Is the Flat Tax Optimal under Income Risk?

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Abstract: We derive testable conditions ensuring that the income tax is optimal when agents are ex-ante identical but face idiosyncratic income risk. The optimal tax depends positively on both absolute risk aversion and risk variance and negatively on labor supply elasticity and absolute prudence. The comparison with the formula of the optimal non-linear income tax provides the restrictions on both the preferences and the income distribution conditional on effort ensuring that the optimal tax is indeed linear. In general it requires that the ratio of absolute prudence to absolute risk aversion be no less than two; if the income density has a linear likelihood ratio, it requires a (generalized) logarithmic consumption utility. Under HARA utility and linear or logarithmic likelihood ratios, explicit solutions for the optimal non-linear income tax are derived.

1 Introduction

Flat taxes have been proposed on simplicity, efficiency, and equity grounds (e.g. Friedman [8] Hall and Rabushka [13]). Eastern Europe has enthusiastically embraced the flat tax after the fall of the iron curtain. 23 countries have adopted the flat tax all over the world and flat tax proposals are currently being considered by influential politicians or political parties in a bunch of other countries. The version of the flat tax we consider in this paper includes a basic income, which allows the tax to be progressive. Still, it is unclear whether they can emerge as the solution of some optimal non-linear tax optimization problem. Many papers have investigated whether they can be optimal in the Mirrlees tradition. Atkinson ([2] and the references therein) show that the conditions are quite demanding. For instance, the flat tax is optimal in case of Ralwsian social objective, constant labor supply elasticity and a Pareto distribution for the distribution of productivities. Recently Werning [23] provides a more optimistic answer: flat taxes, as well as more progressive tax systems, may well be Pareto efficient within Mirrlees' economy. Here we revisit this issue in considering the case when agents are ex-ante identical but face idiosyncratic income risk, that is, the framework considered by Varian [21] and a bunch of other papers such as Tuomala [20], Loom and Maldoom [15]. We follow the traditional approach and assume that households commit (only) to labor supply decisions before the resolution of uncertainty, which introduces moral hazard into the analysis.[1] This vein of the literature on optimal taxation is well suited to question how an optimal income should be designed in the case the purpose of introducing tax system is to provide social insurance. It is quite common that social security contributions which mitigate the effect of uncertainty on disposable income are flat. The question we address

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[1]As pointed out by Cremer and Gavhari [4], if households do not commit to labor supply decisions, then the model corresponds to the standard optimal taxation setting with adverse selection of Mirrlees [17]. Cremer and Gavhari [4] focus on commitment to consumption.

can be reformulated in questioning the linearity of most social security contribution formulae.

We focus on two questions. First, which aspects of the household's preferences towards risk determine the level of the optimal linear tax? Second and most importantly, what are the restrictions on both the preferences for consumption and the income distribution conditional on effort such that the flat tax is optimal when nonlinear schedules are a priori available? Answering these two questions seems quite relevant, in particular, to take a stand on the much debated issue of whether or not governments should replace the ubiquitous piecewise linear income tax by the simpler flat tax. In particular, characterizing the conditions ensuring the optimality of the flat tax is key to assess the potential welfare losses that are incurred when the linear tax is wrongfully imposed.

We first derive a simple formula for the linear income tax under income risk when utility is not assumed to be separable between consumption and effort is not observable. The optimal flat income tax depends, other things equal: (i) positively on both absolute risk aversion and risk variance and (ii) negatively on labor supply elasticity and absolute prudence. This latter coefficient emerges because precautionary motives increase labor supply under risk and call for less distortion. Our simple formula for the flat income tax generalizes Dardanoni [5] and Mirrlees [18], who assume the income disturbance to be additive and multiplicative, respectively. In addition, such a formula relating the income tax rate to prudence sheds some light on the simulations reported in a seminal paper by Eaton and Rosen [7].

Second, we characterize the restrictions on both the preferences and the income distribution conditional on effort ensuring that the linear tax is indeed optimal. For this purpose, we revisit the analysis of the optimal non linear income tax in a moral hazard

context. Varian [21] and Tuomala [20] are early papers studying the insurance effect of income taxation. Our results on the nonlinear income tax are more closely related to those (and help interpreting the simulations) of Low-Maldoom [15], who helpfully provide an intuitive explanation of why prudence matters. However, we depart from the latter authors by assuming sufficient conditions for the first-order approach to be valid. To that effect, we generalize the analysis of Jewitt [14] to preferences over consumption and leisure that are not assumed to be separable and such that preferences over income lotteries are independent of effort as in Grossman and Hart [11]. Alternatively, we could rely on the conditions provided by Alvi [1], notably the convexity of the cumulative density. However, we choose to follow Jewitt's approach because it turns out that the curvature of the optimal nonlinear tax directly depends on the curvature of the likelihood ratio.[2] Therefore, how assumptions on the concavity of the likelihood ratio condition the results on tax progressivity is made transparent. In particular, the beta distributions considered by Low-Maldoom [15] satisfy Jewitt's [14] concavity conditions. We generalize the analysis of Low and Maldoom [15] to a more general class of preferences allowing consumption and leisure to be non-separable. In addition, we ensure the validity of the first-order approach through the appropriate conditions (as opposed to through numerical computations) and derive the restrictions such that the linear tax is optimal under various combinations of utility and density functions. It general requires that the ratio of absolute prudence to absolute risk aversion be no less than two. For example, if the income density has a linear likelihood ratio, then the linear tax is optimal when the ratio of absolute prudence to absolute risk aversion equals two, which is equivalent to requiring a (generalized) logarithmic consumption utility. In the case of a nonlinear likelihood ratio, then the linear tax is optimal under a restrictive condition relating absolute prudence, absolute risk aversion and the concavity of the likelihood ratio. In contrast, a sufficient

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[2]Conlon [3] argues in favor of Jewitt's approach for studying multisignal problems.

condition for the marginal tax rate to be increasing with income is that the prudence-risk aversion ratio is less than two. If consumption utility belongs to the CRRA class, then the optimal income tax is marginally progressive provided that, not implausibly, relative risk aversion is larger than one.

At a more methodological level, our third contribution is to show that under HARA utility and linear or logarithmic likelihood ratios, the standard moral-hazard model of optimal taxation can be solved. These explicit solutions derived in the taxation context might also prove useful in other applications as well.

The remaining of the paper is organized as follows. In Section 2, we derive the optimal income tax among linear schedules under income risk. The optimal nonlinear income tax is studied in a slightly generalized moral hazard setting, in Section 3. Section 4 solves for the optimal nonlinear tax under HARA utility. Finally, Section 5 gathers some concluding remarks while an Appendix states an existence proof.

2   Optimal Linear Income Tax under Risk

In this section, we derive the linear income tax under income risk, when households are ex-ante identical and commit to labor supply decisions. This leads to an unconstrained maximization problem that is easily solved under a particular form introduced by Grossman and Hart [11] in the framework of moral hazard problems.

