimes with sole ownership (e.g., Clark and Munro [9], Plourde and Yeung [46], Quirk and Smith [49], and Smith [50]), while another contrasts socially optimal outcomes with the Nash equilibrium (e.g., Munro [42], Levhari and Mirman [35], and Plourde and Yeung [46]). More recently, the literature has been extended to address other issues, particularly those associated with multispecies and spatial considerations. Such modeling refinements introduce new externalities that give rise to further economic interactions that still lead to a misalignment between socially optimal outcomes and private solutions. This then raises the question of how to regulate harvest to achieve optimal stock levels.

Various instruments have therefore been implemented globally; however, failures appear to increase. Thompson identifies 13 stocks at very low levels despite a reduction in the fishing mortality rate. More recently, in 2009, Worm et *al.* [59] showed that 63% of 166 fish stocks mostly coming from North America, Europe, South Africa, Australia, and New Zealand remained below the management-target levels. In a broader study, Hilborn et *al.* [26] find that 24% of 882 fish stocks worldwide have low biomass but high fishing pressure. This evidence has raised questions about the form of the regulation. Studies have specifically addressed the issue of which instrument is the most efficient (e.g., Hannesson and Kennedy [22] and Weitzman [58]). Rights-based management systems have been largely advocated because these instruments provide incentives to achieve socially optimal outcomes. However, such management tools are also designed to limit fishing pressure to overcome the tragedy of the commons. We may indeed wonder if the design of inappropriate management targets may be among the reasons for management failures, which may raise further issues.

Broadly, property rights attempt to assign exclusive rights to specific areas or fish species. Territorial-use rights for fisheries typically grant rights to individuals to exploit a resource within a given area. Consequently, the resource is fragmented along with the spatial division. Similarly, species-based property rights allocate catch shares for a

given species to several fisheries. Examples of such a system are individual fishing quotas on blue fin tuna in Australia or on surf clam in the United States, as well as the well-known Icelandic system for cod species. Once again, the resource could be considered as fragmented in the sense that management targets concentrate on single-species statuses, with no consideration of the biological interactions and role played by fish species in the ecosystem. Consequently, the implementation of these management systems make anticommon resources appear to incentivize fisheries to underuse the resource (Heller [24] and Buchanan and Yoon [6]), especially when there is one species that is relatively less valuable. The purpose of this study is thus to examine when the tragedy of the anticommons arises and how to resolve it in a prey-predator system.

In this study, we consider two groups of a limited number of fisheries each of which has exclusive rights to harvest a single, specific species. Each fishery thus competes with others within a given fishing industry, but not with those from the other fishing industry and ignores the biological interdependencies between the two species. As usual in the literature, we also consider heterogeneous market prices. Very few studies on multispecies management account for the economic trade-off involved in the exploitation of a prey-predator system (e.g., Mendelssohn [40], Hannesson [20], and Flaaten [14]). Interestingly, Hannesson [20] shows that the joint exploitation of both species is optimal over a specific range of relative prices; otherwise, only the predators should be harvested. Relative prices are interestingly defined based on biomass transfer between trophic levels as prey and predators may actually be considered as a unique resource that is converted into different organisms. Based hereon, we also consider relative prices to account for the different values in equivalent biomass. We specifically assume that the relatively less valuable resource is the predator species as this raises a complex economic question. The conversion of high-priced prey into predators induces a social cost ignored by the predator fisheries. Similarly, prey fisheries ignore the social costs of leaving too many prey to be converted into predators.

We use a standard dynamic approach to analyze the tragedy of the commons and that of the anticommons in the long run. Under the species-specific common-property rights, we first show that the tragedy of the commons arises within both fishing industries while the tragedy of the anticommons results from the interaction between the fishing industries. Second, we compare the long-term effort and stock levels resulting from optimal management with outcomes under species-specific common-property rights. We show that the global effort of the predator-fishing industry is too low and the predator population remains too high because the predator fishers ignore the social benefit of increasing predator harvesting. More surprisingly, we observe a too-low prey stock while the global effort level may also be too low. This result is due to the intense natural predation because the prey fisheries ignore the social costs implied by leaving prey to be converted into predators. Finally, we introduce a system of subsidies to simultaneously overcome all the issues. Nevertheless, we show that regulating only the usual tragedy of the commons may lead to a decrease in the two fisheries' aggregate benefit, and that it generates additional public spending to correct the tragedy of the anticommons.

The rest of the paper is organized as follows: In the next section, we provide some background to models of multispecies management, with a specific focus on the literature that accounts for relative prices of prey-predator populations. Section 3 introduces the model. Section 4 derives harvesting strategies under species-specific property-rights and socially optimal regimes. A comparison of the outcomes under the two regimes is presented in Section 5. We first compute some comparative statics to analyze the effect of fishery size; we then draw a comparison with respect to stock sizes and effort levels. Section 6 discusses the regulation of the tragedies of the commons and anticommons. Section 7 concludes. An appendix contains all the proofs.

2. Background to multispecies management

Fisheries management has been influenced by several stages of evolution in the methodological approaches since the static modeling to contrast open access with the maximum sustainable yield (Gordon [19], Scott [57]). Dynamic modeling was then introduced to contrast the open-access regime with sole ownership (e.g., Clark and Munro [9], Plourde and Yeung [46], Quirk and Smith [49], and Smith [50]). Clark and Munro [9] especially revisit the "golden rule" to characterize the level of natural capital at a steady-state equilibrium. Finally, other studies contrast the socially optimal outcome with the Nash equilibrium. For instance, Munro [42], Levhari and Mirman [35], and Plourde and Yeung [46] compare the non-cooperative solution with the optimal exploitation of competition between a limited number of economic agents who share fish stocks to examine an additional effect resulting from strategic interaction. Hannesson [21] specifically investigates the impact of the number of agents required to achieve cooperation and shows that cooperation is less likely to emerge as the number of agents rises. Broadly, all these studies identify the commons issue and show how unregulated fisheries lead to economic inefficiencies. However, most of the studies restrict their analyses to single-species fisheries.

Recent studies address further issues, including multispecies management (Hannesson [20], Ströbele W.J. and Wacker [54], Doyen et *al.* [11], or recently, Quérou and Tomini [47]-[48]. It is now widely recognized that several fish stocks are embedded in an ecosystem and that all the species interact ecologically in the ecosystem. More precisely, an ecosystem's organisms are distributed according to their feeding positions along a continuum of trophic levels. A series of interconnected trophic levels then forms a food web in which species' interactions involve a transfer of the same biomass from lower to higher levels through consumption. Broadly, we can thus consider that all the species within an ecosystem may be considered as different representations of the same resource. Furthermore, the literature on marine ecosystems outlines that specific organisms play a dominant role in ecosystems' dynamics; however, top-down forces such as fishing activities or changes in environmental conditions may also alter the population sizes of the organisms in lower trophic levels (Lindegren et al. [36]). In a recent economic study, Lai et *al.* [34] develop a multispecies model with three trophic levels to numerically analyze the impact of the abundance of a predator (grey seal) and that of prey (young herring) on the salmon stock and fisheries under different management scenarios in the Baltic Sea. The authors

interestingly discuss the economic losses for fisheries due to a higher level of predator abundance or lower level of prey abundance, emphasizing the trade-off between species and fisheries. Similarly, Blanquist et *al.* [5] consider the empirical implications of the benefits and costs related to grey seals for the harvest and stock size of cod in Nordic Baltic Sea countries. Hoekstra and van den Bergh [31] analyze another type of trade-off considering the economic benefit from prey harvesting against conservation benefits from non-harvested predators and investigate the possible extinction of the non-harvested species. Kellner et *al.*[33] show that temporary or permanent fishing moratoriums can be a solution when accounting for nonfishing values in a multispecies bioeconomic model for the Caribbean area. Quérou and Tomini [47] also show that a ban of prey harvesting is socially acceptable when fishermen harvest both species. Consequently, this evidence consolidates the idea of considering a single species as only one part of a more global resource: the ecosystem. Such a consideration raises several more complex questions on how to allocate effort across species, how to regulate stocks and the ecosystem, but also on stock availability and the consequences of fishing activities. Furthermore, the identification of the desirable stock level within a multispecies model remains an open question as biological interdependencies involve spillovers into all the fisheries even when they specifically target only one species.

Surprising results from a few previous studies on multispecies management illustrate how the characterization of optimal stock levels may be a complex task. Indeed, there may be situations in which the population of one species may be optimally lower than its level under open access. For instance, May et *al.* [39] discuss a specific case in which prey will be almost exclusively harvested when the discount rate is high, even if predators are the most valuable species. Actually, predators are initially heavily exploited; however, the only alternative is then to harvest prey instead of waiting for predator stock to recover. Mendelssohn [40] examines the qualitative properties of optimal policies for stochastic multispecies models and shows that less valuable species should be reduced to very low levels, while the stock of more valuable species should be close to the single-species optimum. Hannesson [20] specifically looks for economic conditions under which both species can be simultaneously harvested. The joint exploitation of both species is optimal over a specific range of relative prices; otherwise, only predators should be harvested. Price limits are characterized based on biomass transfer between trophic levels. Specifically, upper limits are reached when the value of prey rises when they are converted into predators. He provides some intuition using numerical illustrations that subsidies may be required to align the social optimum with free-access. Flaaten [14] provides similar intuitive observations: One species might be exploited at an economic loss and such exploitation might be subsidized because the optimal long-run stock was lower than the stock level under open access based on specific parameter values. Fischer and Mirman [16] also point out a too-low exploitation of predators but a too-high exploitation of prey when economic agents behave non-cooperatively. Interestingly, such studies emphasize the influence of economic variables on the exploitation of resource stocks. However, these studies address neither the reasons for the under-exploitation of one species nor the related issues or policy implications. In contrast, we help to fill this gap by considering a conflicting situation

in which predators' population growth only depends on the prey population, which is also the most valuable species, and both species are under a regime of species-specific common-property rights.

