Tailoring Bank Capital Regulation for Tail Risk

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Abstract

The experience of the 2007-09 financial crisis has showed that the bank capital regulation in place was inadequate to deal with "manufacturing" tail risk in the financial sector. This paper proposes an incentive-based design of bank capital regulation aimed at efficiently dealing with tail risk engendered by bank top managers. It has two specific features: (i) first, it incorporates information on the optimal incentive contract between bank shareholders and bank managers, thereby dealing with the internal agency problem; (ii) second, it relies on the mechanism of mandatory recapitalization to ensure this contract is adopted by bank shareholders.

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1   Introduction

One of the important roles of bank capital regulation is to restrain banks from excessive risk-taking in the context of the explicit and implicit government guarantees they may enjoy. However, the experience of the 2007-2009 financial crisis shows clearly that capital regulation in place at the time failed to perform this role. This failure may be partially related to the fact that the capital regulation framework did not adjust in time to the substantial changes in bank business culture brought about by new techniques in financial engineering, growing securitization and the development of complex financial products. One of the direct results of these changes was the emergence of *tail risk* characterized by rare but devastating losses. There is now extensive empirical evidence that, by focusing on short-term performance, top management in banks was engaged in "manufacturing" tail risk *(Acharya et al.(2010))* in order to generate immediate profits, without regard to the long-term consequences. It seems that risk management strategies in the period preceding the crisis were beyond the control of bank shareholders. In such a context, to deal effectively with such "manufactured" tail risk in the banking sector, bank capital regulation should have taken into consideration the internal agency problem, instead of focusing exclusively on the interests of bank shareholders.

This study proposes an incentive-based design of capital requirements which will induce shareholders to shape executive compensation in such a way as to prevent managers from engendering tail risk. To illustrate these proposals, we build a simple continuous-time model in the principal-agent framework, where a senior bank manager has a reversible choice between prudent and imprudent risk management strategies. An imprudent risk management strategy exposes the bank to *tail risk* characterized by infrequent but devastating losses, while a prudent risk management strategy implies no tail risk but generates a lower expected asset return. We assume that imprudent risk management allows the manager to collect private benefits. Such a set-up corresponds to the double moral hazard problem, since it is not only the manager who may be interested in engaging in tail risk, but bank shareholders as well. To be able to prevent imprudent risk management in this context, incentive-based capital requirements should incorporate information about the optimal incentive contract between bank shareholders and bank manager. This feature differentiates our capital regulation design from existing approaches, which deal with a bank as if it was a black box.

The existing capital regulation literature (see, for instance, *Brattachariya (2002), Décamps et al. (2004), Rochet (2004))* shows that *portfolio risk* can be prevented through the mandatory incentive-based *liquidation* rule. However, the *liquidation* rule appears to be ill-suited to *tail risk*: even in the absence of internal agency problems, it fails to prevent the bank from engaging in tail risk when bank asset value approaches the liquidation point. In fact since, in the neighborhood of the liquidation threshold, the bank default is more likely to be triggered by random fluctuations of asset return rather than by infrequent large losses, the bank will engage in tail risk in order to increase the asset growth rate and to move away from the liquidation point.[1] To specifically deal with tail risk, we design capital requirements in the form of the incentive *recapitalization* rule. Under the enforceable recapitalization rule, portfolio risk cannot cause bank failure. At the same

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[1]The auxiliary result we get, while demonstrating this feature, is the design of equity value in a setting, where the underlying asset's value evolves according to jump-diffusion process.

time, engaging in tail risk will increase the likelihood of further mandatory recapitalizations and thus impose higher expected recapitalization costs on bank shareholders. In fact, it is the fear of bearing the additional recapitalization costs due to tail risk exposure that will induce bank shareholders to promote prudent risk management in their bank.

Although costly recapitalizations are studied in the liquidity management literature *(Décamps et al. (2011), Rochet and Villeneuve (2011))* and the capital regulation literature *(Peura and Keppo (2003), Milne and Walley (2002))*, no one has previously explored the potential incentive effect of mandatory recapitalizations, to the best of our knowledge. However, we are not the first to point out the need to consider the internal agency problem within a capital regulation design. *Bris and Cantale (2004)* address this issue in the context of portfolio risk management, examining the impact of capital requirements on the effort choice of a self-interested risk-averse bank manager in a discrete time framework. They come to the conclusion that capital regulation, which does not take into account the internal agency problem, leads to a socially-unoptimal choice of the *lower* level of risk. Here, in the context of tail risk and with risk-neutral agents, our model shows, in contrast, that the bank will operate at the higher level of risk. Moreover, we explicitly show how to adapt the optimal design of capital regulation to take account of the internal agency problem, linking the literature on incentive capital regulation and the fast-growing literature on optimal contracts in a dynamic framework *(Sannikov(2008), Biais et al. (2010), DeMarzo, Lidvan and Tchistyi (2011))*.

The nature of the moral hazard problem studied in this paper situates it close to *Biais et al. (2010)*. In their model, the agent's effort affects a firm's exposure to a disaster risk, given that lower effort enables the manager to collect private benefits proportional to the firm's size. However, they consider Poisson risk to be the sole type of risk the firm faces, whereas in our setting the bank asset value is also affected by Brownian risk. Both Poisson and Brownian risks are present in the model of *DeMarzo, Lidvan and Tchistyi (2011)*, who design an optimal contract in a two-dimensional setting with Arithmetic Brownian Motion (ABM), when the manager can privately choose between two risk regimes affecting a firm's profit and also has the option of diverting part of the generated cash flow. In our model constructed in a setting with Geometric Brownian Motion (GBM), the manager controls an asset growth, rather than a bank's profitability. Moreover, we restrict our analysis to the class of linear contracts so as to be able to integrate information on the structure of the optimal contract into the capital regulation design.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 explains the incentive effect of mandatory recapitalizations. In Section 4 we design an optimal mandatory recapitalization policy in a setting free of the internal agency problem. Section 5 presents an optimal mandatory recapitalization policy in the context of the internal agency problem. In Section 6 we discuss related regulatory policy issues. Section 7 concludes. All proofs are provided in Appendix B.

2 The model

We consider a risk-neutral bank protected by limited liability. The bank is financed by a constant volume of insured deposits, D, and incurs a continuous payment rD to

depositors,[2] where r denotes a riskless interest rate. Bank assets continuously generate a cash-flow δx_t, where x_t denotes a publicly observable asset value. Thus, an instantaneous cash-flow received by shareholders after paying debt service is given by $\delta x_t - rD$.[3]

The bank is run by a manager. The contract with the manager can be described by a triplet $\{x_T, R(x), R_T\}$, where x_T denotes a contract termination rule,[4] $R(x) \geq 0$[5] for $x > x_T$ is an asset-based remuneration and $R_T \geq 0$ is a terminal pay-off delivered at the contract termination date. The manager is protected by limited liability and has no initial wealth which could be pledged.

The manager has a reversible discretionary choice between 2 risk management strategies: prudent and imprudent. *Imprudent* risk management allows for expected asset return rate μ and involves a bank in tail risk, which implies infrequent but large losses of bank assets. Large losses caused by tail risk materializing follow a Poisson process $\{N_t\}_{t \geq 0}$ with intensity λ. A large loss destroys a fraction $(1 - \alpha)x_t$ of bank assets, where $\alpha \in (0, 1)$ is a constant coefficient reflecting the proportion of assets remaining after the large loss is realized. *Prudent* risk management strategy is free of tail-risk exposure but has a lower expected asset return rate, $(\mu - \Delta\mu)$, where $0 < \Delta\mu < \lambda(1 - \alpha)$. From now on we will assume that $r > \delta + (\mu - \Delta\mu)$.

Let $u_t \in \{0, 1\}$ be a control variable reflecting risk-management strategy, where $u_t = 1$ corresponds to prudent and $u_t = 0$ corresponds to imprudent risk management. Bank asset value follows:[6]

$$dx_t = (\mu - u_t\Delta\mu)x_t dt + \sigma x_t dZ_t - (1 - u_t)(1 - \alpha)x_t dN_t, \qquad (1)$$

where σ is the asset return volatility and $\{Z_t\}_{t \geq 0}$ is a standard Brownian process.

We assume that imprudent risk management allows the manager to collect private benefits $bx_t dt$. Private benefits may represent proceeds from private trading on the financial markets, given that tail-risk management strategies may boost equity growth in the short term.[7] However, materialization of tail risk, will make the choice of management technology verifiable *ex-post*, so that the contract with the manager can stipulate that he will be fired without any terminal pay if a large loss occurs. The threat of being fired after the loss represents a maximum feasible punishment under the limited liability of the manager, allowing shareholders to minimize the cost of creating incentives.

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[2]We do not explicitly model deposit insurance premium, since recapitalization mechanism will eliminate default risk. Otherwise, deposit insurance premium could be deducted from the expected asset return rate.

[3]Note that $\delta x - rD < 0$ corresponds to liquidity injections from the "deep pockets" of bank shareholders as long as they interested in keeping the bank alive.

[4]Since we are dealing with a stationary problem, a termination rule x_T will determine the expected contract duration.

[5]We could consider a non-zero reservation wage for the manager in order to care about the existing competition for top managers in banking sector. However, the focus of this paper is on prudent risk management and not on value creation, which strongly depends on specific managerial talent.

[6]Expression (1) captures a trade-off between faster asset growth and asset safety, formalizing a so-called "search for yield" strategy of banks in the period prior to the crisis. Indeed, practices like aggressive subprime lending, abusive use of securitization, poor trading discipline, creative accounting might allow the bank to create the illusion of high performance in the short-run but will inevitably lead to large losses in the long run.

[7]*Bebchuk at al. (2010)* estimate that the top managers of Bear Stearns and Lehman Brothers were able to realize about \$2 bln, by unloading shares and options during 2000-2008.