2.1   General Case

Suppose that income y is a function of effort and luck: $y = f(l, \varepsilon)$, where l is effort, ε is a random variable, and f is an increasing function of both arguments.[3] In addition, we assume that $E\{f\} = l$, where E denotes the expectation operator over ε.

We follow Grossman and Hart [11] and assume that preferences over consumption and effort are of the following form:

$$U(c, l) = g(l)u[c] - v(l), \tag{1}$$

where l is effort, c is consumption, g is decreasing-concave, u is increasing-concave, v is increasing-convex. The formulation (1) goes beyond the separable case[4] for which $g(l)$ boils down to some parameter.

We now derive the optimal linear tax function $t(y) \equiv ty - d$, where $t \geq 0$ is the marginal tax rate and $d \geq 0$ is the basic income . If one defines $c \equiv 1 - t > 0$ as the retention rate, then optimal effort solves:

$$\max g(l)E\{u[cf(l, \varepsilon) + d]\} - v(l).$$

The first-order condition with respect to l is:

$$g'(l)E\{u[cf(l, \varepsilon) + d]\} + g(l)E\{u'[cf(l, \varepsilon) + d]cf_1(l, \varepsilon)\} = v'(l), \tag{2}$$

where f_1 denotes the partial derivative of f with respect to its first argument (and similarly for f_2).

The government budget constraint is:

$$d = (1 - c)E\{f(l, \varepsilon)\} = (1 - c)l. \tag{3}$$

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[3]This formulation turns out to be more convenient to study linear taxes than that used in Section 3, where the density function depends on y and l, although both are equivalent.

[4]If c designates a vector of consumption goods, the marginal rate of subsitution between each of them will not depend on effort.

Plugging (3) into (2) gives:

$$g'(l)E\{u[l + c(f(l, \varepsilon) - l)]\} + cg(l)E\{u'[l + c(f(l, \varepsilon) - l)]f_1(l, \varepsilon)\} = v'(l),$$

which defines $l = l(c)$ implicitly.

Eliminating the individual first-order condition (2) and the government budget constraint (3), one is left with the unconstrained maximization problem over c:

$$\max g(l(c))E\{u[l(c) + c(f(l(c), \varepsilon) - l(c))]\} - v(l(c)) \tag{4}$$

The first order condition of (4) gives, omitting the arguments to save on notation:

$$\frac{t^*}{1 - t^*} = \frac{-Cov\{f, u'\}}{l^* \epsilon_{l^*} E\{u'\}} \tag{5}$$

using the fact that $E\{f_1\} = 1$, where $Cov\{f, u'\} \equiv E\{fu'\} - E\{f\}E\{u'\}$, $l^* = l(1 - t^*)$, and ϵ_{l^*} denotes the elasticity of $l(c)$. Because f is an increasing function and u' is a decreasing function, one has by Chebyshev's sum inequality that $0 \geq Cov\{f, u'\}$, with a strict inequality under risk. Therefore, $1 \geq t^* > 0$ when risk is present. In contrast, the optimal linear tax rate would be zero absent risk.

Proposition 2.1 (Optimal Linear Income Tax and Income Risk)

Suppose that the tax schedule is linear, i.e. that $t(y) \equiv ty - d$. Then under risk, the optimal tax rate t^* is given by:

$$\frac{t^*}{1 - t^*} = \frac{-Cov\{f(l^*, \varepsilon), u'[x_\varepsilon]\}}{l^* \epsilon_{l^*} E\{u'[x_\varepsilon]\}} \tag{6}$$

with $1 \geq t^* > 0$, where $x_\varepsilon \equiv (1 - t^*)f(l^*, \varepsilon) + t^* l^*$. On the other hand, the optimal basic income $d^* > 0$.

When risk is absent, $t^* = d^* = 0$.

If the variance of ε, μ_2 say, is small, a second-order approximation yields that $Cov\{f, u'\} \approx c\mu_2[f_2]^2 u''$, where both f_2 and u'' are taken at $\varepsilon = 0$, and one gets:

Proposition 2.2 (Approximating the Optimal Linear Income Tax)

Assume that μ_2, the variance of income risk ε, is small. Then the optimal tax rate t^* is given by:

$$\frac{t^*}{(1-t^*)^2} \approx \frac{\chi}{l^*\epsilon_{l^*}} \left(\frac{\mu_2 A[x_0]}{E\{u'[x_\varepsilon]\}/u'[x_0]} \right) \tag{7}$$

where $\chi \equiv [f_2(l^*,0)]^2$, $x_\varepsilon \equiv (1-t^*)f(l^*,\varepsilon) + t^*l^*$ and $A[x_0] \equiv -u''[x_0]/u'[x_0]$ denotes absolute risk aversion.

Formula (7) indicates that the optimal tax rate t^* depends on four elements. It may perhaps help intuition to underline that, other things equal, the tax rate is: a decreasing function of the taxpayer's labor supply elasticity ϵ_l; an increasing function of the risk variance μ_2; an increasing function of the taxpayer's risk aversion A; a decreasing function of the taxpayer's prudence. Whereas, the first three determinants accord well with intuition, the effect of prudence perhaps needs further explanation. Prudence tends to reduce the need of *social* insurance through taxation under risk and calls, in the context of linear taxes, for a lower marginal tax rate. To see this, note that by Jensen's inequality, $E\{u'\}/u' > 1$ when u' is convex (i.e. when utility exhibits prudence), which tends to reduce the optimal tax rate given by (7), other things equal.

In other words, prudence creates a motive for "precautionary effort": the higher prudence, the higher the effort provided by the taxpayer, in which case the larger the cost of taxing income in terms of working incentives.

Remark: the formula in Proposition 2.2 is similar if the income effect on labor supply is assumed away (only the argument of u' changes accordingly).

2.2 Special Cases: Additive and Multiplicative Income Risk

In the simple cases of additive and multiplicative risk, our above formula has corresponding analogues.

(a) Suppose that income risk is additive, that is, $f(l, \varepsilon) = l + \varepsilon$. This case can be interpreted as "pure luck".

Proposition 2.3 (Optimal Linear Income Tax and Additive Risk)

Under the assumptions of Proposition 2.2, suppose that income risk is additive, that is, $f(l, \varepsilon) = l + \varepsilon$. Then the optimal tax rate t^* is given by:

$$\frac{t^*}{1 - t^*} = \frac{-Cov\{\varepsilon, u'[x_\varepsilon]\}}{l^* \epsilon_{l^*} E\{u'[x_\varepsilon]\}}$$

with $1 \geq t^* > 0$, $x_\varepsilon \equiv l^* + (1 - t^*)\varepsilon$. On the other hand, the optimal basic income $d^* > 0$. If μ_2, the variance of income risk ε, is small, then the optimal tax rate t^* is given by:

$$\frac{t^*}{(1 - t^*)^2} \approx \frac{1}{l^* \epsilon_{l^*}} \left(\frac{\mu_2 A[x_0]}{E\{u'[x_\varepsilon]\}/u'[x_0]} \right)$$

where $A[x_0] = -u''[x_0]/u'[x_0]$ denotes absolute risk aversion.