3. The model

We consider a standard Lotka-Volterra model in which the prey population, $x(t)$, follows logistic growth in the absence of predators and harvest, i.e., this population grows at a rate, $r > 0$, but is limited by the carrying capacity, $K > 0$. Prey are harvested in quantity $H_x(t)$ and suffer from predation. A proportion, $a > 0$, of the prey biomass is killed per unit of predator. If $y(t)$ denotes the predator population, an amount, $ax(t)y(t)$, of prey increases the predator population at a biomass conversion rate, $\alpha > 0$. This last population declines given the intrinsic mortality rate, $\delta > 0$, and harvesting, $H_y(t)$. The dynamics of the prey population, $x(t)$, and those of predators, $y(t)$, are thus characterized as follows:

$$\dot{x}(t) = x(t)\left(r\left(1 - \tfrac{x(t)}{K}\right) - ay(t)\right) - H_x(t), \tag{1}$$

$$\dot{y}(t) = y(t)\left(a\alpha x(t) - \delta\right) - H_y(t). \tag{2}$$

We can easily compute a first reference point, $\left(x^N, y^N\right)$, to assess the stock status when the two species coexist but are not harvested. The pristine steady state is given as follows, considering $\frac{\delta}{a\alpha} < K$:[1]

$$x^N = \tfrac{\delta}{a\alpha} \quad \text{and} \quad y^N = \tfrac{r}{a}\left(1 - \tfrac{\delta}{a\alpha K}\right). \tag{3}$$

When harvesting is introduced, we distinguish between two independent fishing industries, $s = x, y$, each comprising a fixed number of fisheries. More precisely, we consider $n_x > 1$ fisheries in the prey-fishing industry and $n_y > 1$ fisheries in the predator-fishing industry. All fisheries are fully specialized, i.e., they target a single-species population.[2] Consequently, the global harvest level, H_x and H_y, is the sum of individual catches within each industry. Assuming a standard Schaefer harvest function, an individual catch linearly depends on the effort level of the j^{th} fishery in industry s, $e_{s,j}$, and stock abundance at each time period:[3]

$$h_{s,j}(t) = e_{s,j}(t)s(t). \tag{4}$$

[1] Two other steady-state equilibria may exist: (i) when the two species are driven to extinction, and (ii) when only the prey population survives and consequently reaches the maximum level, K. However the steady-state equilibrium when the two species coexist is the only stable one.

[2] Following the literature on single-species management (e.g., Kasperski [32]), we consider that fishermen use a perfectly selective harvest technology such that they cannot participate in other fisheries. This assumption is credible when we consider a prey-predator model. For instance, small pelagic gears allow for a better targeting of small coastal pelagic species such as herrings or sardines, or other prey of larger and oceanic pelagic fishes.

[3] The fishing-production function usually depends on a constant catchability parameter, $\theta > 0$, to capture the efficiency of a fishery. We voluntarily omit the parameter for simplicity as it does not affect

Thus, the aggregated effort levels are such that $E_s(t) = \sum_{j=1}^{n_s} e_{s,j}(t)$ and the global harvest for each industry is given by $H_s(t) = \sum_{j=1}^{n_s} e_{s,j}(t)s(t)$ for $s = x, y$.

Considering competitive markets, we have constant and species-specific market prices, p_s, and c_s denotes costs per unit of effort. The profit at time t of the j^{th} fishery in industry s is therefore given by the following:

$$\Pi_{s,j}\left(e_{s,j}(t), s(t)\right) = \underbrace{\left(p_s s(t) - c_s\right)}_{\equiv \pi_s(s(t))} e_{s,j}(t), \tag{5}$$

where $\pi_s\left(s\left(t\right)\right)$ denotes the species-specific profit per unit of effort.

Each fishery will be incentivized to harvest only when profits are non-negative. Specifically, the minimum prey- and predator-stock levels $\left(x^{\min}, y^{\min}\right)$ for which profits are non-negative are respectively as follows:

$$x^{\min} = \frac{c_x}{p_x} \quad \text{and} \quad y^{\min} = \frac{c_y}{p_y}. \tag{6}$$

Furthermore, the minimum stock levels must be lower than the pristine stock levels (3) to ensure the entry of fisheries into the two industries, $x^{\min} < x^N$ and $y^{\min} < y^N$. All fisheries will then maximize the present values of their own current profits by choosing their effort paths, $e_{s,j}(t)$, with respect to the dynamics of the fish population. We moreover assume a constant discount rate, ρ, which is larger than the maximum growth rates for prey and predators, r $(\alpha a K - \delta)$, respectively.

We recall that a prey-predator relationship involves a transfer of biomass from lower to higher trophic levels through the consumption of prey. Consequently, we can contrast the economic value of one unit of biomass at the lower level, p_x, with what we may have harvesting the converted quantity of biomass at the upper level, αp_y. We specifically assume that the value of a species is higher than that of the biomass converted into predators:

$$p_x > \alpha p_y \tag{7}$$

This assumption is quite intuitive. From the biological literature (Lindeman [37]), the ten percent law indicates that most of the energy available at one level in an ecosystem is lost when transferred to an upper level, and only 10% is converted into organisms. This means that this assumption is true as long as the unit price for the predator does not exceed ten times that of the prey.

4. Harvesting strategies

We now analyze harvesting strategies under two property-right regimes: A common-property right with exclusive rights on specific species and a social planner optimizing

the following analysis.

the harvesting of the two species. Under the common-property regime with exclusive rights, all fisheries within a specific industry compete for a single stock and ignore species' interactions. We refer to such a situation as a *species-specific common-property regime.*

4.1. Species-specific common-property regime

In this regime, a fishery, j, chooses its effort levels over time, $e_{s,j}(t)$, to maximize the present value of its stream of profit by accounting for the stock dynamics of the targeted species, Eqs. (1) and (2). Fisheries take as granted the behavior of other fisheries within the same industry and the stock of the other resource.

Formally, the problem of the jth prey fishery is expressed as follows:

$$\begin{aligned} &\max_{e_{x,j}(t)} \int_0^{\infty} \pi_x\left(x\left(t\right)\right) e_{x,j}(t) \exp^{-\rho t} dt \\ &\dot{x}(t) = x(t)\left[r\left(1-\tfrac{x(t)}{K}\right) - ay(t) - e_{x,j}(t) - \textstyle\sum_{k=1,k\neq j}^{n_x} e_{x,k}(t)\right]. \end{aligned} \tag{8}$$

The singular control solution solves the following condition:

$$\pi_x(x) - \lambda_{x,j} x = 0. \tag{9}$$

As usual, we find that the profit per unit of effort and per unit of stock must be equal to the shadow value of the resource, λ_x^j. At the steady state, the shadow value satisfies the following condition:

$$\rho\lambda_{x,j} - \pi_x^{'}(x) e_{x,j} + \lambda_{x,j}\tfrac{rx}{K} = 0. \tag{10}$$

Similarly, for the jth predator fishery, we obtain the following maximization problem:

$$\begin{aligned} &\max_{e_{y,j}(t)} \int_0^{\infty} \pi_y\left(y(t)\right) e_{y,j}(t) \exp^{-\rho t} dt \\ &\dot{y}(t) = y(t)\left(a\alpha x(t) - \delta - e_{y,j}(t) - \textstyle\sum_{k=1,k\neq j}^{n_y} e_{y,k}(t)\right). \end{aligned} \tag{11}$$

The singular control solution solves the following condition:

$$\pi_y(y) - \lambda_{y,j} y = 0. \tag{12}$$

Eq. (12) states that the economic agent, j, will choose an effort level such that the profit per unit of effort is equal to the total shadow value of the resource, $\lambda_{y,j} y$. In the long run, the shadow value of the predators' stock solves for the following:

$$\rho\lambda_{y,j} - \pi_y^{'}(y) e_{y,j} = 0. \tag{13}$$

Henceforth, a steady-state solution under a specifies-specific property regime is defined by the set of conditions (9), (10), (12), and (13), with $\dot{x}(t) = \dot{y}(t) = 0$ respectively in Eqs.(9) and (12). The shadow values must be respectively equal to species-specific profits

per unit of effort and per unit of biomass:

$$\forall j,\, \lambda_{x,j} = \lambda_x = \frac{\pi_x(x)}{x}, \quad \text{and} \quad \forall j,\, \lambda_{y,j} = \lambda_y = \frac{\pi_y(y)}{y}. \tag{14}$$

Using the stationary conditions in (10) and (13), we can then characterize the individual fishing efforts by the following:

$$\forall j,\, e_{x,j} = e_x = \left(\rho + \tfrac{rx}{K}\right) \tfrac{\pi_x(x)}{x\pi_x'(x)} \quad \text{and} \quad \forall j,\, e_{y,j} = e_y = \tfrac{\rho\pi_y(y)}{y\pi_y'(y)}. \tag{15}$$

Plugging (14) and (15) into the two stock dynamics, we obtain a system of equations that provides the two long-run stocks:

$$\rho\tfrac{\pi_x(x)}{x} = \tfrac{\pi_x'(x)}{n_x}\left(r\left(1 - \tfrac{x}{K}\right) - ay\right) - \tfrac{r}{K}\pi_x(x), \tag{16}$$

$$\rho\tfrac{\pi_y(y)}{y} = \tfrac{\pi_y'(y)}{n_y}(\alpha a x - \delta). \tag{17}$$

Basically, the left-hand sides of Eqs. (16) and (17) are the returns on the investment of the rent induced by the last unit of prey and predator harvests, respectively, while the right-hand sides outline the private rates of return of leaving a unit of species s in the sea. This value can be seen as a private conservation value for species s. Specifically, the conservation value for the prey species is given by the stock effect on individual profit net of the loss induced by the decrease in the prey growth rate. The conservation value for the predator species depends only on the stock effect on individual profit.

