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Abstract

What is the role of income polarisation for explaining differentials in public funding of education? To answer this question, we provide a new theoretical modelling for the income distribution that can directly monitor income polarisation. It leads to a new income polarisation index where the middle class is represented by an interval. We implement this distribution in a political economy model with endogenous fertility and public/private educational choices. We show that when households vote on public schooling expenditures, polarisation matters for explaining disparities in public education funding across communities. Using micro-data covering two groups of school districts, we find that both income polarisation and income inequality affect public school funding with opposite signs whether there exist a Tax Limitation Expenditure (TLE) or not.

Keywords: Education Politics, Schooling Choice, Income polarisation, Probabilistic Voting, Bayesian Inference.

JEL codes: I24, D31, D72, H52, C11.

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1 Introduction

The growth period corresponding to the golden sixties was accompanied by the growth of the middle class (see e.g. Mills 1951 for a tentative definition of the middle class). However, from the early 1980s, the income of the upper deciles increased much more rapidly than that of the lower deciles (Piketty and Saez 2003) leading to a large increase of inequality. At the same time, the proportion of US metropolitan families earning middle incomes went down from 28% in 1970 to 22% in 2000 (Booza et al. 2006). The decline in the size of the middle class combined with a sharp rise the income of the upper deciles led to a severe modification of the income distribution that cannot be explained solely by a change in income inequality, but requires considering also income polarisation which is related to the relative importance of the middle class.

The initial expansion of the middle class in the US went together with an increase in local public funding for public schools. The quality of schooling and consequently the importance of (local) public spending became an important issue for the middle class, anticipating its immediate social consequences (Alesina and Glaeser 2004, Chap 7). Atkinson and Stiglitz (2015), for instance, consider the provision of public education as an implicit form of income redistribution. This is however a disputed view (Fernandez and Rogerson 1995, Glomm and Ravikumar 2003), so that the link between income distribution and public educational expenditure has been the topic of many research papers.

While the theoretical literature, as well as its empirical counterpart, have mainly highlighted the effect of income inequality on redistributive expenditure in general (see e.g. Meltzer and Richard 1981, Benabou 2000) and on public educational expenditure in particular (see e.g. Fernandez and Rogerson 1995, Soares 2003, de la Croix and Doepke 2009, Di Gioacchino and Sabani 2009, Corcoran and Evans 2010, Arcalean and Schiopu 2016, Melindi-Ghidi 2018, Uchida and Ono 2020 to quote a few), the role of income polarisation for explaining preferences for public funding of education is still an unsettled matter. Understanding this important issue is the main objective of the present paper.

Even though income inequality and income polarisation are related in the empirical income distribution literature, they correspond to different concepts. Income inequality is related to the spread of the income distribution. A measure of inequality is for instance provided by the coefficient of variation (a particular case of the generalised entropy index) which is just a scaled version of the variance. Income polarisation is more complex to define and to measure than inequality. It characterises the increase of the ends of a distribution at the expense of its center, or in other words to the decline of the middle class as detailed in Wolfson (1994) and Wang and Tsui (2000). The polarisation index proposed in Foster and Wolfson (2010) corresponds to a comparison between two groups defined with respect to the median (which is supposed to be the anchoring point of the middle class). It is so much related to the Lorenz curve that it corresponds to the area between the Lorenz curve and its tangent at the median while the Gini index is twice the area between the Lorenz curve and the 45° line. One way of generalising this approach is to introduce a third group which extends the middle class from a point (the median) to an interval which includes the median, as we do in this paper.

The effect of income polarisation on the provision of public education can be best understood if we consider the presence of two competing systems of education, private and public schools. Indeed, the proportion of private enrolment varies greatly over countries, as does the system of financing for private schools. Epple and Romano (1996) have shown that the political outcome in presence of private alternatives is determined by the conflict between a coalition formed by poor households with a high school dropping-out rate and rich households that opt-out for private schools, and a coalition of middle-income pro-redistribution households that send their children to public schools. This outcome, namely "*the ends against the middle*" implies that the income composition of the school districts is crucial to determine redistribution policies. Moreover, it implies that income polarisation matters to determine the education policy at school district level.

In presence of private education alternatives, however, political economy models with majority voting imply strong assumptions on preferences to allow for the existence of an equilibrium (Epple and Romano 1996). These assumptions can be relaxed when considering probabilistic voting, as shown by de la Croix and Doepke (2009), using a uniform income distribution. Arcalean and Schiopu (2016) extend their model assuming that household income is distributed according to a Pareto distribution. However, both income distributions cannot account for income polarisation.

We extend the seminal model of de la Croix and Doepke (2009) by including a more general income distribution that allows for income polarisation. More precisely, we propose an income distribution which is a mixture of two uniforms corresponding the poor and the middle class to which we add a Pareto component to model the rich group. In this setting, we show that income polarisation creates an income effect, the size of which depends on the relative balance between poor and rich groups in the economy: it reduces the tax base in poor school districts, while it increases the tax base in rich school districts. This income effect might contrast the effect generated by the variation in the tails of the income distribution. While in very poor (respectively rich) districts the model predicts that income polarisation has a negative (respectively positively) effect on public school quality, the results are less clear in districts in which poor and rich cohabit, leading to the need of an empirical investigation.

We confront our theoretical results to recent data at the school district level in the US. Because of a very particular institutional context where public funding has three sources (local, state and federal), we have chosen to oppose two groups of US States. On one side, we have chosen California because it is the largest US State and because it was pioneering in introducing a local *Tax Limit Expenditure* (TLE) in order to preserve equity in the provision of education between poor and rich districts. On the other side, we have collected data for the 11 States that have never issued a limitation on local funding. There will be more reasons to opt-out for private schooling in California because the presence of a TLE amplifies the competition between private and public schools leading clearly to a coalition of *the ends against the middle* (Epple and Romano 1996) as a TLE limits the interest that rich people can have in public schools. On the contrary, when there is no TLE, another type of coalition could appear between the middle and the rich class because both get interest in public schools. A first regression in our empirical investigation shows the existence of two types of opting-out mechanisms, depending on the presence or not of a TLE, but with a common influence of polarisation, the sign of which depends on the position of mean income with respect to an unknown opting-out threshold.

A second regression verifies the impact of polarisation and inequality on public school quality in poor and rich districts for both groups of States. The magnitude of these effects will of course depend on the existence or not of a TLE, but the signs are the same in the two groups. We also reveal the complexity of the political decision process as in poor districts a large compensation is done by state and federal tax revenues. This compensation is more effective in California than in the no-TLE States.

For our empirical implementation, we have to consider switching regressions where the change of regime depends on an unknown threshold. There are strong arguments for adopting a Bayesian approach to make inference in this type of models, as argued in Bauwens et al. (1999, Chap. 8). Essentially, the distribution of the threshold parameter is non-standard in a classical framework and can be multi-modal. This type of situation does not lend itself easily to classical inference: neither for estimation, because we are never sure which maximum is found, nor for testing, because the asymptotic distribution of the threshold parameter is not standard, as detailed in Hansen (2000). With a Bayesian approach, we simply have to integrate the posterior density of the threshold parameter over a given prior range; and for integration, multi-modality is of no practical importance.

The paper is organised as follows. Section 2 presents the institutional background which is essential to understand the political decision problem. In section 3, we introduce our enriched income distribution that allows for income polarisation, measured by our new index. We then extend the theoretical model of de la Croix and Doepke (2009) by including this distribution in order to examine the theoretical impact of income polarisation on public education policies and schooling choice. Section 4 details empirical evidences. Section 5 concludes. Estimation details and proofs of theorems are given in an appendix.

2 Institutional background

In all the US, parents can enroll their children either in private or in public schools. However, the rate of enrolment in public schools is very important, around 90% in all the samples that we have considered. Private school funding comes from a variety of sources, including tuition fees which can be very high while public schooling is offered free of charge. The sources of financing for public schools are local, state and federal tax revenues. Public schools are managed at the level of school districts, which are governed by elected councils. The level of local taxes (property taxes) is voted by the parents living in the school district. It consequently depends on the income distribution of that district. The balance between local, state and federal public funding relies on a complex political process. In 1971, the California supreme Court (*Serrano vs. Priest*) pointed out that the local funding scheme acted as a discrimination against the poor and consequently forced the State of California to supplement the local funding as a compensation system. California was pioneering in this type of policy, followed by many other States as underlined in Jackson et al. (2015). However, eleven States refused to introduce a *Tax Limit Expenditure* (the no-TLE States).

