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Sovereign Risk, Financial Fragility and Debt Maturity

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Abstract

This paper studies the transmission of a sovereign debt crisis in which a shift in default risk generates a recession and gives rise to a doom loop between sovereign distress and bank fragility with important amplification effects. The model is used to investigate the macroeconomic and welfare effects of altering debt maturity during the crisis. Short-term maturities alleviate the bankers' losses on long-term bonds and moderate the recession at the cost of higher levels of debt in the future. In contrast, long-term maturities are more effective to reduce the households' welfare losses as they lower default risk and distortionary taxes.

Keywords: Debt Crisis, Sovereign Default Risk, Financial Fragility, Maturity Dynamics

JEL classifications: E44, E62, H12.

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1 Introduction

At the end of 2011, the ratios of public debt to GDP in the peripheral countries of the Eurozone – Greece, Italy, Spain, Portugal and Ireland – reached their highest levels, raising doubts about the governments' ability to meet their debt obligations. At the same time, the high exposure of domestic banks to their own government's debt in these countries made their equity value dependent on the perceived solvency of the sovereign. The rollover risks associated with the high levels of public debt and the nexus between sovereign and bank credit risk led to a real concern about debt management in the periphery. In particular, some countries (e.g., Greece) lengthened the maturity structure of their liabilities in order to avoid the rollover of debt at very high spreads and the risks of a self-fulfilling crisis. Meanwhile, some other countries (e.g., Italy and Spain) adopted the opposite policy, namely shifting towards short-term maturities. The motives behind these governments' preference for short-term borrowing in the presence of high rollover risks appear ambiguous.

In this paper, I study a model that captures several important features of the Italian debt crisis: a sovereign default risk that is partly driven by factors orthogonal to the economy and partly related to the level of public debt to GDP, a banking sector exposed to risky government debt, distortionary taxation financing the high levels of public debt, and a dynamic debt maturity structure. A primary contribution of the paper is to analyze the transmission mechanism of such a crisis, highlighting not only the disturbances induced by an exogenous shift in default risk but also the amplification effects driven by the interactions between the financial system and the public sector. As in the Italian scenario, news of a potential future default in the model inflicts capital losses for bankers and weakens the government's fiscal situation, thereby leading to a recession. The resulting high levels of public debt to GDP trigger a substantial amplification mechanism of the crisis, which relies on the feedback loop between sovereign risk, bank fragility and distortionary taxation.

A second contribution of this research is to investigate the macroeconomic and welfare effects of altering the maturity structure of public debt in response to the crisis. I show that besides raising the rollover costs of debt, shifting towards short-term maturities reduces the exposure of domestic bankers to the price risk of long-term debt and alleviates their capital losses, which mitigates the recession at the cost of high levels of debt afterwards. In contrast, shifting towards long-term maturities is more effective to reduce the welfare costs incurred by households. By reducing the debt rollover costs, long-term maturities lead to lower levels of public debt, which results in a lower endogenous sovereign risk and lower distortionary taxes dampening households' consumption.

I develop a New Keynesian model with financial intermediaries that collect deposits from households and use them in order to grant loans to firms and buy long-term government bonds. As in Gertler and Karadi (2011), there is an agency problem between bankers and households that introduces an endogenous constraint on the lending ability of banks. This constraint binds only occasionally, when the intermediary leverage ratio is high. Additionally, due to their exposure to long-term government bonds, bankers are sensitive to swings in bond prices. These two features play a key role in the model since they open the door for a link between bank liquidity and sovereign creditworthiness and for a financial accelerator mechanism.

As in Corsetti, Kuester, Meier and Mueller (2014), I define sovereign default risk as a non-linearly increasing function in the debt to GDP ratio, and consider that only the *ex-ante* default probability matters for the pricing of bonds, an actual default being neutral *ex-post*. However, I depart from a purely endogenous default probability by assuming that sovereign risk is partly exogenous to the economy. The exogenous source of risk is motivated by the empirical literature showing that foreign factors were a critical driver of the surge in Italian sovereign spreads during the debt crisis[1]. The model also features distortionary labor taxes used to stabilize the debt-to-GDP ratio.

As for the maturity structure, I model long-term bonds in a way similar to Chatterjee and Eyigungor (2012): average maturity is captured by controlling for the fraction of bonds that does not mature at the end of the period. I differ from Chatterjee and Eyigungor (2012), however, by introducing a dynamic feature to the average maturity of bonds. I do so by allowing for the fraction of surviving bonds to vary in time according to variations in output and the debt-to-GDP ratio, through a rule decided by the government.

I first analyze the transmission of a sovereign debt crisis initiated by a shock to the government's probability of default. The sovereign risk shock is calibrated to match the rise in Italian sovereign spreads during the crisis. When sovereign default risk rises, the market value of long-term government bonds drops. As banks hold these assets on their balance sheets, their net worth declines and their borrowing ability contracts. The resulting fall in investment generates a recession. Furthermore, the collapse of bond prices raises the rollover costs of public debt, which deteriorates the fiscal situation of the government and leads to higher levels of debt. The higher levels of public debt to GDP give rise to a quantitatively important amplification mechanism, relying on endogenous sovereign risk, financial fragility and distortionary labor taxes.

[1] See, for example, Zoli (2013) and Bahaj (2019).

I then study the macroeconomic effects of changing the maturity structure of debt during the crisis by considering two opposite scenarios of a decline and a rise in average maturity. When the government shifts towards short-term maturities, it raises the stock of debt that needs to be rolled over which leads to higher rollover costs for the government. Nevertheless, short-term maturities are profitable for bankers. A greater share of short-term debt on the balance sheets of bankers reduces their exposure to the price risk associated with higher default probabilities, as it only affects the returns of long-term bonds. As such, shortening the maturity of debt dampens the bankers' losses from the crisis and relaxes their leverage constraints, which eventually lowers sovereign and credit spreads and moderates the recession even though public debt rises afterwards. The effect of lengthening the maturity of debt on the recession is ambiguous. Long-term maturities reduce the rollover costs of debt, which leads to lower distortionary labor taxes and a lower sovereign default risk, but they also raise the sensitivity of bankers' net worth to the drop in bond prices, making them more exposed to the (lower) rise in sovereign risk during the crisis.

In terms of welfare, however, the optimized response of the government in order to reduce the households' welfare losses from the crisis is to lengthen the maturity structure of debt. The reason is that the interaction between the endogenous sovereign risk channel and the labor tax channel makes high levels of public debt too costly for households in terms of welfare, which leads to a favoritism for the maturity response that lowers the debt burden. The optimized increase in debt maturity, though, inflicts more losses for financial intermediaries and worsens the economic downturn in general in the medium run.

As a final experiment, I suppose that public spending is used jointly with maturity as policy instruments in order to reduce the welfare costs from the crisis. The optimized joint policy is able to neutralize the welfare losses and to moderate the debt crisis in general. During the economic downturn, a drop in public spending helps stabilize public debt when the government shifts towards short-term maturities, which mitigates the adverse effects of the latter on welfare. When the economy starts recovering, lengthening debt maturity does not hurt bankers, as leverage constraints are no longer binding, and allows to reduce the debt burden. Lower levels of debt reduce distortionary taxation and open more room for fiscal expansion. The latter is valuable for households and enhances private consumption.

This paper relates to some of the recent contributions that emphasize on the feedback loop between sovereign risk and bank crises, such as Acharya, Drechsler and Schnabl (2014); van der Kwaak and van Wijnbergen (2014, 2017); Kirchner and van Wijnbergen (2016); and Auray, Eyquem and Ma (2018), among others. In this literature, the interdependence of sovereigns and banks is analyzed in the context of a financial crisis in which the recession is triggered by a distress in the financial sector that spills over to the public sector. In contrast to these studies, I consider a scenario that is closer to the Italian debt crisis, in which the source of the disturbance is a rise in sovereign default risk. In particular, the crisis in my model is initiated by a shock to the government's probability of default that feeds back into the balance sheets of banks holding long-term bonds. In doing so, the setup bears close resemblance to Bocola (2016) who studies the transmission of exogenous sovereign risk to the banking sector. However, his paper abstracts from the reverse effects that financial distress has on the perceived solvency of the sovereign, which are key for the study of maturity policy in my framework.

A number of papers in the literature discuss the effects of debt maturity during periods of crises. Broner, Lorenzoni and Schmukler (2013) argue that the reason that emerging economies shift towards short-term maturities during crises is that borrowing long-term is costly since foreign investors demand higher bond returns in compensation for the high price risk. In my framework long-term maturities are costly because domestic financial intermediaries demand higher returns not only on government bonds but also on credit assets, which can further hurt the productive sector and exacerbate the crisis. The benefits of short-term maturities during episodes of high sovereign default risk are also highlighted in Arellano and Ramanarayanan (2012), Niepelt (2014), Perez (2017) and Aguiar et al. (2019). These papers point to short-term debt as a commitment device reducing the incentives of the government to default, and therefore lowering the default premia charged by lenders. Rodrik and Velasco (1999) and Cole and Kehoe (2000) show that short-term maturities can also raise the rollover risk of debt, making the country more vulnerable to self-fulfilling crises. Using a DSGE model, van der Kwaak and van Wijnbergen (2014) and Auray and Eyquem (2019) analyze the effects of different steady-state debt maturities on the consequences of a financial shock. They find that the longer the average maturity at the steady-state, the higher the capital losses for financial intermediaries and the lower the rise in public debt. The present analysis focuses on the effects of altering the maturity in the midst of a debt crisis, and the mechanisms through which the maturity change affects the economic dynamics and welfare.

The paper is organized as follows. Sections 2 and 3 respectively describe the model and its calibration. Section 4 discusses the results, and Section 5 concludes.

2 Model

The model is an extension of Gertler and Karadi (2011) in which banks hold risky government bonds on their balance sheets in addition to capital assets. Moreover, the model incorporates a maturity structure for government securities and the possibility of a partial sovereign default. The government issues these securities and raises distortionary and lump sum taxes in order to finance its expenditures and honor its debt obligations. The probability of a sovereign default is increasing in the debt-to-GDP burden and is also affected by an exogenous sovereign risk shock. The setup includes households, banks, intermediate good producers, capital good producers, retailers, a government and a monetary authority. There are two types of households, workers and bankers, who consume and save deposits in banks. Intermediate good producers rent labor from workers and purchase capital from capital producing firms in order to produce intermediate output. They finance their purchases of capital with loans from the banks. Retailers buy the intermediate goods and repackage them into differentiated goods, which are sold with a markup as final goods to households, capital producers, and the government. The monetary authority sets the nominal interest rate on deposits.

2.1 Households

The economy is populated by a unit mass continuum of infinitely lived households. Within each household there are two types of members: workers and bankers. Every period, a fraction $(1-f)$ of workers supply labor to intermediate good producers. The other fraction f consists of bankers running financial intermediaries. At the end of each period, workers transfer their wage back to the household. Bankers reinvest any earnings in the bank's asset holdings over several periods, and give their retained profits to their respective household only when they exit the banking sector. In order to ensure that all households have the same consumption pattern, perfect insurance within the household is assumed. Every period, households earn the wage of labor, the net worth of bankrupt banks, and the profits of firms, which are owned by households. Households use these funds to consume and save deposits in financial intermediaries.

Households derive utility from effective consumption C_t and disutility from labor supply L_t. The household's preferences also exhibit habit formation in effective consumption. The formulation of the utility function is as follows

$$U(C_t, C_{t-1}, L_t) = \log(C_t - h_{t-1}) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \tag{1}$$

where $h \in (0,1)$ measures the degree of habit formation, φ is the inverse of Frisch elasticity and χ is the weight of the disutility from labor.