Since large losses incurred by a single bank may inflict negative externalities on the rest of the banking sector (especially when they lead to the bank's failure), we assume that the objective of the bank regulator is to induce prudent risk-management by using capital regulation tools. Capital regulation in our model is designed in the form of a mandatory recapitalization policy involving two regulatory parameters: mandatory recapitalization rule x_R and recapitalization coefficient $s > 1$, such that sx_R is the target bank asset value after the mandatory recapitalization. Thus, bank shareholders should inject fresh equity capital $(s - 1)x_R$ each time the bank asset value hits the mandatory recapitalization threshold x_R. However, if the bank asset value suddenly falls below the recapitalization threshold, the bank should be liquidated by the regulator (i.e., incumbent shareholders will be expropriated of equity), since a sudden violation of capital requirements shows imprudent risk management.[8]

As in *Décamps et al. (2011)*, we allow for two types of recapitalization costs: proportional costs ξ_1, which are imposed on each unit of capital raised, and lump-sum costs ξ_0. Recapitalization costs can reflect taxes, expert and registration costs of the new equity issue, as well as asset restructuring costs.[9] We denote $K(x_t)$ the total costs shareholders incur when issuing new equity at time t

$$K(x_t) = (\xi_1 + 1)(s - 1)x_t + \xi_0 \tag{2}$$

As we will show in the next section, recapitalization costs represent a driving force of the proposed incentive mechanism.

3   The incentive effect of recapitalizations

To illustrate the need for capital regulation in the above set-up and its value when implemented in the form of mandatory recapitalization policy, let us consider the optimal strategy of bank shareholders when there is no capital regulation.

To focus on the incentive effect of recapitalizations, we will abstract from the internal agency problem, analyzing risk management decisions in the owner-managed bank. Bank shareholders maximize equity value, by instantaneously deciding which risk management strategy to implement. This decision is driven by a trade-off between the instantaneous gain from imprudent risk management and the expected loss of equity value caused by tail risk realization. We introduce a second order differential operator $A_u f(x)$ such that:

$$A_u f(x) = 1/2\sigma^2 x^2 f''(x) + (\mu - u\Delta\mu)x f'(x) - rf(x), \tag{3}$$

where $u \in \{0, 1\}$ and $f(x)$ is any contingent claim.

Then, the shareholders' maximization problem can be written as follows:

$$\max_{u_t \in \{0,1\}} \{A_{u_t} E(x_t) - (1 - u_t)\lambda(E(x_t) - E(\alpha x_t)) + \delta x_t - rD\} = 0, \tag{4}$$

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[8]Equity expropriation represents a maximum feasible penalty under the limited liability of bank shareholders, allowing for the most efficient design of the incentive mechanism. However, this penalty will never be applied ex-post, since the incentive-based recapitalization mechanism will completely eliminate imprudent risk management.

[9]Note that liquidity injections realized by shareholders in order to ensure the continuity of debt service are assumed to be costless, since they do not affect asset value and thus do not require asset restructuring. In contrast, a recapitalization which leads to asset expansion will incur asset restructuring costs.

where x_t is given by (1) and $E(x)$ denotes bank equity value in the absence of the internal agency problem.

Consequently, bank shareholders have an interest in implementing prudent risk management, so long as the expected negative jump of equity value caused by a realization of tail risk exceeds the instantaneous gain from imprudent risk management:

$$\lambda(E(x) - E(\alpha x)) \geq \Delta \mu x E'(x) \tag{5}$$

Let x_u^* be a critical threshold which makes the incentive constraint (5) binding and let $x_L^S < x_u^*$ denote a threshold:

$$x_L^S = \frac{-\gamma_2}{1 - \gamma_2} \frac{r}{r + \lambda} \frac{D}{\nu}, \tag{6}$$

where $\gamma_2 < 0$ is a root of $1/2\sigma^2 \gamma(\gamma - 1) + \mu \gamma = r + \lambda$ and $\nu = \frac{\delta}{r - \mu + \lambda} < 1$.

Result 1 *Without any regulatory control, the owner-managed bank will implement prudent risk management for $x_t \geq x_u^*$, will opt for imprudent risk management for $x_t < x_u^*$ and will be liquidated by shareholders when $x_t = x_L^S$.*

To see the intuition of the above result, consider first the optimal choice between a costly recapitalization and bank liquidation. The fact that $r > \delta + (\mu - \Delta\mu)$ means that the return on a one-dollar investment in riskless security would be higher than that on a one-dollar investment in the bank asset portfolio. Then, whatever the effort strategy chosen, in the absence of regulatory control bank shareholders will never recapitalize the bank on their own and will strategically default at the optimal liquidation point. However, in the neighborhood of the liquidation point, a moral hazard problem emerges as a consequence of the conflict between portfolio risk and tail risk. In fact, when bank asset value becomes relatively low, bank liquidation is more likely to be triggered by a continuous decline of bank asset value rather than by a sudden negative jump. As a result, the bank will "gamble for resurrection", optimally engaging in tail risk in order to increase asset growth and to move away from the liquidation point. Here, capital regulation is needed in order to induce the bank to maintain prudent risk management.

A capital regulation literature dealing with the asset substitution problem has shown that it is possible to prevent a bank from choosing a riskier portfolio by imposing an appropriate incentive *liquidation* rule (see, for example, *Brattachariya (2002), Décamps et al. (2004), Koziol and Lowrenz (2012)*). The next question is then: why can't we prevent the bank from engaging in tail risk in a similar way? Why do we need, instead, to resort to mandatory recapitalizations?

Let us consider the incentives of bank shareholders when the regulator imposes any arbitrary *liquidation* rule $x_L \geq \widehat{x}_L$, where[10]

$$\widehat{x}_L = \frac{-\beta_2}{1 - \beta_2} \frac{D}{\eta} \tag{7}$$

where $\beta_2 < 0$ is a root of $1/2\sigma^2 \beta(\beta - 1) + \mu\beta = r$ and $\eta = \frac{\delta}{r - \mu + \Delta\mu} < 1$.

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[10]Note that $\widehat{x}_L$ represents the optimal shareholders' closure rule, provided that the bank sticks to prudent risk management.

A verification of necessary condition (5) shows that it doesn't hold when bank asset value approaches x_L. Thus, in the neighborhood of the mandatory *liquidation* threshold the bank will engage in tail risk for the same "gambling-for-resurrection" reasons we described above.

Result 2 *Any mandatory **liquidation** rule $x_L \geq \widehat{x}_L$ would be unable to prevent tail risk.*

It follows from this result that a mere increase in capital requirements would be insufficient to discourage banks from "manufacturing" tail risk. As pointed out by several recent studies (see, for example, *Perotti et al. (2011)*), new regulatory tools are required in order to maintain control over tail risk. Incentive-based mandatory recapitalization policy could be viewed as one of them. Indeed, the effect of the *recapitalization* rule will be different from the effect of the *liquidation* rule. A crucial point is that, *under the mandatory recapitalization rule, asset return volatility cannot lead to the bank's failure.* Thus, the mandatory *recapitalization* rule will eliminate a conflict between portfolio risk and tail risk. At the same time, realization of tail risk under an imprudent risk management strategy will raise the likelihood of mandatory recapitalizations, thereby increasing the total expected recapitalization costs. It is the very threat of suffering the additional recapitalization costs under tail risk exposure that will induce bank shareholders to promote prudent risk management in their bank.

Thus, in order to induce shareholders to promote prudent risk management, mandatory recapitalization policy should be designed in such a way that: (i) the expected loss of equity value caused by realized tail risk exceeds the instantaneous gain from imprudent risk-management; (ii) equity value at the mandatory recapitalization point is sufficiently high for shareholders to optimally prefer to recapitalize the bank rather than to be punished by equity expropriation.

4 Capital regulation when there is no internal agency problem: a benchmark

In order to track the impact of the internal agency problem on capital regulation, we first design the incentive mandatory recapitalization policy in a setting where the interests of the bank manager are perfectly aligned with the interests of the bank shareholders.

Let us formally define the regulatory problem. The regulator is looking for the optimal combination of recapitalization rule x_R and recapitalization coefficient $s > 1$ which will prevent imprudent risk management at the bank for $\forall x \geq x_R$, maximizing bank social value. Bank social value can be computed as a sum of bank equity value and the market value of deposits net of any social costs. However, in contrast to mandatory liquidation or a public bail-out, all costs associated with mandatory recapitalizations will be entirely borne by bank shareholders. At the same, under the incentive recapitalization mechanism, the bank will never default, so that the market value of deposits will remain constant. As a result, if the bank chooses prudent risk management, maximization of bank social value will be equivalent to maximization of bank equity value. Thus, for any current bank asset value $x_t > x_R$, the regulatory problem can be stated as follows:

$$\underset{x_R > 0, s > 1}{\text{Max}} \quad E(x) \geq 0 \qquad \text{s.t.}$$

$$\lambda(E(x) - \mathbb{1}_{x \geq x_R/\alpha} E(\alpha x)) \geq \Delta \mu x E'(x) \text{ for } \forall x \geq x_R,$$

where $E(x)$ is given by

$$E(x) = \mathbb{E}\left[\int_t^{+\infty} e^{-r(\tau - t)}(\delta x_\tau - rD - \mathbb{1}_{\{x_\tau = x_R\}} K(x_\tau))d\tau\right] \tag{8}$$

Let $\overline{\xi}_1$ denote a critical level of marginal recapitalization costs, such that for $\xi \geq \overline{\xi}_1$ a solution of the above problem does not exist:[11]

$$\overline{\xi}_1 = \left(\frac{1 - s^{\beta_2}}{s - 1}\right)\left(\frac{\lambda - \Delta \mu}{\lambda - \beta_2 \Delta \mu}\right)\eta + \eta - 1 \tag{9}$$

Proposition 1 *For $\xi_1 < \overline{\xi}_1$, the regulator can prevent an owner-managed bank from engaging in tail risk, by imposing the optimal mandatory recapitalization policy $\{s^*, x_R^B(s^*)\}$ which implies:*

- *(i) a recapitalization rule*

$$x_R^B(s) = \frac{(1 - s^{\beta_2})\lambda D + \xi_0(\lambda - \beta_2 \Delta \mu)}{(1 - s^{\beta_2})(\lambda - \Delta \mu)\eta - (1 + \xi_1 - \eta)(s - 1)(\lambda - \beta_2 \Delta \mu)} \tag{10}$$

- *(ii) a recapitalization coefficient $s^* = \arg \min x_R^B(s)$.*

It can easily be shown that bank equity value is decreasing on x_R. Thus, for any given recapitalization coefficient $s > 1$, the maximum equity value can be attained under the minimum feasible recapitalization rule $x_R^B(s) > D$, such that $\lambda E(x) = \Delta \mu x E'(x)$ at $x = x_R$.[12] Note that $E(x_R^B(s)) > 0$. This makes our recapitalization mechanism enforceable, since shareholders will prefer to recapitalize the bank rather than to be expropriated of equity. We also verify that, faced with mandatory recapitalization rule $x_R^B(s)$, shareholders will not undertake voluntary recapitalizations at any $x > x_R^B(s)$, since recapitalization costs would always exceed the expected growth of equity value resulted from capital injections.[13] Then, given that bank equity value is decreasing on recapitalization rule $x_R^B(s)$, a complete solution of the above regulatory problem will be delivered by $s^* = \arg \min x_R^B(s)$.