We have collected data at the school level district, as explained in Appendix C, covering the period 2015-2019. We have retained California on one side because it is the most populated State in the US and because it has implemented a strict TLE. On the other side, we have collected data for the eleven States with no TLE. We have 655 observations for California and 1,180 observations for the no-TLE States. It becomes an empirical question to know what is the impact of TLE on public school funding, knowing that the TLE constraint need not be active in all school districts. Table 1 provides the list of no-TLE States coming from Yuan et al. (2009) together with financial details and a comparison with California. The last line corresponds to the $p-$value of a $t-$test of equality between the means.

A clear effect of the *Serrano vs. Priest* for limiting the local tax revenue per pupil can be seen in Table 1 ($9,007 in no-TLE States versus $5,568 in California). There is consequently an induced difference in total tax revenue per pupil ($17,020 versus $13,921) while federal tax revenue and state revenues are roughly the same between the two groups as seen from a $t-$test of equality of the means.

However, the absence of TLE corresponds also to a strong heterogeneity among the eleven concerned States. Three States (Connecticut, Massachusetts and New Hampshire) have local revenues which are twice above that of California while some other States are well below California. Among

Table 1: Annual tax revenues per pupil

State	Total tax revenue	Local rev.	State rev.	Federal rev.
Connecticut	23,729	15,251	7,827	651
Delaware	16,645	4,869	10,571	1,205
Maine	16,555	9,666	5,897	991
Maryland	15,788	6,939	7,691	1,158
Massachusetts	19,717	12,334	6,703	680
New Hampshire	20,186	13,338	5,983	865
North Carolina	10,481	2,427	6,755	1,301
North Dakota	18,096	6,658	9,886	1,552
Tennessee	9,810	3,238	5,421	1,151
Vermont	23,183	2,312	19,591	1,280
Virginia	12,474	5,600	5,850	1,024
Average no TLE	17,020	9,007	7,027	986
California	13,921	5,568	7,310	1,044
$t-$test $p-$value	0.00	0.00	0.0389	0.201

the eleven States, the case of Vermont is very particular. It has the lowest local revenue and the highest state revenue, at odds with the logic involved by refusing a TLE. So there are reasons for eliminating Vermont if we want to illustrate the effect of TLE on public school funding. This is what we do in the sequel, keeping 1,168 observations and ten no-TLE States.

In a dynamic setting, Fernandez and Rogerson (1998) have shown that the TLE reform had a positive impact on average income, welfare and school spending. However, TLE did not manage to reduce heterogeneity in local

Table 2: Proportion of local funding for public schools

State	Average	Min	Max	Gini local	Gini total
California	0.378	0.033	0.911	0.389	0.119
Connecticut	0.636	0.191	0.874	0.191	0.092
Delaware	0.278	0.165	0.442	0.276	0.097
Maine	0.562	0.238	0.912	0.257	0.093
Maryland	0.440	0.204	0.720	0.203	0.049
Massachusetts	0.614	0.093	0.852	0.208	0.096
New Hampshire	0.654	0.277	0.866	0.157	0.100
North Carolina	0.230	0.117	0.517	0.194	0.077
North Dakota	0.363	0.010	0.643	0.255	0.136
Tennessee	0.323	0.117	0.698	0.233	0.051
Vermont	*0.088*	*0.006*	*0.333*	*0.554*	*0.096*
Virginia	0.431	0.163	0.864	0.251	0.080

funding in California which has the highest Gini of 0.389 as reported in Table 2. On the contrary, total tax revenue is much less dispersed and California is no exception in the list. So there is a large after-tax redistribution in coherence with Yuan et al. (2009). Nevertheless, redistribution does not means equalisation, due to the increase in income inequality (see e.g. Jackson et al. 2015). If we now compute the proportion of local funding in total funding, it can vary between 28% and 65% on average and California is just in the middle of the list.

Local taxes can thus still have an important impact on public school quality. Public school quality can be measured in different ways. The number of pupils per class has for long been used as an indicator, starting with the Coleman report (Coleman et al. 1966). More recent studies tend to favour *total public expenditure per pupil* (de la Croix and Doepke 2009, Arcalean and Schiopu 2012, Melindi-Ghidi 2018). The message conveyed by our data is very clear. Figure 1 represents the impact of local tax limitation

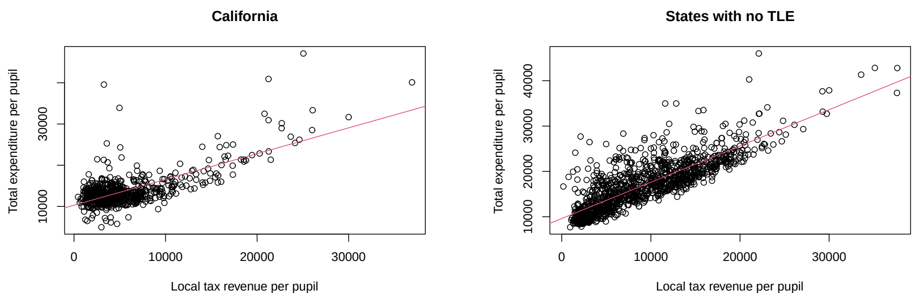

Figure 1: The impact of local tax revenue on school quality with and without TLE

on total expenditure per pupil. For both groups, there is a positive and significant slope coefficient in the regression of total expenditure on local tax revenue. But the slope is steeper for the States without a TLE (0.800 against 0.626 for California) while most observations for California are concentrated in the bottom left of the plot. This empirical evidence illustrates the overall impact of a TLE policy.

3 A political model with income polarisation

In this section we propose a new enough flexible income model that we integrate in a theoretical political economy model to explain public school quality as a function of local taxes voted by school district residents in order to measure the impact of income polarisation on public school quality.

3.1 A new model for the income distribution

To analyse the effect of income polarisation on public school quality, we propose an income model that is a mixture of two uniform densities for the poor and the middle class while the rich class is represented by a Pareto member. Single distributions like the uniform or the Pareto are perfectly adequate to model income inequality, but they cannot be used for modelling polarisation. For instance the Gini index and the Wolfson polarisation index are equal for the uniform distribution while they evolve in a parallel way for the Pareto distribution.

The first member of our income model represents the poor households and thus can start at zero. Its upper limit is x_1 and so can be interpreted as a poverty line. The second member is defined between two bounds, x_1 and x_2. It represents the middle class. The households above x_2 corresponds to the rich people with no upper bound and their income is modelled with a Pareto tail. We then have to define the proportions of each class. We introduce a first parameter g to monitor the size of the middle class measured as $1-g$ and a second parameter β to monitor the relative balance between the poor (β) and the rich ($1-\beta$). The obtained mixture is as follows:

$$\begin{aligned} f(x) &= g\beta\frac{1}{x_1}\mathbb{I}(x \leq x_1) + (1-g)\frac{1}{x_2-x_1}\mathbb{I}(x_1 < x < x_2) \\ &\quad + g(1-\beta)\frac{\alpha x_2^{\alpha}}{x^{\alpha+1}}\mathbb{I}(x \geq x_2), \end{aligned} \tag{1}$$

where $\mathbb{I}(\cdot)$ is the indicator function. The cumulative distribution corresponds to:

$$\begin{aligned} F(x) &= g\beta\frac{x}{x_1}\mathbb{I}(x \leq x_1) + \left(g\beta + (1-g)\frac{x-x_1}{x_2-x_1}\right)\mathbb{I}(x_1 < x < x_2) + \\ &\quad (1-g(1-\beta)\left(\frac{x}{x_2}\right)^{-\alpha})\mathbb{I}(x \geq x_2), \end{aligned} \tag{2}$$

while the mean is given by:

$$\mu = g\beta\frac{x_1}{2} + (1-g)\frac{x_1+x_2}{2} + g(1-\beta)x_2\frac{\alpha}{\alpha-1}, \tag{3}$$

with the restriction $\alpha > 1$.