Let us now further discuss the system of Eqs. (16) and (17) and observe that the two solutions can be expressed as two implicit functions of the number of fisheries in the two industries, respectively $x(n_x, n_y)$ and $y(n_x, n_y)$. We now want to restrict our analysis to a situation in which the two species coexist while they are jointly harvested in the long run. This specifically requires an upper bound on the number of prey fisheries. A high number of prey fisheries actually depletes too much of the stock of prey. Due to biological interaction, this restricts food for predators. As such, the predator population may reach the minimum level under which harvesting is not profitable, y^{min}. From Eq. (17), such a situation with non-active predator fisheries, $E_y = 0$, leads to a long-run stock level of prey equal to the natural level, x^N. To avoid obtaining a steady-state solution (x^N, y^{min}), we assume that the marginal return on the last unit of prey harvest is lower than the return from leaving it in the sea. This incentive to spare the resource increases the prey population. Formally, the assumption is as follows:

$$\rho\tfrac{\pi_x(x^N)}{x} < \tfrac{\pi_x'(x^N)}{n_x}\left(r\left(1 - \tfrac{x^N}{K}\right) - ay^{min}\right) - \tfrac{r}{K}\pi_x(x^N), \tag{18}$$

which provides the following upper bound on n_x:

$$n_x < \frac{\pi_x'(x^N)\left(r\left(1-\frac{x^N}{K}\right)-ay^{\min}\right)}{\left(\rho+\frac{rx^N}{K}\right)\frac{\pi_x(x^N)}{x^N}} \equiv \bar{n}. \tag{19}$$

In the remainder of the paper, we thus restrict the number of prey fishers below this threshold $\bar{n}$. The following proposition summarizes the main properties of the stationary solution under a specifies-specific common-property regime.

Proposition 1. *If $1 < n_x < \bar{n}$, we can say the following:*
(i) There exists a unique steady-state solution for the system of Eqs.(16) and (17) in which both species coexist, $x(n_x, n_y) > 0$ and , $y(n_x, n_y) > 0$.
(ii) The long-run harvest strategies are characterized by the two effort levels, $e_x(n_x, n_y) > 0$ and $e_y(n_x, n_y) > 0$, given by Eq.(15).
(iii) Profits per unit of effort are positive in each industry, $\pi_x(n_x, n_y), \pi_y(n_x, n_y) > 0$.

4.2. Sole ownership

Thus far, we have considered that the property rights to a species-specific stock are exclusive to a community of fisheries. We now assume a sole owner who aims at managing the collective use of the stocks of the two species. Basically, they will choose the effort levels, $e_{s,j}(t)$, for all fisheries j in both industries $s = x, y$ to maximize the discount value of the sum of the profit streams, accounting for the two population dynamics given by Eqs. (1) and (2). For consistency, we maintain the assumption that $n_x < \bar{n}$. As fisheries remain symmetric within the industry, the sole owner's problem is defined as follows:

$$\begin{aligned}
&\max_{e_x(t),e_y(t)\geq 0} \int_0^\infty \left(n_x \pi_x\left(x(t)\right) e_x(t) + n_y \pi_y\left(y(t)\right) e_y(t)\right) \exp^{-\rho t} dt, \\
&\dot{x}(t) = x(t)\left(r\left(1-\tfrac{x(t)}{K}\right) - ay(t) - n_x e_x(t)\right), \\
&\dot{y}(t) = y(t)\left(a\alpha x(t) - \delta - n_y e_y(t)\right).
\end{aligned} \tag{20}$$

As previously, considering $\dot{x}(t) = \dot{y}(t) = 0$, we only highlight the steady-state conditions:

$$\pi_x(x) - \lambda_x x = 0, \tag{21}$$
$$\pi_y(y) - \lambda_y y = 0, \tag{22}$$
$$\rho\lambda_x - n_x \pi_x^{'}(x) e_x + \lambda_x \tfrac{rx}{K} - \lambda_y a\alpha y = 0, \tag{23}$$
$$\rho\lambda_y - n_y \pi_y^{'}(y) e_y + a\lambda_x x = 0. \tag{24}$$

The first two optimality conditions, (21) and (22), define the shadow values as in Eqs.(9) and (12) but those values are now evaluated at different points. Eqs. (23) and (24), in contrast to Eqs.(10) and (13), now account for the number of fisheries as well as the

biological interaction. Specifically, in Eqs. (23) and (24), we account for the effect of stock variation on the industry's aggregate profit and the marginal value of the other species.

Hereafter, we add superscript s to denote the long-term optimal solutions. From the stock dynamics, given $\dot{x}(t) = \dot{y}(t) = 0$, we immediately obtain the steady-state values of effort levels as functions of the steady-state stock levels $\{x^s, y^s\}$:

$$e_x^s = \frac{1}{n_x}\left(r\left(1 - \frac{x^s}{K}\right) - ay^s\right) \quad \text{and} \quad e_y^s = \frac{1}{n_y}\left(a\alpha x^s - \delta\right). \tag{25}$$

We also directly observe from Eqs. (21) and (22) that the shadow prices, λ_x^s and λ_y^s, are equal to the long-run rent per unit of stock:

$$\lambda_x^s = \frac{\pi(x^s)}{x_s^s} \quad \text{and} \quad \lambda_y^s = \frac{\pi(y^s)}{y^s}. \tag{26}$$

Finally, using Eqs.(25) and (26), we obtain the conditions required for the long-run levels of the two population stocks:

$$\rho\frac{\pi_x(x^s)}{x^s} = \pi_x^{'}(x^s)\left(r\left(1 - \frac{x^s}{K}\right) - ay^s\right) - \frac{r}{K}\pi_x(x^s) + \alpha a\pi_y(y^s), \tag{27}$$

$$\rho\frac{\pi_y(y^s)}{y^s} = \pi_y'(y^s)(\alpha a x^s - \delta) - a\pi_x(x^s). \tag{28}$$

The left-hand side of each equation is again the return on the investment of the rent induced by the last unit of harvest; however, the right-hand side is now the social rate of return on leaving a unit of a species in the sea. Considering all the terms on the right-hand side, we refer to them as the social conservation value for species s. Specifically, in Eq.(27), the first two terms reflect the stock effect of prey on the aggregated profits net of the loss induced by a decrease in the prey growth rate. The last term measures the positive trophic externality of the population of prey on that of predators. Indeed, leaving one unit of prey in the sea increases the predator population by a rate of αa. Thus, without changing the predator-fishing effort, the profitability of predator harvesting increases by $\alpha a\pi_y(y^s)$. In Eq.(28), the first term again depicts the externality of the stock of predators while the second term captures the negative trophic externality. An additional predator decreases the population of prey by a rate of a, hence, at a given effort, the profitability of harvesting prey.

In contrast to our previous species-specific common-property regime, profits do not need to be positive in both industries because the compensation principle applies. In the prey industry, we observe that the steady-state stock of prey should be greater than the pristine level, $x^s > x^N$, due to the predator dynamics (see Eq. 20). Because we have assumed that $x^{\min}$ is lower than x^N, we can immediately conclude that the prey fisheries' profit, $\pi_x(x^s)$, is positive. However, this is not the case for predator fisheries. Indeed, observing the right-hand side of Eq.(28), we find that the social conservation value, $\pi_y'(y)(\alpha a x - \delta) - a\pi_x(x)$, is negative. This value decreases with x due to the price structure, $p_x > \alpha p_y$, and reduces to $-a\pi_x(x^N) < 0$ when $x = x^N$. As $x^s \geq x^N$ when predators subsist at the steady state, this suggests that the predator fisheries' profit, $\pi_y(y^s)$, is negative. The next proposition describes the sole owner's equilibrium.

Proposition 2. *If $1 < n_x < \bar{n}$, we can say the following:*
(i) There exists a unique steady-state solution given by the system of Eqs.(27) and (28) in which both species coexist, $x^s, y^s > 0$.
(ii) Both species are harvested and the fishing efforts, $e_x^s, e_y^s > 0$, are given by Eq.(25).
(iii) The profit per unit of effort is positive for prey, $\pi_x(x^s) > 0$, and negative for predators, $\pi_y(y^s) < 0$.

Proposition 2 shows that the sole owner's harvest strategies are to exploit both species even if harvesting predators is costly. This result is typically driven by biological externality, given that the species at the lower trophic level is more valuable at that level than when converted into the upper level. By contrast, we outline that no fishing effort will be engaged when profits are negative under a species-specific property regime, and this case specifically arises for a high number of prey-specific fisheries.

5. Commons versus anticommons

Thus far, we have characterized the species-specific common-property regime and sole owner's outcome. We can now analyze how the tragedies of the commons and anticommons arise in this model. The first tragedy is related to the competition between fisheries within industries because of the common property, while the second tragedy is a consequence of the design of property rights induced by an exclusive specialization that neglects the biological interaction. We essentially proceed in two steps. We first analyze the effects of a change in the number of fisheries in each industry when the resource is managed under our species-specific common-property regime. Second, we contrast private property-rights management with socially optimal management.