In order to illustrate the optimal mandatory recapitalization policy, we resort to numerical simulations. For the parameter set $D = 1$, $r = 0.04$, $\mu = 0.035$, $\Delta \mu = 0.005$, $\delta = 0.01$, $\sigma = 0.2$, $\lambda = 0.05$, $\alpha = 0.7$, $\xi_0 = [0.1 \times 10^{-4}, 0.1 \times 10^{-3}]$, $\xi_1 = [0.01, 0.1]$,[14] $x_R^B(s^*)$ varies in the range of $(1.12, 1.20)$ and s^* takes values between $(1.01, 1.08)$. For example, for $\xi_0 = 0.1 \times 10^{-3}$ and $\xi_1 = 0.1$ we obtain $x_R^B(s^*) = 1.20$ and $s^* = 1.03$, which

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[11]For $\xi_1 > \overline{\xi}_1$ the incentive condition never holds in the neighborhood of x_R.

[12]We show in Appendix B that, if the incentive constraint is respected for $x \in [x_R, x_R/\alpha)$, it will automatically be respected for any $x > x_R/\alpha$.

[13]In practice, shareholders are unwilling to undertake voluntary recapitalizations not only because of the private costs they incur, but also for fear of sending a negative signal about the bank's financial health. Making a recapitalization mandatory, however, might partially mitigate this signaling effect.

[14]Empirical estimations realized for the set of U.S. firms provide the following values of average marginal issuance costs: 2.8% in *Gomes (2001)*, 5.1% in *Altinkiliç and Hansen (2000)*, 10.7% for small firms and 5% for large firms in *Hennessy and Whited (2007)*.

corresponds to the minimum capital ratio of 16.7% and post-recapitalization capital ratio of 19, 1% respectively.

The optimal recapitalization coefficient results from the trade-off between two opposite effects generated by fixed and proportional recapitalization costs: s^* is increasing on ξ_0 and decreasing on ξ_1. Fixed recapitalization costs encourage shareholders to raise a maximum of funds in order to postpone further mandatory recapitalizations, whereas proportional costs reduce recapitalization capacity.[15] The optimal recapitalization rule $x_R^B(s^*)$ is increasing on both fixed and proportional recapitalization costs. Indeed, given significant recapitalization costs, the bank equity must be strong enough to persuade shareholders to choose costly recapitalizations over bank liquidation. Moreover, $x_R^B(s^*)$ is increasing on asset volatility, since higher σ exacerbates a trade-off between tail risk and portfolio risk, aggravating a moral hazard problem.

5 Capital regulation under the internal agency problem

Now we turn to the set-up which allows for the internal agency problem. The manager has a different perception of tail risk exposure from that of shareholders, since his objective deviates from the maximization of equity value. Thus, besides the instantaneous loss $\Delta \mu x_t dt$ in terms of asset growth, the real cost of prudent risk management for shareholders will be increased by the amount of incentive compensation. In such a context, shareholders have to make two strategic decisions: (i) whether to promote prudent risk management in their bank and (ii) if so, how to create the appropriate incentives for the manager at minimum cost. We start by answering the second question, defining the optimal incentive contract with the manager. Then we incorporate a structure of the optimal incentive contract into the initial problem of bank shareholders and build the optimal incentive-based recapitalization policy which will induce shareholders to promote prudent risk management in their bank.

5.1 The optimal incentive contract

Assume there is some incentive recapitalization policy $\{x_R, s\}$ for which shareholders find it optimal to promote prudent risk management in their bank for $\forall x \geq x_R$. Then, we need to find an incentive contract which will induce the manager to maintain prudent risk management at a minimum cost for bank shareholders. We restrict our analysis to the class of linear contracts, looking for asset-based compensation in this form:

$$R(x) = w_0 + w_1 x, \tag{11}$$

where $w_0 \geq 0$ and $w_1 \geq 0$.

We analyze the manager's incentives for prudent risk management. Given any contract $\{x_T, R(x), R_T\}$, the manager maximizes contract continuation value, $W(x)$, which

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[15]*Décamps et al. (2011)* point out the similar effects of recapitalization costs in the context of liquidity management.

is contingent on bank asset value and represents the current expected value of total future gains from managerial position, including any private benefits:

$$W(x) = \mathbb{E}\left[\int_t^{\tau \wedge \tau_T} e^{-r(\tau - t)}(R(x_\tau) + (1 - u_\tau)bx_\tau)d\tau + e^{-r(\tau_T - t)}R_T\right], \qquad (12)$$

where $u_\tau \in \{0, 1\}$, x_τ follows (1) and $\tau_T = \inf\{t \geq 0 \; s.t \; x_t \leq x_T\}$.

Imprudent risk management has an ambiguous effect on the manager's wealth. On the one hand, it increases contract continuation value due to higher expected asset return and private benefits. On the other hand, the manager risks losing his position (and, consequently, the expected value of further payoffs) with probability λdt in a short period of time dt. Then, the manager's maximization problem can be stated as follows:

$$\max_{u_t \in \{0,1\}} \{A_{u_t}W(x_t) + R(x_t) - (1 - u_t)(\lambda W(x_t) - bx_t)\} = 0 \qquad (13)$$

The manager will choose prudent risk management as long as the expected loss of contract continuation value under tail-risk exposure exceeds the instantaneous gain from imprudent risk management. Then, the optimization problem of bank shareholders looking to minimize the costs of creating incentives for the manager, $MC(x)$, can be stated as follows:

$$\min_{x_T \geq x_R, w_0 \geq 0, w_1 \geq 0, R_T} MC(x) = \mathbb{E}\left[\int_t^{+\infty} e^{-r(\tau - t)}(\mathbb{1}_{x_\tau \neq x_T}R(x_\tau) + \mathbb{1}_{x_\tau = x_T}R_T)d\tau\right]$$

$$\text{s.t.}$$

$$\lambda W(x) \geq \Delta\mu x W'(x) + bx, \quad \forall x \geq x_R \qquad (14)$$

We can have either $x_T = x_R$, which means that the manager will be replaced at the recapitalization point, or $x_T = \emptyset$, which means that the manager will be allowed to keep his position forever if no loss occurs. In Appendix B we compare the minimum value of shareholders' costs under these alternatives. We show that, faced with $x_T = x_R$, the manager has to be provided with a terminal payoff R_T equal to the expected value of the further contract payoffs he could obtain from continuation. Conversely, letting the manager stay forever provided that no loss occurs would allow shareholders to avoid these payments.

Proposition 2 *The optimal incentive contract which will induce prudent risk management at minimum cost implies*[16]

$$R(x) = \frac{\delta}{\eta}\frac{b}{\lambda - \Delta\mu}x \equiv w_1^* x \qquad (15)$$

The manager is never fired provided no loss occurs, i.e., $x_T = \emptyset$ and $R_T = \emptyset$.

The total shareholders' costs associated with the implementation of the optimal incentive contract will coincide with the minimum incentive-compatible contract continuation value for the manager:

$$MC^*(x) \equiv W^*(x) = \frac{\eta}{\delta}w_1^*\left(x + x_R\left(\frac{s - 1}{1 - s^{\beta_2}}\right)\left(\frac{x}{x_R}\right)^{\beta_2}\right) \qquad (16)$$

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[16]Note that $w_1^* > 0$, since $\lambda < \Delta\mu(1 - \alpha) < \Delta\mu$.

It is easy to see that $MC^*(x)$ are increasing with the manager's ability to generate private gains on the financial market, b. In practice, top executives are partially remunerated via equity-based compensation. In the context of our model, this would create a self-amplifying mechanism: by rewarding the manager with stock options and stocks, shareholders would increase what he stands to gain from the financial markets. In order to induce prudent risk management, shareholders will have to increase a variable proportion of the incentive compensation, which will again raise risk-taking incentives and so forth. Thus, it would be useful to maintain control over the proportions of equity-based compensation within compensation packages, in order to make managers less sensitive to the short-term reactions of the financial markets to their performance.

5.2   The optimal mandatory recapitalization policy

Given the optimal incentive contract with the manager, we are now in a position to address the initial problem of bank shareholders faced with a decision on whether to induce prudent risk management or to assume tail risk exposure. Given the internal agency problem, bank shareholders have an interest in promoting prudent risk management in their bank when the following incentive condition is satisfied:

$$\lambda(E_{W^*}(x) - \mathbb{1}_{x \geq x_R/\alpha} E_{W^*}(\alpha x)) \geq \Delta \mu x E'_{W^*}(x) + w_1^* x, \tag{17}$$

where

$$E_{W^*}(x) = \mathbb{E}\left[\int_t^{+\infty} e^{-r(\tau - t)}((\delta - w_1^*)x_\tau - rD - \mathbb{1}_{\{x_\tau = x_R\}}K(x_\tau))d\tau\right] \tag{18}$$

In contrast to the benchmark case, the incentive condition of bank shareholders now incorporates information on the optimal incentive contract. Then, the regulatory problem in the context of the internal agency problem becomes:

$$\underset{x_R > 0, s > 1}{\text{Max}} \quad E_{W^*}(x) \geq 0 \quad \text{s.t. (17) for} \quad x \geq x_R \tag{19}$$

Let $\bar{b}$ denote a critical value of private benefits such that for $b \geq \bar{b}$ the solution of the above problem does not exist:

$$\bar{b} = \eta\left(\frac{\bar{\xi} - \xi}{1 + \bar{\xi}}\right)(\lambda - \Delta\mu), \tag{20}$$

where $\bar{\xi}_1$ is given by (9). Indeed, for $\bar{b} < b < \delta$ the incentive condition never holds in the neighborhood of x_R, whereas for $b > \delta$ the optimal incentive contract would be too costly for shareholders.