We can now build a polarisation index which compares the population share of the middle to the population share of the two extremes and thus can be seen as an extension of the polarisation index of Wang and Tsui (2000) (see also Scheicher 2010). The population share of the middle class is monitored by the value of $g \in [0, 1]$. Its relative size is maximum for $g = 0$ while an increasing g models the collapse of the middle class. The balance between the poor and the rich is monitored by β. For $\beta = 0$ we have no poor households, and for $\beta = 1$ we have no rich households. For these two extreme cases, we cannot have polarisation, because one of the extreme groups disappears. For $\beta = 0.5$, the distance between the poor and the rich is maximum. Building on these remarks, we propose the following polarisation index at value between $[0, 1]$:

$$Pol = 4g\beta(1-\beta). \tag{4}$$

This index is maximum and equal to 1 when $g = 1$ (no middle class) and $\beta = 0.5$ (equal number of rich and poor). It represents the maximum distance between the poor and the rich. It is 0 when either $g = 0$ or when either the rich or the poor group disappears ($\beta = 0$ or $\beta = 1$). This index depends only on population shares and not on the income shares, so it is not related to the Gini index. Note however that β and g depend indirectly on the values chosen for x_1 and x_2.

3.2 The household's problem

Our theoretical analysis is based on the model developed on the seminal work of de la Croix and Doepke (2009). A representative household composed of two parents initially chooses its consumption level c and decides for its number of children n. At the same time, parents decide whether to send their children to a public or a private school. Public education is free of charge, while private education involves a tuition fee, normalised to one for simplicity. As long as fertility is decided beforehand, households are locked in a certain school regime depending on the size of the family and on its income level. When fertility and education decisions are taken, households vote for a rate of income tax rate to finance public education spending.

The representative household is endowed with an additive and separable utility function where $\gamma \in \mathbb{R}^+$ is the overall weight attached to the number of children n and $\eta \in (0, 1)$ is the relative weight of human capital quality:

$$u = \ln(c) + \gamma[\ln(n) + \eta \ln(h)]. \tag{5}$$

In this equation, h represents the level of child human capital, i.e., the quality of education acquired by each child. When parents chose public education, $h = s$ and s represents the quality of public schooling, proxied by public spending per child. When parents chose private education, $h = e$, where e represents private investment on education. Public and private education are mutually exclusive. Assuming private education spending tax deductible, the budget constraint is simplified into:

$$c = (1-\tau)[x(1-\phi n) - n\, e], \tag{6}$$

where τ is the income tax rate, x the exogenous wage rate and ϕ the proportion of time allocated to raising one child, $(1 - \phi n)$ representing labour supply.[1] When maximizing the utility function (5) under the budget constraint (6), it is possible to derive the desired number of children n^* and the

[1] There is an implicit constraint in the model so that the maximum number of children n is bounded by $1/\phi$.

optimal education financial investment e^*, for each choice of schooling type:

$$\begin{array}{lll} Public: & e_s^* = 0, & n_s^* = \dfrac{\gamma}{\phi\,(1+\gamma)}, \\ Private: & e_e^* = \dfrac{x\eta\phi}{1-\eta}, & n_e^* = \dfrac{\gamma(1-\eta)}{\phi(1+\gamma)}, \end{array} \tag{7}$$

implying $n_s^* > n_e^*$ which means that parents choosing the public system have more children.[2]

We define the indirect utility function V^s, corresponding to choosing public education, and V^e, corresponding to choosing private education, by replacing the budget constraint (6) and the optimal decisions (7) in the utility function (5). The final schooling choice is made by comparing the two indirect utility functions V^s and V^e. The possibility that $V^e > V^s$ depends on the expected quality of public schooling $E(s)$ and only arises if the agent has an income greater than a given threshold $\tilde{x}$ determined by:

$$x > \tilde{x} \equiv \frac{(1-\eta)}{\phi\eta\delta} E(s), \qquad \text{with: } \delta = (1-\eta)^{1/\eta}, \tag{8}$$

which is Lemma 2 (opting-out decision) in de la Croix and Doepke (2009). At a given wage x, the higher the expected quality of public schooling $E(s)$, the lower the probability of opting-out from the public education system. So the expected quality of the public school is one of the key variables for the opting-out decision, the other one being the income distribution.

3.3 Equilibrium with income polarisation and probabilistic voting

We now introduce our mixture income model of two uniforms and a Pareto with the objective to analyse the role of income polarisation in explaining policy outcomes. The income distribution enters at the level of the definition of the participation rate in the public school system Ψ which is given by the integral of that income distribution between 0 and the opting-out threshold $\tilde{x}$:

$$\Psi = \int_0^{\tilde{x}} f(x)\,dx = F(\tilde{x}) - F(0) = F(\tilde{x}).$$

Using (1) and its corresponding CDF (2), we find that:

$$\begin{aligned} \Psi \;=\; & g\beta \frac{\tilde{x}}{x_1} \mathbb{I}(\tilde{x} \le x_1) \\ & + \left[(1-g)\frac{\tilde{x}-x_1}{x_2-x_1} + g\beta\right] \mathbb{I}(x_1 < \tilde{x} < x_2) \\ & + \left[1-(1-\beta)g\left(\frac{\tilde{x}}{x_2}\right)^{-\alpha}\right] \mathbb{I}(\tilde{x} \ge x_2). \end{aligned} \tag{9}$$

[2]Since $e_s^* = 0$, we shall simplify our notation into $e_e = e$ in the remaining part of the paper.

We now derive the political equilibrium in order to study how the voted policies are modified when varying β and g, two parameters monitoring directly income polarisation. For a given school district, the total spending for public schools with an enrolment of n_s^* pupils is given by:

$$\int_0^{\tilde{x}} s\, n_s^* f(x)dx = s\, n_s^* F(\tilde{x}) = s\, n_s^* \Psi = s\frac{\gamma}{\phi(1+\gamma)}\Psi, \tag{10}$$

assuming perfect foresight with $\mathbf{E}(s) = s$. The local budget balanced rule requires that spending must be equal to the total local income tax revenue. As all households pay taxes, the local tax revenue is:

$$\tau \int_0^{\tilde{x}} [x(1-\phi\, n_s^*)]f(x)dx + \tau \int_{\tilde{x}}^{\infty} [x(1-\phi\, n_e^*) - e\, n_e^*]f(x)dx.$$

Since education spending is assumed tax deductible and as fertility is endogenous, taxable income is the same whether parents choose public or private education. Using (7), it is easy to verify that $x\,(1-\phi\, n_s^*) = x(1-\phi\, n_e^*) - e\, n_e^* \equiv x/(1+\gamma)$. Therefore the local tax revenue can be simplified into:

$$\frac{\tau}{1+\gamma}\int_0^{\infty} x f(x)dx = \frac{\tau}{1+\gamma}\mu, \tag{11}$$

where μ is the mean income. We can rewrite the balanced budget rule of the local government as follows, equating (10) and (11):

$$s\frac{\gamma}{(1+\gamma)\phi}\Psi = \frac{\tau}{1+\gamma}\mu. \tag{12}$$

Rearranging (12), we are able to rewrite the local government budget constraint so as to express the quality of public schooling as a function of the participation rate Ψ, the tax rate τ and the mean income μ:

$$s = \frac{\phi}{\gamma\Psi}\tau\mu. \tag{13}$$

The equilibrium choice under probabilistic voting is equivalent to maximizing a weighted sum of the indirect utilities of individuals (see de la Croix and Doepke 2009), noted $\Omega(\tau)$:

$$\Omega(\tau) = \int_0^{\tilde{x}} V^s(x, n_s^*, 0, s, \tau) f(x)\, dx + \int_{\tilde{x}}^{\infty} V^e(x, n_e^*, e, 0, \tau) f(x)\, dx. \tag{14}$$

Using (5), (6) and (7) in order to implicitly define the two indirect utility functions V^s and V^e, and using the local government budget constraint (13), after some algebraical manipulations, the above social welfare function writes as follows:

$$\begin{aligned}\Omega(\tau) &= \ln\left(\frac{1-\tau}{1+\gamma}\right) + \gamma\ln\left(\frac{\gamma}{\phi(1+\gamma)}\right) \\ &+ \gamma\eta\ln\left(\frac{\mu\tau\phi}{\gamma\Psi}\right)\int_0^{\tilde{x}} f(x)\,dx + \int_0^{\infty}\ln(x)f(x)\,dx \\ &+ \int_{\tilde{x}}^{\infty}\left[\gamma\ln(1-\eta) + \gamma\eta\ln\left(\frac{x\eta\phi}{1-\eta}\right)\right] f(x)\,dx. \end{aligned} \tag{15}$$

The maximum in τ of this welfare function is found by equating to zero its first-order derivative, so as to express the optimum voted local tax rate in terms of participation rate in the public education system Ψ:

$$\tau^* = \frac{\gamma\eta\Psi}{1+\gamma\eta\Psi} \equiv \tau(\Psi), \tag{16}$$

to which corresponds an expected level of public education spending per pupil:

$$s^* = \frac{\phi\eta}{1+\gamma\eta\Psi}\mu \equiv s(\Psi,\mu). \tag{17}$$

We thus arrive to the definition of a political equilibrium:

Definition 1 *A political-economic equilibrium, under perfect foresight ($E[s] = s$) and balanced local government budget rule, is defined by an income threshold $\tilde{x}$, a vector of fiscal policies (τ^* and s^*), a set of private decisions on consumption, fertility and private education $(c^s, n^s, 0)$ if $x \leq \tilde{x}$ or (c^e, n^e, e) if $x > \tilde{x}$, such that both household utility and social welfare are maximised.*

Proposition 1 *A probabilistic voting equilibrium when the income distribution is modelled as a three member mixture exists and is unique.*

Proof, see Appendix A.1.