(b) Suppose now that income risk is multiplicative, that is, $f(l, \varepsilon) = l(1 + \varepsilon)$. This case can be seen as representating an idiosyncratic shock to the marginal product of labor.

Proposition 2.4 (Optimal Linear Income Tax and Multiplicative Risk)

Under the assumptions of Proposition 2.2, suppose that income risk is multiplicative, that is, $f(l, \varepsilon) = l(1 + \varepsilon)$. Then the optimal tax rate t^* is given by:

$$\frac{t^*}{1 - t^*} = \frac{-Cov\{\varepsilon, u'[x_\varepsilon]\}}{\epsilon_{l^*} E\{u'[x_\varepsilon]\}}$$

with $1 \geq t^* > 0$, $x_\varepsilon \equiv l^*[1 + (1 - t^*)\varepsilon]$. *On the other hand, the optimal basic income* $d^* > 0$.

If μ_2, the variance of income risk ε, is small, then the optimal tax rate t^* is given by:

$$\frac{t^*}{(1 - t^*)^2} \approx \frac{l^*}{\epsilon_{l^*}} \left(\frac{\mu_2 A[x_0]}{E\{u'[x_\varepsilon]\}/u'[x_0]} \right)$$

where $A[x_0] = -u''[x_0]/u'[x_0]$ denotes absolute risk aversion.

Remark: here again, formulas in Proposition 2.3 and 2.4 are similar if the income effect is assumed away (only the argument of u' changes accordingly).

2.3   The Effect of Risk on the Linear Tax Rate: an Example

In this section, we provide a simple example showing that the optimal linear tax rate is an increasing function of risk. More precisely, we study the impact of a mean-preserving spread of income. To that end, we assume that risk is additive and we rule out any income effect on labor supply. The level of utility derived by a tax-payer with before-tax income y and after-tax income $cy + d$ can be written $E\{u[c(l + \varepsilon) + d - v(l)]\}$.   Then the chosen effort is the solution of $c = v'(l)$. The government budget constraint is again $d = (1 - c)l$ so that the indirect utility writes, choosing l as control variable instead of c:

$$E\{u(l - v(l) + v'(l)\varepsilon\}. \tag{8}$$

Now suppose that $\varepsilon = kx$ where x is a pure standard risk : $E\{x\} = 0$, $V\{x\} = 1$, where k is a positive real. For a given k the optimal income l^* (and then tax $= 1 - c^* = 1 - v'^*$)) is the solution of:

$$\max_l E\{u[l - v(l) + v'(l)kx). \tag{9}$$

We seek a condition on u insuring that when k increases, the optimal tax rate increases, that is (as v is convex) l^* decreases. First of all the FOC gives, defining $z(l) \equiv l - v(l) +$

$v'(l)kx$:

$$(1 - v'(l)) = v''(l)k\frac{-E\{u'[z(l)]x\}}{E\{u'[z(l)]\}}. \tag{10}$$

The left-hand side of the above equation is a decreasing function of l. The solution l^* is a maximum (interior) if the RHS is a positive increasing function at least in the neighborhood of the solution. It follows that a sufficient condition for l^* to be decreasing with k is that: $\frac{-E\{u'[z(l)]x\}}{E\{u'[z(l)]\}}$ is increasing with k at least in the neighborhood of l^*. In fact, $\frac{E\{u'[z(l)]x\}}{E\{u'[z(l)]\}}$ is the expectation of x with a changed probability (risk neutral probability) which overweights low values of x. The claim amounts to show that this changed expectation is decreasing with k.

The derivative of $\frac{E\{u'[z(l)]x\}}{E\{u'[z(l)]\}}$ with respect to k has the same sign as:

$$E\{u''^2\}E\{u'[z]x\} - E\{u''[z]x\}E\{u'[z]x\}.$$

This has negative sign if, after dividing by $E\{u'[z]\}E\{u''[z]\}$ (which is negative):

$$\frac{E\{u''^2\}}{E\{u''[z]\}} \geq \frac{E\{u''[z]x\}}{E\{u''[z]\}}\frac{E\{u'[z]x\}}{E\{u'[z]\}},$$

which gives, subtracting $\left(\frac{E\{u''[z]x\}}{E\{u''[z]\}}\right)^2$:

$$\frac{E\{u''^2\}}{E\{u''[z]\}} - \left(\frac{E\{u''[z]x\}}{E\{u''[z]\}}\right)^2 \geq \left(\frac{E\{u'[z]x\}}{E\{u'[z]\}} - \frac{E\{u''[z]x\}}{E\{u''[z]\}}\right)\frac{E\{u''[z]x\}}{E\{u''[z]\}}.$$

The left-hand side of the latter inequality is the expression of the variance of x computed with the density $\frac{u''[z]}{E\{u''[z]\}}$. This is essentially positive. Therefore, we can state:

Proposition 2.5 (Optimal Linear Tax and Additive Risk)

Assume that utility exhibits no income effect on labor supply. Moreover, suppose that income risk is additive and such that $\varepsilon = kx$ where x is a pure standard risk ($E\{x\} = 0$, $V\{x\} = 1$), where k is a positive real. Then the optimal linear tax rate is an increasing function of k:

(*i*) *if the tax payer is not prudent (that is, $0 \geq u'''[c]$ for all c).*

(*ii*) *when the tax payer is prudent, if:*

$$0 \geq \frac{E\{u'[z]x\}}{E\{u'[z]\}} \geq \frac{E\{u''[z]x\}}{E\{u''[z]\}},$$

which is satisfied if $-u'$ is more concave than u, that is, if consumption utility u has decreasing absolute risk aversion (DARA).

3   Optimal Nonlinear Income Tax under Risk

As in Low and Maldoom [15], we now introduce risk in the optimal taxation problem when households are ex-ante identical and we derive the optimal nonlinear tax. The setting is a bit more general than in Low and Maldoon [15], as we do not assume preferences to be separable.

Under risk, the planner chooses a consumption schedule $c(y)$, as a function of realized income y, that offers (partial) social insurance against income risk. As a constraint, the planner internalizes the first-order condition which states that effort l should be optimal from the household's viewpoint (equivalently, the incentive constraint in the first-order approach to moral hazard):

$$\max_{c(.),l} \int g(l)u[c(y)]dF(y,l) - v(l)$$

subject to

$$g'(l)\int u[c(y)]dF(y,l) + g(l)\int u[c(y)]dF_l(y,l) - v'(l) = 0 \text{ and } \int [y - c(y)]dF(y,l) = 0,$$

$$(11)$$

where y is random income, l is effort. The first-order condition with respect to $c(y)$ is:

$$\frac{1}{u'[c(y)]} = \lambda g(l) + \mu[g'(l) + g(l)h(y,l)] \tag{12}$$

where $h(y, l) \equiv f_l(y, l)/f(y, l)$ is the likelihood ratio and λ, μ are Lagrange multipliers associated with the constraints (11).