The analysis of the effects of a change in the number of fisheries in each industry draws mainly on the implicit function theorem in the system represented by (16) and (17). These results are proven in Appendix C. Tables 1 and 2) summarize the results for the prey- and predator-fishing industries, respectively..

	$x(n_x, n_y)$	$E_x(n_x, n_y)$	$e_x(n_x, n_y)$	$\pi_x(n_x, n_y)$
Nb of prey fisheries (n_x)	$-$	$+$	$-$	$-$
Nb of predator fisheries (n_y)	$+$	$+$	$+$	$+$

Table 1: Number of fisheries and prey harvesting

	$y(n_x, n_y)$	$E_y(n_x, n_y)$	$e_y(n_x, n_y)$	$\pi_y(n_x, n_y)$
Nb of predator fisheries (n_x)	$-$	$-$	$-$	$-$
Nb of prey fisheries (n_y)	$-$	$+$	$-$	$-$

Table 2: Number of fisheries and predator harvesting

Tables 1 and 2 show that both stocks are affected by the sizes of the fisheries. This indicates direct and indirect effects of a change in the size of a given industry on the steady-state stocks. The direct effect depicts the tragedy of the commons while the indirect effect results from the anticommons issue induced by specialization.

For instance, if we consider the prey-fishing industry, a larger number of prey fisheries decreases the long-term fish stock due to a higher aggregated harvesting-effort level while the individual effort and profit per unit of effort decrease. These are the key components of the tragedy of the commons. Typically, a rise in the number of prey fisheries diminishes the private conservation value for prey (see Eq.(16)). This mechanically leads to a new steady state with less prey and a higher aggregated catch effort. This also induces a negative externality on all the fisheries in the industry as they individually end up with a lower catch effort and smaller profits. A very similar observation can be made about the predator-fishing industry (see Table 2).

The observed effects due to the tragedy of the commons within an industry spill over into the other industry due to biological interactions. Fewer prey due to an additional prey fishery mechanically reduces the predator population and results in a negative externality on that industry. This reduces not only the aggregated and individual fishing efforts but also each predator fishery's profit (see the second line of Table 2). In contrast, fewer predators save prey and induce a positive externality on the prey-fishing industry. As depicted in the second line of Table 1, the higher the number of predator fisheries, the smaller the prey population, and the smaller the individual and aggregated prey-harvesting efforts and profit of each prey fishery. It should even be noticed that these results hold independently of the size of each industry. This means that these externalities remain even if a regulator solves the tragedy of the commons in each of the industries. This is typically because of the anticommons effect. Due to the specialization and subsequent exclusivity, the property-rights system still shares a single ecological system.

For a better understanding of the commons and anticommons effects, let us now contrast the outcomes of the two property regimes. To isolate each of them, let us first address the case in which each industry is controlled by a single specialized fishery, i.e., $n_x = n_y = 1$. In this case, the sole owner only accounts for the externalities induced by the biological interaction. In a second step, we extend the comparison to all fisheries in both industries.

In predator-prey models, it is well known that fewer predators improve the prey population, the high-priced species in our model. This induces a social benefit due to this release of predation. To benefit from this advantage, the sole owner has therefore an incentive to raise the harvest effort above the level chosen by predator fisheries. This suggests predator underfishing under our species-specific property regime, i.e., $E_y(1,1) < E_y^s$. Consequently, the predator population is higher in this case than in the sole owner's situation, i.e., $y(1,1) > y^S$. Moreover, the sole owner's decision leads the predator-fishing industry to operate at a negative profit, according to Proposition 2. Hence, $y(1,1) > y^{\min} > y^S$ because profits per effort are increasing in stock.

The comparison of outcomes for the prey industry is not as straightforward. Let us

recall the private and social conservation values for prey respectively introduced in Eqs. (16) and (27). Under the assumption on prices, i.e., $p_x > \alpha p_y$, both are decreasing with the size of the predator population and are, in the case of a single prey fishery, identical when $y = y^{\min}$. Because $y(1,1) > y^{\min} > y^S$, this means that the private conservation value is always lower than the social conservation value independently of the size of the prey population. This clearly suggests a lower prey stock under the species-specific property regime than in the sole owner's case, i.e., $x(1,1) < x^S$. However, in contrast to the intuition, we observe prey underfishing, i.e., $E_x(1,1) < E_x^s$. Prey fisheries ignore the social benefit they generate by increasing the harvest of high-priced prey. To derive this social benefit, the sole owner must raise the harvesting effort above the level chosen by the prey fishery.

Finally, it remains to extend these results to any possible size of the two industries, integrating the usual effect from the tragedy of the commons. This is fairly straightforward for the ranking of predator stocks. In this industry, no fishery is willing to operate at a negative profit independently of both industry sizes. It follows that $y(n_x, n_y) > y^{\min} > y^S$. From Table 2, we nevertheless know that $\frac{\partial y}{\partial n_x} < 0$ and $\frac{\partial y}{\partial n_y} < 0$. Even if the stock gap decreases when the size of at least one industry increases, it never disappears. Thus, the tragedy of the commons in both industries never overcomes the tragedy of the anticommons in the predator-fishing industry. The results are less evident for the prey stocks. From the single owner's case, we already know that $x(1,1) < x^S$. Thus, an additional prey fishery simply increases this gap by the usual effect of the tragedy-of-the-commons mechanism, i.e., $x(n_x, 1) < x(1,1) < x^S$. However, from Table 1, we know that $\frac{\partial x}{\partial n_y} \geq 0$ because the tragedy of the commons in the predator industry has a positive externality on the prey population. We nevertheless show that this opposite effect does not compensate for the anticommons effect, i.e., $x(n_x, n_y) < x^S$.

Let us now compare the effort levels. Whatever the management regime, the stationary predator's effort deduced from the predator dynamics is always given by $E_y = \alpha a x - \delta$. Because $x(n_x, n_y) < x^S$, it follows that $E_y(n_x, n_y) < E_y^s$. The result is less obvious for the prey effort. Even if $E_x(1,1) < E_x^S$, Table 1 shows that $\frac{\partial E_x}{\partial n_x} > 0$ and $\frac{\partial E_x}{\partial n_y} > 0$. Thus, if the upper bound of $E_x(n_x, n_y)$ is larger than E_x^s, the tragedy of the commons offsets the tragedy of the anticommons. Now, observe that this upper bound is reached for $n_x = \bar{n}$ (see Eq. (19)) and $n_y = +\infty$. As $n_x = \bar{n}$ is the size of the prey industry for which the predator fishing effort becomes zero, $x(\bar{n}, +\infty)$ must be equal to the pristine level, x^N. Meanwhile, $y(\bar{n}, +\infty) = y^{\min}$ is the quantity for which the profit in the predator industry becomes zero due to open access. Thus,

$$E_x(\bar{n}, +\infty) = r\left(1 - \tfrac{x^N}{K}\right) - a y^{\min} \tag{29}$$

The next proposition summarizes this discussion.

Proposition 3. *By comparing the species-specific common-property regime to the sole-owner solution, we observe the following:*

(i) There is a global underfishing effort in the predator industry, i.e., $E_y(n_x, n_y) < E_y^s$, which contributes to the emergence of an excessive predator population, i.e., $y(n_x, n_y) > y^s$.
(ii) This excessive predation leads to a lower prey population, i.e., $x(n_x, n_y) < x^s$, which is enforced by a global under-fishing effort, i.e., $E_x(n_x, n_y) < E_x^s$, if and only if $E_x(\bar{n}, +\infty) < E_x^s$.

6. Multispecies management and fishery policies

Thus far, we have outlined two sources of issues: the tragedy of the commons and that of the anticommons. Now, we move to the characterization of ecosystem-based fishery management (EBFM). We first derive the optimal regulation scheme to align the species-specific common-property regime with the social optimum. In a second step, we want to highlight the implications of regulating the tragedy of the anticommons when fishery cooperatives already exist.

6.1. A species-specific global regulation system

We consider that the government aims at implementing two species-specific policy tools, one tool for each fishing industry. From Proposition 3, we know that the government will be faced with underfishing and thus may decide to grant a subsidy, σ_s with $s = x; y$, per unit of harvest.