In this equilibrium, the voted tax rate is an increasing function of the participation rate in public school while the corresponding public spending per student is a decreasing function of it. Note that, in our theoretical setting public school funding is a function of the tax base so that school quality depends on tax revenue and it is correlated with average income.[3]

3.4 Opting-out, public school quality and the importance of the middle class

We now concentrate on the effects of the decrease of the importance of the middle-class on the public school spending. Because $Pol = 4g\beta(1-\beta)$,

[3]In the original model of de la Croix and Doepke (2009) school quality depends on a balance between Ψ and τ, because mean income is normalized to one.

we can express g as a function of Pol for a given β. We can thus study the impact of polarisation on the optimal levels of taxation and of public educational spending per pupil.[4]

First of all, polarisation has a direct effect on mean income and consequently on the tax base because we consider x_1 and x_2 as fixed. An increase of g corresponding to the collapse of the middle class implies that the people leaving the middle class have to go to one of the other two groups. Where they go depends on the value of β.

Lemma 1 *There exists a threshold value $\bar{\beta}$ such that the mean income and the tax base increase in g if and only if beta is smaller than $\bar{\beta}$. The threshold value is given by:*

$$\bar{\beta} = \frac{x_1(1-\alpha) + x_2(1+\alpha)}{x_1(1-\alpha) + 2\alpha x_2} \tag{18}$$

Proof, see Appendix A.2.

This income effect is essential to understand the differences between our model and that of de la Croix and Doepke (2009) where the mean income is normalised to one.

The next lemma relates tax rate to public school participation rate:

Lemma 2 *When polarisation changes, the optimal taxation rate and the public school participation rate evolve in the same direction.*

Proof, see Appendix A.3.

The intuition behind Lemma 2 is straightforward. The larger the number of pupils enrolled in public schools, the greater the number of voters for a higher taxation rate.

Finally, note that the effect of g on the participation rate Ψ is ambiguous and depends on the relative position of the opting-out threshold $\tilde{x}$ with respect to the exogenous thresholds x_1 and x_2 as shown in the next lemma:

Lemma 3 *Given β, the participation rate in public education Ψ is a decreasing function of g if the opting-out threshold $\tilde{x}$ is greater than $(1-\beta)x_1 + \beta x_2$ and an increasing function of g in the reverse case.*

For a proof, see Appendix A.4.

Lemma 3 details the effect of changing the proportion of the middle class on the participation rate in public school when β is considered as given. When either $\tilde{x} < x_1$ or $\tilde{x} > x_2$, the educational choice made by the middle class is straightforward, they respectively choose either private school or public school. The interesting case is when $\tilde{x} \in [x_1, x_2]$. Then, the

[4] Of course, since $Pol = 4g\beta(1-\beta)$, Pol and g evolve in the same direction.

relative proportion of rich and poor matters to switch from one choice to the other. This means that the income composition of each school district is crucial to explain participation to public school. Since rich households are more demanding in terms of education they opt-out from the public school system with a higher probability, especially when there is a TLE. When $\tilde{x}$ is high, low- and middle-income parents are not able to enroll their children in private schools so an increase in the middle class, i.e. a decrease in g, generates an increase in the participation rate in the public education system. Put differently, polarisation tends to have a negative impact on public school participation when $\tilde{x}$ is high enough. Conversely, when $\tilde{x}$ is low, some low- and middle-income parents are now able to enroll their children in private schools. It follows that participation rate in public school goes down when the size of the middle-class $(1 - g)$ becomes more important.

We have now our main proposition explaining the impact of income polarisation on school quality:

Proposition 2 *Income polarisation has a negative impact on public school quality in a poor regime characterised by* $\beta > \max\{\Psi; \bar{\beta}\}$. *Income polarisation has a positive impact on public school quality in a rich regime characterised by* $\beta < \min\{\Psi; \bar{\beta}\}$. *The effect is ambiguous in the other cases.*

Proof, see Appendix A.5

Because an income distribution integrates to one, the decrease of the middle class entails an increase of the two other groups in a proportion which depends on the value of β. Consequently, Proposition 2 highlights two possible regimes which are essentially distinguished by the value of β.

In a poor regime ($\beta > \max\{\Psi; \bar{\beta}\}$), polarisation implies an increase of the size of the poor group compared to the rich group. Since the poor group increases proportionally more than the rich group, overall tax revenue does not compensate the larger participation rate to public schools, even though the optimal tax rate increases, as indicated by Lemma 2. This is due to the fact that the average income of the school district decreases when g increases and β is sufficiently high. In other words, in the poor regime the revenue effect and the tax revenue are dominated by the higher participation rate of poor households to the public school system. Therefore, polarisation generates lower public school quality in districts where the share of poor families is important.

In a rich regime ($\beta < \min\{\Psi; \bar{\beta}\}$), polarisation plays the same role as inequality in de la Croix and Doepke (2009). Since a decrease in the middle class implies an increase in the ends, when β is low, the rich group increases more proportionally than the poor group. The tax base increases strongly, public schools receive more funds and school quality increases because participation rate increases slower than overall tax revenue. In other words, the revenue effect and the tax revenue dominate the higher participation rate generated by higher public school quality.

3.5 The respective roles of inequality and polarisation

What is the relationship between income inequality and polarisation in our theoretical setting? The literature has analysed the effect of income inequality on public school quality in the form of a mean-preserving spread. This method implies movements in the second moment of the income distribution, while the mean income does not change. Therefore, to isolate the effect of inequality in our setting, we shall neutralise the effect of increasing polarisation on the mean income μ. Consider the particular case $\beta = \bar{\beta}$, as defined in Lemma 1. In this scenario, the effects of income inequality and income polarisation are the same because $\partial\mu/\partial g = 0$. Proceeding as in Proposition 2, we can observe that when $\beta = \bar{\beta}$, a mean preserving spread has a negative (respectively positive) impact on public school quality in a poor (respectively rich) regime characterised by $\bar{\beta} > \Psi$ (respectively $\bar{\beta} < \Psi$). This result, echoes the main theoretical results of Arcalean and Schiopu (2016): when inequality increases, the per student spending in public education decreases if the tax base is low enough, otherwise it increases.

However, in our setting, whenever $\beta \neq \bar{\beta}$ income polarisation and income inequality become different concepts. Income polarisation generates a decrease in the size of the middle class. This can be done at the benefit of the importance of the poor group when β is high enough, implying a decrease in the mean income. Or at the benefit of the importance of the rich group when β is low enough, implying thus an increase in the mean income. Thus, when $\beta \neq \bar{\beta}$, varying g implies a non-mean-preserving spread. Put differently, compared to the standard effect of income inequality generated by a mean-preserving spread, income polarisation creates an income effect the size of which depends on the relative balance between the poor and rich groups in the economy. This income effect might have important consequences both on the opting-out and on the voted level of total schooling expenditures.

The latter claim can be better understood at the light of the results provided in Proposition 2. Consider an economy where β is large compared to Ψ, that is a district in which the share of poor is important and the participation to public school is low because the rich and the middle class enrol their children in private school. In this case, if $\beta > \bar{\beta} > \Psi$, then an increase in income polarisation decreases public school spending per student. At the same time, a spread of the income distribution would have the same negative effect on public school quality. However, when the participation to public school is larger, that is $\beta > \Psi > \bar{\beta}$, then an increase in income polarisation has a negative effect on public school spending per student while inequality should positively impacts it.