When utility is additively separable (that is, $g(l) = 1$ for all l), then (12) simplifies to equation (2.5) in Jewitt [14, p. 1179] (and equation (6) in Low and Maldoom [15, p. 446]). We now state our main assumptions.

Assumption 3.1

(i) *The distribution function F is such that both $\int_{-\infty}^{x} F(y, l)dy$ is nondecreasing-convex in l for each value of x and $\int y dF(y, l)$ is nondecreasing-concave in l.*

(ii) *The density function f is such that the likelihood ratio $h(y, l) \equiv f_l(y, l)/f(y, l)$ is nondecreasing-concave in y for each value of l.*

(iii) *The utility function u satisfies $3A[c] \geq P[c]$ for all c, where $P[c] \equiv -u'''[c]/u''[c]$ is absolute prudence and $A[c] \equiv -u''[c]/u'[c]$ is absolute risk aversion, with $A[c] > 0$.*

The above assumptions are those stated in Jewitt [14, Thm 1, p. 1180]. It is not difficult to show that condition (iii) in Assumption 3.1 is an equivalent formulation of Jewitt's [14] condition (2.12) (stating that $u[c]$ is a concave transformation of $1/u'[c]$). It is an important assumption that prevents the level of prudence from being too large relative to risk aversion. In the CRRA case with relative risk aversion γ, condition (iii) is equivalent to $\gamma > 1/2$.

The first lemma provides a slight generalization of Jewitt [14, Thm 1, p. 1180] to the case of preferences (1). Then we derive and sign both the gradient and the curvature of the optimal tax schedule, the existence of which follows from the arguments given in the Appendix. Finally, we build on such a characterization to derive conditions ensuring either that the linear tax is optimal or that the marginal tax rate is an increasing function of income.

Lemma 3.1 (Gradient and Curvature of Optimal Consumption)

Under Assumption 3.1, the first-order approach is valid and it yields that the optimal consumption schedule is such that:

$$c'(y) > 0 \quad and \quad \frac{c''(y)}{c'(y)} = \frac{h_{yy}(y,l)}{h_y(y,l)} + (P[c(y)] - 2A[c(y)])c'(y), \quad for \ all \ y.$$

Proof: As in Jewitt [14], we need to show that Assumption 3.1 ensures that the first-order approach is valid, i.e., that the relaxed moral hazard problem characterizes the optimal solution. The first step is to show that $\mu \geq 0$. To that end, using the fact that $\int h(y,l)dF(y,l) = 0$, one gets from (12) that:

$$\lambda g(l) = \int \frac{dF(y,l)}{u'[c(y)]} - \mu g'(l), \tag{13}$$

so that $\lambda \geq 0$ if $\mu \geq 0$. From (12), one gets that $f_l(y,l) = [f(y,l)/(g(l)u'[c(y)]) - \lambda f(y,l)]/\mu - f(y,l)g'(l)/g'(l)$. Plugging the latter expression of $f_l(y,l)$ into the first-order condition with respect to l, that is, the first equation in (11), gives:

$$\int \frac{u[c(y)]}{u'[c(y)]}dF(y,l) - \lambda g(l) \int u[c(y)]dF(y,l) = \mu v'(l), \tag{14}$$

Replacing in (14) the expression of $\lambda g(l)$ in (13) delivers:

$$Cov_y(u[c(y)], 1/u'[c(y)]) = \mu\{v'(l) - g'(l) \int u[c(y)]dF(y,l)\}. \tag{15}$$

From the fact that the left-hand side is positive, as both u and $1/u'$ are increasing functions, and that the right-hand side is positive, we get that $\mu \geq 0$. That $\mu = 0$ is excluded follows from the conclusion that this would imply full insurance and hence violates the first equation in (11), that is, the incentive constraint. But $\mu > 0$ implies, in view of (12), that $1/u'[c(y)]$ is a nondecreasing-concave function of y under condition (ii) in Assumption 3.1. Finally, the fact that the transformation $\psi \mapsto \psi^*$, defined by $\psi^*(l) = g(l) \int \psi(y)dF(y,l)$, preserves concavity follows from (i) and (iii) in Assumption

3.1 because $g(l)$ is decreasing-concave in l and $\int \psi(y)dF(y,l)$ is non-decreasing concave in l. In summary, the first-order approach is valid. The final steps consist in differentiating (12) twice with respect to y to get $c'(y) = \mu g(l)h_y(y,l)u'[c(y)]/A[c(y)] > 0$ and $c''(y)/c'(y) = h_{yy}(y,l)/h_y(y,l) + (P[c(y)] - 2A[c(y)])c'(y)$. $\qquad \square$

The fact that the concavity of the likelihood ratio appears in the expression of $c''(y)$, and therefore affects the shape of the optimal tax, justifies our use of the approach advocated in Jewitt [14]. We now use Lemma 3.1 to characterize the optimality of either the flat income tax or of the marginally progressive income tax. More precisely, we focus first on the restrictions related to preferences and then go on to exhibit the joint conditions on utility and conditional density.

Theorem 3.1 (Marginally Progressive Optimal Income Tax)

Under the assumptions of Lemma 3.1, the marginal income tax rate is increasing with income if:

$$P[c] < 2A[c] \qquad (16)$$

for all c.

Condition (16) is met if $0 \geq P[c]$ (that is, if consumption utility $u[c]$ does not exhibit prudence), if $P[c] = A[c]$ (that is, if consumption utility $u[c]$ has CARA) and it is compatible with DARA preferences (that is, such that $P[c] > A[c]$).

It follows that the marginal income tax rate is either decreasing with income or constant only if $P[c] \geq 2A[c]$.

Proof: From Lemma 3.1, $c''(y)/c'(y) = h_{yy}(y,l)/h_y(y,l) + (P[c(y)] - 2A[c(y)])c'(y)$ holds for all y. As $c'(y) > 0$ and $0 \geq h_{yy}(y,l)/h_y(y,l)$ under (ii) in Assumption 3.1, it follows

that $c''(y) < 0$ if $P[c] < 2A[c]$. Finally, the tax function is $t(y) \equiv y - c(y)$ so that $t''(y) > 0 > c''(y)$ under condition (16), that is, the optimal marginal tax rate is an increasing function of income. In addition, it follows that $0 \geq t''(y)$ only if $P[c] \geq 2A[c]$. $\square$

The intuitive explanation stated in Low and Maldoom [15] applies equally to our setting: if absolute prudence is small enough relative to absolute risk aversion, the self-insurance motive is weak and it is optimal to have an increasing marginal tax rate. Theorem 3.1 states that if consumption utility belongs to the CARA class, then it is optimal to have a marginally progressive tax schedule, regardless of the output density conditional on effort. On the other hand, if consumption utility belongs to the CRRA class, then we have the following:

Corollary 3.1 (Optimal Income Tax under CRRA Utility)

Suppose that consumption utility $u[c]$ belongs to the CRRA class, with relative risk aversion $\gamma \geq 0$. Then under the assumptions of Lemma 3.1, the marginal income tax rate is increasing with income if $\gamma > 1$. In addition, the marginal income tax rate is decreasing with income or constant only if $1 \geq \gamma$.