Under that landing regulation, each fishery's profit (5) becomes:

$$\Pi_{s,j}(e_{s,j}(t), s(t)) = ((p_s + \sigma_s)s(t) - c_s)\, e_{s,j}(t). \tag{30}$$

The long-run equilibrium conditions in (16) and (17) required for the level of the two population stocks at the steady states consequently become:

$$\rho\tfrac{\pi_x(x)}{x} = \tfrac{\pi_x'(x)}{n_x}\left(r\left(1-\tfrac{x}{K}\right) - ay\right) - \tfrac{r}{K}\pi_x(x) - \sigma_x\left(\rho + \tfrac{r}{K} - \tfrac{1}{n_x}(r(1-\tfrac{x}{K}) - ay)\right), \tag{31}$$

$$\rho\tfrac{\pi_y(y)}{y} = \tfrac{\pi_y'(y)}{n_y}(\alpha a x - \delta) - \sigma_y(\rho - \tfrac{1}{n_y}(\alpha a x - \delta)). \tag{32}$$

As the purpose is to implement the optimal stock levels, we observe that the right-hand sides of the conditions in (31) and (32) should be equal to the right-hand sides of conditions (27) and (28), respectively. We then deduce that the long-run optimal levels of the subsidy granted to each fishery in the two fishing industries are respectively as follows:

$$\sigma_x(n_x) = \frac{-\pi_x'(x^s)\frac{(n_x-1)}{n_x}E_x^s - \alpha a \pi_y(y^s)}{\rho + \frac{r}{K}x^s - \frac{E_x^s}{n_x}} \text{ with } E_x^s = r(1-\frac{x^s}{K}) - ay^s, \tag{33}$$

$$\sigma_y(n_y) = \frac{-\pi_y'(y^s)\frac{(n_y-1)}{n_y}E_y^s + a\pi_x(x^s)}{\rho - \frac{E_y^s}{n_y}} \text{ with } E_y^s = \alpha a x^s - \delta. \tag{34}$$

A perusal of the numerators of Eqs. (33) and (34) shows that the subsidy scheme must be designed to account for the tragedies of the commons and anticommons. A single instrument, even species-specific, thus allows us to capture all the externalities. Indeed, the first term in each of the two equations captures the social costs associated with the tragedy of the commons, i.e., the marginal impact of stock variation from the other fisheries in competition. Recall that from Proposition 2, we know that $\pi_y(y^s) < 0$ and that $\pi_x(x^s) > 0$. Then, the second term reflects the social benefit associated with the tragedy of the anticommons. The sign of the policy instrument therefore depends on the balance between the magnitudes of the two effects.[4]

Under a species-specific common-property regime, from Proposition 3, we know that predators are overpopulated and underfished despite the presence of the usual tragedy of the commons. Given a single predator fishery, the tragedy of the anticommons requires that the predator catch be subsidized. As the number of predator fisheries increases, the amount of the subsidy decreases, i.e., $\partial_{n_y}\sigma_y(n_y) < 0$, suggesting that a situation in which the tragedy of the commons occurs leads to a reduction in the fishing gap between a common-property regime and the socially optimal scenario. Nevertheless, from Condition (28), we can easily observe that the externalities resulting from the anticommons offset the tragedy of the commons in the predator-fishing industry, i.e., $a\pi_x(x^s) > \pi_y^{'}(y^s)\frac{(n_y-1)}{n_y}E_y^s$. This result is due to the fact that predator fisheries do not account for the opportunity benefit they will generate in the prey-fishing industry by increasing their catch. Consequently, the government will always grant a subsidy to the predator fisheries.

Similarly, for the prey-fishing industry, we know that the stock is underpopulated although there is underfishing. Without competition among fisheries, when $n_x = 1$, only the tragedy of the anticommons remains and the government must subsidize the prey harvest to internalize the opportunity cost of letting prey be converted into predators[5]. Moreover, as the number of prey fisheries increases, the amount of the subsidy monotonically decreases. The nature of the policy may even change if the term associated with the tragedy of the commons, $-\pi_x^{'}(x^s)\frac{(n_x-1)}{n_x}E_x^s$, exceeds the term for the tragedy of the anticommons, $\alpha a\pi_y(y^s)$. This happens over a given number of prey fisheries, n_x^*, defined as follows: [6]

$$n_x^* = \frac{\pi_x^{'}(x^s)E_x^s}{\pi_x^{'}(x^s)E_x^s + \alpha a\pi_y(y^s)}. \tag{35}$$

Proposition 4 summarizes the discussion.

Proposition 4. *A landing regulation entails implementing a system of species-specific*

[4]AS we have assumed that ρ is greater than the maximum growth rate of each of the two species, the two denominators, those of Eqs. (33) and (34), are positive.

[5]Subsidizing the prey sector does not necessarily mean decreasing the stock as interaction matters and the regulator collectively controls the exploitation of the whole ecosystem.

[6]Using Condition (27), we know that the denominator of the threshold in (35) is strictly positive.

subsidies for a harvest, defined by Eqs. (33) and (34). The regulator may, however, implement a tax on the prey harvest when $n_x > n_x^*$.

6.2. The cost of cooperative fishing rights

This section examines a situation in which there exist cooperative fishing rights. Such organization generally involves a group of fisheries that share common characteristics, including target species and fishing technology. In our framework, this corresponds to our species-specific common-property regime when a single fishery manages the industry, i.e., when $n_s = 1$ for $s = x, y$.

This type of agreement that neutralizes the tragedy of the commons neglects the tragedy of the anticommons and even induces some additional social costs. In a first step, we characterize the social benefit induced by stopping cooperation in the predator industry. Second, we assess the additional regulation costs to solve the tragedy of the anticommons in the presence of a cooperative.

Let us now define the social benefit as the discounted sum of profits at the steady state:

$$\Pi(n_x, n_y) = \frac{1}{\rho}\Big(\pi_x(n_x, n_y)E_x(n_x, n_y) + \pi_y(n_x, n_y)E_y(n_x, n_y)\Big) \tag{36}$$

We aim to contrast the joint profit in the presence of cooperative, i.e., $\Pi(1, 1)$, with that in a situation in which predator fisheries behave uncooperatively, i.e., $\Pi(1, n_y)$. This corresponds to examining the impact of an additional predator fishery on the social benefit. Recall that profits and efforts actually depend on the number, n_y, only because of stocks. Thus, the joint profit, Π, also depends on n_y because of stocks. By applying the usual chain rule, we obtain the effect of an increase in the number of predator fisheries on the joint profit:

$$\frac{1}{\rho}\left[\frac{\partial \Pi}{\partial x}\frac{\partial x}{\partial n_y} + \frac{\partial \Pi}{\partial y}\frac{\partial y}{\partial n_y}\right] > 0 \tag{37}$$

From Eqs. (16)-(17) and $\pi_y(y) > 0$, we have $\frac{\partial \Pi}{\partial x} = p_x E_x(x, y) - \frac{r}{K}\pi_x(x) + \pi_y(y)\alpha a > 0$; thus, from Table 1, $\frac{\partial \Pi}{\partial x}\frac{\partial x}{\partial n_y} > 0$. Concerning the second term, recall that the long-run prey stock is higher than the natural level, x^N, and the derivative, $\frac{\partial \Pi}{\partial y}$, is decreasing in x under Assumption 7. Moreover, because $\frac{\partial \Pi}{\partial y} = \pi_x(x^N)(-a) + p_y E_y(x^N) < 0$, then, using Table 1, we obtain $\frac{\partial \Pi}{\partial y}\frac{\partial y}{\partial n_y} > 0$. The following proposition summarizes this discussion.

Proposition 5. *The tragedy of the commons that occurs in the predator-fishing industry contributes to achieving a second-best equilibrium even in the presence of the tragedy of the anticommons, compared with a situation in which both fishing industries are organized as cooperatives.*

Therefore, there exists a social benefit of deregulating the predator industry[7]. Now, let us show that there is an additional cost of solving the tragedy of the commons in both industries without considering the tragedy of the anticommons. To do this, consider that cooperatives are organized within each industry, and let us now characterize the system of subsidies required to solve the tragedy of the anticommons. From Eqs. (33) and (34), these subsidies no longer depend on the industry sizes and are equal to $\sigma_s(n_s)$ when $n_s = 1$, with $s = x, y$:

$$\sigma_x(1) = \frac{-a\alpha\pi_y\left(y^s\right)}{\rho + \frac{r}{K}x^s - E_x^s} > 0, \qquad \sigma_y(1) = \frac{a\pi_x(y^s)}{\rho - E_x^s} > 0 \tag{38}$$

Moreover, we know from Eqs. (33) and (34) that the level of the subsidy decreases with n_x and n_y, i.e., $\partial_{n_x}\sigma_x(n_x) < 0$ and $\partial_{n_y}\sigma_y(n_y) < 0$. Then, we obtain the following:

$$\sigma_x(n_x) \leq \sigma_x(1), \qquad \sigma_y(n_y) \leq \sigma_y(1) \tag{39}$$

As the objective is to implement the socially optimal outcome, this means that the total amount of subsidies granted by the regulator to fisheries is higher when cooperatives are organized. We can therefore say the following:

Proposition 6. *The cost of public spending is greater when only the tragedy of the commons is internalized than when both the tragedy of the commons and that of the anticommons are globally regulated.*

7. Conclusion

This analysis specifically contributes to the literature on multispecies management and complements the literature on the economic tradeoff implied by the exploitation of species in interaction. In this study, we specifically analyze the economic and ecological implications of a species-specific common-property rights system in a prey-predator model when the prey species is more valuable when harvested than when converted into a predator. We find that the predator-fishing industry should operate at economic losses under sole ownership as the losses would be offset by the opportunity benefit for the prey-fishing industry. We also find that the tragedy of the commons traditionally arises within the two separate fishing industries; however, the exclusive rights system implies the tragedy of the anticommons as the two species are underexploited. A combination of the two issues leads to (i) a too-low prey stock but an abundant predator population under a species-specific common-property right system and (ii) too-low effort levels for the two fishing industries. We finally discuss how to regulate these issues and analyze the implications of regulating only the tragedy of the commons as it may be done in fishery cooperatives. We especially find that a system of subsidies allows the two issues to be

[7] We can also consider the deregulation of the prey-fishing industry; however, we do not obtain clear-cut results.

overcome, while the regulation of the tragedy of the anticommons entails additional costs when the tragedy of the commons has already been overcome.