When a non-mean preserving spread is allowed, the income effect can prevail or not on the effect created by the spread of the distribution. Which effect dominates crucially depends on the participation to public school and on the relative size of poor and rich households in the economy. Therefore, the main effect of income polarisation and income inequality may differ between school districts and thus its determination becomes an empirical question.

4 Empirical Evidences

We now confront our model to school district data observed over 2015-2019 for California and for ten no-TLE States as detailed in Appendix C. We restrict our data to households with children enrolled either in public or private schools.[5]

4.1 Testable assumptions

The theoretical model and the institutional background have pointed out a certain number of salient facts and ambiguities that have to be tested. We could summarise them in the form of three hypothesis:

1. *Hypothesis 1*: The participation rate of richer people to public schooling is a complex function of β and g which depends on the position of the opting-out threshold in the income distribution.

2. *Hypothesis 2*: Income polarisation has opposed effects on public school quality in poor and rich districts depending on the proportion of poor in the school district (Proposition 2). Inequality and polarisation can have different effects on school quality.

3. *Hypothesis 3*: Public school funding is the result of a complex political process where the balance between local, state and federal funding is modified by the existence or not of a TLE.

4.2 Overall income distributions

Information on household income (for households with children at school) is provided at the school district level in the form of ten unequal classes, possibly with top-coding for the highest class. The lowest class corresponds

[5]There are very few empirical studies using school district data for analysing public school quality, except Arcalean and Schiopu (2012). They explain total public funding at the school district level by characteristics of the local income distribution (mean and variance) in a single equation with different samples including or not no-TLE States. Our empirical model is quite different.

to households with a yearly income plus benefits lower than \$10,000, while the highest class corresponds to households with a yearly income plus benefits above \$200,000. These group boundaries were maintained over time, taking into account price indices. The data correspond to a household with two adults and two children, use being made of the new OECD equivalence scale.

From the Census, the poverty threshold for a family of four including two children was \$25,465 in 2018.[6] Consequently, we took \$25,000 as the poverty line which means that the first three income classes (lower than \$10,000, \$15,000 and \$25,000) correspond to the poor group. The upper limit of the middle class is more prone to discussion. Among the various of definitions of the middle class reported in Renwick and Short (2014), those referring to quantiles use the range between $Q_{0.25}$ and $Q_{0.75}$ of the income distribution. The definitions involving fractions of the median income take a lower bound between 0.50 and 0.75 of the median income and an upper bound between between 1.25 and 2.00 of the median income. When we compute the median income of the two groups of States, an upper bound for the middle class fixed at \$100,000 would corresponds to 1.66 times the median for California and to 1.77 times the median for the no-TLE States, well within the range given in Renwick and Short (2014). With these assumptions, the two overall aggregated income distributions lead to Figure 2.

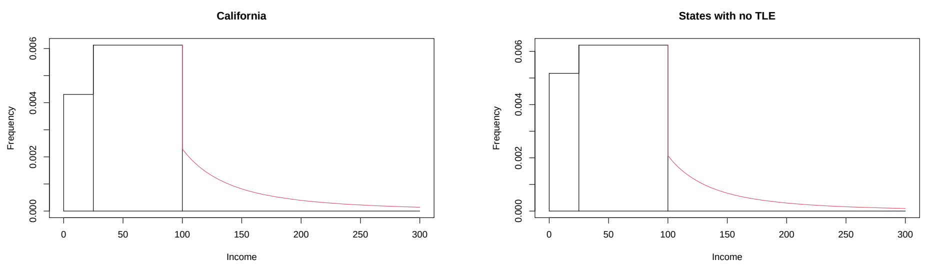

Figure 2: Stylised income distributions from aggregated income classes

For estimating our model, we need values of β, g, Pol and the Gini of the $655 + 1,168 = 1,823$ different income distributions, one for each school district, using the class boundaries defined above. The method is detailed in Appendix B. The main results are that the middle class dominant in 51% of the school districts in California, but this figure goes down to 47% for the ten no-TLE States. The rich class is dominant in 33% of school districts of California, but this figure goes up to 38% for the ten no-TLE States.

Conditionally on these estimates, we shall specify two switching regressions, one for determining the opting-out income level $\tilde{x}$, one for explaining

[6] `https://www.census.gov/library/publications/2019/demo/p60-266.html`, Poverty Thresholds 2018.

school quality. A prior information is needed on the threshold for identification reasons.[7] We have chosen a uniform prior density defined on a range 15% and 95% quantiles of the variable explaining the change of regime. We shall be non-informative on the other parameters. We have chosen a specification strategy called from general to particular, which means that we shall sequentially eliminate variables with a low statistical contribution from a large list provided by the economic theory explained above. This is the reason why there might appear more variables in one regime.

4.3 Income polarisation and public school participation

Hypothesis 1 concerns the non-linear relation between the rate of public enrolment Ψ and the parameters β, g or Pol, where the non-linearity depends on the position of the opting-out income threshold $\tilde{x}$. This suggests a two regime regression model with β and g or Pol as explanatory variables, the switching mechanism being determined by comparing the mean income of a district to an unknown threshold $\tilde{x}$ to be estimated. We have chosen to scale the mean income by 1,000 and to divide it by the average number of children per household. We have estimated our model separately for the two groups of States in order to point out the differences between two possible mechanisms, depending on the presence or not of a TLE. Estimation results are reported in Table 3.

First, the most striking fact is that the opting-out threshold is pretty high, well within the rich class in both cases. This is coherent with a high public school enrolment rate.

Second, we have clearly two mechanisms at work, depending on the existence or not of a TLE. In California, a State with a strong TLE mechanism, the opting-out mechanism is well defined as the opting-out threshold has a very concentrated posterior density (see Figure 3). This mechanism induces a strong difference in public school enrolment rate between the two cases (see Table 3, $\Psi = 0.93$ versus $\Psi = 0.84$). In the absence of a TLE, this difference in enrolment rates vanishes. The posterior density of the opting-out threshold is more dispersed. In fact, when there is no-TLE, rich people have less incentives to opt-out for a private school, because they can freely vote for higher local taxes so that public schools can match the quality of private schools. In California, because of the TLE mechanism, rich people have on average more incentives to enrol children in private schools because they cannot decide for higher local public funding.

Third, polarisation has a negative impact on public school participation when the mean income is lower that $\tilde{x}$ and a positive impact in the reverse case. The impact of polarisation is more important when there is no TLE. The second effect of an absence of a TLE is on the impact of the relative

[7] There should be at least as many observations as there are parameters in each regime.

Table 3: Determining the opting-out threshold

	California					
	Mean income $< \tilde{x}$			Mean income $> \tilde{x}$		
	Coef.	S.D.	$t-$value	Coef.	S.D.	$t-$value
Intercept	0.959	0.014	66.69^{***}	0.713	0.054	13.29^{***}
β	0.076	0.014	5.63^{***}	0.398	0.117	3.40^{***}
g	-0.111	0.024	-4.65^{***}	0.138	0.064	2.15^{*}
Implicit impact of polarisation	*-0.038*			*0.047*		
Observations		553			102	
$\tilde{x}$			$179,969 ($14,040)			
σ^2			0.00396			
Pseudo R^2			0.308			
Average Ψ	0.93			0.84		
	no-TLE States					
	Mean income $< \tilde{x}$			Mean income $> \tilde{x}$		
	Coef.	S.D.	$t-$value	Coef.	S.D.	$t-$value
Intercept	0.917	0.005	205.16^{***}	0.915	0.007	136.21^{***}
β	0.049	0.013	3.92^{***}	-1.920	0.515	-3.71^{***}
Pol	-0.058	0.019	-3.03^{**}	0.598	0.198	3.01^{**}
State dummies	Yes					
Observations		1001			179	
$\tilde{x}$			$167,477 ($7,712)			
σ^2			0.00362			
Pseudo R^2			0.049			
Average Ψ	0.91			0.91		

The reader used to significance codes with stars can apply the following conversion scale: *** or $P \leq 0.001$, ** or $P \leq 0.01$, * or $P \leq 0.05$ corresponding to a Student ratio greater than 3.29, 2.58 and 1.96 respectively under a normality assumption. The standard deviation of the opting-out threshold is given between parentheses. State dummies were Maine and Virginia for the poor regime and none for the rich regime, selected after a specification search.

proportion of poor, β. When $\mu < \tilde{x}$, there is no TLE effect because β has the same positive sign for California and for the no-TLE States. When β increases, households go increasingly to public schools. When $\mu > \tilde{x}$, households continue to enroll children to public schools when β increases in California. But when there is no TLE, households send less their children to public schools because an increase in β means in this case that the tax base is going to be reduced so that there will be no longer a sufficient level of local funding for public schools and consequently a private alternative becomes favourable. This effect is very strong in the no-TLE States with an estimated coefficient of -1.920.