One important implication of the above result is that in the CRRA case, income tax progressivity is likely in view of the bulk of evidence from microeconomic data showing that relative risk aversion is larger than one. Therefore, in the CRRA configuration, it is optimal to have a flat or regressive tax only if the household's relative risk aversion is (perhaps unrealistically) lower than one. As in the CARA case, this result holds independently of the output density and it suggests that the optimality of the linear tax is a knife-edge result which implies strong restrictions. To make this claim more

precise, we now have to be more specific about the output density $f(y, l)$ and assume the following. As emphasized by Varian [21] and Tuomala [20] (in the case of normal and gamma distributions, respectively), some results about the curvature of the optimal tax schedule obtain if one further assumes that $h_{yy}(y, l) = 0$ for all y, that is, when the likelihood ratio is linear in y. This holds true generally, as pointed out in Low and Maldoom [15].

Theorem 3.2 (Optimal Income Tax Under Linear Likelihood Ratio)

Under Assumption 3.1, suppose that $h_{yy}(y, l) = 0$ for all y. Then it follows that the optimal income tax is:

(i) marginally progressive if $P[c] < 2A[c]$ for all c,

(ii) linear if $P[c] = 2A[c]$ for all c,

(iii) marginally regressive if $P[c] > 2A[c]$ for all c.

In particular, if consumption utility $u[c]$ belongs to the CRRA class with relative risk aversion $\gamma \geq 0$, then conditions $(i) - (iii)$ are, respectively, $\gamma >, =, < 1$.

In addition, It follows that the linear tax is optimal if and only if consumption utility is $u[c] = \log(\alpha + c)$, for some real number α. Such utility function exhibits: (a) strictly decreasing absolute risk aversion, and (b) nonincreasing relative risk aversion if and only if $0 \geq \alpha$.

Proof: Proving (i)-(iii) follows from the expression of c'' in Lemma 3.1. In the CRRA class with relative risk aversion $\gamma \geq 0$, $A[c] = \gamma/c$ and $P[c] = (1 + \gamma)/c$ so that $P[c]/A[c] = 1 + 1/\gamma$ and conditions $(i) - (iii)$ are, respectively, $\gamma >, =, < 1$. Finally, from (ii) in Theorem 3.2, one learns that the linear income tax is optimal under linear likelihood ratio if and only $P[c] = 2A[c]$ for all c. Noticing that $P[c]/A[c] = 1 + dT[c]/dc$, where $T[c] = 1/A[c]$ is risk tolerance, one has that $P[c] = 2A[c]$ if and only if $T[c] = \alpha + c$,

for some real number α, which is equivalent, up to a constant, to $u[c] = \log(\alpha + c)$. Therefore, absolute risk aversion $A[c] = 1/(\alpha + c)$ is strictly decreasing in c, while relative risk aversion $cA[c]$ is nonincreasing in c if and only if $0 \geq \alpha$. □

The proof of Theorem 3.2 depends on the linearity of h but this feature is not very restrictive. As pointed out in Jewitt [14], the linearity of the likelihood ratio is not as strong an assumption as it may seem, as gamma and Poisson distributions satisfies it. We give further examples below for the normal and exponential distributions. In that case, the optimality of the linear income tax turns out not to be robust to small changes in relative risk aversion, if consumption utility has CRRA. Outside the CRRA case, the linear tax is optimal if and only if consumption utility is a (generalized) logarithmic function, which belongs to the HARA class. Note that relative risk aversion is decreasing if and only if utility is a particular form of the Stone-Geary preferences, as α has then to be negative. In that case, relative risk aversion is larger than one and can be large if c is close to (but larger than) α. Here again, the linear income tax is unrobust.

Theorem 3.2 generalizes the discussion in Low and Maldoom [15, p. 448] to the case of preferences (1). Intuition suggests that such a generalization is made possible by the fact that the household's behavior towards income lotteries does not depend on effort under the assumed utility function in (1), just as in the additively separable case.

In the microeconomics of uncertainty, the condition that the absolute prudence is larger or smaller than twice the absolute risk aversion emerges in different contexts. For instance, Gollier and Kimball have shown that $P < 2A$ is necessary and sufficient for the property that risk-taking reduces the willingness to save (see Gollier [9]). Gollier, Jullien and Treich [10] have proved that the reverse condition $P > 2A$ is necessary for scientific progress to induce an early prevention effort when consumption may produce damages in the future. The condition $P < 2A$ is met in the CRRA hypothesis if relative

risk aversion is larger than unity. However the CRRA hypothesis is challenged by the empirical study of Guiso and Paiella [12], in the case of Italy, although their estimates of absolute prudence and aversion still verify the above condition. However, Ventura and Eisenhauer [22] obtain quite large values of relative prudence (around 4) while Merrigan and Normandin [16] get estimates that range from less than 1 to slightly above 2 for a British sample. Up to now, the empirical evidence is not sufficient to settle this issue. The results of this paper call for more investigations in that direction. But one of its key contribution is to connect the issue of the optimality of the flat tax in presence of income risk to a simple and testable condition.

Interestingly, a natural case is covered by Theorem 3.2: assume that y has Gaussian distribution with mean l and variance σ^2. Then it is not difficult to show that h is linear in y.[5] A particular case is additive risk, that is, $y = l + \varepsilon$, as in Proposition 2.3. Therefore, in the Gaussian "pure luck" case as well, Theorem 3.2 implies that linear income taxation is optimal if and only if utility is logarithmic.

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[5]The normal distribution satisfies condition (i) in Assumption 3.1. See Jewitt [14, p. 1183] and our discussion below on the exponential family. Moreover, the likelihood ratio is not affected if one truncates the normal distribution on either one or both ends.

Proposition 3.1 (Optimal Linear Income Tax Under Gaussian Risk)

Suppose that income y has a Gaussian distribution with mean l and variance $\sigma^2 > 0$. Then $f(y, l)$ is the normal density and the conditions of Theorem 3.2 are fulfilled, hence the linear tax is optimal if and only if $P[c] = 2A[c]$ for all c.