This study provides interesting insights into how the tragedy of the anticommons arises in the context of fisheries. We nevertheless adopt a very simple representation of the ecosystem as we only consider two species and even one type of interactions. An immediate question arises as to whether we may observe the same issues considering other types of biological interactions. For instance, competition between species should entail reciprocal opportunity benefits for other fisheries when one of the competitive species is harvested. In contrast, mutualistic relationships generate reciprocal opportunity costs. More broadly, we may improve the analysis by considering more than two species in interaction. This should specifically help us to analyze how the spillovers of harvesting one species spread over the entire ecosystem, especially with species that are not directly linked with the targeted species. Finally, we have considered a regulation system based on the intervention of the government. We may instead analyze a unitization system of the ecosystem based on cooperation between the two fishing industries. However, this would drive the analysis of the characterization of a profits-sharing rule, especially of how the prey-fishing industry would offset the economic losses of the predator fishing. Nevertheless, all these considerations would be interesting to develop in future studies.

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Appendix A. Proof of Proposition 1

We essentially have to check that the system comprising Eqs.(16) and (17) admits, for $1 < n_x < \bar{n}$, a unique solution $(x(n_x, n_y), y(n_x, n_y))$ that belongs to the interior of the following set:

$$\Delta = \left\{(x, y) \in \mathbb{R}^2_+ : r(1 - \tfrac{x}{K}) - ay \geq 0 \text{ and } \alpha\alpha x - \delta \geq 0\right\}, \tag{A.1}$$

and satisfies $y(n_x, n_y) > y_{\min}$. The interiority condition ensures that, at the steady state, (i) both species coexist, i.e., $(x(n_x, n_y), y(n_x, n_y)) \gg 0$, (ii) prey are harvested, i.e., $e_x(n_x, n_y) = \frac{1}{n_x}(r(1 - \frac{x(n_x, n_y)}{K}) - ay(n_x, n_y)) > 0$, at a positive profit level, $\pi_x(n_x, n_y) > 0$, because $x(n_x, n_y) > \frac{\delta}{\alpha\alpha} \geq x^{\min}$, and (iii) with the additional result that $y(n_x, n_y) > y_{\min}$, profits are strictly positive in the predator sector, $\pi_y(n_x, n_y) > 0$, and predators are harvested, i.e., $e_y(n_x, n_y) = \frac{1}{n_y}(\alpha\alpha x(n_x, n_y) - \delta) > 0$.

Let us now note that a steady state that solves Eqs.(16) and (17) is a zero of $\tilde{\varphi} : \Delta \to \mathbb{R}^2$ given by the following:

$$\tilde{\varphi}(x, y) = \begin{bmatrix} \tilde{\varphi}_1(x, y) \\ \tilde{\varphi}_2(x, y) \end{bmatrix} = \begin{bmatrix} (\rho + \frac{r}{K}x)\frac{\pi_x(x)}{x} - \frac{p_x}{n_x}\left(r(1 - \frac{x}{K}) - ay\right) \\ \rho\frac{\pi_y(y)}{y} - \frac{p_y}{n_y}(\alpha\alpha x - \delta) \end{bmatrix} \tag{A.2}$$

and that the condition, $n_x < \bar{n}$, is equivalent to assuming that $\tilde{\varphi}_1(x^N, y^{\min}) < 0$. Moreover, if the unique solution is in the interior of Δ, such that $x(n_x, n_y) > \frac{\delta}{\alpha\alpha}$, the second equation ensures that $y(n_x, n_y) > y_{\min}$.

(o) Method

The proof is based on a homotopy argument. An intuitive presentation can be found in Eaves and Schmedders [13] (for a more detailed argument, see also Villanacci et al. [55] ch.7 or Hirsch [27] ch.5). Following their presentation, a complex equation system, $\tilde{\varphi}(x, y) = 0$, has a unique solution in the interior of Δ if there exists (i) a simple equation system, $\tilde{\psi}(x, y) = 0$, and (ii) a homotopy, $\tilde{H} : [0, 1] \times \Delta \to \mathbb{R}^2$, given by $\tilde{H}(x, y, \lambda) = (1 - \lambda)\tilde{\varphi}(x, y) + \lambda\tilde{\psi}(x, y)$, with the following properties:

- $\tilde{\psi}(x, y) = 0$ admits a unique solution and 0 is a regular value of both $\tilde{\psi}$ and $\tilde{H}$ (i.e., $\partial_{x,y}\tilde{\psi}$ and $\partial_{x,y,\lambda}\tilde{H}$ are of full rank)
- $\tilde{H}^{-1}(0)$ is a compact subset of $int(\Delta) \times [0, 1]$, where int denotes the interior
- The $\det(\partial_{x,y}\tilde{\varphi})$ is, at equilibrium, always of the same sign

(i) The choice of $\tilde{\psi}(x, y)$

To construct this function, let us first observe that there exists a unique $x_1 \in \left(x^N, K\right)$ with the property that $\tilde{\varphi}_2\left(x_1, \frac{r}{a}(1 - \frac{x_1}{K})\right) = 0$ as (i) $\lim_{x \to x^N} \tilde{\varphi}_2\left(x, \frac{r}{a}(1 - \frac{x}{K})\right) = \tilde{\varphi}_2\left(x^N, y^N\right) = \rho\frac{\pi_y(y^N)}{y^N} > 0$ because, by the assumption that $y^{\min} < y^N$, (ii) $\lim_{x \to K} \tilde{\varphi}_2\left(x, \frac{r}{a}(1 - \frac{x}{K})\right) = \tilde{\varphi}_2(K, 0) = -\infty$ and (iii) $\frac{d}{dx}\tilde{\varphi}_2\left(x, \frac{r}{a}(1 - \frac{x}{K})\right) = \partial_x\tilde{\varphi}_2 - \frac{r}{aK}\partial_y\tilde{\varphi}_2 = -\frac{p_y\alpha a}{N_y} - \frac{r\rho c_y}{aKy^2} < 0$. Now, let us introduce $\tilde{\psi} : \Delta \to \mathbb{R}^2$ given by $\tilde{\psi}(x, y) = \begin{bmatrix} x - \bar{x} \\ y - \bar{y} \end{bmatrix}$, with $x^N < \bar{x} < x_1$ and $0 < \bar{y} < y_{\min}$. This function admits a unique solution $(x, y) = (\bar{x}, \bar{y})$ that is regular because $\partial\tilde{\psi}(x, y) = I$, the identity of $\mathbb{R}^2$. We can now construct $\tilde{H}(x, y, \lambda) = (1 - \lambda)\tilde{\varphi}(x, y) + \lambda\tilde{\psi}(x, y)$. Because this function takes as parameters $(\bar{x}, \bar{y})$, the generic transversality theorem (see Eaves and Schmedders [13]) affords us the opportunity to choose $(\bar{x}, \bar{y})$ such that 0 is a regular value of $\tilde{H}$.

(ii) $\tilde{H}^{-1}(0)$ is a compact subset of $int(\Delta) \times [0, 1]$

To verify that $\tilde{H}^{-1}(0)$ is a compact subset of $int(\Delta) \times [0, 1]$, let us assume the contrary, i.e., there exists a sequence $(x_n, y_n, \lambda_n) \to (x_\infty, y_\infty, \lambda_\infty)$ with the property that $\forall n, (x_n, y_n, \lambda_n) \in \tilde{H}^{-1}(0)$ and $(x_\infty, y_\infty, \lambda_\infty) \in \partial\Delta \times [0, 1]$. If $y_\infty = 0$, we observe that $\tilde{\varphi}_2(x_n, y_n) \to -\infty$ as x_∞ finite and that $\tilde{\psi}_2(x_\infty, y_\infty) = -\bar{y}$, hence $\tilde{H}_2(x_\infty, y_\infty, \lambda_\infty) < 0$. We can therefore say $\exists N > 0$, $\forall n > N$, $\tilde{H}(x_n, y_n, \lambda_n) \neq 0$, which is the desired contradiction. Now, assume that $x_\infty = x^N$ and $y_\infty > y^{\min}$. It follows that $\tilde{\varphi}_2(x_\infty, y_\infty) > 0$ as $(\alpha a x^N - \delta) = 0$ and $\pi_y(y_\infty) > 0$ for $y_\infty > y^{\min}$. Moreover, $\tilde{\psi}_2(x_\infty, y_\infty) = y_\infty - \bar{y} > 0$ because $\bar{y} < y^{\min}$. This is again a contradiction because $\tilde{H}_2(x_\infty, y_\infty, \lambda_\infty) > 0$. If $x_\infty = x^N$ and $y_\infty \leq y^{\min}$, our assumption says that $\tilde{\varphi}_1(x^N, y^{\min}) < 0$. Because $\partial_y\tilde{\varphi} = \frac{ap_x}{N} > 0$, we deduce that $\tilde{\varphi}_1(x_\infty, y_\infty) < 0$. The desired contradiction is achieved by observing that $\psi_1(x_\infty, y_\infty) = x^N - \bar{x} < 0$ so that $\tilde{H}_1(x_\infty, y_\infty, \lambda_\infty) < 0$. Let us finally consider the case in which $r(1 - \frac{x_\infty}{K}) - ay_\infty = 0$. Let us first observe that $\tilde{\varphi}_1(x_\infty, y_\infty) > 0$ because $\pi(x_\infty) > 0$. Thus, as long as $x_\infty \geq \bar{x}$, $\tilde{\psi}_1(x_\infty, y_\infty) \geq 0$, and the contradiction is achieved. If $x_\infty < \bar{x}$, we know by the choice of $\bar{x} < x_1$ (see point (i)) that $\tilde{\varphi}_2\left(\bar{x}, \frac{r}{a}(1 - \frac{\bar{x}}{K})\right) > 0$, and as the function is decreasing $\tilde{\varphi}_2\left(x_\infty, \frac{r}{a}(1 - \frac{x_\infty}{K})\right) = \tilde{\varphi}_2(x_\infty, y_\infty) > 0$. Moreover, by construction of $\tilde{\varphi}_2$ this also implies, when $x_\infty > x^N$, that $y_\infty > \bar{y}$, hence $\tilde{\psi}_2(x_\infty, y_\infty) > 0$, a contradiction.