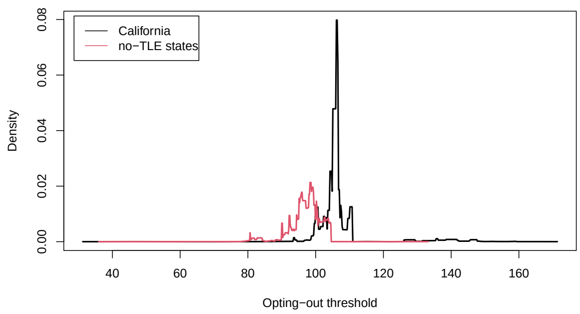

Figure 3: Opting-out threshold for California and for no-TLE States

4.4 Income polarisation and public school quality

For explaining public school quality, we have chosen *Total Expenditure per Pupil* which covers a broad spectrum of expenditures and which is presumably very representative of the differences between rich and poor school districts.[8]

Hypothesis 2 suggests a two-regime switching regression model, where the change of regime is determined by the value of β, the relative proportion of poor. We have however to add several other variables to this regression model on top of income polarisation. First of all, we need to add income and a measure of inequality in order to be able to measure separately the impact polarisation and inequality and be able to compare our results with those of de la Croix and Doepke (2009). Second, following *Hypothesis 3*, we have to consider the impact of the three sources of financing (local, state

[8]The web site of the National Center for Education Statistics provides the following definitions. *Instructional Expenditure per Pupil* covers mainly wages and activities related to the interaction between teachers and students. *Total Expenditure per Pupil* includes the previous expenditure and adds maintenance, investment, interest payments, student support, food, administration, etc.

and federal) and their balance.[9] Inference results are provided in Table 4.

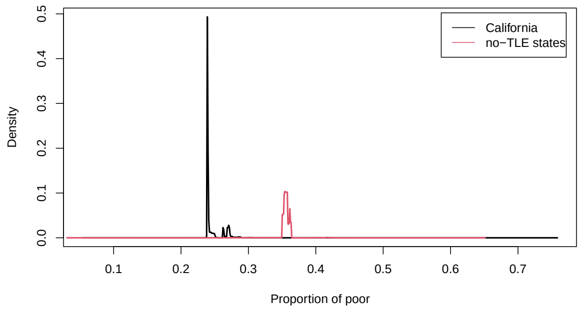

Figure 4: Posterior sample separation according to the relative proportion of poor households

The posterior density of the threshold is very concentrated for California and less for the no-TLE States as shown in Figure 4, with respective posterior means of 0.246 and of 0.356. With this threshold, 55% of the school districts are in the rich regime in California while this number goes up to 70% for the no-TLE States. Imposing a TLE in California had the consequence that the posterior difference in school expenditure is on average only \$476 and it is in favour of poor districts. So we indeed measured a significant *Serrano vs. Priest* effect. In the no-TLE States, there is a difference of \$4,062 in public spending per pupil in favour of rich districts, revealing the consequences of an absence of any TLE.

With Proposition 2, we expect a positive effect of polarisation on school quality in the rich regime. This is what we have for California (0.325) when the relative proportion of poor is lower than 0.246 and also for the no-TLE States (0.107) when the proportion of poor is lower than 0.356. The negative impact of polarisation in the poor regime could be measured only for California (-0.180).

Hypothesis 2 also assumes that inequality and polarisation can have different effects on public school quality. This is what we find in Table 4 where in the rich regime and independently of a TLE, polarisation has a positive effect while the effect of inequality is negative. This also shows that inequality and polarisation cannot be analysed separately from an empirical point of view.

Hypothesis 3 concerns the complexity of the political process. In our model, tax revenues have a local source, depending strictly on the local tax base. In reality three different sources of tax revenues serve to finance *Public expenditures per Pupil*, and their balance operates a kind of redistribution.

[9]Note that there is no accounting identity in the data between these three reported sources of financing and total expenditure per pupil.

Table 4: Explaining total expenditure per pupil

	California					
	Rich regime $\beta < \bar{\beta}$			Poor regime $\beta > \bar{\beta}$		
	Coef.	S.D.	$t-$value	Coef.	S.D.	$t-$value
Intercept	3.520	0.422	8.29***	4.320	0.424	10.19***
Pol	0.325	0.072	4.54***	-0.180	0.074	−2.44*
Gini	-0.823	0.112	−7.36***			
log(Inc/Hsize)	0.074	0.024	3.10**			
Local Tax rev	0.347	0.018	19.39***	0.174	0.016	10.70***
State Tax rev	0.161	0.026	6.11***	0.263	0.041	6.46***
Federal Tax rev	0.131	0.016	8.42***	0.186	0.018	10.57***
$\bar{\beta}$	0.246 (0.012)					
σ^2	0.0152					
Pseudo R^2	0.649					
School districts	364			291		
Mean expenditure difference	-$476					
	no-TLE States					
	Rich regime $\beta < \bar{\beta}$			Poor regime $\beta > \bar{\beta}$		
	Coef.	S.D.	$t-$value	Coef.	S.D.	$t-$value
Intercept	3.630	0.278	13.05***	2.900	0.229	12.70***
Pol	0.107	0.036	2.95**			
Gini	-0.166	0.078	−2.14*			
log(Inc/Hsize)	0.049	0.017	2.89**			
Local Tax rev	0.388	0.010	38.74***	0.287	0.0105	27.37***
State Tax rev	0.147	0.013	10.91***	0.251	0.022	11.30***
Federal Tax rev	0.100	0.010	9.72***	0.266	0.016	12.48***
State dummies	Yes					
$\bar{\beta}$	0.356 (0.005)					
σ^2	0.0118					
Pseudo R^2	0.873					
School districts	821			347		
Mean expenditure difference	$4,062					

Tax variables are taken in logs. State variables were found after a specification search. They are Connecticut, Maine, Tennessee and Virginia for the rich regime and Tennessee and Virginia for the poor regime. Average characteristics of the two sub-samples were computed as a byproduct of integration. As most readers are familiar with significance codes using stars, we have adopted the following conversion scale: *** or $P \leq 0.001$, ** or $P \leq 0.01$, * or $P \leq 0.05$ corresponding to a Student ratio greater than 3.29, 2.58 and 1.96 respectively under a normality assumption.

The structure of the tax revenues which serve at covering school expenditures is totally different between the two regimes as shown in Table 5 and also between the two groups of States, illustrating again the impact of a TLE. Independently of the TLE status, state and federal sources provide more

Table 5: Estimated total public spending in rich and poor school districts

	California		no-TLE	
	Rich	Poor	Rich	Poor
Local	\$7,152	\$3,611	\$11,182	\$1,544
State	\$6,015	\$8,908	\$6,772	\$7,194
Federal	\$747	\$1,409	\$744	\$1,544
Total	\$13,914	\$13,928	\$18,698	\$10,282

revenue to poor districts while of course local funding is more important in rich districts. The effect of a TLE is extremely sensitive on local funding. In California local funding is just double in rich districts when this proportion goes up 8 times in no-TLE States.

When we look at the regression coefficients which represent elasticities, there is also a large difference between California and the no-TLE States. In California, local funding has the highest elasticity in the rich regime when state funding has highest elasticity in the poor regime. In no-TLE States, local funding has an even highest elasticity in the rich regime. However, in the poor regime, the three sources of funding have an equal impact, underlying again the lack of redistribution in the no-TLE States. This contrast is coherent with the fact that in California polarisation and inequality have strong elasticities in the rich regime while in no-TLE States these elasticities, despite having equivalent signs, are much weaker. This contrast is an illustration of the importance of *Hypothesis 3*. As already noted, the political process in California manages to erase differences in public financing between rich and poor districts as shown in the last line of Table 5 when this difference remains rather strong in no-TLE States.

4.5 Checking robustness

We have checked the robustness of our results by considering an alternative indicator of school quality with *instructional expenditure*.We report in Table 6 empirical results only for the three key variables: Pol, $Gini$ and $\bar{\beta}$. We can confirm the main results we had, but in a less clear-cut way: the importance of polarisation and inequality in the rich regime with opposite signs and the impact of TLE for limiting difference in public spending between poor and rich school districts.