Proof: If y is normal with mean l and variance $\sigma^2 > 0$, then:

$$f(y, l) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{[y-l]^2}{2\sigma^2}\}.$$

It follows that f satisfies Assumption 3.1, with $h_y(y, l) = 1/\sigma^2 > 0$ and $h_{yy}(y, l) = 0$ for all y. Therefore, Theorem 3.2 applies. $\square$

Note that in the case of multiplicative risk, that is, $y = \varepsilon l$ (as in Proposition 2.4) with ε Gaussian, the likelihood ratio is not monotone in y and therefore violates our condition (ii) in Assumption 3.1. This is not innocuous because the assumption of likelihood monotonicity cannot be dispensed with in principal-agent problems, since the early work by Mirrlees [18]. However, Proposition 3.1 shows that non-monotonicity is not a general feature of the conditional normal density. In addition, there is nothing special to multiplicative risk. For example, the conditions of Theorem 3.2 are satisfied if risk is multiplicative and if the density is exponential with mean l, that is, if $f(y, l) = \exp[-y/l]/l$, as $h_y(y, l) = 1/l^2 > 0$ and $h_{yy}(y, l) = 0$ for all y.

We now turn to configurations such that the likelihood ratio is not linear. Although there are many densities that satisfy such property, we follow Jewitt [14, p. 1183] by focussing on the exponential family, which in fact includes the normal distributions.

Assumption 3.2

The density function $f(y,l)$ belongs to the class of exponential family, that is, it can be written as:

$$\log f(y,l) = \theta(y) + \psi(l) + \sum_{i=1}^{k} \alpha_i(l)\beta_i(y), \tag{17}$$

for some $k \geq 1$. Moreover, the functions α_i, $i = 1, \cdots, k$ are nondecreasing. Then the likelihood ratio $h(y,l) = \partial \log f(y,l)/\partial l = \psi'(l) + \sum_{i=1}^{k} \alpha_i'(l)\beta_i(y)$.

This set of functions encompasses many densities that are used in statistics and economics, including the normal (or lognormal), gamma, Pareto, Poisson, Chi-square, exponential. Note that (ii) in Assumption 3.1 requires $\sum_{i=1}^{k} \alpha_i'(l)\beta_i(y)$ to be nondecreasing-concave in y for each l.

Proposition 3.2 (Optimal Linear Tax Under Exponential Distributions)

Under Assumption 3.1 and 3.2, the linear tax is optimal if and only if:

$$c = \frac{\sum_i^k \alpha_i'(l)\beta_i''(y)}{(\sum_i^k \alpha_i'(l)\beta_i'(y))(2A[cy+d] - P[cy+d])} \tag{18}$$

is independent of y, where c is the optimal retention rate and $d = (1-c)E[y]$ is the optimal basic income .

The proof follows from setting the expression of $c''(y)$ in Lemma 3.1 to zero. Condition (18) characterizes the optimality of the linear tax within a fairly general class of densities. It appears to be a strong, joint restriction on utility and conditional density, the economic sense of which is not easily intuited. Further results can be obtained with particular densities that are elements of the exponential family.

Proposition 3.3 (Optimal Linear Tax Under Lognormal and Pareto Distributions)

Under Assumption 3.1 and 3.2, suppose that $f(y,l)$ is either:

(*i*) *lognormal:*

$$f(y,l) = \frac{1}{\sigma y \sqrt{2\pi}} \exp\{-\frac{[\ln(x) - \mu(l)]^2}{2\sigma^2}\},$$

with $\mu'(l) > 0$, $\sigma^2 > 0$,

(*ii*) *Pareto:*

$$f(y,l) = \frac{k(l)y_m^{k(l)}}{y^{k(l)+1}},$$

with $x_m > 0$, $k(l) > 0$ *and* $k'(l) < 0$.

Then it follows that $h_{yy}(y,l)/h_y(y,l) = -1/y$ so that the linear tax is optimal if and only if:

$$c = \frac{1}{(P[cy + d] - 2A[cy + d])y} \tag{19}$$

is independent of y, *where* c *is the optimal retention rate and* $d = (1 - c)E[y]$ *is the optimal basic income .*

In particular, condition (19) is violated if consumption utility belongs to the HARA class.

Proof: The lognormal and Pareto densities belong to the exponential family, hence satisfies Assumption 3.2. It is straightforward to show that $h_y(y,l) = \mu'^2 y]$ in the lognormal case and $h_y(y,l) = -k'(l)/y$ in the Pareto case so that h is increasing-concave in y under assumptions (*i*)-(*ii*). It follows that $h_{yy}(y,l)/h_y(y,l) = -1/y$ in either configuration, hence, from Lemma 3.1, that $c = 1/\{(P[cy + d] - 2A[cy + d])y\}$ should then be independent of y. The latter condition is violated in the HARA case, as c is then shown to be a hyperbolic function of y, hence not constant. □

Note that assumptions (*i*)-(*ii*) in Proposition 3.3 imply that the mean of the distribution is an increasing function of effort in both cases. For lognormal or Pareto distributions, the optimality of the linear income tax is ruled out under the fairly class of HARA utility functions. Finally, it is not difficult to show that condition (19) also

arises if the density is Chi-squared with degree of freedom l, as $h_y(y, l) = 2/y$ then.

4 Solving for the Optimal Marginal Tax Rate under HARA Utility

The purpose of this section is to solve explictly for the optimal marginal tax when consumption utility belongs to the HARA class, i.e. when $A[c] = \gamma/(\phi + c)$, with the restrictions that $\gamma > 1$ and $\phi + c > 0$. It follows that $P[c] = (1 + \gamma)/(\phi + c)$ and $P[c] - 2A[c] = (1 - \gamma)/(\phi + c)$.

We know from Lemma 3.1 that the optimal consumption shedule satisfies:

$$\frac{c''(y)}{c'(y)} = \frac{h_{yy}(y, l)}{h_y(y, l)} + (P[c(y)] - 2A[c(y)])c'(y), \quad \text{for all } y.$$

Therefore, $c(y)$ is increasing-concave, that is, the marginal rax rate is an increasing function of income under Assumptions 3.1 and $\gamma > 1$. We now focus on two examples, within the exponential family, which contain a fairly large class of usual density functions and we further show that the asymptotic marginal rax rate is unity.

Assumption 4.1

Consumption utility $u[c]$ belongs to the HARA class, with $A[c] = \gamma/(\phi + c)$, $\gamma > 1$ and $\phi + c > 0$.

4.1 Linear Likelihood Ratio

The case such that $h(y, l)$ is linear in y arises when the conditional density $f(y, l)$ is either gaussian or exponential. Then $h_{yy}(y, l) = 0$ and one has to solve $c''(y)/c'(y) =$

$c'(y)(1-\gamma)/(\phi+c(y))$. Integrating the latter equation gives $c'(y) = \psi(\phi+c(y))^{1-\gamma}$, for some $\psi > 0$, which can be integrated again to yield $c(y) = [\gamma(\psi y + \eta)]^{\frac{1}{\gamma}} - \phi$, for some $\eta > 0$.