Proposition 3 *If $\xi_1 < \bar{\xi}_1$ and $b < \bar{b}$, the regulator can prevent the bank from engaging in tail risk, by imposing the optimal mandatory recapitalization policy $\{s^{**}, x_R^A(s^{**})\}$ which implies:*

- *(i) a recapitalization threshold*

$$x_R^A(s) = \frac{(1 - s^{\beta_2})\lambda D + \xi_0(\lambda - \beta_2 \Delta\mu)}{(1 - s^{\beta_2})(\lambda - \Delta\mu)(1 - \frac{w_1^*}{\delta})\eta - (1 + \xi_1 - \eta(1 - \frac{w_1^*}{\delta}))(s - 1)(\lambda - \beta_2 \Delta\mu)}$$

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- *(ii) a recapitalization coefficient $s^{**} = \arg\min x_R^A(s)$.*

It can be shown that $x_R^A(s) > x_R^B(s)$ for the same recapitalization coefficient s, so that *an incentive recapitalization policy which does not allow for the internal agency problem would be unable to prevent the bank from engaging in tail risk.* This proves that the internal agency problem matters and should be taken into account by bank regulators when designing capital regulation.

It is worth noting that shareholders' costs of creating incentives for the manager can be reduced through internal random audits. If an audit uncovers imprudent risk management, the manager should be fired without receiving any terminal pay. This will increase his risk of losing contract continuation value should there be imprudent risk management, allowing shareholders to reduce the amount of incentive compensation. If instantaneous random audit costs are not too high, random audits could increase shareholders' incentives to promote prudent risk management in their bank, enabling the regulator to reduce a stringency of capital regulation. Moreover, recall that true bank asset value must be observable in order to implement the optimal incentive contract. Since the manager may have the opportunity to manipulate financial statements in order to enjoy higher rewards, random audits could also involve verifying bank asset value, which would discourage the manager from misreporting.

6 Discussion

6.1 The impact of bonus taxes on risk management

In the aftermath of the 2007-09 financial crisis, several European countries (UK, France, Italy, Greece) introduced a tax on the performance bonuses of bank top management. One of the official purposes of this measure was to improve the risk-management culture in the banking sector. We now use our model to examine the effect of bonus taxes on shareholders' incentives to promote prudent risk management in their bank.

Let a tax rate τ be applied to the variable proportion of managerial compensation $w_1 x$ which might be interpreted as a performance bonus. Assume first that taxes are paid by bank shareholders. In this case, bonus taxes have no impact on the manager's incentives, so that bank shareholders can induce prudent risk management by using the same optimal incentive contract as in a tax-free world. However, total shareholders' costs of creating incentives will be increased by the amount of taxes and will be equal to $MC(x) = (1 + \tau)W^*(x)$, where $W^*(x)$ is defined in (16). Consider now the alternative setting, where bonus taxes are paid by the bank manager.[17] To be motivated to maintain prudent risk management, the manager should have at least the same level of wealth after the tax levy as in a tax-free world. Then, the minimum incentive contract continuation value that should be offered to the manager will be given by $MC(x) = 1/(1-\tau)W^*(x)$.

The first conclusion that can be drawn from this analysis is that bonus taxes are inappropriate for dealing with excessive risk-taking in banks. Under both scenarios, they increase the real shareholders' cost of promoting prudent risk management and thus would lead to a situation requiring tougher capital regulation.[18] It is also easy to see that

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[17] We assume that bonus taxes are paid immediately after receiving compensation.

[18] In fact, the effect of bonus taxes is equivalent to the effect produced by increasing b.

total shareholders' costs of creating incentives when bonus taxes are paid by the manager would be higher than when bonus taxes are paid by the shareholders themselves. Thus, the lesser of two evils would be to collect bonus taxes from bank shareholders rather than from bank managers, which is consistent with the bonus tax policies adopted in UK and France in 2009-2010.

6.2   Implicit *vs* explicit regulation of managerial pay

There is now convincing empirical evidence that equity-based compensations have made bank executives willing to increase bank equity value at any price in order to reap gains *(Chen et al. (2006), Williams et al. (2008), Vallascas and Hagendorff (2010))*. In order to improve the risk-management culture in the banking sector, many academics and policymakers call for the *explicit* regulation of executive pay*(Bebchuk and Spamann (2010), Bolton et al. (2010))*. However, this proposal raises a range of concerns. The first problem is that regulators do not dispose of all the information needed for the efficient design and enforcement of executive compensation policy. Second, it is still unclear what form the optimal incentive compensation structure should take. Finally, experience suggests that economic agents always find a way to get around regulations if their incentives diverge from regulatory purposes. Thus, explicitly regulating executive pay without regulating shareholders' incentives would probably be a waste of regulatory resources. These reasons argue for implicit control over managerial incentives. The detailed design of managerial compensation should be left to bank shareholders; the role of the bank regulator is to ensure that shareholders have sufficient incentives to promote prudent risk management in their banks.

6.3   Capital requirements and insurance protection

The last relevant question is whether capital requirements should be reduced if a bank acquires an insurance policy against tail risk. Actually, only the advanced approach of the Basel II capital requirements considers an insurance policy as a risk mitigation tool and authorizes banks holding such policies to operate with reduced mandatory capital. However, it seems that greater reliance on insurance protection may aggravate the problem of moral hazard. The point is that an insurance policy allows banks to transfer risk without tackling it at source, i.e., it helps to reallocate risks but cannot prevent their accumulation within the financial system. Moreover, in the context of a systemic crisis, insurance companies themselves may experience serious financial problems,[19] being therefore unable to provide loss coverage. Thus, even though recourse to insurance may be beneficial for bank shareholders (i.e., it might be cheaper to buy an insurance policy than to create appropriate incentives for the manager), prudent risk management would be *the only durable solution* from the perspective of social welfare. Banks can be allowed to buy insurance protection against some external risks (like external fraud, hacking attacks, natural disasters), since an insurance policy will not promote moral hazard and risk accumulation in these cases. At the same time, regulators should induce banks to

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[19]This happened to AIG, one of the biggest players on the world insurance market, bailed out by the Federal Reserve Bank and the U.S. Treasury in 2008.

tackle the sources of internal risk. As we have shown, this can be realized by means of the incentive mandatory recapitalization policy which allows for the internal agency problem.

7   Conclusion

This study is an attempt to rethink the approach to bank capital regulation in response to the huge incentive distortions revealed by the 2007-09 financial crisis. We design a mandatory recapitalization policy, which deals with "manufactured" tail risk in banking. We show how, through the appropriate choice of mandatory recapitalization parameters, the regulator can induce bank shareholders to put in place an incentive compensation scheme that will deter bank managers from engaging in tail risk. We restrict our analysis to the class of linear contracts and derive the optimal incentive compensation in explicit form. It appears to be optimal to offer the manager a contract of indefinite duration. Otherwise, at the contract termination date, the manager would have to be granted a positive terminal payoff equal to the remaining contract continuation value, which would impose additional costs on bank shareholders.

Since the internal agency problem makes it costly for bank shareholders to promote prudent risk-taking behavior by bank managers, shareholders should be required to maintain a larger stake in the game. This might be viewed as a justification for more stringent capital requirements for systemically important banks, which typically have both severe agency problems and a predisposition to large losses. However, as we have shown, merely increasing capital requirements would not prevent banks from engaging in tail risk. For this reason, we suggest implementing capital regulation in the form of the incentive-based recapitalization policy.

Appendix A. Valuation of contingent claims

A.1. Equity value in the benchmark case

Let x_R be any arbitrary recapitalization rule. Let $K(x)$ denote the total costs shareholders incur when issuing new equity:

$$K(x) = (\xi_1 + 1)(s - 1)x + \xi_0$$

Equity value in the absence of the internal agency problem is given by

$$E(x) = \mathbb{E}\left[\int_t^{+\infty} e^{-r(\tau - t)}(\delta x_\tau - rD - \mathbb{1}_{\{x_\tau = x_R\}}K(x_\tau))d\tau\right] \tag{A1}$$

Solving a corresponding ODE

$$1/2\sigma^2 x^2 E''(x) + (\mu - \Delta\mu)xE'(x) - rE(x) + \delta x - rD = 0, \tag{A2}$$

under the boundary condition

$$E(x_R) = E(sx_R) - K(x_R), \tag{A3}$$

we obtain

$$E(x) = -\left[\frac{\xi_0 + (\xi_1 + 1 - \eta)(s - 1)x_R}{1 - s^{\beta_2}}\right]\left(\frac{x}{x_R}\right)^{\beta_2} + \eta x - D, \tag{A4}$$

where $\eta = \frac{\delta}{r - \mu + \Delta\mu} < 1$ and $\beta_2 < 0$ is a root of the characteristic equation

$$1/2\sigma^2\beta(\beta - 1) + (\mu - \Delta\mu)\beta = r$$

A.2. Equity value under the optimal incentive contract

Given the optimal incentive contract $\{x_T = \emptyset, R(x) = w_1^* x, R_T = \emptyset\}$ where $w_1^* = \frac{\delta}{\eta}\frac{b}{\lambda - \Delta\mu}$, bank equity value follows

$$E_{W^*}(x) = \mathbb{E}\left[\int_t^{+\infty} e^{-r(\tau - t)}((\delta - w_1^*)x_\tau - rD - \mathbb{1}_{\{x_\tau = x_R\}}K(x_\tau))d\tau\right] \tag{A5}$$

Solving a corresponding ODE

$$1/2\sigma^2 x^2 E_{W^*}''(x) + (\mu - \Delta\mu)xE_{W^*}'(x) - rE_{W^*}(x) + (\delta - w_1^*)x - rD = 0, \tag{A6}$$

under the boundary condition $E_{W^*}(x_R) = E_{W^*}(sx_R) - K(x_R)$, we obtain

$$E_{W^*}(x) = -\left[\frac{\xi_0 + (1 + \xi_1 - (1 - \frac{w_1^*}{\delta})\eta)(s - 1)x_R}{1 - s^{\beta_2}}\right]\left(\frac{x}{x_R}\right)^{\beta_2} + \left(1 - \frac{w_1^*}{\delta}\right)\eta x - D \tag{A7}$$

A.3. Equity value in a jump-diffusion framework

Let x_L be any arbitrary liquidation rule. Consider the case when only the imprudent risk-management technology is available and let us design equity value under permanent tail risk exposure.