Table 6: Instructional expenditure for public school quality

	California					
	Rich regime $\beta < \bar{\beta}$			Poor regime $\beta > \bar{\beta}$		
	Coef.	S.D.	$t-$value	Coef.	S.D.	$t-$value
Pol	0.355	0.079	4.50***	0.042	0.100	0.42
Gini	-0.777	0.125	-6.22***	-0.155	0.158	-0.98
$\bar{\beta}$			0.256 (0.017)			
School districts	375			280		
Mean difference	$34					
	no-TLE States					
	Rich regime $\beta < \bar{\beta}$			Poor regime $\beta > \bar{\beta}$		
	Coef.	S.D.	$t-$value	Coef.	S.D.	$t-$value
Pol	0.188	0.040	4.69***	0.153	0.093	1.64
Gini	-0.113	0.085	-1.32***	-0.060	0.151	-0.39
$\bar{\beta}$			0.356 (0.003)			
School districts	820			348		
Mean difference	$2,506					

The reader used to significance codes with stars can apply the following conversion scale: *** or $P \leq 0.001$, ** or $P \leq 0.01$, * or $P \leq 0.05$ corresponding to a Student ratio greater than 3.29, 2.58 and 1.96 respectively under a normality assumption.

5 Conclusion

In this paper we have studied the important relationship between income polarisation, schooling choice and education politics. From a theoretical perspective, we extended the model developed by de la Croix and Doepke (2009) by considering an income distribution which is a mixture of two uniforms corresponding the poor and the middle class and of a Pareto to represent the rich group. This extension was necessary to measure the impact of income polarisation on public school quality.

A first theoretical result is that having a substantially higher proportion of low-income than of high-income families in a community negatively impacts the quality of public schooling, a finding confirmed by the evidence that low quality public schools are mainly concentrated in poor areas. A second theoretical result is that the effects of income inequality and polarisation might differ because an increase in income polarisation modifies both the spread and the mean of the income distribution. This result suggests that income polarisation, as well as income inequality, should be taken into account in the analysis of education politics.

We have then confronted our theoretical results to recent data at the school district level in California and in ten States that have never issued a limitation on local funding. Regression results have shown that polarisation positively (respectively negatively) impacts school quality in rich (respectively poor) districts in both groups. They have also revealed that a TLE led to a different opting-out mechanisms and possibly to different types of political coalitions when voting for local taxes. The local political decision process becomes very complex as in districts where poor households are dominant a large compensation is done by state and federal tax revenues. This compensation seems more effective in California than in the other group with no TLE.

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A Proofs

A.1 Proof of Proposition 1

We follow the analytical proof of Proposition 1 in Arcalean and Schiopu (2016). We start from the equation $\tilde{x} = s\,(1-\eta)/(\phi\eta\delta)$, with: $\delta = (1-\eta)^{1/\eta}$. We replace s by its optimal value defined in (17). We get:

$$\tilde{x} = \frac{1-\eta}{\delta(1+\eta\gamma\Psi)}\mu.$$

The left hand side of the above equation is increasing between zero and infinity. Because Ψ is a cumulative probability function, monotone increasing in $\tilde{x}$ at value in $[0,1]$, the right hand side is decreasing between $(1-\eta)\mu > 0$ and $(1-\eta)\mu/(\delta(1+\gamma\eta))$ with $\gamma > 0$ and $0 < \eta < 1$. Because of monotonicity and continuity, it necessarily exists an unique intersection point.

A.2 Proof of Lemma 1

First of all, note that the taxable income is given by $\mu/(1+\gamma)$ in each school district. Let us differentiate μ given in (3) wrt g:

$$\frac{\partial\mu}{\partial g} = \beta\frac{x_1}{2} - \frac{x_1 + x_2}{2} + (1-\beta)x_2\frac{\alpha}{\alpha-1}.$$

This derivative is positive if and only if:

$$\beta < \bar{\beta} = \frac{x_1(1-\alpha) + x_2(1+\alpha)}{x_1(1-\alpha) + 2\alpha x_2},$$

negative otherwise, and zero when $\beta = \bar{\beta}$, with $\bar{\beta} > 0$.

A.3 Proof of Lemma 2

Since $Pol = 4g\beta(1-\beta)$, we can rewrite $g \equiv g[Pol, \beta]$ and $\Psi \equiv \Psi[Pol, \beta]$. We have:

$$\frac{\partial\tau[\Psi]}{\partial Pol} \equiv \frac{\partial\tau[\Psi]}{\partial g}\Big/\frac{\partial Pol}{\partial g} = \frac{\gamma\eta}{1+\gamma\eta\Psi}\frac{\partial\Psi}{\partial g}.$$

Because $\partial Pol/\partial g > 0$ for all $\beta \in (0,1)$, the effect of polarisation on taxation depends on the effect of g on the participation rate.

A.4 Proof of Lemma 3

Because fertility and schooling choices are determined before the political process takes place, households are locked in a certain school regime depending on the chosen number of children. Consequently, the opting-out threshold $\tilde{x}$ can be taken as given. We derive equation (9) with respect to g. We obtain:

if $\tilde{x} < x_1$, $\frac{\partial\Psi}{\partial g} = \beta\frac{\tilde{x}}{x_1} > 0$;

if $x_1 < \tilde{x} < x_2$, $\frac{\partial\Psi}{\partial g} = \beta - \frac{\tilde{x}-x_1}{x_2-x_1}$;

if $\tilde{x} > x_2$, $\frac{\partial\Psi}{\partial g} = (\beta-1)(\frac{\tilde{x}}{x_2})^{-\alpha} < 0$.

The sign of the derivative is ambiguous only when $x_1 < \tilde{x} < x_2$. It is positive if $\tilde{x} < (1-\beta)x_1 + \beta x_2$ and negative otherwise. Given parameter restrictions, it follows directly that $\partial\Psi/\partial g > 0$ if $\tilde{x} < (1-\beta)x_1 + \beta x_2$ and negative otherwise.

A.5 Proof of Proposition 2

The sign of the derivative of $s^*[\Psi,\mu]$ with respect to Pol is the same as the sign of the derivative with respect to g since:

$$\begin{aligned}\frac{\partial s[\Psi,\mu]}{\partial Pol} &\equiv \frac{\partial s[\Psi,\mu]}{\partial g}\Big/\frac{\partial Pol}{\partial g}\\ &= \frac{\eta\phi}{(1+\gamma\eta\Psi)^2}\left((1+\gamma\eta\Psi)\frac{\partial\mu}{\partial g}-\gamma\eta\mu\frac{\partial\Psi}{\partial g}\right),\end{aligned}$$

with $\partial Pol/\partial g > 0$. The total sign will depend on the respective signs of $\partial\mu/\partial g$ and $\partial\Psi/\partial g$. The sign of $\partial\mu/\partial g$ is discussed in Lemma 1 and depends on $\bar{\beta}$. The sign of $\partial\Psi/\partial g$ depends on the position of the opting-out threshold wrt the two class boundaries. Therefore, we look at three different scenarios.

Assume first that $\tilde{x} \leq x_1$. In this case $\Psi = g\beta\tilde{x}/x_1$. Replacing $\tilde{x} = E[s](1-\eta)/\delta\phi\eta$ into Ψ, with $\delta = (1-\eta)^{1/\eta}$, we can express the participation rate as a function of expected school quality. Since in equilibrium, $E[s] = s$, we get:

$$\Psi = \frac{g\beta}{x_1}\frac{s(1-\eta)}{\delta\phi\eta}.$$

Rearranging in term of s and deriving with respect to g, we observe that $\partial s/\partial g = 0$ when:

$$\frac{\eta x_1\delta\phi}{\beta(1-\eta)g^2}\left(\frac{\partial\Psi}{\partial g}g-\Psi\right)=0.$$

It follows that $\partial s/\partial g = 0$ iff $\partial\Psi/\partial g = \Psi/g$, which is always positive, meaning that in equilibrium an increase in g always increases the participation rate. Since $\partial s[\Psi,\mu]/\partial Pol$ depends on the sign of both derivatives $\partial\Psi/\partial g$ and $\partial\mu/\partial g$, we can observe that when $\beta > \bar{\beta}$, then $\partial\mu/\partial g < 0$. Thus, $\partial s[\Psi,\mu]/\partial Pol < 0$. When $\beta < \bar{\beta}$, then $\partial\mu/\partial g > 0$. The sign of the derivative $\partial s[\Psi,\mu]/\partial Pol$ is therefore ambiguous.