Proposition 4.1 (MTR under HARA Utility and Linear Likelihood)

Under Assumption 4.1, if the likelihood ratio is linear in income so that $h_{yy}(y,l) = 0$ for all y, then the optimal consumption schedule is $c(y) = [\gamma(\psi y + \eta)]^{\frac{1}{\gamma}} - \phi$, where $\psi > 0$, so that $c'(y) = \psi[\gamma(\psi y + \eta)]^{\frac{1}{\gamma}-1}$ and $c''(y) < 0$ for all $y \geq 0$.

It follows that the optimal marginal tax rate $t'(y) = 1 - c'(y)$ is such that:

$$\lim_{y\to\infty} t'(y) = 1.$$

A simple example arises under CARA utility, that is, when $\phi \equiv \gamma/\alpha$. Then $A[c] = P[c] = \alpha > 0$ when $\gamma = \infty$ and one gets $t'(y) = 1 - 1/(\alpha y + \nu)$, where $\nu = 1/(1 - t'(0))$ depends on the free initial condition $t'(0)$. If, for example, $t'(0) = 0$ then $\nu = 1$ and $t'(y) = 1 - 1/(\alpha y + 1)$, t''^{-2}, with $t''(0) = \alpha$, and $t'''(y) < 0$ for all y. Figure 1 plots the graph of the marginal tax rate when $\alpha = 1$.

Insert Figure 1 about here

4.2 Logarithmic Likelihood Ratio

This case arises when $f(y,l)$ is, e.g., lognormal, Pareto, beta, chi-squared, or gamma. Then $h(y,l)$ is logarithmic in y so that $h_{yy}(y,l)/h_y(y,l) = -1/y$ for all $y \geq \underline{y} > 0$ and the marginal retention rate satisfies $c''(y)/c'(y) = -1/y + c'(y)(1-\gamma)/(\phi+c(y))$.

Proposition 4.2 (MTR under HARA Utility and Logarithmic Likelihood)

Under Assumption 4.1, if the likelihood ratio is logarithmic in income so that $h_{yy}(y,l)/h_y(y,l) = -1/y$, then the optimal consumption schedule is $c(y) = [\gamma(\psi \ln(y) + \eta)]^{\frac{1}{\gamma}} - \phi$, where $\psi > 0$, so that $c'(y) = \psi[\gamma(\psi \ln(y) + \eta)]^{\frac{1}{\gamma}-1}/y$ and $c''(y) < 0$ for all $y \geq \underline{y} > 0$. It follows that the optimal marginal tax rate $t'(y) = 1 - c'(y)$ is such that:

$$\lim_{y \to \infty} t'(y) = 1.$$

For example, suppose that consumption utility is logarithmic, that is, $\gamma = 1$ and $\phi = 0$. Then the marginal tax rate is $t'(y) = 1 - \psi/y$, where ψ is determined by the initial condition at minimal income $\underline{y} > 0$. If one assumes that, e.g., $t'(\underline{y}) = 0$ then $\psi = \underline{y}$ and $t'(y) = 1 - \underline{y}/y$, $t''(y) = \underline{y}y^{-2}$, with $t''(\underline{y}) = 1/\underline{y}$, and $t'''(y) < 0$ for all y. Figure 2 plots the graph of the marginal tax rate when $\underline{y} = 1$.

Insert Figure 2 about here

Under HARA utility and either linear or logarithmic likelihood ratio, therefore, explicit solutions for the optimal tax can be derived. As can be seen from Figures 1-2, the optimal marginal tax rate that obtains differs markedly from the linear tax rate. This is most obvious when risk is small, in which case the analysis in Section 2 predicts that the flat tax rate is small when the risk's variance is small (see e.g. the expression in (7) when the tax is restricted to be linear). This is in contrast with some simulations reported in Varian [21] for the CRRA-Gaussian case, which take as given the Lagrange multipliers and conclude that the nonlinear marginal tax rates are small when risk is small. Our examples above suggest that this is not generally the case.

The result that the asymptotic optimal marginal tax rate is unity arises because, in our model, only risk may lead to very high income levels. When agents are ex-ante

identical, it is optimal, from a social insurance point of view, to tax heavily high incomes so as to compensate for unlucky agents.

5   Conclusion

This paper reexamines the well-known intuition that income risk might be a powerful force pushing towards income tax schedules that exhibit marginal progressivity. We have shown that under an utilitarian social welfare function, not only is the optimality of the linear tax very restrictive, but marginal progressivity is likely to arise for reasonable assumptions on the household's behavior towards risk. Therefore, income risk is a fundamental dimension to take account of if one is to speculate about optimal tax schedules and desirable tax reforms.

At a more methodological level, this paper underlines that under HARA utility and linear (or logarithmic) likelihood ratio, the standard moral-hazard model can be explicitly solved for the optimal agent's transfer. Whereas we have focused on applications pertaining to taxation issues, our analysis might prove useful for other uses as well. For instance, managerial compensation and other insurance problems may be cast into the setting studied here.

Finally, the effect of income risk on the marginal progressivity of optimal income taxes underlined in this paper is likely to appear in more general settings. Most importantly, it remains to be studied how it interacts with redistribution purposes when households are ex-ante heterogenous. We believe this calls for further research.

A Existence of the Optimal Nonlinear Income Tax

The purpose of this appendix is to prove the existence of the optimal tax $t(y) = y - c(y)$ defined from Lemma 3.1. The strategy is to show that the problem in Section 3 can be recast as an optimal control problem for which Varian [21, p. 66-67] provides an existence theorem. Essentially, this class of problems is such that (a) the Hamiltonian does not depend on the state variables and (b) the maximization takes place over a function $c(y)$, the consumption schedule, and a parameter l, the effort supplied by the household.

Our original problem is:

$$\max_{c(.), l} \int g(l)u[c(y)]f(y, l)dy - v(l) \tag{20}$$

subject to:

$$g'(l) \int u[c(y)]f(y, l)dy + g(l) \int u[c(y)]f_l(y, l)dy - v'(l) = 0, \tag{21}$$

$$\int [y - c(y)]f(y, l)dy = 0 \quad \text{and} \quad \bar{l} \geq l \geq 0, \ \bar{l} \geq c(y) \geq 0. \tag{22}$$

As in Varian [21, p. 60], we introduce two dummy variables:

$$M(y) \equiv \int_{\underline{y}}^{y} [t - c(t)]f(t, l)dt,$$

where $\underline{y}$ is the minimum value of all income realizations, and:

$$N(y) = g'(l) \int_{\underline{y}}^{y} u[c(t)]f(t, l)dt + g(l) \int_{\underline{y}}^{y} u[c(t)]f_l(t, l)dt - v'(l).$$

Then the maximization program (20)-(22) can be restated as:

$$\max_{c(.), l} \int g(l)u[c(y)]f(y, l)dy - v(l) \tag{23}$$

subject to:

$$M'(y) = [y - c(y)]f(y, l) \quad \text{and} \quad N'(y) = g'(l)u[c(y)]f(y, l) + g(l)u[c(y)]f_l(y, l) - v'(l), \tag{24}$$

with:

$$M(\underline{y}) = M(\overline{y}) = N(\underline{y}) = N(\overline{y}) = 0 \ \text{ and } \ \overline{l} \geq l \geq 0, \ \overline{c} \geq c(y) \geq \underline{c}, \tag{25}$$

where $\overline{y}$ is the maximum value of all income realizations.