A.3.1. General solution

Given $x_i = x_L/\alpha^i, i = 0..+\infty$, let $E_i(x)$ be bank equity value on the interval $[x_i, x_{i+1})$.

a) First, we construct equity value on the interval $[x_0, x_1)$. Since a large loss on this interval will lead to the bank default, equity value satisfies the equation:

$$\frac{1}{2}\sigma^2 x^2 E_0''(x) + \mu x E_0'(x) - (r+\lambda)E_0(x) + \delta x - rD = 0 \qquad (A8)$$

Let $\gamma_1 > 0$ and $\gamma_2 < 0$ be the roots of

$$1/2\sigma^2\gamma(\gamma-1) + \mu\gamma = r + \lambda \qquad (A9)$$

A general solution of equation (A8) will be given by:

$$E_0(x) = C_{0,1}x^{\gamma_1} + C_{0,2}x^{\gamma_2} + A_{0,1}x + A_{0,0} \qquad (A10)$$

where $C_{0,1}$, $C_{0,2}$ are any arbitrary constants and

$$A_{0,1} = \frac{\delta}{r-\mu+\lambda} \equiv \nu, \quad A_{0,0} = -\frac{r}{r+\lambda}D \qquad (A11)$$

Note that, from initial assumptions $r > \delta + \mu - \Delta\mu$ and $\Delta\mu < \lambda(1-\alpha)$, it follows that $\delta < r - \mu + \lambda$ and, therefore, $\nu < 1$.

b) On each interval $[x_i, x_{i+1}), i = 1.. + \infty$, equity value satisfies:

$$\frac{1}{2}\sigma^2 x^2 E_i''(x) + \mu x E_i'(x) - (r+\lambda)E_i(x) + \delta x - rD = -\lambda E_{i-1}(\alpha x) \qquad (A12)$$

A general solution of the above equation is given by:

$$E_i(x) = C_{i,1}(x)x^{\gamma_1} + C_{i,2}(x)x^{\gamma_2} + A_{i,1}x + A_{i,0} \qquad (A13)$$

By substituting (A13) into (A12) and by equating identical terms, we get 4 equations which iteratively define coefficients $C_{i,1}(x)$, $C_{i,2}(x)$, $A_{i,1}$, $A_{i,0}$:

$$1/2\sigma^2 x^2 C_{i,1}''(x) + (\sigma^2\gamma_1 + \mu)x C_{i,1}'(x) = -\lambda\alpha^{\gamma_1}C_{i-1,1}(\alpha x) \qquad (A14)$$

$$1/2\sigma^2 x^2 C_{i,2}''(x) + (\sigma^2\gamma_2 + \mu)x C_{i,2}'(x) = -\lambda\alpha^{\gamma_2}C_{i-1,2}(\alpha x) \qquad (A15)$$

$$A_{i,1} = A_{0,1}\left(1 + \frac{\alpha\lambda A_{i-1,1}}{\delta}\right) = A_{0,1}\frac{\delta}{\delta-\alpha\lambda}\left[1 - \left(\frac{\alpha\lambda}{\delta}\right)^{i+1}\right] \qquad (A16)$$

$$A_{i,0} = -\frac{r}{r+\lambda}D + \frac{\lambda}{r+\lambda}A_{i-1,0} = A_{0,0}\frac{r+\lambda}{r}\left[1 - \left(\frac{\lambda}{r+\lambda}\right)^{i+1}\right] \qquad (A17)$$

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A.3.2. Recursive algorithm to compute unknown coefficients $C_{i,1}(x)$ and $C_{i,2}(x)$

Consider the following non-homogeneous second-order ODE:

$$f''(x) + q(x)f'(x) = q_0 g(x) \tag{A18}$$

where $f(x) \in \mathcal{C}^2$, $q(x) \in \mathcal{C}^0$, $g(x) \in \mathcal{C}^0$ and q_0 is any arbitrary constant.

A general solution of the above equation is given as follows:

$$f(x) = a_1 f_1(x) + a_2 f_2(x) + q_0 f_p(x), \tag{A19}$$

where $f_1(x)$ and $f_2(x)$ are two solutions of the homogeneous equation, a_1, a_2 are any arbitrary coefficients and $f_p(x)$ is a particular solution which can be defined by using the method of the variation of parameters:

$$f_p(x) = -f_1(x) \left(\int \frac{f_2(x)g(x)}{W(f_1(x), f_2(x))}dx + c_1 \right) + f_2(x) \left(\int \frac{f_1(x)g(x)}{W(f_1(x), f_2(x))}dx + c_2 \right), \tag{A20}$$

where

$$W(f_1(x), f_2(x)) = f_1(x)f_2'(x) - f_1'(x)f_2(x) \neq 0$$

is a Wronskian and c_1, c_2 are any arbitrary constants of integration.

Thus, (A19) can be rewritten as follows:

$$f(x) = k_1 f_1(x) + k_2 f_2(x) - q_0 \left(f_1(x) \int \frac{f_2(x)g(x)}{W(f_1(x), f_2(x))}dx - f_2(x) \int \frac{f_1(x)g(x)}{W(f_1(x), f_2(x))}dx \right), \tag{A21}$$

where $k_1 = a_1 - q_0 c_1$ and $k_2 = a_2 + q_0 c_2$.

We can rewrite equations (A14) and (A15) in the form of (A18):

$$C_{i,1}''(x) + \frac{2(\gamma_1 + \mu\sigma^{-2})}{x}C_{i,1}'(x) = -\frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\frac{C_{i-1,1}(\alpha x)}{x^2} \tag{A22}$$

$$C_{i,2}''(x) + \frac{2(\gamma_1 + \mu\sigma^{-2})}{x}C_{i,2}'(x) = -\frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\frac{C_{i-1,2}(\alpha x)}{x^2} \tag{A23}$$

Let

$$\theta_j = 1 - 2(\gamma_j + \mu\sigma^{-2}), \quad j \in \{1, 2\}$$

A general solutions of (A22) is given by:

$$C_{i,1}(x) = k_{i,0} \cdot 1 + k_{i,1}x^{\theta_1} + \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\phi_i(x), \tag{A24}$$

where $k_{i,0}$, $k_{i,1}$ are any arbitrary constants and

$$\phi_i(x) = \frac{1}{\theta_1} \left(\int \frac{C_{i-1,1}(\alpha x)}{x}dx - x^{\theta_1} \int \frac{C_{i-1,1}(\alpha x)}{x}x^{-\theta_1}dx \right) \tag{A25}$$

A general solutions for (A23) is given by:

$$C_{i,2}(x) = p_{i,0} \cdot 1 + p_{i,1}x^{\theta_2} + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\varphi_i(x), \tag{A26}$$

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where $p_{i,0}$, $p_{i,1}$ are any arbitrary constants and

$$\varphi_i(x) = \frac{1}{\theta_2} \left(\int \frac{C_{i-1,2}(\alpha x)}{x} dx - x^{\theta_2} \int \frac{C_{i-1,2}(\alpha x)}{x} x^{-\theta_2} dx \right) \qquad \text{(A27)}$$

Note that, for each $x_i = x_L/\alpha^i$, $i = 1..+\infty$, the following conditions should be satisfied:

$$E_{i-1}(x_i) = E_i(x_i) \qquad \text{(A28)}$$

$$E'_{i-1}(x_i) = E'_i(x_i) \qquad \text{(A29)}$$

We denote

$$\Delta A_{i,1} = A_{i,1} - A_{i-1,1} = A_{0,1} \left(\frac{\alpha\lambda}{\delta} \right)^i \qquad \text{(A30)}$$

$$\Delta A_{i,0} = A_{i,0} - A_{i-1,0} = A_{0,0} \left(\frac{\lambda}{r+\lambda} \right)^i \qquad \text{(A31)}$$

$$\Delta C'_{i,1}(x) = C'_{i,1}(x) - C'_{i-1,1}(x) = \theta_1 x^{\theta_1-1}(k_{i,1} - k_{i-1,1}) + \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}[\phi'_i(x) - \phi'_{i-1}(x)] \quad \text{(A32)}$$

$$\Delta C'_{i,2}(x) = C'_{i,2}(x) - C'_{i-1,2}(x) = \theta_2 x^{\theta_2-1}(p_{i,1} - p_{i-1,1}) + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}[\varphi'_i(x) - \varphi'_{i-1}(x)] \quad \text{(A33)}$$

Given (A13), equations (A28) and (A29) enable us to establish a link between coefficients on contiguous regions:

$$C_{i,1}(x_i) = C_{i-1,1}(x_i) - \left[\frac{(1-\gamma_2)\Delta A_{i,1}x_i^{1-\gamma_1} - \gamma_2\Delta A_{i,0}x_i^{-\gamma_1} + x_i(\Delta C'_{i,1}(x_i) + \Delta C'_{i,2}(x_i)x_i^{\gamma_2-\gamma_1})}{\gamma_1 - \gamma_2} \right] \qquad \text{(A34)}$$

and

$$C_{i,2}(x_i) = C_{i-1,2}(x_i) + \left[\frac{(1-\gamma_1)\Delta A_{i,1}x_i^{1-\gamma_2} - \gamma_1\Delta A_{i,0}x_i^{-\gamma_2} + x_i(\Delta C'_{i,1}(x_i)x_i^{\gamma_1-\gamma_2} + \Delta C'_{i,2}(x_i))}{\gamma_1 - \gamma_2} \right] \qquad \text{(A35)}$$