The representative household maximizes expected life-time utility subject to its budget constraint

$$E_t \sum_{s=0}^{\infty} \beta^s U(C_{t+s}, C_{t+s-1}, L_t)$$

$$s.t. \qquad C_t + D_t = (1-\tau_t)W_t L_t + \Upsilon_t^f + \Upsilon_t^b + R_{t-1}D_{t-1} - T_t$$

Deposits D_{t-1} saved at $t-1$ earn a gross riskless interest rate R_{t-1}. W_t denotes the real wage, τ_t a distortionary tax on labor income, and T_t are lump-sum taxes. Υ_t^f and Υ_t^b are respectively the net profits from firms and bankrupt banks. The household's first-order conditions for consumption, labor supply and deposits write

$$U_{c,t} = \frac{1}{C_t - hC_{t-1}} - \frac{\beta h}{E_t(C_{t+1}) - hC_t} \tag{2}$$

$$\frac{\chi L_t^{\varphi}}{U_{c,t}} = (1-\tau_t)W_t \tag{3}$$

$$\beta E_t \Lambda_{t,t+1} R_t = 1 \tag{4}$$

with $\Lambda_{t,t+1} = \frac{U_{c,t+1}}{U_{c,t}}$.

2.2 Firm Sectors

The model contains three types of firms: capital producers, intermediate good producers and retailers. Intermediate good producers operate under perfect competition and borrow funds from financial intermediaries to purchase the capital necessary to the production process. At the end of each period, the firms pay workers and repay their loans to banks with the earnings from the sale of intermediate goods and the sale of the used capital. A capital producing sector buys up the used capital from intermediate good producers and transforms it, along with investment goods, into new capital which is sold again to intermediate good producers for the production of the next period. Monopolistically competitive retailers buy a continuum of intermediate goods and repackage them into a differentiated retail good. Aggregate final output is a composite of a continuum of retail goods.

Intermediate Good Producers

In this setup, intermediate good producers and capital producers are distinct agents. This assumption allows to isolate the dynamic investment decision, that is carried out by capital producers, from the static borrowing decision belonging to intermediate good producers. The production function of intermediate good producers takes a standard Cobb Douglas form given by

$$Y_{mt} = A_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{5}$$

where A_t is the total factor productivity. In period t, the intermediate good producer hires labor L_t and uses capital K_{t-1} in order to produce an intermediate output, which is sold for a relative price P_{mt} to retail firms. The purchase of capital in period $t-1$ at a price Q_{t-1} per unit is financed by issuing a claim for each unit of capital to banks, which trade at the same price. Given that firms operate under perfect competition and profits are zero in equilibrium, the gross interest rate paid on loans ($R_{k,t}$) is equal to the realized ex-post return on capital. The intermediate good producers sell back what is left of the capital stock to capital producers for the end-of-period price Q_t, and thus receive $(1-\delta)Q_t K_{t-1}$. Hence, period t profits are

$$\mathcal{P}_t^m = P_{mt} Y_{mt} - W_t L_t - R_{kt} Q_{t-1} K_{t-1} + (1-\delta) Q_t K_{t-1} \tag{6}$$

Each period, the firm maximizes expected current and future profits using the household's stochastic discount factor $\beta \Lambda_{t,t+1}$ and taking all prices as given

$$\max E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \mathcal{P}_{t+s}^m \tag{7}$$

The first-order condition with respect to labor is given by

$$W_t = (1-\alpha) P_{mt} \frac{Y_{mt}}{L_t} \tag{8}$$

In equilibrium profits are zero. Hence, the ex-post return on capital in period t can be found by substituting the first-order condition for wage into the zero-profit condition $\mathcal{P}_t^m = 0$:

$$R_{k,t} = \frac{\alpha P_{m,t} \frac{Y_{m,t}}{K_{t-1}} + (1-\delta) Q_t}{Q_{t-1}} \tag{9}$$

Capital Producers

The role of capital producers in the model is to isolate the investment decision that adds a dynamic feature to the optimization problem because of the introduction of investment adjustment costs. At the end of each period, capital producers buy the remaining stock of effective capital from intermediate good producers at a price Q_t. They combine the used capital with final goods, i.e. investment I_t, in order to produce the capital that will be used in the next period's production process K_t. Capital producers trade the new and the used capital at the same price Q_t. Hence, the profits returning to the households owning these firms are determined by the amount of investment. The profit at the end of period t writes

$$\mathcal{P}_t^c = Q_t K_t - (1-\delta) Q_t K_{t-1} - I_t - f\left(\frac{I_t}{I_{t-1}}\right) I_t \tag{10}$$

where investment adjustment costs are defined as follows

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta_i}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \ , \ \eta_i > 0 \tag{11}$$

and capital evolves according to the following law

$$K_t = (1-\delta) K_{t-1} + I_t \tag{12}$$

Capital producers choose the amount of investment maximizing expected current and discounted future profits

$$\max E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s} - 1) I_{t+s} - f\left(\frac{I_{t+s}}{I_{t+s-1}}\right) I_{t+s} \right\} \tag{13}$$

The first-order condition for investment gives

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \left(\frac{I_t}{I_{t-1}}\right) f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \beta \Lambda_{t,t+1} \left[\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \right] \tag{14}$$

Retailers

Retailers operate under monopolistic competition and face nominal rigidities *à la* Calvo (1983). They buy intermediate goods $Y_{m,t}$ for a relative price $P_{m,t}$ and repackage them into differentiated retail goods $Y_{f,t}$, which are sold at a nominal price $P_{f,t}$. It takes one intermediate goods unit to produce one retail good. Aggregate final output Y_t is a CES composite of a continuum of retail goods

$$Y_t = \left(\int_0^1 Y_{f,t}^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}} \tag{15}$$

where $\epsilon > 1$ is the elasticity of substitution between the differentiated goods of retailers. The demand that retail firms face for their goods is given by

$$Y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon} Y_t \tag{16}$$

and the aggregate price index is

$$P_t = \left(\int_0^1 P_{f,t}^{1-\epsilon} df \right)^{\frac{1}{1-\epsilon}} \tag{17}$$

Nominal rigidities are introduced by assuming that in each period the monopolistic retailer is able to reset his price with a probability $(1-\gamma)$. During the periods in which the firm cannot re-optimize, it indexes its price to the inflation of the foregoing period. The probability that the second event happens for i periods is γ^i. Hence, the retailers' problem is to choose the optimal price P_t^* to solve

$$\max E_t \sum_{i=0}^{\infty} (\gamma\beta)^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} - P_{m,t+i} \right] Y_{f,t+i}$$

$$s.t. \qquad Y_{f,t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon} Y_t$$

where $P_t^* \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} = P_{f,t+i}$ is the nominal price of retail goods in period $t+i$, $\Pi_t = P_t/P_{t-1}$ is gross inflation, and γ_p denotes the parameter of price indexation. As retailers' only input is intermediate goods sold under perfect competition, the marginal cost of retail firms equals the relative intermediate output price $P_{m,t}$. The first-order condition for the optimal price setting writes

$$E_t \sum_{i=0}^{\infty} (\gamma\beta)^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} (\Pi_{t+k-1})^{\gamma_p} - \frac{\epsilon}{\epsilon-1} P_{m,t+i} \right] Y_{f,t+i} = 0 \tag{18}$$

The above necessary condition implies

$$\Pi_t^* = \frac{\epsilon}{\epsilon-1} \frac{F_t}{Z_t} \Pi_t \tag{19}$$

where F_t and Z_t are defined as

$$F_t = P_{m,t} Y_t + \gamma\beta\Lambda_{t,t+1} \Pi_t^{-\gamma_p \epsilon} \Pi_{t+1}^{\epsilon} F_{t+1} \tag{20}$$

$$Z_t = Y_t + \gamma\beta\Lambda_{t,t+1} \Pi_t^{\gamma_p(1-\epsilon)} \Pi_{t+1}^{\epsilon-1} Z_{t+1} \tag{21}$$

Due to specific assumptions on nominal rigidity, the aggregate price index can be defined as

$$\Pi_t^{1-\epsilon} = (1-\gamma)(\Pi_t^*)^{1-\epsilon} + \gamma \Pi_{t-1}^{\gamma_p(1-\epsilon)} \tag{22}$$

Aggregate final output is related to the aggregate intermediate output in the following way

$$Y_{m,t} = \Delta_{p,t} Y_t \tag{23}$$

where $\Delta_{p,t} = \int_0^1 \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon} df$ measures the distortion introduced by the dispersion in individual relative prices. It evolves according to the following law of motion

$$\Delta_{p,t} = (1-\gamma)\left[\frac{1-\gamma\Pi_{t-1}^{\gamma_p(1-\epsilon)}\Pi_t^{\epsilon-1}}{1-\gamma}\right]^{\frac{\epsilon}{\epsilon-1}} + \gamma\Delta_{p,t-1}\Pi_{t-1}^{-\gamma_p\epsilon}\Pi_t^{\epsilon} \tag{24}$$

2.3 Banks

There is a continuum of banks that lend funds obtained from households to intermediate good producers and government. In particular, they collect deposits D_t from households and combine them to their net worth N_t. The funds are used to purchase the claims issued by intermediate good producers for the acquisition of capital, and government bonds B_t at a price Q_t^b. The balance-sheet of bank j has the following structure

$$Q_t K_{j,t} + Q_t^b B_{j,t} = N_{j,t} + D_{j,t} \tag{25}$$

and its net worth evolves as follows

$$N_{j,t} = R_{k,t} Q_{t-1} K_{j,t-1} + R_{b,t} Q_{t-1}^b B_{j,t-1} - R_{t-1} D_{j,t-1} + T_t^b \tag{26}$$

where $R_{k,t}$ and $R_{b,t}$ are, respectively, the gross interest rates on capital assets and government bonds, determined after the realization of shocks at the beginning of period t, R_t is the real deposit rate and T_t^b is a lump-sum transfer from the government that covers the bankers' losses in the case of a sovereign default.[2] By substituting the balance sheet (eq. (28)) into equation (26), the law of motion for net worth reads

$$N_{j,t} = (R_{k,t} - R_{t-1}) Q_{t-1} K_{j,t-1} + (R_{b,t} - R_{t-1}) Q_{t-1}^b B_{j,t-1} + R_{t-1} N_{j,t-1} + T_t^b \tag{27}$$

[2] As I focus on the effects of an increased sovereign risk prior to a default, I follow Corsetti et al. (2014) and abstract from the *ex post* consequences of an actual default.

The financial intermediary keeps accumulating net worth until it exits the sector. Each period, the probability that the banker exits and becomes a worker the next period is $(1-\theta)$, in which case he gives his net worth $N_{j,t+1}$ to the household. Thus, the probability that the banker will be allowed to continue his activity the next period equals θ. Taking prices as given, the banker chooses the quantity of capital assets and government bonds maximizing the expected discounted terminal wealth

$$V_{j,t} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \beta^{i+1} \Lambda_{t,t+1+i}(N_{j,t+1+i}) \tag{28}$$

which can be defined in the following recursive form

$$V_{j,t} = \max E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\theta) N_{j,t+1} + \theta V_{j,t+1} \right] \right\} \tag{29}$$

As in Gertler and Karadi (2011), I introduce an agency problem between households and financial intermediaries. Bankers can divert a fraction of the assets at the beginning of the period, and transfer the funds back to their respective households. If that happens, lenders will withdraw their remaining funds and force the bank into bankruptcy. Therefore, depositors accept to supply their resources only if the bank's continuum value (i.e. the bankruptcy cost) is higher than the amount that the bank can divert. Accordingly, the incentive constraint on bankers is given by

$$V_{j,t} \geqslant \lambda (Q_t K_{j,t} + Q_t^b B_{j,t}) \tag{30}$$

where λ denotes the fraction of loans that the bank can divert. The banker's optimization problem is thus formulated as follows

$$V_{j,t} = \max_{K_{j,t}, B_{j,t}} E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\theta) N_{j,t+1} + \theta V_{j,t+1} \right] \right\}$$

$$s.t. \qquad \lambda (Q_t K_{j,t} + Q_t^b B_{j,t}) \leqslant V_{j,t}$$

The initial guess of the value function is

$$V_{j,t} = v_{n,t} N_{j,t} \tag{31}$$

where $v_{n,t}$ is the shadow value of net worth. Hence, the incentive constraint can be rewritten as

$$\frac{Q_t K_{j,t} + Q_t^b B_{j,t}}{N_{j,t}} \leqslant \frac{v_{n,t}}{\lambda} \tag{32}$$