Assume now that $\tilde{x} \geq x_2$. In this case $\Psi = 1-(1-\beta)g(\tilde{x}/x_2)^{-\alpha}$. Proceeding as in the previous scenario, we observe that $\partial s/\partial g = 0$ iff $\partial\Psi/\partial g = (\Psi-1)/g$. Since $0 < \Psi < 1$, it follows that $\partial\Psi/\partial g < 0$ when $\partial s/\partial g = 0$. Therefore, when $\beta < \bar{\beta}$ we observe that $\partial s[\Psi,\mu]/\partial Pol > 0$, because $\partial\mu/\partial g > 0$. Of course, when $\beta > \bar{\beta}$ the sign of $\partial s[\Psi,\mu]/\partial Pol$ is again ambiguous.

Finally, assume $\tilde{x} \in]x_1, x_2[$, so that $\Psi = (1-g)(\tilde{x}-x_1)/(x_2-x_1)+g\beta$. Proceeding as in the previous cases, we obtain that $\partial s/\partial g = 0$ iff $\partial\Psi/\partial g = (\beta-\Psi)/(1-g)$. Thus, the sign depends on the level of the participation rate to public school. In this scenario, if $\beta > \max\{\Psi;\bar{\beta}\}$, we get that $\partial\Psi/\partial g > 0$, $\partial\Psi/\partial\mu < 0$ and $\partial s[\Psi,\mu]/\partial Pol < 0$; if $\beta < \min\{\Psi;\bar{\beta}\}$, we get that $\partial\Psi/\partial g < 0$, $\partial\Psi/\partial\mu > 0$ and $\partial s[\Psi,\mu]/\partial Pol > 0$. When, however, $\beta \in [\Psi,\bar{\beta}]$ or $\beta \in [\bar{\beta},\Psi]$, the sign of $\partial s[\Psi,\mu]/\partial Pol$ is ambiguous.

Thus, it necessarily follows that if $\beta > \max\{\Psi; \bar{\beta}\}$ (poor regime) we observe $\partial s[\Psi, \mu]/\partial Pol < 0$; if $\beta < \min\{\Psi; \bar{\beta}\}$ (rich regime) we observe $\partial s[\Psi, \mu]/\partial Pol > 0$. In all other cases, the sign is ambiguous.

B Gini and parameter estimation for the income distribution

For each school district, let us call nc_i, $i = 1, \cdots, 10$, the number of households in each of the original ten income classes, n the total number of households, $\pi_i = nc_i/n$ the frequency of each class, while x_i are the class boundaries and μ_i the mean income inside each class. We need to estimate two types of parameters; first β and g for each school district from which we deduce Pol; and second the Gini coefficient which causes specific problems.

Let us first define n_j, $j = 1, 2, 3$ as the number of households in each of our three aggregated income groups: $n_1 = \sum_{i=1}^{3} nc_i$, $n_2 = \sum_{i=4}^{7} nc_i$, $n_3 = \sum_{i=8}^{10} nc_i$. For each school district, we estimate β and g as:

$$\hat{g} = 1 - n_2/n, \tag{19}$$

$$\hat{\beta} = n_1/(n\,\hat{g}), \tag{20}$$

from which we can estimate our polarisation index.

In order to get the most precise estimate of the Gini for each school district, we have to use the information contained in the original ten classes, together with the school district mean income which is provided in the data set. Cowell (1995, page 110) and many authors suggest to use the empirical Lorenz curve and compute the surface between this curve and the 45 degree line. This gives a lower bound for the Gini, assuming that in each class everybody has got the same income μ_i:

$$\hat{G} = \frac{1}{\mu} \sum_{i=1}^{10} \sum_{j=i}^{10} \pi_i \pi_j |\mu_i - \mu_j|.$$

An upper bound is found by adding a correcting term, based on the assumption of maximum inequality within each class. When the number of classes tends to infinity this correcting factor tends to zero. The delicate question is that the data base provides the mean total income μ, but not the mean inside each class μ_i. We can estimate the μ_i using the midpoint approximation. Then the mean of the open-ended group can be obtained by difference:

$$\mu_{10} = (\mu - \sum_{i=1}^{9} \pi_i \mu_i)/\pi_{10}.$$

However, for some school districts, we obtained a value which is lower than the upper bound x_{10}, even if $n_{10} > 0$. So, this method cannot be used with our data set.

We have assumed that the income of the aggregated rich class (over x_8) followed a Pareto distribution with coefficient α. Extending the suggestions made in Quandt (1966) and in von Hippel et al. (2015), α can be estimated as follows:

$$\hat{\alpha} = \frac{\log(nc_8 + nc_9 + nc_{10}) - \log(nc_{10})}{\log(x_{10}) - \log(x_8)}, \tag{21}$$

with $x_{10} = 200$ and $x_8 = 100$. The mean of the last open class is given by $\mu_{10} = x_{10}\hat{\alpha}/(\hat{\alpha} - 1)$. But this method fails for some very rich districts for which the usual classes are not enough detailed entailing that $nc_{10} > nc_8 + nc_9$. This happens for 13% of cases in California and for 8% in the ten no-TLE States. One solution is to split the last class and create a new class between say $x_{10} = \$200,000$ and $x_{11} = \$300,000$ with $\widetilde{nc}_{10} = nc_{10}/1.4$ so that the last top open class contains $\widetilde{nc}_{11} = (1 - 1/1.4)nc_{10}$. Of course the choice of 1.4 is arbitrary. The resulting estimator for α becomes:

$$\hat{\alpha} = \frac{\log(nc_8 + nc_9 + \widetilde{nc}_{10} + \widetilde{nc}_{11}) - \log(\widetilde{nc}_{11})}{\log(x_{11}) - \log(x_8)}. \tag{22}$$

In some districts $\hat{\alpha} \leq 1$, meaning that the mean of the last open class is not defined. Because the data base provides the mean income for each school district, we can incorporate this information to provide a correction for α, forcing the overall mean of our three member mixture to be as close as possible to the overall mean provided in the data set. Let us call $\mu(\alpha, \hat{g}, \hat{\beta})$ the function giving the mean of our income distribution model conditionally on the estimated β and g. We then minimise in α the loss function:

$$(\mu(\alpha, \hat{g}, \hat{\beta}) - ms)^2 + (\alpha - \hat{\alpha})^2,$$

where $\hat{\alpha}$ is the initial estimator and ms the empirical mean provided in the data set. With this method, the mean α is 2.66 (minimum 1.17) in California and 4.09 (minimum 1.10) in the ten no-TLE States.

C Data bases

The *Elementary/Secondary Information System* (ELSI) collects detailed information on public and private schools.[10] The *American Community Survey* (ACS) provides demographic, social, economic, and housing data for the US.[11] The particularity of these two data bases is that they provide information at the level of school districts within a selected US State. Data in the ACS are available at the school district level only with the five-year estimates since 2009. The last available period is 2015-2019. We took a selection of US States, California on one side as a representative of the States

[10]Available on: https://nces.ed.gov/ccd/elsi/.

[11]Available on: https://nces.ed.gov/programs/edge/Demographic/ACS.

which have implemented a TLE and the group of 11 States with no TLE as provided by Yuan et al. (2009). We later eliminated Vermont from this list for reasons explained in the text.

Data in the ELSI are available since 1994-1995 on a yearly basis. In order to be coherent with the ACS data base, we collected 5 years of observations between 2015 and 2019, retained the year 2019 as the most complete one. Then, we imputed missing values and computed the mean over the five previous years. We have eliminated from the ELSI file school districts where information was missing in the last year together with abnormal observations. This means school districts with less than 11 students, with zero instructional expenditure or with zero total revenue. In total, there remains 3,817 observations for the ELSI file. After imputation and taking the mean, we eliminated observations when the pupil teacher ratio was smaller than 2 and larger than 42, when the total expenditure per pupil was greater than \$100,000, when total local revenue per student greater than \$60,000 and when the Total number of Students per school was greater than 40,000, suspecting these observations of being outliers or resulting from undue calculations. This led to eliminate 103 extra observations. Finally, there remains 3,714 observations in our ELSI file.

The two files were merged according to their LEAID (school district identifier). After merging, there remains 1,823 common observations when Vermont is excluded, 655 for California and 1,168 for the ten States without TLE.