Given the recursive nature of (A34), we can rewrite coefficient $C_{i,1}(x_i)$ as a function of coefficient $C_{0,1}$:

$$C_{i,1}(x_i) = C_{0,1} - \left[\frac{(1-\gamma_2)a_{i,1}x_i^{1-\gamma_1} - \gamma_2 a_{i,2}x_i^{-\gamma_1} + G_{i,1}(k_{i,1}; p_{i,1}; x_i)}{\gamma_1 - \gamma_2} \right] \qquad \text{(A36)}$$

where

$$a_{i,1} \equiv \sum_{n=1}^{i} \Delta A_{n,1} = A_{0,1} \frac{\alpha\lambda}{\delta - \alpha\lambda} \left[1 - \left(\frac{\alpha\lambda}{\delta} \right)^i \right] = A_{i-1,1} \frac{\alpha\lambda}{\delta} \qquad \text{(A37)}$$

$$a_{i,2} \equiv \sum_{n=1}^{i} \Delta A_{n,0} = A_{0,0} \frac{\lambda}{r} \left[1 - \left(\frac{\lambda}{r+\lambda} \right)^i \right] = A_{i-1,0} \frac{\lambda}{r+\lambda} \qquad \text{(A38)}$$

$$G_{i,1}(k_{i,1}, p_{i,1}, x_i) = x_i \sum_{n=1}^{i} \Delta C'_{n,1}(x_i) + x_i^{\gamma_2-\gamma_1+1} \sum_{n=1}^{i} \Delta C'_{n,2}(x_i), \qquad \text{(A39)}$$

where $\Delta C'_{n,1}(x_i)$ and $\Delta C'_{n,2}(x_i)$ are given by (A32) and (A33) respectively.

In a similar way we can express $C_{i,2}(x_i)$ as a function of $C_{0,2}$:

$$C_{i,2}(x_i) = C_{0,2} + \left[\frac{(1-\gamma_1)a_{i,1}x_i^{1-\gamma_2} - \gamma_1 a_{i,2}x_i^{-\gamma_2} + G_{i,2}(k_{i,1}; p_{i,1}; x_i)}{\gamma_1 - \gamma_2}\right] \tag{A40}$$

where

$$G_{i,2}(k_{i,1}; p_{i,1}; x_i) = x_i^{\gamma_1-\gamma_2+1}\sum_{n=1}^{i}\Delta C'_{n,1}(x_i) + x_i\sum_{n=1}^{i}\Delta C'_{n,2}(x_i) \tag{A41}$$

Equating the right parts of (A36) and (A24) evaluated at x_i, as well as the right parts of (A40) and (A26), we can set

$$k_{i,0} = C_{0,1} \tag{A42}$$

and

$$p_{i,0} = C_{0,2} \tag{A43}$$

for $i = 1..+\infty$.

Then, for $\forall i = 1..+\infty$, $k_{i,1}$ and $p_{i,1}$ will be given by the following system of equations:

$$\mathcal{P}_i = \begin{cases} \frac{G_{i,1}(k_{i,1};p_{i,1};x_i)}{\gamma_1-\gamma_2} + k_{i,1}x_i^{\theta_1} = -\left[\frac{(1-\gamma_2)a_{i,1}x_i^{1-\gamma_1}+\gamma_2 a_{i,2}x_i^{-\gamma_1}}{\gamma_1-\gamma_2}\right] - \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\phi_i(x_i) \\[3mm] \frac{G_{i,2}(k_{i,1};p_{i,1};x_i)}{\gamma_1-\gamma_2} - p_{i,1}x_i^{\theta_2} = -\left[\frac{(1-\gamma_1)a_{i,1}x_i^{1-\gamma_2}-\gamma_1 a_{i,2}x_i^{-\gamma_2}}{\gamma_1-\gamma_2}\right] + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\varphi_i(x) \end{cases}$$

$$\tag{A44}$$

A.3.2. A choice of $C_{0,1}$ and $C_{0,2}$

We make a guess that

$$C_{0,1} = 0, \quad C_{0,2} = \left(\frac{r}{r+\lambda}D - \nu x_L\right)x_L^{-\gamma_2} \tag{A45}$$

Let $E_{\alpha_0}(x)$ denote the bank equity value when $\alpha = 0$, i. e., when a large loss will lead to a complete destruction of bank asset. It follows ODE:

$$\frac{1}{2}\sigma^2 x^2 E''_{\alpha_0}(x) + \mu x E'_{\alpha_0}(x) - (r+\lambda)E_{\alpha_0}(x) + \delta x - rD = 0 \tag{A46}$$

Let $E_{\lambda_0}(x)$ denote bank equity value when $\lambda = 0$, i.e., there is no tail risk exposure. It follows ODE:

$$\frac{1}{2}\sigma^2 x^2 E''_{\lambda_0}(x) + \mu x E'_{\lambda_0}(x) - rE_{\lambda_0}(x) + \delta x - rD = 0 \tag{A47}$$

We are going to verify that, given $C_{0,1}$ and $C_{0,2}$ defined in (A45), the following limit conditions are satisfied:

- (i) $\lim_{\alpha\to 0} E_0(x) \to E_{\alpha_0}(x)$

- (ii) $\lim_{\lambda \to 0} E_0(x) \to E_{\lambda_0}(x)$

A general solution for $E_{\alpha_0}(x)$ will be given by

$$E_{\alpha_0}(x) = a_{\alpha_0} x^{\gamma_1} + b_{\alpha_0} x^{\gamma_2} + \nu x - \frac{r}{r+\lambda} D, \qquad (A48)$$

Given terminal condition at x_L and no-bubble condition for $x \to +\infty$, we have

$$a_{\alpha_0} = 0, \quad b_{\alpha_0} = \left(\frac{r}{r+\lambda} D - \nu x_L \right) x_L^{-\gamma_2} \qquad (A49)$$

Thus, limit condition (i) is satisfied.
A solution for $E_{\lambda_0}(x)$ will be given by

$$E_{\lambda_0}(x) = a_{\lambda_0} x^{\hat{\gamma}_1} + b_{\lambda_0} x^{\hat{\gamma}_2} + \frac{\delta}{r-\mu} x - D, \qquad (A50)$$

where $\hat{\gamma}_1 > 0$, $\hat{\gamma}_2 < 0$ are the roots of characteristic equation

$$1/2\sigma^2 \hat{\gamma}(\hat{\gamma} - 1) + \mu\hat{\gamma} = r$$

and

$$a_{\lambda_0} = 0, \quad b_{\lambda_0} = \left(D - \frac{\delta}{r-\mu} x_L \right) x_L^{-\hat{\gamma}_2} \qquad (A51)$$

Given that $\lim_{\lambda \to 0} \gamma_2 \to \hat{\gamma}_2$, it can be easily shown that condition (ii) is satisfied as well.

A.3.3. The optimal shareholders' liquidation rule

In order to avoid time consistency problem, the choice of the optimal shareholders' liquidation rule should be realized for $x \in [x_0, x_1)$, where equity value follows:

$$E_0(x) = \left(\frac{r}{r+\lambda} D - \nu x_L \right) \left(\frac{x}{x_L} \right)^{\gamma_2} + \nu x - \frac{r}{r+\lambda} D, \qquad (A52)$$

Then, from $\frac{\partial E_0(x)}{\partial x_L} = 0$, we get

$$x_L^S = \frac{-\gamma_2}{1 - \gamma_2} \frac{r}{r+\lambda} \frac{D}{\nu} \qquad (A53)$$

A.3.4. A summary of the recursive algorithm

Given the couple $\{C_{0,1}, C_{0,2}\}$, the sequence of coefficients $\{C_{i,1}(x), C_{i,2}(x)\}, i = 1..+\infty$, can be defined according to the following iterative procedure:

- for each i, use $C_{i-1,1}(x_i)$ and $C_{i-1,2}(x_i)$ (which are already known) to define $\phi_i(x_i)$ and $\varphi_i(x)$ according to (A25) and (A27) respectively;

- using the series of $\{k_{n,1}^*, p_{n,1}^*\}, n = 1..i-1$, calculate $G_{i,1}(k_{i,1}; p_{i,1}; x_i)$ and $G_{i,2}(k_{i,1}; p_{i,1}; x_i)$ which will be the functions of $k_{i,1}$ and $p_{i,1}$;

- after solving the system $\mathcal{P}_i$, which is linear on $k_{i,1}^*$ and $p_{i,1}^*$, compute coefficients $\{C_{i,1}(x), C_{i,2}(x)\}$ according to:

$$C_{i,1}(x) = C_{0,1} + k_{i,1}^* x^{\theta_1} + \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\phi_i(x) \tag{A54}$$

$$C_{i,2}(x) = C_{0,2} + p_{i,1}^* x^{\theta_2} + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\varphi_i(x) \tag{A55}$$

Example 1 *Let us define coefficients $C_{1,1}(x)$ and $C_{1,2}(x)$. Given $\{C_{0,1}, C_{0,2}\}$, we obtain*

$$\phi_1(x) = \frac{C_{0,1}}{\theta_1}\left(\ln(x) + \frac{1}{\theta_1}\right), \quad \varphi_1(x) = \frac{C_{0,2}}{\theta_2}\left(\ln(x) + \frac{1}{\theta_2}\right) \tag{A56}$$

Then, given that $k_{0,1} = 0$ and $p_{0,1} = 0$,

$$\Delta C_{1,1}'(x) = C_{1,1}'(x) = x^{-1}\left(\theta_1 x^{\theta_1} k_{1,1} + \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\frac{C_{0,1}}{\theta_1}\right) \tag{A57}$$

$$\Delta C_{1,2}'(x) = C_{1,2}'(x) = x^{-1}\left(\theta_2 x^{\theta_2} p_{1,1} + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\frac{C_{0,2}}{\theta_2}\right) \tag{A58}$$

$$a_{1,1} = A_{0,1}\frac{\alpha\lambda}{\delta}, \qquad a_{1,2} = A_{0,0}\frac{\lambda}{r+\lambda} \tag{A59}$$

$$G_{1,1}(k_{1,1}; p_{1,1}; x_1) = \theta_1 x_1^{\theta_1} k_{1,1} + \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\frac{C_{0,1}}{\theta_1} + x_1^{\gamma_2-\gamma_1}\left(\theta_2 x_1^{\theta_2} p_{1,1} + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\frac{C_{0,2}}{\theta_2}\right) \tag{A60}$$

$$G_{1,2}(k_{1,1}; p_{1,1}; x_1) = x_1^{\gamma_1-\gamma_2}\left(\theta_1 x_1^{\theta_1} k_{1,1} + \frac{2\lambda\alpha^{\gamma_1}}{\sigma^2}\frac{C_{0,1}}{\theta_1}\right) + \theta_2 x_1^{\theta_2} p_{1,1} + \frac{2\lambda\alpha^{\gamma_2}}{\sigma^2}\frac{C_{0,2}}{\theta_2} \tag{A61}$$

Then, unknown parameters $k_{1,1}$, $p_{1,1}$ can be uncovered from system $\mathcal{P}_1$ defined in (A44), and coefficients $C_{1,1}(x)$, $C_{1,2}(x)$ will be provided by expressions (A54) and (A55) respectively.