A higher leverage ratio of a banker j raises his incentive to divert funds. Thus, this equation shows that the banker's leverage ratio is limited by a threshold $\frac{v_{n,t}}{\lambda}$. The latter depends negatively on λ as depositors supply less funds when the banker is expected to divert a higher fraction of assets, and positively on $v_{n,t}$ as a higher shadow value of net worth implies a higher bankruptcy cost for the banker, making him less willing to cheat. By substituting the conjectured formulation into the Bellman equation, one can write the continuum value of the banker as

$$\begin{aligned} V_{j,t} &= \beta E_t \Omega_{t+1} N_{j,t+1} \\ &= \beta E_t \Omega_{t+1} \Big\{ (R_{k,t+1} - R_t) Q_t K_{j,t} + (R_{b,t+1} - R_t) Q_t^b B_{j,t} + R_t N_{j,t} \Big\} \end{aligned} \tag{33}$$

where $\Omega_t = \Lambda_{t-1,t} \{(1-\theta) + \theta v_{n,t}\}$ denotes the stochastic discount factor of the banker. Accordingly, solving the banker's optimization problem yields[3]

$$\beta E_t \Omega_{t+1} (R_{k,t+1} - R_t) = \frac{\lambda \mu_t}{1 + \mu_t} \tag{34}$$

$$\frac{E_t (R_{k,t+1} - R_t)}{E_t (R_{b,t+1} - R_t)} = 1 \tag{35}$$

$$v_{n,t} = (1 + \mu_t) \beta E_t \Omega_{t+1} R_t \tag{36}$$

The Lagrange multiplier on the incentive constraint, μ_t, is defined as

$$\mu_t = \max \left\{ \frac{\lambda (Q_t K_t + Q_t^b B_t)}{\beta E_t \Omega_{t+1} R_t N_t} - 1 \quad , \quad 0 \right\} \tag{37}$$

where K_t and B_t are, respectively, the aggregate bankers' holdings of capital assets and government bonds, and N_t is the aggregate net worth.[4] Equations (34) and (35) show that the bankers' demand for capital assets and government bonds is such that the corresponding discounted spreads are increasing in the shadow price of the financial constraint. When $\mu_t = 0$, asset spreads are zero and the model is frictionless. Indeed, as bankers are not financially constrained, they keep building assets in order to arbitrage away differences between asset returns and funding costs. When $\mu_t > 0$, the binding financial constraint limits the ability of bankers to exploit such arbitrage opportunities, which leads to expected excess returns on the financial market. In this circumstance, any decline in the bankers' net worth relatively to their asset holdings increases the shadow price of funds (see eq. (37)). In order to meet their leverage requirements, bankers cut their demand for capital assets and government bonds, thereby leading to a rise in asset spreads.

[3] Detailed derivations can be found in Appendix A.

[4] When the financial constraint is binding, the Lagrange multiplier μ_t is the same across financial intermediaries (see Appendix A).

Aggregation of financial variables

The aggregate balance-sheet is given by

$$Q_t K_t + Q_t^b B_t = N_t + D_t \tag{38}$$

At the end of the period, only a fraction θ of bankers will remain a banker. The net worth of these bankers is then carried to the next period. Hence, the aggregate net worth of continuing intermediaries at the end of period $t-1$ equals

$$N_{e,t} = \theta \left[(R_{k,t} - R_{t-1}) Q_{t-1} K_{t-1} + (R_{b,t} - R_{t-1}) Q_{t-1}^b B_{t-1} + R_{t-1} N_{t-1} + T_t^b \right] \tag{39}$$

The fraction $(1-\theta)$ leaving the banking sector is replaced by the same fraction of households who enter the financial industry the next period. As in Gertler and Karadi (2011), the new bankers bring with them a starting net worth proportional to the assets of exiting bankers in the following way

$$\begin{aligned} N_{n,t} &= \frac{\omega}{1-\theta}(1-\theta)(Q_{t-1} K_{t-1} + Q_{t-1}^b B_{t-1}) \\ &= \omega (Q_{t-1} K_{t-1} + Q_{t-1}^b B_{t-1}) \end{aligned} \tag{40}$$

The total net worth is then defined as

$$N_t = N_{e,t} + N_{n,t} \tag{41}$$

2.4 Government

Maturity structure. The government issues long-term bonds B_t in period t, which are held by banks at a market price Q_t^b. Following Chatterjee and Eyigungor (2012), maturity is introduced by assuming that a fraction $(1-\rho_{c,t})$ of bonds matures each period and the government pays back the corresponding principal to bondholders. For the rest of bonds $\rho_{c,t}$, the government pays a coupon r_c and the principal payment is due in the future. Hence, $\rho_{c,t}$ governs the average maturity of government bonds. When $\rho_{c,t} = 1$, the bonds are consols paying a coupon r_c every period. When $\rho_{c,t} = 0$, then government debt is entirely short-term. For intermediate values of $\rho_{c,t}$, the bonds mature on average in $1/(1-\rho_{c,t})$ periods. While Chatterjee and Eyigungor (2012) consider a constant average maturity of debt, I consider that maturity is time-varying and use it as policy instrument by assuming that the fraction of surviving bonds $\rho_{c,t}$ is set according to the following rule

$$\log(\rho_{c,t}/\rho_c) = d_\rho \log(\rho_{c,t-1}/\rho_c) + (1-d_\rho)\left[\kappa_{\rho b}\log(b_{t-1}/b) + \kappa_{\rho y}\log(Y_{t-1}/Y)\right] \tag{42}$$

Here and in what follows, the variables without time subscript denote the steady-state values of their counterparts with time subscript. The parameters $\kappa_{\rho b}$ and $\kappa_{\rho y}$ measure the responsiveness of debt maturity to the deviation of respectively the debt-to-GDP ratio (b_t) and output (Y_t) from their initial steady-state values.

Bond return and budget constraint. Given the above assumptions, the realized gross return on government debt at the end of period $t-1$ is defined as

$$R_{b,t} = (1-d_t D)\left[\frac{(1-\rho_{c,t-1}) + \rho_{c,t-1}(r_c + Q_t^b)}{Q_{t-1}^b}\right] \tag{43}$$

where d_t is a default indicator variable and $D \in [0,1]$ is the haircut. Every period, the government can default and write off a fraction D of its outstanding debt; see further discussion on the default scheme below. When government bonds are not entirely short-term (i.e. $\rho_{c,t-1} > 0$), the realized return on bonds at the end of the period ($R_{b,t}$) is sensitive to variations in the bond price (Q_t^b). In particular, a decline in the resale price of long-term bonds at the end of the period induces a lower return as bankers experience capital losses on those bond holdings. Therefore, the fraction of surviving bonds $\rho_{c,t-1}$ captures the sensitivity of the return on government bonds to the bond price. The higher is the average maturity of bonds, the higher is the exposure of bankers to the price risk of long-term debt.

The budget constraint of the government is given by

$$Q_t^b B_t = (1-d_t D)\left[(1-\rho_{c,t-1}) + \rho_{c,t-1}(r_c + Q_t^b)\right] B_{t-1} + G_t - \tau_t W_t L_t - T_t + T_t^b \tag{44}$$

As in Corsetti et al. (2014), the *ex post* consequences of an actual sovereign default are neutralized by assuming that the government's transfers to bankers (T_t^b) fully compensate the imposed haircut in the case of a default. Therefore, a sovereign default *ex post* does not affect the level of public debt. Accordingly, transfers T_t^b are set as

$$T_t^b = d_t D\left[(1-\rho_{c,t-1}) + \rho_{c,t-1}(r_c + Q_t^b)\right] B_{t-1} \tag{45}$$

The consolidated government budget constraint then writes

$$Q_t^b \underbrace{(B_t - \rho_{c,t-1} B_{t-1})}_{\text{Newly issued bonds}} = \underbrace{[(1 - \rho_{c,t-1}) + r_c \rho_{c,t-1}]\, B_{t-1}}_{\text{Payments of principals and coupons}} + \underbrace{G_t - \tau_t W_t L_t - T_t}_{\text{Primary deficit}} \tag{46}$$

The labor tax rule is given by

$$\log(\tau_t/\tau) = \rho_\tau \log(\tau_{t-1}/\tau) + (1 - \rho_\tau)\kappa_\tau \log(b_{t-1}/b) \tag{47}$$

When used as a policy instrument, public spending is set according to the following rule

$$\log(G_t/G) = d_g \log(G_{t-1}/G) + (1 - d_g) \left[\kappa_{gb} \log(b_{t-1}/b) + \kappa_{gy} \log(Y_{t-1}/Y)\right] \tag{48}$$

In order to grant the stability of public debt in the medium run, I set a lump-sum tax that follows

$$T_t = T + \kappa_b (B_{t-1} - B) \tag{49}$$

Sovereign default risk. In the model, a sovereign default does not actually occur, but the government faces every period a time-varying probability of default that agents fully integrate in their optimizing decisions. The *ex ante* probability of a future default is key for the pricing of government bonds, and subsequently for the bankers' return on long-term bond holdings and the rollover costs of public debt. I consider sovereign default risk as partly driven by the country's economic fundamentals and partly exogenous. I model the endogenous probability of default in a way similar to Bi and Traum (2012). Sovereign default depends on a fiscal limit $b_{t,max}$, which is the maximum level of debt to GDP that can be sustained. If the debt-to-GDP ratio (b_t) is higher than the fiscal limit, the government partially defaults on its debt. The default process is summarized as

$$d_t = \begin{cases} 1 & \text{if } b_t \geqslant b_{t,max} \\ 0 & \text{if } b_t < b_{t,max} \end{cases} \tag{50}$$

The fiscal limit is stochastic and follows a logistic distribution. Hence, the probability of hitting the fiscal limit next period is given by the cumulative density function of the logistic distribution

$$p_t^d = Prob(d_t = 1) = \frac{\exp\left(\eta_0 + \frac{\eta_1}{4}(b_t - b)\right)}{1 + \exp\left(\eta_0 + \frac{\eta_1}{4}(b_t - b)\right)} \tag{51}$$

where $\eta_1 > 0$. As such, endogenous sovereign risk is non-linearly increasing in the level of debt to GDP. This approach to sovereign default falls under what is known as *non-strategic default*, where unanticipated large shocks can raise debt to such a level that the tax rate reaches the peak of the

Laffer curve and the government will be unable to fully repay its debt (see Davig et al. (2010), Bi (2012) and Bi and Traum (2012)). In contrast, Eaton and Gersovitz (1981), Arellano (2008) and others model sovereign default as the consequence of an optimal and strategic decision by the government. Under both approaches, the probability of sovereign default is a non-linearly increasing function of the level of debt to GDP.

I further assume that at the first period the economy can be hit by an exogenous sovereign risk shock which would increase the probability of default next periods. Accordingly, I define the effective *ex ante* probability of sovereign default as

$$\Delta_t^d = \exp(s_t) p_t^d \tag{52}$$

where $s_t = \rho_s s_{t-1} + \epsilon_t^s$. The exogenous source of risk is motivated by the empirical literature showing that foreign factors (e.g., news related to the Euro-area crisis) were a critical driver of the surge in Italian sovereign spreads during the debt crisis (see, for example, Zoli (2013) and Bahaj (2019)).

Intuitions on the effects of sovereign risk. Even though a sovereign default does not actually take place, the mere increase in the *ex ante* probability of default has important consequences on real and financial activity. A rise in sovereign default risk leads to a fall in the price of government bonds. First, a fall in the bond price induces capital losses for bankers on their long-term bond holdings, which leads to a decline in their net worth. If the financial constraint is binding, these losses result in a contraction of lending and a rise in both credit and sovereign spreads. Second, the drop in the bond price, driven by the increased sovereign risk and distressed banking system, translates into higher rollover costs of debt for the government, which raises the stock of total debt. A higher level of public debt over output distorts the economy through two channels: on the one hand, it leads to a higher distortionary tax burden that eventually magnifies the economic downturn; on the other hand, a higher ratio of debt to output further raises the probability of default because of the endogenous feature of the latter. This second effect gives rise to an amplification mechanism of the sovereign debt crisis, relying on sovereign risk, bank fragility and distortionary taxation. Therefore, the mere anticipation of a future sovereign default can be the trigger of a recession and have substantial negative effects on the economic activity.