(iii) The properties of $\partial\tilde{\varphi}$

By computation,

$$\det\left(\partial_{(x,y)}\tilde{\varphi}\right)=\begin{vmatrix} \underbrace{\rho\frac{c_x}{x^2}+\frac{rp_x}{K}\left(\frac{1+n_x}{n_x}\right)}_{>0} & \underbrace{a\frac{p_x}{n_x}}_{>0} \\ \underbrace{-\alpha a\frac{p_y}{n_y}}_{<0} & \underbrace{\rho\frac{c_y}{y^2}}_{>0} \end{vmatrix}>0 \tag{A.3}$$

Appendix B. Proof of Proposition 2

We simply have to prove that the system comprising Eqs. (27) and (28) admits, for $n_x<\bar{n}$, a unique solution that belongs to the interior of Δ and now has the property that $y^s<y^{\min}$. This again ensures that (i) both species coexist, i.e., $(x^s,y^s)\gg 0$, (ii) both species are harvested, i.e., $\left(e_x^s,e_y^s\right)\gg 0$, and (iii) $\pi_x^s(x^s)>0$ as $x^{\min}<x^N<x^s$, and if $y^s<y^{\min}$, $\pi_y^s(y^s)<0$.

This is equivalent to studying the zero of the function, $\hat{\varphi}:\Delta\to\mathbb{R}^2$, given by the following:

$$\hat{\varphi}(x,y)=\begin{bmatrix}\hat{\varphi}_1(x,y)\\ \hat{\varphi}_2(x,y)\end{bmatrix}=\begin{bmatrix}(\rho+\frac{r}{K}x)\frac{\pi_x(x)}{x}-p_xr(1-\frac{x}{K})+p_xay-\alpha a\pi_y(y)\\ \rho\frac{\pi_y(y)}{y}-p_y(\alpha ax-\delta)+a\pi_x(x)\end{bmatrix} \tag{B.1}$$

As $\tilde{\varphi}_1(x^N,y^{\min})=\hat{\varphi}_1(x^N,y^{\min})$ because $\pi_y(y^{\min})=0$, the assumption that $n_x<\bar{n}$ now becomes $\hat{\varphi}_1(x^N,y^{\min})<0$. We first verify that $\hat{\varphi}(x,y)=0$ admits a unique solution in $int(\Delta)$, and then that $y^s<y^{\min}$. For the first part, the method, as in the proof of Proposition 1, is based on a homotopy argument.

(i) Choice of $\hat{\psi}(x,y)$ and regularity of $\hat{H}$

To define this function, let us first introduce x_0 given by $r\left(1-\frac{x_0}{K}\right)-ay^{\min}=0$ and observe that $x_0>x^N$ because we have assumed that $y^N>y^{\min}$. Now, construct $\hat{\psi}:\Delta\to\mathbb{R}^2$ given by $\hat{\psi}(x,y)=\begin{bmatrix}x-\bar{x}\\ y-\bar{y}\end{bmatrix}$ with $x^N<\bar{x}<x_0$ and $0<\bar{y}<y^{\min}$. Obviously, $\hat{\psi}(x,y)$ admits a unique and regular solution, $(x,y)=(\bar{x},\bar{y})$, as $\partial\psi(x,y)=I$, the identity of $\mathbb{R}^2$. We can then define $\hat{H}(x,y,\lambda)=(1-\lambda)\,\hat{\varphi}(x,y)+\lambda\hat{\psi}(x,y)$ and use the generic transversality theorem to choose $(\bar{x},\bar{y})$ such that 0 is a regular value of $\hat{H}$.

(ii) $\hat{H}^{-1}(0)$ is a compact subset of $int\,(\Delta)\times[0,1]$

Let us assume the contrary, i.e., there exists a sequence, $(x_n,y_n,\lambda_n)\to(x_\infty,y_\infty,\lambda_\infty)$, with the properties that $\forall n$, $(x_n,y_n,\lambda_n)\in\hat{H}^{-1}(0)$ and $(x_\infty,y_\infty,\lambda_\infty)\in\partial\Delta\times[0,1]$. Let us first assume that $y_\infty=0$. It follows that $\hat{\varphi}_2(x_n,y_n)\to-\infty$ as x_∞ finite and $\psi(x_\infty,y_\infty)=-\bar{y}$. We can therefore say $\exists N>0$, $\forall n>N$, $\hat{H}\,(x_n,y_n,\lambda_n)\neq 0$, which is the desired contradiction. Now, assume that $x_\infty=x^N$. If $y_\infty\geq y^{\min}$, $\hat{\varphi}_2(x_\infty,y_\infty)>0$ as $(\alpha ax^N-\delta)=0$, $\pi_x(x^N)>0,\pi_y(y_\infty)\geq 0$ because $y_\infty\geq y^{\min}$. Moreover, $\hat{\psi}_2(x_\infty,y_\infty)=y_\infty-\bar{y}>0$ because $\bar{y}<y^{\min}$. Hence, $\exists N>0$, $\forall n>N$, $\hat{H}\,(x_n,y_n,\lambda_n)\neq 0$. If $y_\infty<y^{\min}$, we know that $\hat{\varphi}_1(x^N,y^{\min})<0$ and as $\partial_y\hat{\varphi}_1=a\,(p_x-\alpha p_y)>0$, it follows that $\hat{\varphi}_1(x_\infty,y_\infty)<0$. The desired contradiction is achieved by observing that $\hat{\psi}_1(x_\infty,y_\infty)=x^N-\bar{x}<0$. Let us finally consider the case in which $r(1-\frac{x_\infty}{K})-ay_\infty=0$. If $y_\infty\leq y^{\min}$, we can say that $\pi_y(y_\infty)\leq 0$, hence that $\hat{\varphi}_1(x_\infty,y_\infty)=(\rho+\frac{r}{K}x_\infty)\frac{\pi_x(x_\infty)}{x_\infty}-\alpha a\pi_y(y_\infty)>0$. Moreover, $\hat{\psi}_1(x_\infty,y_\infty)=K\left(1-\frac{a}{r}y_\infty\right)-\bar{x}>K\left(1-\frac{a}{r}y^{\min}\right)-\bar{x}$ since $y_\infty\leq y^{\min}$. We also observe that $K\left(1-\frac{a}{r}y^{\min}\right)=x_0$ (see part (i) of the proof). We can therefore say that $\hat{\psi}_1(x_\infty,y_\infty)>x_0-\bar{x}>0$ and obtain the desired contradiction. If $y_\infty>y^{\min}$, let us first observe that $\hat{\varphi}_2(x^N,y_\infty)>0$ as $(\alpha ax^N-\delta)=0$ and, in this case, $\pi_x(x^N)>0,\pi_y(y_\infty)>0$. Moreover, $\partial_x\hat{\varphi}_2=a\,(p_x-\alpha p_y)>0$, hence $\hat{\varphi}_2(x_\infty,y_\infty)>0$. The contradiction is obtained by observing that $\hat{\psi}_2(x_\infty,y_\infty)=y_\infty-\bar{y}>0$.

(iii) The properties of $\partial\hat{\varphi}$

It remains to show that $\det(\partial\hat{\varphi})$ is sign-invariant at each solution. By computation and the definition of $\pi_x(x)$ and $\pi_y(y)$, the following is immediate:

$$\partial\hat{\varphi} = \begin{bmatrix} \rho\frac{c_x}{x^2} + 2\frac{r}{K}p_x & a\,(p_x - \alpha p_y) \\ a\,(p_x - \alpha p_y) & \rho\frac{c_y}{y^2} \end{bmatrix} \tag{B.2}$$

Because we evaluate the determinant of the Jacobian matrix for each (x, y) that satisfies $\hat{\varphi}(x, y) = 0$, we also observe, after computation, that the diagonal terms can be written as follows:

$$\begin{cases} \rho\frac{c_x}{x^2} + 2\frac{r}{K}p_x = \frac{y}{x}\left(\underbrace{\left(\frac{p_x(\rho - r + \frac{rx}{K}) + \frac{r}{K}\pi_x(x) + a\alpha c_y + 2\frac{r}{K}p_x x}{y}\right)}_{=A} + \underbrace{a(p_x - \alpha p_y)}_{=C}\right) \\ \rho\frac{c_y}{y^2} = \frac{x}{y}\left(\underbrace{\left(\frac{(\rho+\delta)p_y - ac_x}{x}\right)}_{=B} + \underbrace{a(p_x - \alpha p_y)}_{=C}\right) \end{cases} \tag{B.3}$$

Because we have assumed that $\rho > r$, $p_x > \alpha p_y$ and under an additional technical sufficient condition, $(\rho + \delta)p_y > ac_x$, it is immediate that $A, B, C > 0$; thus, it follows that

$$\det\left(\partial\hat{\varphi}|_{\hat{\varphi}(x,y)=0}\right) = \begin{vmatrix} \frac{y}{x}(A + C) & C \\ C & \frac{x}{y}(B + C) \end{vmatrix} = AB + AC + BC > 0 \tag{B.4}$$

(iv) Negativity of the predator fishery's profit, $\pi_y^s(y^s) < 0$

To verify that $y^s < y^{\min}$, let us observe that $\hat{\varphi}_2(x^N, y^{\min}) = a\pi_x(x^N) > 0$ because $x^N > x^{\min}$. Moreover, we know that $x^s > x^N$ (interiority) and $\partial_x\hat{\varphi}_2 > 0$ as $p_x > \alpha p_y$. It follows that $\hat{\varphi}_2(x^s, y^{\min}) > 0$. As $\partial_y\hat{\varphi}_2 = \rho\frac{c_y}{y^2} > 0$, we deduce that $y^s < y^{\min}$; otherwise, $\hat{\varphi}_2(x^s, y^s) > 0$, a contradiction.