Appendix B. Proofs

Lemma 1 *For any $\theta < 0$, $\alpha \in (0,1)$ and $\Delta\mu < \lambda(1-\alpha)$, it follows that*

$$\lambda(1-\alpha^\theta) - \theta\Delta\mu < 0$$

Proof of Lemma 1

Given that $\Delta\mu < \lambda(1-\alpha)$ and $\theta < 0$, we have:

$$\lambda(1-\alpha^\theta) - \theta\Delta\mu < \lambda(1-\alpha^\theta) - \theta\lambda(1-\alpha) \equiv f(\alpha) \tag{B1}$$

Since $f(1) = 0$ and $f'(\alpha) = \theta\lambda(1-\alpha^{\theta-1}) > 0$ for $\alpha \in (0,1)$, we have $f(\alpha) < 0$ for $\alpha \in (0,1)$. Hence, $\lambda(1-\alpha^\theta) - \theta\Delta\mu < 0$.

Proof of Result 1

Consider the optimal strategy of bank shareholders when there is nether internal agency problem nor capital regulation. Bank shareholders maximize equity value $E(x)$, by choosing (i) a liquidation/recapitalization rule; (ii) risk management strategy, $u_t \in \{0,1\}$.

Let $E_J(x)$ denote bank asset value constructed in Appendix A.3. Consider the shareholders' liquidation rule derived in Appendix A.3.3:

$$x_L^S = \frac{-\gamma_2}{1-\gamma_2}\frac{r}{r+\lambda}\frac{D}{\nu} \tag{B2}$$

Let u^* denote risk management strategy such that:

$$u^* = \begin{cases} 0 & x \in [x_L^S, x_u^*) \\ 1 & x \geq x_u^* \end{cases}$$

where x_u^* is a critical threshold given by:

$$\lambda(E_J(x) - E_J(\alpha x)) = \Delta\mu x E_J'(x) \tag{B3}$$

Equity value constructed under $\{x_L^S, u^*\}$ is given as follows:

$$E^*(x) = \begin{cases} E_J(x) & x \in [x_L^S, x_u^*) \\ (E_J(x_u^*) - \eta x_u^* + D)\left(\frac{x}{x_u^*}\right)^{\beta_2} + \eta x - D & x \geq x_u^* \end{cases}$$

where $\beta_2 < 0$ is a root of $1/2\sigma^2\beta(\beta-1) + (\mu - \Delta\mu)\beta = r$.

Let us show that $E^*(x) = \sup_{x_L, u_t \in \{0,1\}} E(x)$, so that $\{x_L^S, x_u^*\}$ is the optimal shareholders' strategy in the absence of regulatory control.

Recall that by the initial assumption, $r > \delta + \mu - \Delta\mu$. This means that a one-dollar investment in riskless security will bring more that a one-dollar investment in bank asset portfolio, so that recapitalization is never optimal.

Let us now show that u^* is the optimal risk management strategy. First, we verify that $u_t = 1$ for $x > x_u^*$. The corresponding necessary condition implies:

$$\lambda(E(x) - E(\alpha x)) > \Delta\mu x E'(x) \text{ for } x > x_u^* \tag{B4}$$

Replacing equity value into (B4), we get:

$$(E_J(x_u^*) - \eta x_u^* + D)\left(\lambda(1 - \alpha^{\beta_2}) - \beta_2\Delta\mu\right)\left(\frac{x}{x_u^*}\right)^{\beta_2} + (\lambda(1 - \alpha) - \Delta\mu)\eta x > 0 \qquad \text{(B5)}$$

By Lemma 1, we have $\lambda(1 - \alpha^{\beta_2}) - \beta_2\Delta\mu < 0$, while $\lambda(1 - \alpha) - \Delta\mu > 0$ by the initial assumption. Then, using the fact that (B5) is binding at x_u^*, we conclude that $E_J(x_u^*) - \eta x_u^* + D > 0$. Then, a first derivative of the left part of (B5) is increasing on x, so that condition (B5) will hold for $\forall x > x_u^*$.

Since x_u^* is unique, $u_t = 0$ for $x < x_u^*$ and equity value on this region will be defined as described in Appendix A.3. Then, by construction, x_L^S will be the optimal liquidation rule.

Proof of Result 2

Consider any arbitrary liquidation threshold x_L. Assume that the bank will never switch to the imprudent risk-management strategy for $x \geq x_L$. Then, bank equity value would follow:

$$E(x) = (D - \eta x_L)\left(\frac{x}{x_L}\right)^{\beta_2} + \eta x - D \qquad \text{(B6)}$$

Let us show that, given bank equity value (B6), incentive condition (5) doesn't hold for $\forall x_L > \widehat{x}_L$. For $x \in [x_L, x_L/\alpha)$ a large loss would trigger bank liquidation, so that incentive condition (5) can be rewritten as follows:

$$\lambda E(x) \geq \Delta\mu x E'(x) \qquad \text{(B7)}$$

Thus, we have:

$$(\lambda - \beta_2\Delta\mu)(D - \eta x_L)\left(\frac{x}{x_L}\right)^{\beta_2} + (\lambda - \Delta\mu)\eta x - \lambda D \geq 0 \qquad \text{(B8)}$$

Let $f(x)$ denote the left-hand side of (B8). Then, we have:

$$\lim_{x \to x_L} f(x) = \Delta\mu(\eta x_L(\beta_2 - 1) - \beta_2 D) < 0, \qquad \text{(B9)}$$

for $\forall x_L > \widehat{x}_L$, where $\widehat{x}_L$ is given by

$$\widehat{x}_L = \frac{\beta_2}{\beta_2 - 1}\frac{D}{\eta} \qquad \text{(B10)}$$

Therefore, condition (B7) doesn't hold for $\forall x_L > \widehat{x}_L$.

Proof of Proposition 1

Consider the regulatory problem in the case of the owner-managed bank:

$$\underset{x_R > 0, s > 1}{\text{Max}} \quad E(x) \geq 0 \text{ s.t.}$$

$$\lambda(E(x) - \mathbb{1}_{x \geq x_R/\alpha} E(\alpha x)) \geq \Delta\mu x E'(x) \text{ for } \forall x \geq x_R$$

where $E(x)$ is given by (A4).

For any $s > 1$, consider a minimum incentive-compatible recapitalization rule $x_R^B(s)$ such that $\lambda E(x_R^B(s)) = \Delta\mu x_R^B(s) E'(x_R^B(s))$:

$$x_R^B(s) = \frac{(1 - s^{\beta_2})\lambda D + \xi_0(\lambda - \beta_2\Delta\mu)}{(1 - s^{\beta_2})(\lambda - \Delta\mu)\eta - (\xi_1 + 1 - \eta)(s - 1)(\lambda - \beta_2\Delta\mu)} \tag{B11}$$

Let us show that the pair $s^* = \arg\min x_R^B(s)$ and $x_R^B(s^*)$ is a solution of the above maximization problem if

$$\xi_1 < \left(\frac{1 - s^{\beta_2}}{s - 1}\right)\left(\frac{\lambda - \Delta\mu}{\lambda - \beta_2\Delta\mu}\right)\eta + \eta - 1 \equiv \bar{\xi}_1 \tag{B12}$$

P1.1. Incentive-compatibility

For any arbitrary recapitalization coefficient $s > 1$, let us check that $x_R^B(s)$ ensures the incentive constraint of bank shareholders for $\forall x \geq x_R^B(s)$. Replacing equity value (A4) into the incentive condition, for $x \in [x_R^B(s), x_R^B(s)/\alpha)$ we must have:

$$-(\lambda - \beta_2\Delta\mu)\left(\frac{\xi_0 + (\xi_1 + 1 - \eta)(s - 1)x_R^B(s)}{1 - s^{\beta_2}}\right)\left(\frac{x}{x_R^B(s)}\right)^{\beta_2} + (\lambda - \Delta\mu)\eta x - \lambda D \geq 0 \text{ (B13)}$$

Since the above condition is binding at $x = x_R^B(s)$ and its left-hand side is increasing on x, it holds for $\forall x \in [x_R^B(s), x_R^B(s)/\alpha)$. For $x \geq x_R^B(s)/\alpha$, we must have:

$$-(\lambda(1 - \alpha^{\beta_2}) - \beta_2\Delta\mu)\left(\frac{\xi_0 + (\xi_1 + 1 - \eta)(s - 1)x_R^B(s)}{1 - s^{\beta_2}}\right)\left(\frac{x}{x_R^B(s)}\right)^{\beta_2} + (\lambda(1 - \alpha) - \Delta\mu)\eta x \geq 0 \tag{B14}$$

By Lemma 1, $\lambda(1 - \alpha^{\beta_2}) - \beta_2\Delta\mu < 0$ and $\Delta\mu < \lambda(1 - \alpha)$ by the initial assumption. Therefore, (B14) holds for $x \geq x_R^B(s)/\alpha$.

P1.2. Feasibility

Condition (B12) ensures that $x_R^B(s) > 0$. Now, we verify that (i) it is optimal to re-capitalize the bank at $x_R^B(s)$, rather than to lose equity; (ii) given $x_R^B(s)$, a voluntary recapitalization at any $x > x_R^B(s)$ is unoptimal.