2.5 The Monetary Authority and Market Clearing

The monetary authority controls the nominal interest rate on deposits i_t according to a standard Taylor rule

$$\log(i_t/i) = \rho_i \log(i_{t-1}/i) + (1-\rho_i)\left[\kappa_\pi \log(\Pi_t/\Pi) + \kappa_y \log(Y_t/\tilde{Y}_t)\right] \tag{53}$$

where $\tilde{Y}_t$ is the natural level of output.[5] In practice, the short-term interest rate in Italy is determined by the ECB, which considers inflation and output of the whole Euro-area, and not only Italy. The above specification then relies on a rather strong assumption. However, inflation and output in a number of Euro-area countries, including Italy, are correlated with those of the Eurozone (see, for instance, Cavallo and Ribba (2015)). Therefore, as explained in Fernández-Villaverde and Ohanian (2010), it is plausible to approximate the Taylor rule evaluated at the Euro-area level by a rule at the country level. By assuming that the deviations of inflation and the output gap in Italy are equal to those of the Eurozone plus idiosyncratic shocks at the national level, variations in the interest rate i_t can be interpreted as a stand-in for variations in the ECB policy rate plus variations driven by the idiosyncrasies of the Italian economy.

The nominal rate and the real interest rate on deposits are linked via the Fisher equation

$$i_t = R_t E_t(\Pi_{t+1}) \tag{54}$$

The good market clearing condition reads

$$Y_t = C_t + I_t + f\left(\frac{I_t}{I_{t-1}}\right) I_t + G_t \tag{55}$$

3 Calibration

The model is calibrated to match the Italian economy at a quarterly frequency. The calibration also follows Gertler and Karadi (2011) in many aspects. Table 1 summarizes the parameter values.

The annual debt-to-GDP ratio is set to 119% and the steady-state ratio of public spending over GDP is equal to 19.66%. I take these values from Bi and Traum (2012), based on Italian data from 1999 to 2010. The steady-state fraction of surviving bonds is calibrated to capture the average maturity of government bonds in Italy between 1998 and 2008, which is 6.12 years according to the OECD Stats database. I set the coupon on long-term bonds r_c to 4.6% annually, which is the 1998–2008 average of the interest rate on Italian government bonds with a maturity of 10 years found in the Statistical Data Warehouse of the ECB. The haircut D is equal to 0.55, in line with the estimates on the Greek debt restructuring reported in Zettelmeyer, Trebesch, and Gulati (2013). For the parameters of the fiscal stress function, I follow Bi and Traum (2012) and calibrate them to $\eta_0 = -3.8918$ and $\eta_1 = 10$. The value of η_0 is equivalent to a steady-state default probability of 2%. I use estimates from Forni, Monteforte and Sessa (2009) on Euro-area data to obtain values for

[5] Variations in the markup will serve as a proxy for variations in the output gap.

Table 1: Baseline parameter values.

Households		
β	0.99	Discount factor
h	0.81	Habit parameter
φ	1	Inverse of the Frisch elasticity
L	0.25	Fraction of time spent working
Firms		
α	0.33	Effective capital share
δ	0.025	Depreciation rate
η_i	1.728	Investment adjustment parameter
ϵ	4.167	Elasticity of substitution
γ	0.779	Calvo parameter
γ_p	0.241	Price indexation parameter
Financial intermediaries		
μ	0.001	Steady-state Lagrange multiplier
λ	0.2604	Fraction of assets that can be diverted
θ	0.975	Survival rate of the bankers
Φ	4	Leverage ratio
Government		
ρ_c	0.9592	Steady-state fraction of surviving bonds
r_c	0.011	Coupon
η_0	-3.8918	Default probability parameter
η_1	10	Default probability parameter
D	0.55	Haircut
G/Y	0.1966	Share of gov. spending
$B/4Y$	1.19	Steady-state debt-to-GDP ratio
ρ_τ	0.91	Persistence of labor tax
κ_τ	0.28	Response of labor tax to the debt-to-GDP ratio
κ_b	0.15	Response of lump sum tax to debt
ρ_s	0.95	Persistence of exogenous sovereign risk
Monetary policy		
κ_π	1.5	Inflation coefficient of the Taylor rule
κ_y	0.125	Output gap coefficient of the Taylor rule
ρ_i	0.8	Smoothing parameter of the Taylor rule

the parameters of the labor tax rule: $\rho_\tau = 0.91$, $\kappa_\tau = 0.28$ and $\tau = 0.45$. The feedback parameter of the lump sum tax rule is $\kappa_b = 0.15$ and the persistence of exogenous sovereign risk is $\rho_s = 0.95$. The parameters of the policy rules for debt maturity ($d_\rho, \kappa_{\rho b}$ and $\kappa_{\rho y}$ in eq. (42)) and government spending (d_g, κ_{gb} and κ_{gy} in eq. (48)) are fixed to zero in the baseline.

The steady-state Lagrange multiplier on the financial constraint μ is taken from Bocola (2016), who calculates it using Italian Flow of Funds and interbank spreads over the period 2002 to 2012. This results in an average multiplier of 0.001, implying small agency costs in the model and an annualized liquidity premium equal to 10 basis points on bonds and capital assets. The steady-state leverage ratio Φ is calibrated to 4, following Gertler and Karadi (2011). The steady-state Lagrange multiplier and leverage constraint pin down the fraction of assets that the banker can divert λ and the proportional transfer to the entering bankers ω. I set the survival probability of bankers at $\theta = 0.975$ to match an average survival period of a decade.

The discount factor β is set such that the risk-free rate matches the sample average of its empirical counterpart (4.1 percent annually), implying $\beta = 0.99$. The degree of habit formation in effective consumption is $h = 0.81$ and the inverse of the Frisch elasticity on labor supply is $\varphi = 1$. I calibrate the fraction of time spent working to 0.25 as in Bi and Traum (2012). As for the production sector, the effective capital share is $\alpha = 0.33$ and the depreciation rate is $\delta = 0.025$. The elasticity of substitution among differentiated goods is set such that the steady-state gross markup $\epsilon/(\epsilon - 1)$ is 1.315, which implies $\epsilon = 4.167$. The Calvo parameter γ is calibrated by taking the duration for which prices are expected to remain unchanged to be equal to 4.5 quarters, which gives $\gamma = 0.779$. The price indexation parameter is $\gamma_p = 0.241$ and the investment adjustment parameter is $\eta_i = 1.728$. Finally, the parameters of the Taylor rule are set to conventional values: $\rho_i = 0.8$, $\kappa_y = 0.125$ and $\kappa_\pi = 1.5$.

4 Results

In order to analyze the effects of a debt maturity policy during a sovereign debt crisis, I first set the stage for the intervention by simulating the crisis through a shock to the government's probability of default. I investigate the propagation mechanisms of the shock and highlight the important contribution of endogenous sovereign risk and distortionary taxation. Then, I evaluate the effectiveness of a debt maturity policy in moderating the crisis. To that end, I consider two policy scenarios, one in which the government lengthens the maturity structure of its debt in response to the crisis, and one in which it shortens debt maturity. Next, I derive an optimized maturity policy that minimizes the welfare losses generated by the shock and discuss its effects on the economy. Finally, I consider an optimized policy in which debt maturity is used jointly with public spending and explore the interaction between the two instruments.

The model is simulated non-linearly under perfect foresight using a non-linear Newton-Raphson method over 600 periods.[6] This solution method allows to handle non-linearities in the model since it does not rely on linearization, as opposed to perturbation methods. The non-linearity problem in this model stems from the occasionally binding leverage constraint on financial intermediaries. Under perfect foresight, agents know from the first period about future exogenous shocks, however, the shock that hits the economy at the first period is still a surprise to agents, as in stochastic simulations.

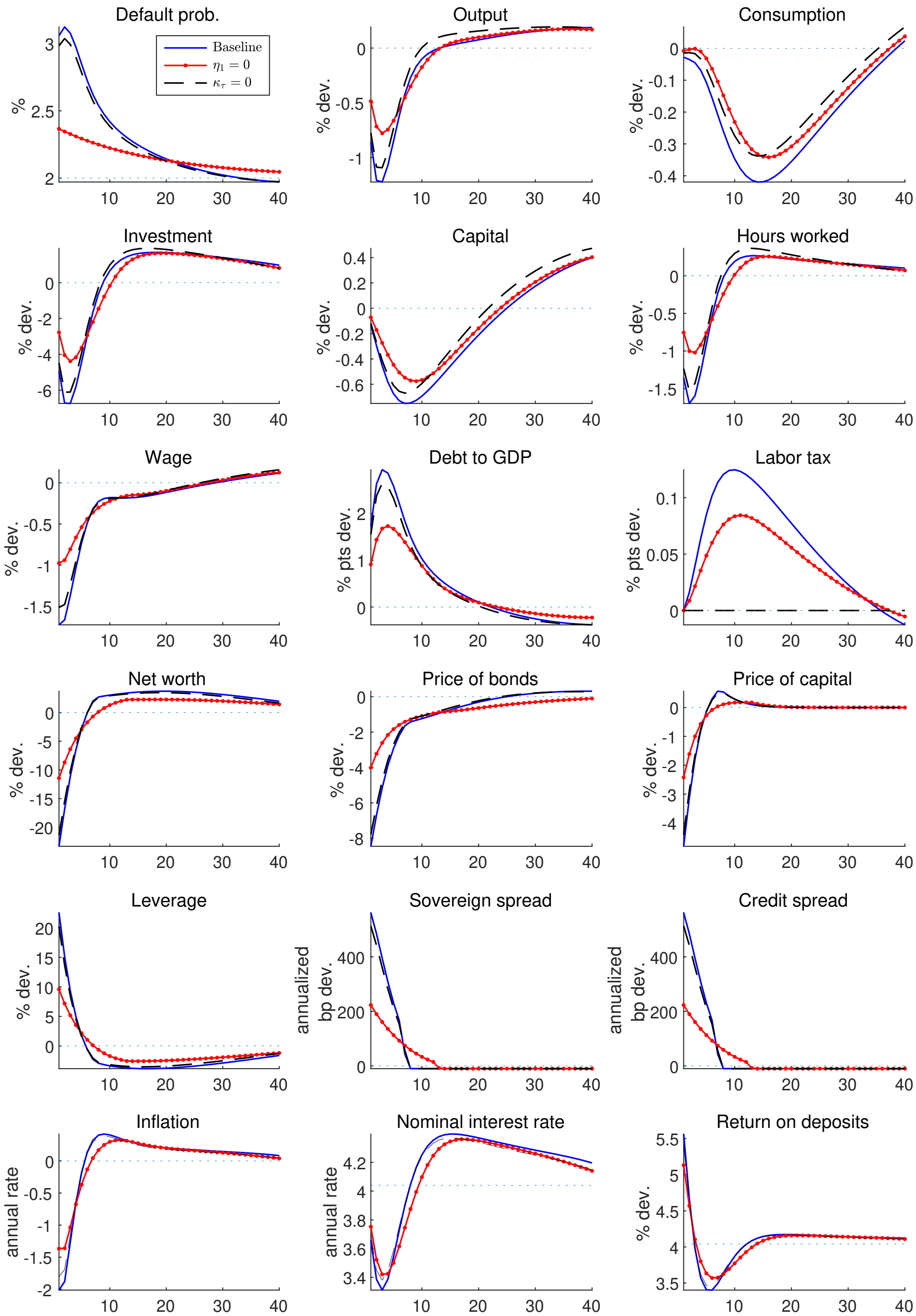
4.1 The sovereign debt crisis

As a preliminary step, I analyze the propagation mechanisms of a sovereign debt crisis without debt maturity policy. I model the Italian debt crisis as an exogenous sovereign risk shock raising the government's probability of default. The initiating shock is calibrated so that the rise in sovereign spreads is of a roughly similar magnitude to the one observed for Italian spreads in 2012, which is about 550 annual basis points. Both debt maturity and government spending are assumed to be fixed for now. Figure (1) shows the response of the baseline model to the shock and compares it to two variants of the model: one with a probability of default that is purely exogenous (i.e. $\eta_1 = 0$), and one without labor income tax (i.e. $\kappa_\tau = 0$). The goal is to highlight the contribution of the endogeneity of sovereign risk and of distortionary labor taxation in the transmission of the shock.