Appendix C. Numbers of fisheries and steady state

(i) Effects of n_x and n_y on the fish stock

By applying the implicit function theorem to Eq.(A.2), we know the following:

$$\begin{bmatrix} \frac{\partial x}{\partial\theta} \\ \frac{\partial y}{\partial\theta} \end{bmatrix}_{\theta\in\{n_x,n_y\}} = -\left(\partial_{(x,y)}\tilde{\varphi}\big|_{\tilde{\varphi}(x,y)=0}\right)^{-1}\left(\partial_{(n_x,n_y)}\tilde{\varphi}\big|_{\tilde{\varphi}(x,y)=0}\right) \tag{C.1}$$

Moreover, using Eq.(A.3), we can say the following:

$$-\left(\partial_{(x,y)}\tilde{\varphi}\big|_{\hat{\varphi}(x,y)=0}\right)^{-1} = -\left(\underbrace{\det\left(\partial\tilde{\varphi}|_{\tilde{\varphi}(x,y)=0}\right)}_{>0}\right)^{-1}\begin{bmatrix} \rho\frac{c_y}{y^2} & -a\frac{p_x}{n_x} \\ \alpha a\frac{p_y}{n_y} & \rho\frac{c_x}{x^2} + \frac{rp_x}{K}\left(\frac{1+n_x}{n_x}\right) \end{bmatrix} = \begin{bmatrix} - & + \\ - & - \end{bmatrix} \tag{C.2}$$

and by differentiating Eq.(A.2) with respect to n_x, n_y, we obtain the following:

$$\left(\partial_{(n_x,n_y)}\tilde{\varphi}\big|_{\hat{\varphi}(x,y)=0}\right) = \begin{bmatrix} \frac{p_x}{n_x^2}\left(r(1 - \frac{x}{K}) - ay\right) & 0 \\ 0 & \frac{p_y}{n_y^2}(\alpha a x - \delta) \end{bmatrix} = \begin{bmatrix} + & 0 \\ 0 & + \end{bmatrix} \tag{C.3}$$

The signs directly follow from the non-negativity of the harvesting efforts. From these observations, we

immediately deduce the following:

$$\frac{\partial x}{\partial n_x} \leq 0,\ \frac{\partial x}{\partial n_y} \geq 0,\ \frac{\partial y}{\partial n_x} \leq 0 \text{ and } \frac{\partial y}{\partial n_y} \leq 0 \tag{C.4}$$

(ii) Effects of n_x and n_y on the aggregated efforts

Because the aggregated efforts are, at the steady state, given by $E_x = r(1-\frac{x}{K}) - ay$ and $E_y = \alpha a x - \delta$, we know that for all $\theta \in \{n_y, n_y\}$,

$$\frac{\partial E_x}{\partial \theta} = -\frac{r}{K}\frac{\partial x}{\partial \theta} - a\frac{\partial y}{\partial \theta} \text{ and } \frac{\partial E_y}{\partial \theta} = \alpha a \frac{\partial x}{\partial \theta} \tag{C.5}$$

From (C.4), we directly conclude that $\frac{\partial E_x}{\partial n_x} \geq 0$, $\frac{\partial E_y}{\partial n_x} \leq 0$ and $\frac{\partial E_y}{\partial n_y} \geq 0$. To compute the last derivative, let us observe, from the definition of the steady state (see Eq.(A.2)), that the prey-harvesting effort can also be written as $E_x = (\rho + \frac{r}{K}x)\frac{\pi_x(x)n_x}{p_x x}$. It follows that $\frac{\partial E_x}{\partial n_y} = \frac{\partial E_x}{\partial x}\frac{\partial x}{\partial n_y} > 0$, with $\frac{\partial E_x}{\partial x} = \left(\rho \frac{n_x c_x}{p_x x^2} + \frac{r n_x}{K}\right) > 0$.

(iii) Effects of n_x and n_y on the individual efforts

From Eq.(A.2), we know that the long-run effort levels are given by $e_x = (\rho + \frac{r}{K}x)\frac{\pi_x(x)}{p_x x}$ and $e_y = \rho\frac{\pi_y(y)}{p_y y}$. It follows that for $\theta \in \{n_x, n_y\}$,

$$\frac{\partial e_x}{\partial \theta} = \frac{d}{dx}\left((\rho + \frac{r}{K}x)\frac{\pi_x(x)}{p_x x}\right)\frac{\partial x}{\partial \theta} = \underbrace{\left(\rho\frac{c_x}{p_x x^2} + \frac{r}{K}\right)}_{>0}\frac{\partial x}{\partial \theta} \text{ and } \frac{\partial e_y}{\partial \theta} = \frac{d}{dy}\left(\rho\frac{\pi_y(y)}{p_y y}\right)\frac{\partial y}{\partial \theta} = \underbrace{\rho\frac{c_x}{p_x x^2}}_{>0}\frac{\partial y}{\partial \theta} \tag{C.6}$$

From point (i), we can therefore conclude that $\frac{\partial e_x}{\partial n_x} \leq 0$, $\frac{\partial e_x}{\partial n_y} \geq 0$, $\frac{\partial e_y}{\partial n_x} \leq 0$ and $\frac{\partial e_y}{\partial n_y} \leq 0$.

(iv) Effects of n_x and n_y on the profits per unit of effort

Because these profits are given by $\pi_s(s)$ with $s = x, y$, we can say that $\frac{\partial \pi_s(s)}{\partial \theta} = \frac{\partial \pi_s(s)}{\partial s}\frac{\partial s}{\partial \theta}$ with $\theta \in \{n_x, n_y\}$. If we now note that $\frac{\partial \pi_s(s)}{\partial s} = p_s > 0$ for $s = x, y$, we can, by Eq.(C.4), obtain $\frac{\partial \pi_x}{\partial n_x}$, $\frac{\partial \pi_x}{\partial n_x}$.

Appendix D. Proof of proposition 3

(i) $y(n_x, n_y) \geq y^{\min} \geq y^s$

From Propositions 1 and 2, we respectively know that $\pi_y\left(y(n_x, n_y)\right) \geq 0 \geq \pi_y\left(y^s\right)$. The result then follows from the fact that $\pi_y(y)$ is increasing and $\pi_y(y^{\min}) = 0$.

(ii) $x(n_x, n_y) \leq x^s$

From (i) of Appendix Appendix C, we know that $\frac{\partial x}{\partial n_x} \leq 0$ and $\frac{\partial x}{\partial n_y} \geq 0$; it therefore remains to show, roughly, that $x(1, \infty) \leq x^s$. More precisely, we know, from Eq. (A.2) with $n_x = 1$, that $\tilde{\varphi}\left(x(1, n_y), y(1, n_y)\right) = 0$ and $\left(x(1, n_x), y(1, n_y)\right) \in \Delta$, a compact set. $\left(x(1, \infty), y(1, \infty)\right)$ is therefore the limit of a converging subsequence. Moreover, when $n_y \to \infty$, the second equation, $\tilde{\varphi}_2\left(x(1, n_y), y(1, n_y)\right) = 0$, becomes $\rho\frac{\pi_y(y(1,\infty))}{y(1,\infty)} = 0$, such that $y(1, \infty)$ corresponds to the open access solution, i.e., $y(1, \infty) = y^{\min}$. If we recall the first equation, $\hat{\varphi}_1(x, y)$, of the sole owner's system (see Eqs. (27) and (28)), we can say the following:

$$0 = \tilde{\varphi}_1\left(x(1, n_y), y^{\min}\right) = \hat{\varphi}_1\left(x(1, n_y), y^{\min}\right) \geq \hat{\varphi}_1\left(x(1, n_y), y^s\right) \tag{D.1}$$

The last inequality follows from the fact that $y^{\min} \geq y^s$ (see (i) of this appendix) and that $\partial_y \hat{\varphi}_1(x, y) = a\left(p_x - \alpha p_y\right) > 0$. Now, let us observe that $\partial_x \hat{\varphi}_1(x, y) = \rho\frac{c_x}{x^2} + 2\frac{r}{K}p_x > 0$. Because $\hat{\varphi}_1\left(x(1, n_y), y^s\right) \leq 0$ and $\hat{\varphi}_1\left(x^s, y^s\right) = 0$, we can say that $x(1, \infty) \leq x^s$ and conclude that $x(n_x, n_y) \leq x^s$.

(iii) $E_y(n_x, n_y) \leq E_y^s$

Recall that a stationary predator-harvesting effort level is always given by $E = \alpha a x - \delta$. Thus, if $x(n_x, n_y) \leq x^s$, then $E_y(n_x, n_y) \leq E_y^S$.

(iv) $E_x(n_x, n_y) \leq E_x^S$ iff $E_x^s \leq E_x(\bar{n}, +\infty)$
Follows from our discussion.