The problem (23)-(25) belongs to the class of optimal control problems for which the appendix in Varian [21, p. 66-67] proves that a solution exists.

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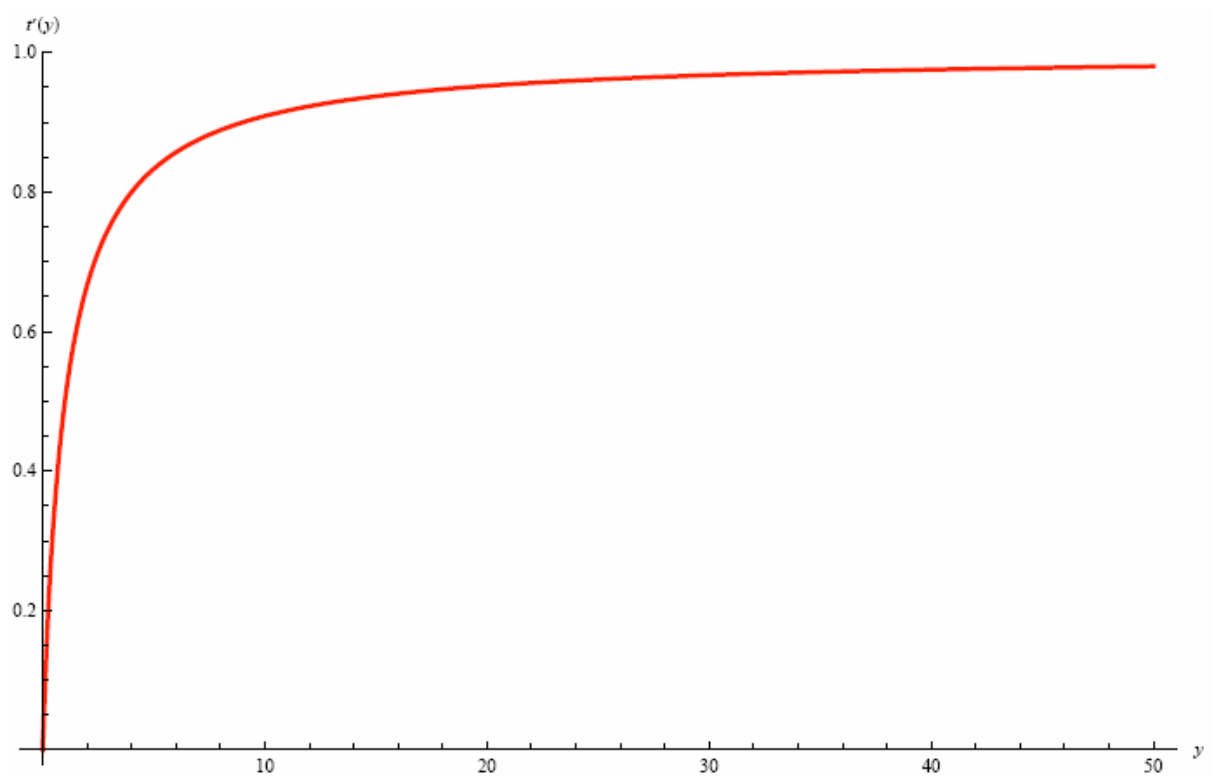[23] Werning, I. (2007). "Pareto efficient income taxation" WP. NBER.

Figure 1: the optimal marginal tax rate as a function of income under CARA utility (with $\alpha = 1$) and linear likelihood ratio
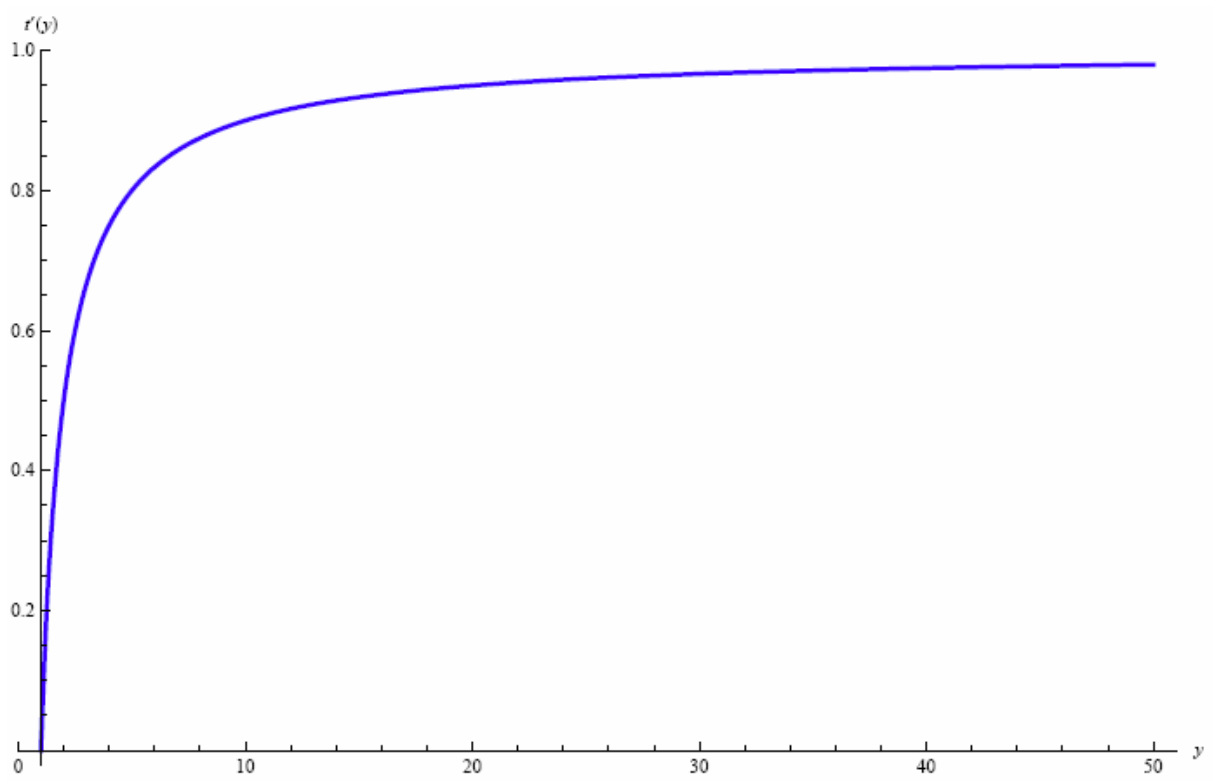
Figure 2: the optimal marginal tax rate as a function of income under logarithmic utility and logarithmic likelihood ratio