The sovereign risk shock raises the probability of a government default next period from 2% to 3.1%. The rise in the default probability induces a decline in the bond price because agents perceive the higher risk of experiencing future losses in their holdings of government debt. The decline in the resale value of government bonds feeds back into the balance sheets of banks, resulting in a drop in their net worth. This effect stems from the long-term maturity structure of government bonds that makes bankers sensitive to variations in the bond price. When bonds are entirely short-term, a rise in sovereign risk that results in a decline in the bond price does not generate capital losses for bankers unless the government actually defaults on its debt. Long-term maturity of public debt is therefore a key ingredient without which financial intermediaries would not be affected by an increase in the *ex ante* probability of default.

[6] The algorithm is implemented in Dynare and detailed in Juillard (1996), who builds on previous work by Laffargue (1990) and Boucekkine (1995).

Figure 1: Effects of a sovereign risk shock.



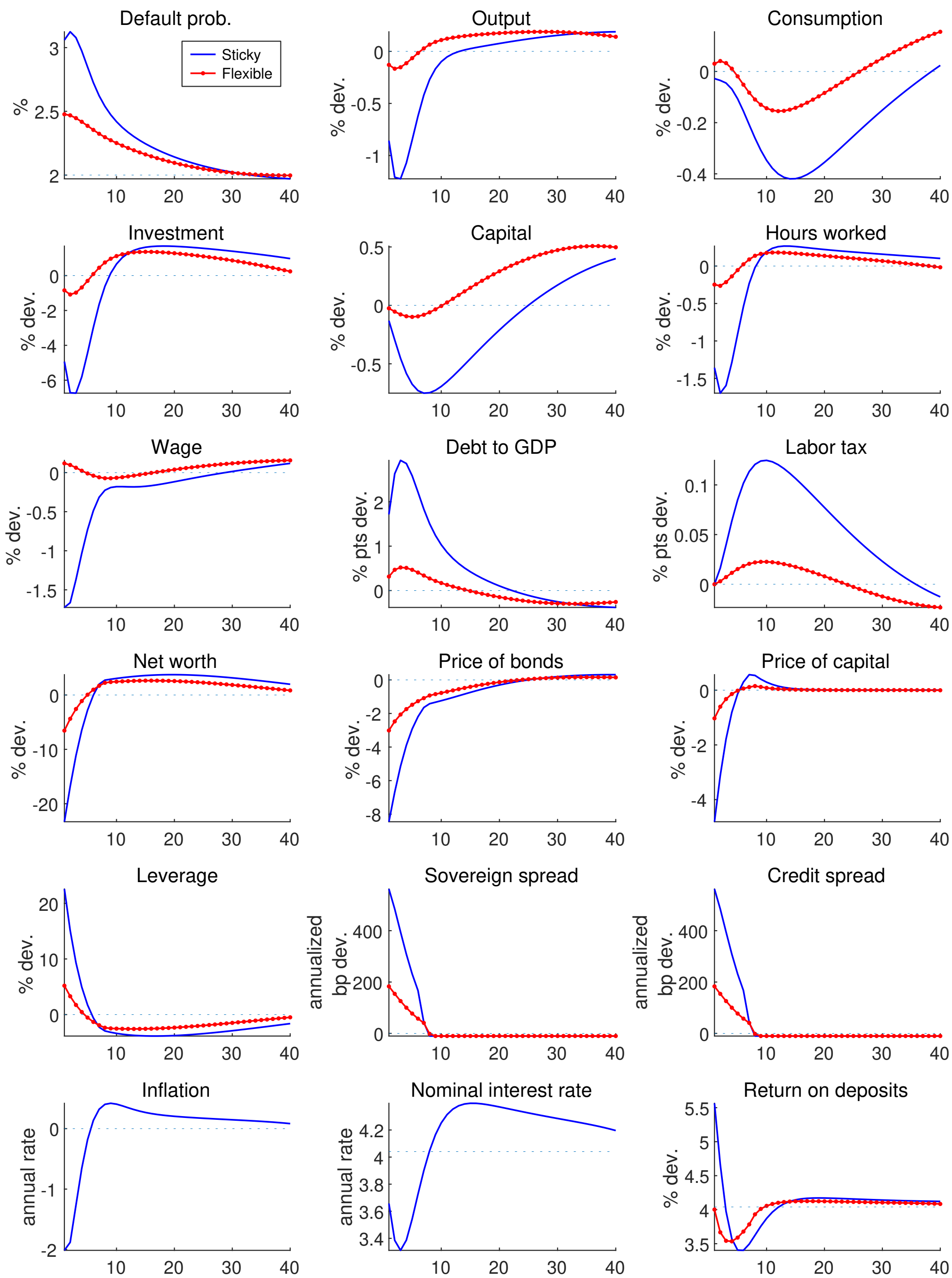
Blue solid line: baseline. Black dash-dotted line: model without labor tax.
Red solid line with dots: model with purely exogenous sovereign risk.

As financial constraints are binding at the steady state, a decline in the bankers' net worth makes them more balance sheet constrained by increasing the shadow price of funds. The higher funding costs reduce the bankers' ability to lend to non-financial firms and government, triggering fire sales of both capital assets and government bonds. As a consequence, the price of firms' assets falls down and the bond price further deteriorates, which translates into a rise in both sovereign and credit spreads. The collapse of asset prices inflicts further capital losses on bankers and triggers a financial accelerator mechanism. The fall in the demand for capital assets by financial intermediaries and the rise in credit spreads discourage investment in the productive sector. This results in a drop in output of 1.2%, thereby leading to a fall in hours worked, consumption and wages.

When the price of bonds falls down, newly-issued debt becomes more onerous. Consequently, the government needs to issue more debt securities in order to meet the higher rollover costs of debt. The rise in the stock of public debt, along with the fall in output induced by the economic downturn, lead to a higher debt-to-GDP ratio. The increase in the level of debt to GDP affects the economy through two channels. First, it raises labor income tax in order to stabilize government debt in the medium run, which amplifies the decline in hours worked, consumption and output, thereby leading to a further higher debt-to-GDP ratio. In Figure 1 we can observe that when public debt is stabilized using only lump sum taxation, the contraction in real activity is less pronounced than in the baseline. Second, a rise in the debt-to-GDP ratio increases the endogenous probability of default, which in turn further reduces the bond price and causes additional losses for bankers. Subsequently, the disturbances induced by a sovereign risk shock are magnified compared to the case where the default probability is purely exogenous. Figure 1 shows that the amplification effect associated with this channel is quantitatively important, as can be seen from a rise in the sovereign spread that is more than twice larger in the baseline. Therefore, the level of public debt to output is a key variable in the model, through which the (endogenous) sovereign risk channel and the labor tax channel interact with each other and generate a substantial amplification mechanism of the crisis.

The higher stock of government debt in the balance sheets of banks, as well as the increased spreads on their asset holdings, raise the bankers' profits from financial intermediation and restore their net worth after approximately 5 quarters. The resulting decline in the leverage ratio of banks is large enough to reduce the shadow price of funds to zero, which makes financial constraints no longer binding. Asset spreads fall down to zero as a result and investment rises by around 2% above its steady-state value, which helps foster the recovery of the economy.

Figure 2: Effects of a sovereign risk shock: the role of nominal rigidities.



Blue solid line: baseline. Red solid line with dots: model without nominal rigidities.

Nominal rigidities in the model constitute an additional important feature that amplifies the macroeconomic effects of a sovereign risk shock.[7] Figure (2) highlights the role played by this friction in

[7]A similar finding is reported by Del Negro et al. (2017) in the context of a financial crisis. They show that a

shaping the responses of the economy to the shock. When prices are sticky, a sovereign risk shock has larger consequences because it induces a rise in firms' markups that allows for a fall in consumption and a more pronounced drop in hours worked, investment and output. Absent nominal rigidities, the fall in investment driven by bank distress does not lead to a drop in output that is as high as in the baseline, due to the low response of hours worked. Indeed, given that capital is predetermined, the decline in output is mainly driven by the behavior of hours worked. However, when markups are constant, a fall in hours worked increases the marginal product of labor and reduces the marginal rate of substitution, equilibrium requires then consumption to increase. As a result, the fall in investment is offset by the rise in consumption and, eventually, the impact on aggregate output is very small. With nominal rigidities, the fall in the marginal cost (i.e. the rise in the markup) allows for a higher decrease in hours worked following the shock without raising consumption. The adjustment mechanism is also affected by the behavior of the return on deposits. The latter declines under flexible prices because of the increased agency costs, which stimulates consumption. In the presence of nominal rigidities, the fall in current and future marginal costs reduces expected inflation, thereby leading to a decline in the nominal rate of interest. Because of interest rate smoothing, though, the nominal interest rate decreases less than expected inflation. Consequently, the real rate rises on impact, putting downward pressures on consumption.

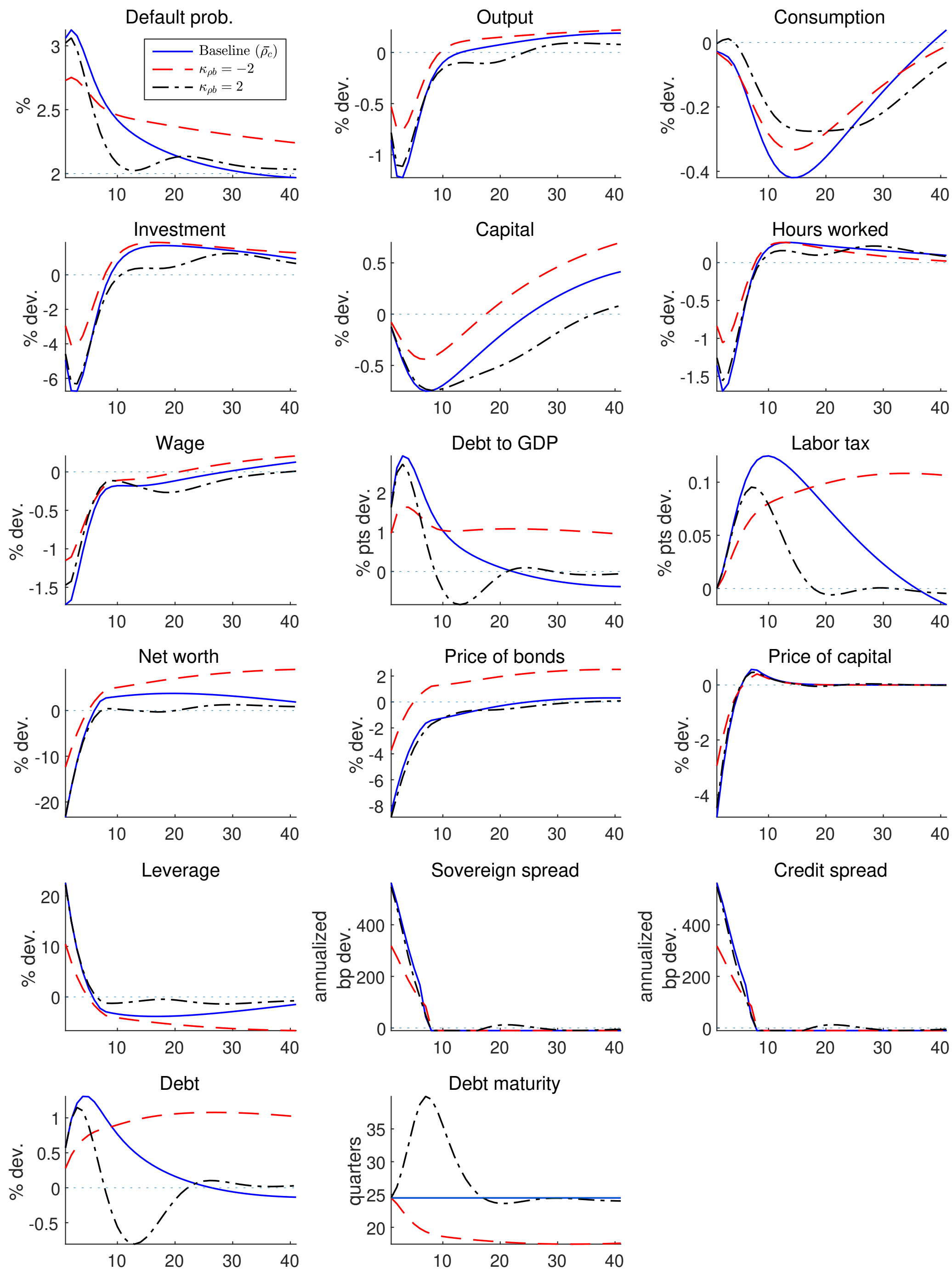
4.2 The effects of a debt maturity policy

In this section, I consider that the government responds to the sovereign debt crisis by altering the maturity structure of its bonds. My aim is to analyze how a debt maturity policy affects the fiscal situation of the government as well as financial intermediaries. Figure (3) reports the effects of two different interventions. In the first case (black dash-dotted line), the government lengthens the average maturity of debt in response to the crisis. The parameters of the policy rule for $\rho_{c,t}$ (eq. (42)) are calibrated to $\{d_\rho, \kappa_{\rho b}, \kappa_{\rho y}\} = \{0.9, 2, 0\}$. As such, average maturity responds positively to the rise in the debt-to-GDP ratio during the crisis. In the second (red dashed line), the government shortens average maturity in response to the crisis. The feedback parameter with respect to the debt-to-GDP ratio in the policy rule for $\rho_{c,t}$ is negative: $\{d_\rho, \kappa_{\rho b}, \kappa_{\rho y}\} = \{0.9, -2, 0\}$. Finally, for comparison, the blue solid line corresponds to the baseline with a constant debt maturity (i.e. $\{d_\rho, \kappa_{\rho b}, \kappa_{\rho y}\} = \{0, 0, 0\}$).

When the government lengthens the maturity structure of its debt, it increases the fraction of surviving bonds that entails coupon payments at the end of each period and reduces the fraction of

shock to the liquidity of private assets has very little effect on aggregate activity in the absence of nominal rigidities.

Figure 3: Effects a sovereign risk shock with debt maturity policy.



Blue solid line: baseline. Black dash-dotted line: rise in maturity.
Red dashed line: fall in maturity.

maturing bonds that it has to roll over every period (see eq. (46)). The net effect on the costs of debt financing, and therefore the stock of public debt, depends on the value of the coupon paid on long-term debt and on the price of newly-issued bonds. Essentially, for a one percentage point increase in the fraction of surviving bonds to induce a decline in new issuances, it requires that $1 - Q_t^b > r_c$. This condition has a simple interpretation. Public debt decreases after a one percentage point rise in $\rho_{c,t}$ if the rollover cost spared on the one percentage point decline in the fraction of maturing bonds is higher than the extra coupon paid on the fraction of surviving bonds. Following a sovereign risk shock, the rollover of maturing bonds becomes particularly costly because of the collapse of bond prices. Borrowing long term then allows to avoid rolling over its debt at very low prices. Thus, during the sovereign debt crisis, the fall in the rollover cost on short-term bonds after a rise in debt maturity largely outweighs the rise in coupon payments on long-term bonds. The resulting decline in debt servicing costs leads to a reduction in the level of total debt. Figure (3) confirms this prediction. The increase in average maturity from 6 years to around 10 years is followed by a decline in the stock of debt of around 0.8% under its steady-state value.

Altering the maturity structure of government bonds affects financial intermediaries as well. When the government shifts towards long-term maturities, it raises the share of long-term debt that bankers hold on their balance sheets. Subsequently, they become more exposed to the price risk of long-term debt. A deterioration of the bond price inflicts larger capital losses on bankers because of the higher sensitivity of the bond return to the resale value of long-term bonds. In this sense, since bankers anticipate the impact of the policy from the first period, the collapse of bond prices driven by the sovereign risk shock should result in higher sovereign and credit spreads, a higher contraction of lending, and ultimately a more severe crisis. However, the pricing of government bonds is also affected by future default probabilities, which in turn depend on the government's fiscal situation. In particular, the fall in public debt induced by the rise in average maturity reduces the ratio of debt-to-GDP, which lowers sovereign default risk and dampens the decline in the bond price. As a result, a rise in the maturity of government debt raises the sensitivity of bankers to the collapse of bond prices on the one hand, but dampens the price decline on the other hand by lowering the debt rollover costs and thus sovereign risk. The net impact of the policy on the balance sheets of banks is ambiguous, but for a wide range of parametrizations used for the policy rule, the net effect on bankers' net worth is either negligible or negative.

Figure (3) shows that the rise in average maturity has a small stabilizing effect on real activity along the first periods. This effect is mainly due to the lower rise in labor tax. Since long-term maturities reduce the rollover costs of public debt and therefore the total stock of debt, the resulting decline in the debt-to-GDP ratio leads to lower labor taxes, which moderates the fall in consumption, hours worked and output. During the first quarters, banks' net worth is relatively not impacted which implies that the effects of the policy on the balance sheets of banks roughly compensate each other. However, after approximately 10 quarters, the effects of the shock are magnified as the debt level rises and returns to its steady-state value, thereby raising endogenous sovereign risk.

Shortening the average maturity of debt is not symmetric to lengthening it. When the government shortens the maturity structure of its debt, it raises the stock of debt that needs to be rolled over every period. Again, in a context of a sovereign debt crisis in which bond prices are low and rollover costs are high, this induces a rise in the level of total debt even if coupon payments on long-term bonds decline. On the bankers' side, reducing the average maturity of government debt translates into a decline in the share of long-term debt on their balance sheets and a rise in the share of short-term debt. As a result, since bankers don't experience capital losses on their holdings of short-term bonds, they become less sensitive to the decline in bond prices during the crisis. At the same time, reducing the average maturity of debt has also an impact on the pricing of government bonds through endogenous sovereign risk. The rise in public debt associated with the higher rollover costs raises the government's probability of default, which puts downward pressures on the price of bonds. However, contrary to the previous case, this second effect has little impact on the balance sheets of bankers. Indeed, because lower maturities decrease their exposure to price risk, they suffer less from any decline in the bond price. Ultimately, shortening maturity alleviates the net worth losses for financial intermediaries despite its negative effect on the rollover risk of public debt. As we can see in Figure (3), this results in a sovereign debt crisis that is substantially less severe than in the baseline: sovereign spreads rise by around 300 basis points (against 550 basis points in the baseline), output falls by 0.7% (against 1.2%) and hours worked by 1% (against 1.8%). The stabilizing effect of the policy on consumption is not important, though, due to the higher levels of public debt afterwards that translate into higher labor taxes.

The asymmetry between the two maturity policies stems mainly from the policy's impact on the exposure of bankers to the price risk of long-term debt, which in turn determines the extent to which the policy-induced variations in endogenous sovereign risk affect bankers and thus the economy as a whole. One way to discern the contribution of the endogenous probability of default in the transmission of the two policies is to shut down this channel by assuming that sovereign risk is purely exogenous. As Figure 6 in Appendix D illustrates, the responses of the economy to the two policy rules are more symmetric without endogenous sovereign risk. When agents neglect the economy's fundamentals in the evaluation of the government's probability of default, lengthening debt maturity essentially amplifies the crisis and raises the levels of debt because of the higher exposure of bankers to price risk, even though it improves the government's fiscal situation in the medium run. In contrast, shortening debt maturity has a stabilizing effect on the economy in general, whether agents integrate or not the associated higher rollover risk in the pricing of bonds. This is due to the lower sensitivity of banks' returns to the deterioration of the bond price.

4.3 Optimized maturity rules

In the previous section, we have seen that shifting towards short-term maturities during the crisis has a significant stabilizing effect on bankers' net worth, and therefore output and most macroeconomic variables. Lower debt maturities, however, also imply higher levels of public debt and therefore higher distortionary taxes, which eventually dampen private consumption. This leads to the question of whether shortening the maturity structure of government debt is the best policy to enhance households' welfare during a sovereign debt crisis. To answer this question, I derive an optimized debt maturity policy that minimizes the welfare losses associated with the crisis. To do so, I compute the welfare costs generated by the shock as the Hicksian consumption equivalent ϵ solving the following equation

$$E_t \left\{ \sum_{s=t}^{\infty} \beta_g^{s-t} \left[U(C_s, L_s) - U(C(1-\epsilon/100), L) \right] \right\} = 0$$

ϵ measures the steady-state percentage of consumption loss associated with experiencing the crisis, and β_g denotes the discount factor of the government. The policy maker is assumed to be more short-sighted than households, i.e. it has a lower discount factor which is set to $\beta_g = 0.97$. Government myopia can be explained as the result of an expected finite planning horizon of the government that corresponds to the longevity of its survival in power (see Grossman and Van Huyck (1988), Acemoglu et al. (2011) and Rieth (2014)). The parameters of the debt maturity rule $\{d_\rho, \kappa_{\rho b}, \kappa_{\rho y}\}$ are chosen to minimize the Hicksian consumption equivalent ϵ to zero. The results are reported in Table (2). In order to shed light on the importance of the labor tax and the endogeneity of sovereign risk in the welfare effect of the maturity policy, Table (2) also reports the optimized policy rules when those two channels are shut down. Figure (4) depicts the responses of the economy under the optimized maturity rule in the baseline model.

The results show that the optimized response of the government in the baseline is to lengthen the

Table 2: Optimized policy rules.

	ϵ	ϵ^p	$\%\Delta\epsilon$	d_ρ	$\kappa_{\rho b}$	$\kappa_{\rho y}$	d_g	κ_{gb}	κ_{gy}
Baseline	0.0852	0.0483	43.3%	0.940	5.002	-5.440	-	-	-
Exog. sov. risk ($\eta_1 = 0$)	0.0588	0.0358	39.11%	0.919	-3.311	0.019	-	-	-
No labor tax ($\kappa_\tau = 0$)	0.0589	0.0290	50.76%	-0.511	-2.044	-1.703	-	-	-
Active G policy	0.0854	0	100%	0.842	-0.966	0.412	0.871	-2.997	2.444

Note: ϵ is the welfare loss in consumption-equivalent units from the crisis with fixed maturity and public spending. ϵ^p is the welfare loss from the crisis with optimized policy rules. $\%\Delta\epsilon$ denotes the welfare gain in percentage from the optimized policies. The remaining parameters correspond to the optimized coefficients of the policy rules.

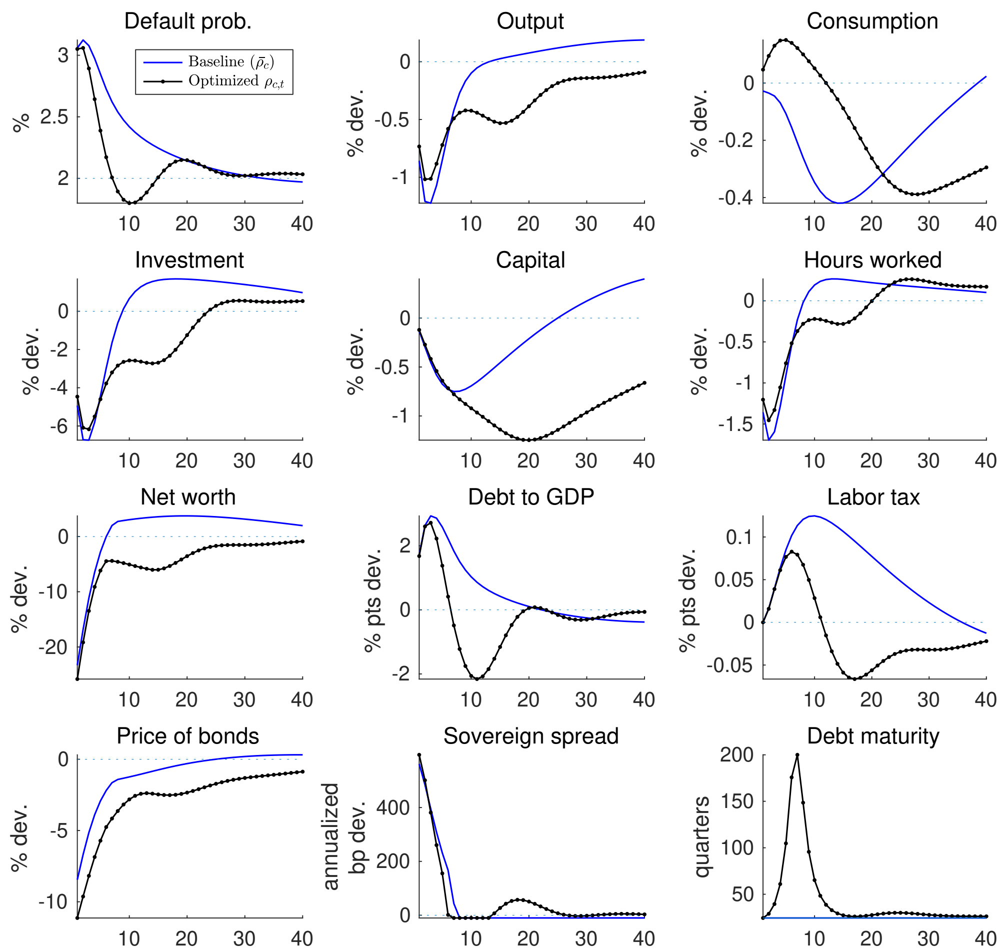
maturity structure of its debt. The welfare costs associated with the crisis decline from 0.0852 to 0.0483% of permanent consumption. That is, the shock under the optimized policy rule induces a welfare loss that is 43.3% lower than in the baseline with constant debt maturity. In order to understand this result, a useful step is to consider the optimized maturity rules when distortionary taxation or endogenous sovereign risk are absent in the model. Figures (7) and (8) in Appendix C report the responses of maturity and some key economic variables for these cases.

As shown in Table (2) and Figure (7), when sovereign risk is purely exogenous, shortening the maturity of public debt leads to a welfare loss from the crisis that is 39.11% lower than in the model without policy intervention. Lower debt maturities are welfare-enhancing in this case because the rise in the level of government debt caused by the policy does not feed back to the probability of default. Hence, even though shortening maturity makes bankers less sensitive to variations in endogenous sovereign risk, the policy has a larger stabilizing effect on the economy when the default probability is entirely exogenous. As a result, public debt rises less, inducing a lower increase in the labor tax, and the positive effect of the policy on real wages is greater as well. Subsequently, the policy is more effective in stimulating consumption and therefore welfare.

Likewise, when government debt is stabilized using only lump sum taxes, the optimized policy response is, again, to shorten debt maturity (see Fig. (8) in Appendix C). The reason is simple: decreasing the maturity structure of debt raises the level of public debt in the medium run, but the latter does not induce a higher labor taxation and thus a distortionary effect on private consumption. At the same time, the stabilizing impact of the policy on bankers' net worth moderates the crisis in general, which implies a lower decline in consumption.

Even though the labor tax channel or the endogeneity of sovereign risk *per se* do not offset the pos-

Figure 4: Effects of the shock with optimized maturity policy and constant public spending.



Blue solid line: baseline. Black solid line with dots: optimized maturity rule.

itive impact of short-term maturities on consumption and therefore on welfare, it is the interaction between these two channels that makes long-term debt maturities more effective in terms of welfare. Indeed, shortening debt maturity when the default probability is partly endogenous results in a lower stabilizing effect on the balance sheets of financial intermediaries and higher levels of public debt. Those provoke a more pronounced increase in the labor tax which dampens consumption. In contrast, while lengthening debt maturity magnifies the capital losses for financial intermediaries, it provides significant welfare gains for households by reducing the stock of government debt and therefore labor taxes. Figure (4) shows that a rise in average maturity from 6 to around 50 years during the debt crisis reduces substantially the labor tax, which raises private consumption and slightly stabilizes real activity as well during the first periods. However, this comes at the cost of a more distressed financial sector and a more severe crisis in general in the medium run. Indeed, the policy magnifies the deterioration of bankers' net worth, which leads to a higher rise in sovereign spreads to around 600 basis points and a larger drop in bond prices. When the debt-to-GDP ratio recovers, the negative effects of the policy on bankers' net worth, due to their higher exposure to price risk, spread to the rest of the economy thereby amplifying the crisis.

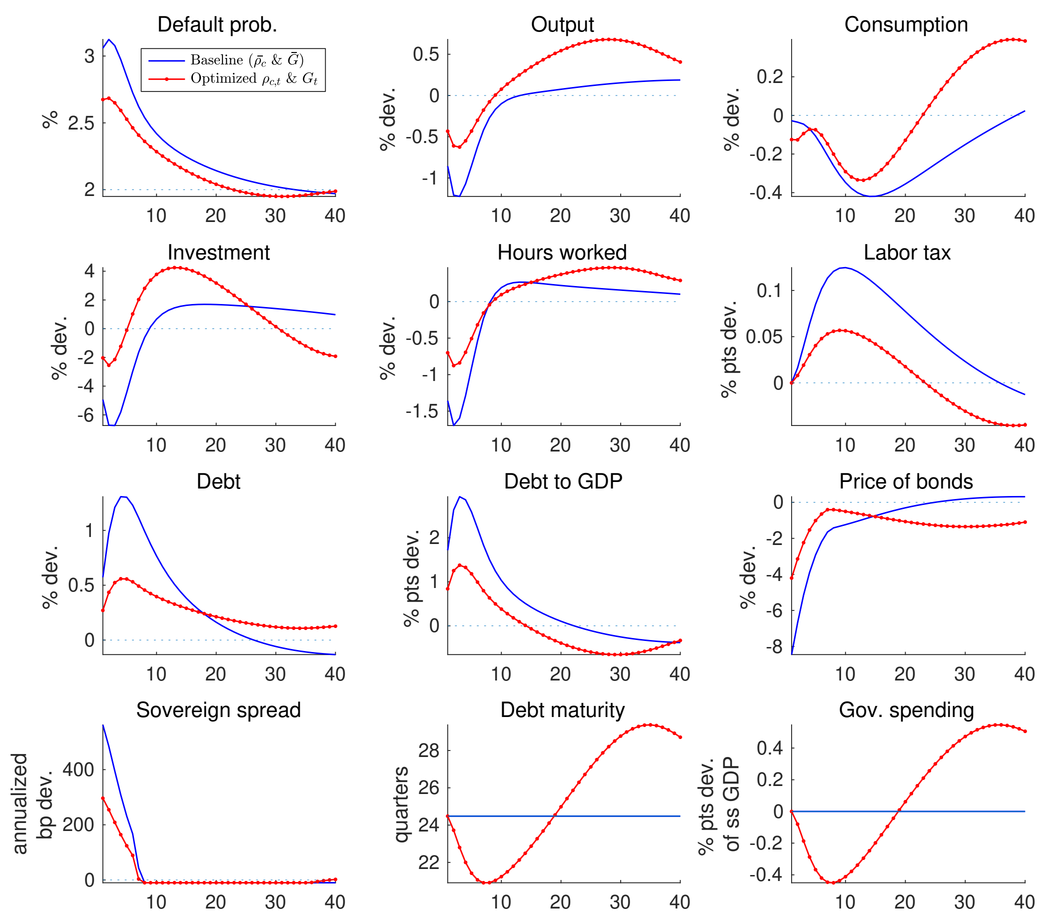
As a final experiment, I assume that public spending is valuable for households and I use it jointly with debt maturity as a policy instrument to minimize the welfare losses associated with the crisis. I consider that the effective consumption of households $\tilde{C}_t$ is a CES composite of private consumption C_t and public spending G_t in the following way

$$\tilde{C}_t = \left(\kappa_c C_t^{\frac{\nu-1}{\nu}} + (1-\kappa_c) G_t^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}}, \quad \nu > 0 \tag{56}$$

where κ_c scales the weight of private consumption in the effective consumption index $\tilde{C}_t$ and ν governs the degree of substitutability between private and public goods. When $\nu = 0$, private and public goods are pure complements. The substitutability between the two types of goods increases with ν. When $\nu \to \infty$, public and private consumption become pure substitutes. The preference parameter κ_c is calibrated such that the marginal utility of private consumption equals that of public spending ($U_c = U_G$), which gives $\kappa_c = G^{-1/\nu}/(C^{-1/\nu} + G^{-1/\nu})$. The elasticity of substitution between private and public goods is set to $\nu = 0.45$. This value indicates a complementarity between private and public consumption and is consistent with the estimates of Leeper, Traum, and Walker (2017).

I analyze the effects of a joint policy response by optimizing the parameters of the policy rules for both maturity and government spending. Table (2) reports the parameter values and Figure (5) displays the impacts of the optimized joint policy on the economy. As shown in figure (5), when debt maturity policy is used jointly with public spending policy, the optimized response of the government is to shorten average maturity and decrease public expenditures during the first periods when the debt-to-GDP ratios are high and output is low, then to lengthen maturities and raise public expenditures when the economy recovers. The joint policy is able to neutralize the welfare losses generated by the shock, and to moderate the macroeconomic effects of the shock. During the first quarters, the drop in government spending helps stabilize public debt when the government shifts towards short-term maturities. This alleviates the adverse impact of short-term maturities on sovereign risk and labor taxation. As a result, the maturity policy provides a larger stabilizing effect on the economy in general. Consumption is not substantially stimulated, though, because of the complementarity between public and private goods. After approximately 20 quarters, the increase in debt maturity in response to output and the debt-to-GDP ratio does not hurt the financial system because leverage constraints are no longer binding. At the same time, it allows to reduce the debt burden, thereby lowering labor taxation and opening more room for fiscal expansion. The following rise in government spending stimulates consumption on the one hand, due to their complementarity; and provides direct utility gains on the other, as public expenditures enter the utility function of households.

Figure 5: Effects of the shock with optimized maturity and public spending rules.



Blue solid line: baseline. Red solid line with dots: optimized joint policy rules.

5 Conclusion

In this paper, I investigate the interactions between sovereign risk, bank fragility and distortionary taxes during a sovereign debt crisis. I show that the mere increase in the government's probability of default is sufficient to trigger a crisis in which the three channels interact with each other and give rise to a substantial amplification mechanism. I use the framework to study the macroeconomic and welfare effects of altering debt maturity in response to the crisis. I find that short-term maturities alleviate the bankers' losses by reducing their exposure to price risk, which ultimately mitigates the crisis but at the cost of higher debt levels afterwards because of the rise in rollover costs. In contrast, long-term maturities provide significant welfare gains by reducing the stock of government debt and thus sovereign risk and labor taxes, which enhances private consumption. An optimized joint policy of debt maturity and public spending is able to neutralize the welfare losses and to stabilize the economy in general, through an interaction between the two instruments.

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A Derivation of the banking sector equations

The banker's optimization problem is given by

$$V_{j,t} = \max_{K_{j,t},B_{j,t}} E_t \left\{ \beta \Lambda_{t,t+1} \left[(1-\theta) N_{j,t+1} + \theta V_{j,t+1} \right] \right\}$$

$s.t.$

$$\lambda (Q_t K_{j,t} + Q_t^b B_{j,t}) \leqslant V_{j,t}$$

$$V_{j,t} = \beta E_t \Omega_{t+1} \Big\{ (R_{k,t+1} - R_t) Q_t K_{j,t} + (R_{b,t+1} - R_t) Q_t^b B_{j,t} + R_t N_{j,t} \Big\}$$

which yields the following first-order conditions

$$\beta E_t \Omega_{t+1} (R_{k,t+1} - R_t) = \frac{\lambda \mu_t}{1+\mu_t} \tag{A-1}$$

$$\beta E_t \Omega_{t+1} (R_{b,t+1} - R_t) = \frac{\lambda \mu_t}{1+\mu_t} \tag{A-2}$$

$$\mu_t \Big(v_{n,t} N_{j,t} - \lambda (Q_t K_{j,t} + Q_t^b B_{j,t}) \Big) = 0 \tag{A-3}$$

Combining equations (A-1) and (A-2) gives the no-arbitrage condition between sovereign and credit spreads

$$\frac{E_t(R_{k,t+1} - R_t)}{E_t(R_{b,t+1} - R_t)} = 1 \tag{A-4}$$

Using the first-order equations, we can rewrite the banker's continuum value in the following way

$$\begin{aligned}
V_{n,t} &= \beta E_t \Omega_{t+1} \Big[(R_{k,t+1} - R_t) Q_t K_{j,t} + (R_{b,t+1} - R_t) Q_t^b B_{j,t} + R_t N_{j,t} \Big] \\
&= \frac{\lambda \mu_t}{1+\mu_t} (Q_t K_{j,t} + Q_t^b B_{j,t}) + \beta E_t \Omega_{t+1} R_t N_{j,t} \\
&= \frac{\mu_t v_{n,t} N_{j,t}}{1+\mu_t} + \beta E_t \Omega_{t+1} R_t N_{j,t}
\end{aligned}$$

which allows to determine the shadow value of net worth, $v_{n,t}$:

$$v_{n,t} = (1+\mu_t) \beta E_t \Omega_{t+1} R_t$$

By plugging the expression of $v_{n,t}$ into equation (A-3), we can define the Lagrange multiplier on the incentive constraint, μ_t :

$$\mu_t = \max \left\{ \frac{\lambda (Q_t K_{j,t} + Q_t^b B_{j,t})}{\beta E_t \Omega_{t+1} R_t N_{j,t}} - 1 \ \ , \ \ 0 \right\}$$

When the constraint is binding (i.e. $\mu_t > 0$), the leverage ratio does not depend on bank specific factors

$$\frac{v_{n,t}}{\lambda} = \frac{\lambda(Q_t K_{j,t} + Q_t^b B_{j,t})}{N_{j,t}} = \frac{\lambda(Q_t K_t + Q_t^b B_t)}{N_t}$$

Therefore, μ_t can be rewritten as

$$\mu_t = \max\left\{ \frac{\lambda(Q_t K_t + Q_t^b B_t)}{\beta E_t \Omega_{t+1} R_t N_t} - 1 \quad , \quad 0 \right\}$$

B Steady-state

The gross riskless interest rate is $R = 1/\beta$ and the steady-state markup is equal to $\mathcal{M} = \epsilon/(\epsilon - 1) = 1/P_m$ (as inflation is zero). The capital price Q, the capital quality ξ, the level of technology A, and the level of the utilization U are normalized to one. By imposing μ and Φ, we can solve for the shadow value of net worth and the diversion parameter

$$v_n = \frac{(1+\mu)(1-\theta)}{1-(1+\mu)\theta}$$
$$\lambda = \frac{v_n}{\Phi}$$

It is then straightforward to get R_k and R_b

$$R_k = R_b = \frac{\lambda \mu R}{v_n} + R$$

Accordingly, the parameters of the depreciation rate function are calibrated as follows

$$b = R_k - (1 - \delta(U))$$
$$\delta_c = \delta(U) - \frac{b}{1+\xi}$$

Defining the ratio of output to capital as $Y/K = \frac{R_k - (1-\delta(U))}{\alpha P_m}$, we can obtain the real wage, output and investment taking $Y = Y_m$

$$W = (1-\alpha) P_m \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}$$
$$Y = \frac{WL}{(1-\alpha)P_m}$$
$$I = \delta(U) K$$

Public spending is computed through its imposed share in output. Consumption can then be deducted from the good market clearing equation

$$C = Y - G - I$$

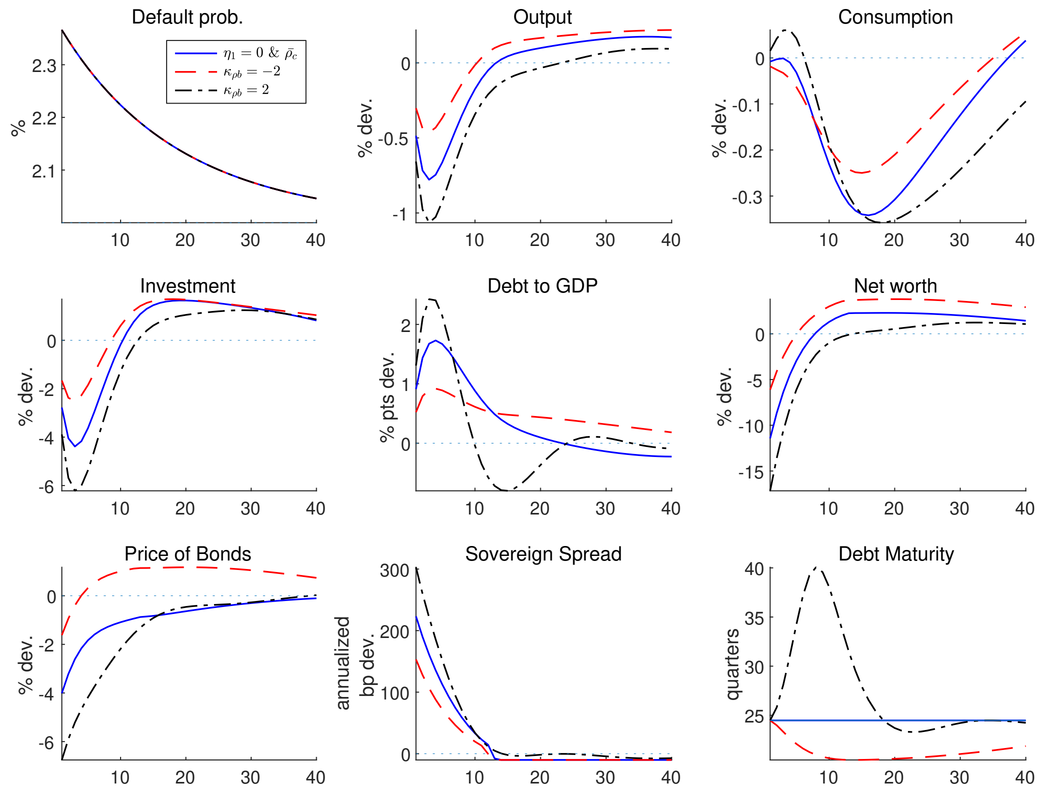
The relative utility weight of labor is adjusted such that the households' first-order condition for labor supply is satisfied, which gives $\chi = (1-\tau)WU_c/L^{\phi}$.

The remaining steady-state values for government and the banking sector can be solved in the following way

$$\begin{aligned}
\Delta^d &= \frac{\exp(\eta_0)}{1+\exp(\eta_0)} \\
Q^b &= \frac{(1-\Delta^d D)(1-\rho_c+\rho_c r_c)}{R_b-(1-\Delta^d D)\rho_c} \\
B &= bY \\
N &= \frac{K+Q^b B}{\Phi} \\
T &= G + \Big((1-\rho_c)+\rho_c r_c + Q^b(\rho_c-1)\Big) B - \tau W L \\
\omega &= \frac{N-\theta\left[(R_k-R)K + \left(1-\rho_c+\rho_c(r_c+Q^b)-RQ^b\right)B + RN\right]}{K+Q^b B}
\end{aligned}$$

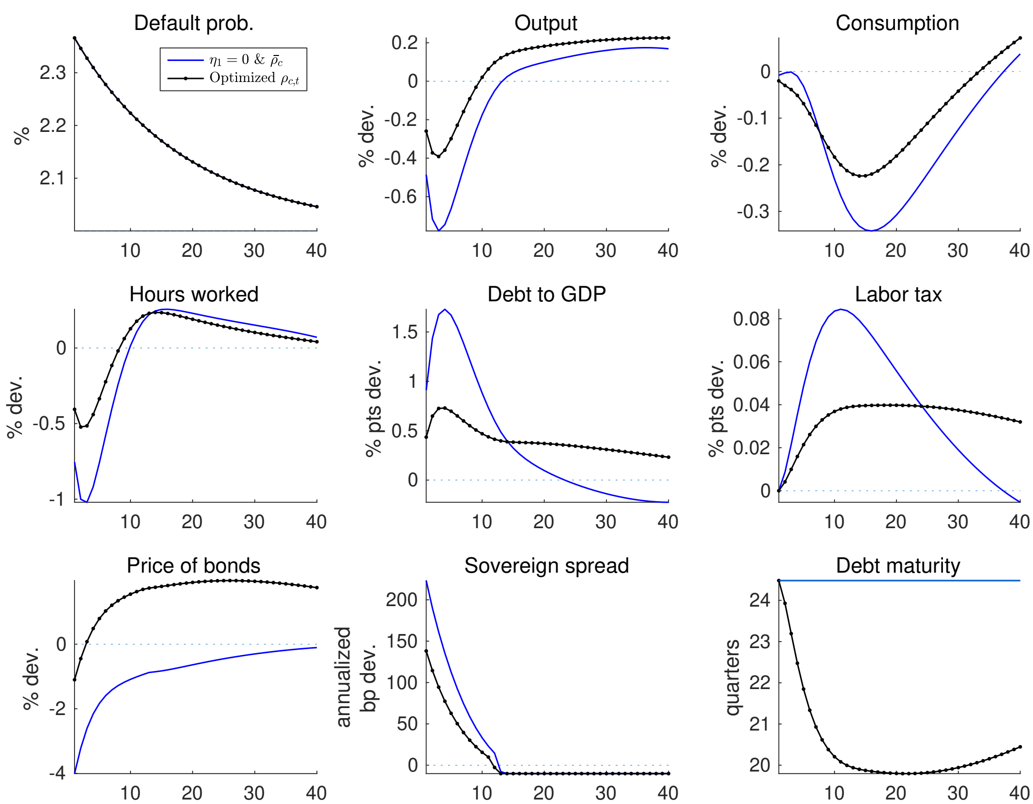
C Additional figures

Figure 6: Effects of debt maturity policy on the crisis – exogenous sovereign risk.



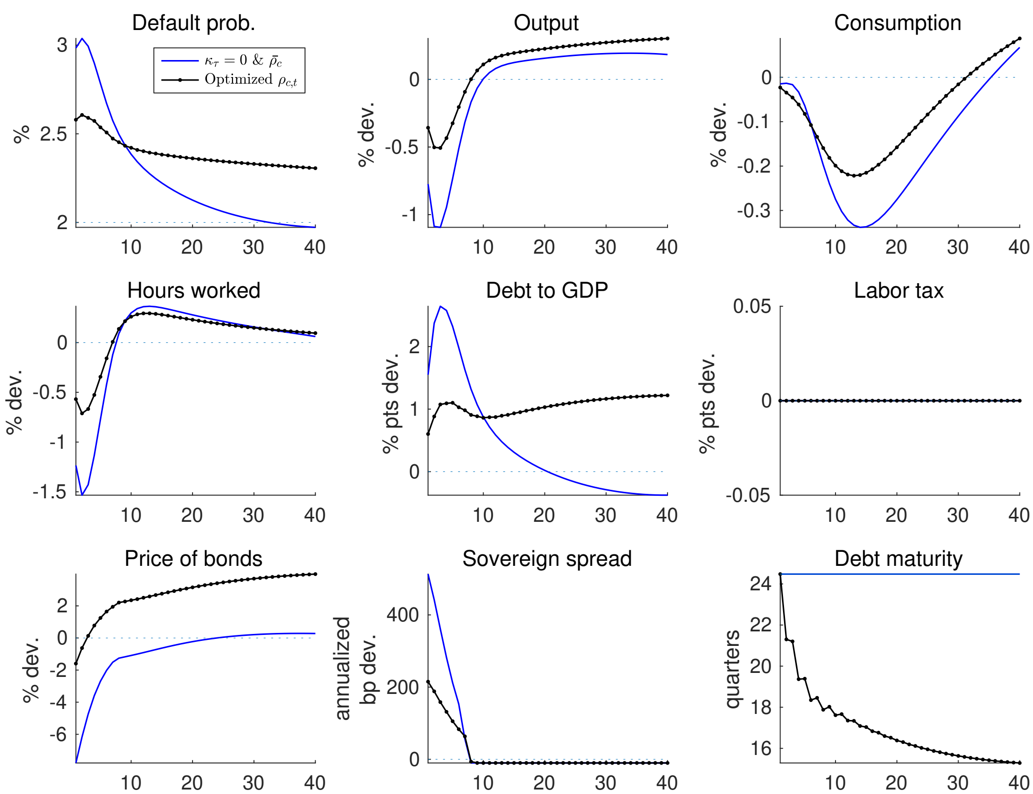
Blue solid line: model with purely exogenous sovereign risk and constant maturity. Black dash-dotted line: rise in maturity. Red dashed line: fall in maturity.

Figure 7: Effects of the shock with optimized maturity policy – exogenous sovereign risk.



Blue solid line: model with purely exogenous sovereign risk and constant maturity.
Black solid line with dots: optimized maturity rule.

Figure 8: Effects of the shock with optimized maturity policy – no labor tax.



Blue solid line: model without labor tax and with constant maturity.
Black solid line with dots: optimized maturity rule.