First, we check that $E(x_R^B(s)) > 0$. Let $x_R^O(s)$ denote a critical recapitalization threshold such that $E(x_R^O(s)) = 0$:

$$x_R^O(s) = \frac{(1 - s^{\beta_2})D + \xi_0}{(1 - s^{\beta_2})\eta - (\xi_1 + 1 - \eta)(s - 1)} > D \tag{B15}$$

In can be shown that $x_R^B(s) > x_R^O(s)$, so that $E(x_R^B(s)) > 0$.

In order to show that there is no other recapitalization rule that could strictly increase equity value, it is sufficient to verify the following condition for $x > x_R^B(s)$:

$$g(x) \equiv E(sx) - E(x) - K(x) < 0 \tag{B16}$$

We have $g'(x) < 0$, $g''(x) > 0$ and $g(x_R^B(s)) = 0$. Thus, (B16) holds for $x > x_R^B(s)$.

P1.3. Optimality

Let us show that $\{s^*, x_R^B(s^*)\}$ is the optimal recapitalization policy. Note that $E(x)$ is decreasing on x_R. Under $\xi_1 < \bar{\xi}_1$, $x_R^B(s)$ is the optimal incentive-compatible recapitalization rule for any given $s > 1$. Then, a solution of the maximization problem will be delivered by $s^* = \arg\min x_R^B(s)$.

Proof of Proposition 2

Assume there exists some incentive recapitalization policy $\{x_R, s\}$, such that shareholders want to induce prudent risk management in their bank for $\forall x \geq x_R$. Then, the shareholders' maximization problem is reduced to minimization of the total expected amount of incentive managerial compensation:

$$\min_{x_T \geq x_R, w_0 \geq 0, w_1 \geq 0, R_T \geq 0} MC(x) = \mathbb{E}\left[\int_t^{+\infty} e^{-r(\tau-t)}(\mathbb{1}_{x_\tau \neq x_T} R(x_\tau) + \mathbb{1}_{x_\tau = x_T} R_T)d\tau\right]$$

s.t.

$$\lambda W(x) \geq \Delta\mu x W'(x) + bx \tag{B17}$$

where $R(x_\tau) = w_0 + w_1 x_\tau$ and

$$W(x) = \mathbb{E}\left[\int_t^{\tau \wedge \tau_T} e^{-r(\tau-t)} R(x_\tau)d\tau + e^{-r(\tau_T-t)} R_T\right] \tag{B18}$$

Contract continuation value $W(x)$ follows ODE:

$$1/2\sigma^2 x^2 W''(x) + (\mu - \Delta\mu)x W'(x) - rW(x) + w_0 + w_1 x = 0 \tag{B19}$$

Under the no-bubble condition, a general solution of (B19) is given by:

$$W(x) = C_0 x^{\beta_2} + \frac{\eta}{\delta} w_1 x + \frac{w_0}{r}, \tag{B20}$$

where C_0 is any constant depending on the boundary condition at x_T.

Let consider two alternative cases: $x_T = x_R$ and $x_T = \emptyset$.

1) Case 1: $x_T = x_R$. Under the boundary condition $W(x_R) = R_T$, we get

$$W(x) = \frac{w_0}{r} + \frac{\eta}{\delta} w_1 x + \left(R_T - \frac{w_0}{r} - \frac{\eta}{\delta} w_1 x_R\right)\left(\frac{x}{x_R}\right)^{\beta_2} \tag{B21}$$

Given that $MC(x_R) = MC(sx_R) + R_T$ in this case, the shareholders' optimization problem can be rewritten as follows:

$$\min_{w_0 \geq 0, w_1 \geq 0, R_T \geq 0} MC(x) = \left\{\frac{w_0}{r} + \frac{\eta}{\delta} w_1 x + \frac{R_T + \frac{\eta}{\delta} w_1 x_R(s-1)}{(1 - s^{\beta_2})}\left(\frac{x}{x_R}\right)^{\beta_2}\right\}$$

s.t.

$$(\lambda - \Delta\mu\beta_2)\left(R_T - \frac{w_0}{r} - \frac{\eta}{\delta} w_1 x_R\right)\left(\frac{x}{x_R}\right)^{\beta_2} + (\lambda - \Delta\mu)\frac{\eta}{\delta} w_1 x + \lambda\frac{w_0}{r} \geq bx \tag{B22}$$

The solution of the above problem is given by

$$w_0^* = 0, \quad w_1^* = \frac{\delta}{\eta}\frac{b}{\lambda - \Delta\mu}, \quad R_T^* = \frac{\eta}{\delta}w_1^* x_R$$

The optimal value of $MC(x)$ is given by:

$$MC_1^*(x) = \frac{\eta}{\delta}w_1^*\left(x + x_R\frac{s}{(1-s^{\beta_2})}\left(\frac{x}{x_R}\right)^{\beta_2}\right) \tag{B23}$$

2) Case 2: $x_T = \emptyset$. Note that, in this case, $R_T = \emptyset$ and $MC(x) = W(x)$, so that the shareholders' problem takes the following form:

$$\min_{w_0 \geq 0, w_1 \geq 0} \quad MC(x) = \left\{\frac{w_0}{r} + \frac{\eta}{\delta}w_1 x + \frac{\eta}{\delta}w_1\frac{(s-1)}{(1-s^{\beta_2})}x_R\left(\frac{x}{x_R}\right)^{\beta_2}\right\}$$

s.t.

$$(\lambda - \Delta\mu\beta_2)\frac{\eta}{\delta}w_1\frac{(s-1)}{(1-s^{\beta_2})}x_R\left(\frac{x}{x_R}\right)^{\beta_2} + (\lambda - \Delta\mu)\frac{\eta}{\delta}w_1 x + \lambda\frac{w_0}{r} \geq bx \tag{B24}$$

The couple $w_0^* = 0$ and $w_1^* = \frac{\delta}{\eta}\frac{b}{\lambda - \Delta\mu}$ solves the above problem. The corresponding value of shareholders' costs is given by:

$$MC_2^*(x) = \frac{\eta}{\delta}w_1^*\left(x + x_R\frac{(s-1)}{(1-s^{\beta_2})}\left(\frac{x}{x_R}\right)^{\beta_2}\right) \equiv W^*(x) \tag{B25}$$

Since $MC_2^*(x) < MC_1^*(x)$, the optimal incentive-compatible contract will be defined by the triple

$$\left\{x_T = \emptyset, \ R(x) = \frac{\delta}{\eta}\frac{b}{\lambda - \Delta\mu}x \equiv w_1^* x, \ R_T = \emptyset\right\}$$

Proof of Proposition 3

Allowing for the internal agency problem between bank shareholders and the bank manager, the regulatory problem transforms to:

$$\underset{x_R > 0, s > 1}{\text{Max}} \quad E_{W^*}(x) \geq 0 \text{ s.t.}$$

$$\lambda(E_{W^*}(x) - \mathbb{1}_{x \geq x_R/\alpha}E_{W^*}(\alpha x)) \geq \Delta\mu x E'_{W^*}(x) \text{ for } \forall x \geq x_R$$

where $E_{W^*}(x_0)$ is given by (A7).

For any $s > 1$, consider the minimum incentive-compatible recapitalization rule $x_R^A(s)$ such that $\lambda E_{W^*}(x_R^A(s)) = \Delta\mu x_R^A(s)E'_{W^*}(x_R^A(s))$:

$$x_R^A(s) = \frac{(1-s^{\beta_2})\lambda D + \xi_0(\lambda - \beta_2\Delta\mu)}{(1-s^{\beta_2})(\lambda - \Delta\mu)(1-\frac{w_1^*}{\delta})\eta - (1+\xi_1 - \eta(1-\frac{w_1^*}{\delta}))(s-1)(\lambda - \beta_2\Delta\mu)}, \tag{B26}$$

given that

$$b < \eta\left(\frac{\bar{\xi} - \underline{\xi}}{1 + \bar{\xi}}\right)(\lambda - \Delta\mu) \equiv \bar{b}, \tag{B27}$$

where $\bar{\xi}_1$ is defined by (9).

Let us show that the pair $s^{**} = \arg\min x_R^A(s)$ and $x_R^A(s^{**})$ is a solution of the above maximization problem.

P3.1. Incentive-compatibility

For any arbitrary recapitalization coefficient $s > 1$, let us check that $x_R^A(s)$ ensures the incentive constraint of bank shareholders for $\forall x \geq x_R^A(s)$. Replacing equity value (A7) into the incentive condition, for $x \in [x_R^A(s), x_R^A(s)/\alpha)$ we must have:

$$-(\lambda - \beta_2 \Delta\mu)H(s)\left(\frac{x}{x_R^A(s)}\right)^{\beta_2} + (\lambda - \Delta\mu)\left(1 - \frac{w_1^*}{\delta}\right)\eta x \geq \lambda D \qquad \text{(B28)}$$

where $H(s)$ denote:

$$H(s) = \frac{\xi_0 + \left(1 + \xi_1 - \eta(1 - \frac{w_1^*}{\delta})\right)(s-1)x_R^A(s)}{1 - s^{\beta_2}}$$

Since condition (B28) is binding for $x = x_R^A(s)$ and its left-hand side is increasing on x when $w^* < \delta$, it holds for $\forall x \in [x_R^A(s), x_R^A(s)/\alpha)$. For $x \geq x_R^A(s)/\alpha$, we must have:

$$-(\lambda(1 - \alpha^{\beta_2}) - \beta_2 \Delta\mu)H(s)\left(\frac{x}{x_R^A(s)}\right)^{\beta_2} + (\lambda(1 - \alpha) - \Delta\mu)\left(1 - \frac{w_1^*}{\delta}\right)\eta x \geq 0 \quad \text{(B29)}$$

By Lemma 1, $\lambda(1 - \alpha^{\beta_2}) - \Delta\mu\beta_2 < 0$ and thus (B29) holds $x \geq x_R^A(s)/\alpha$.

P3.2. Feasibility

Condition (B27) ensures that $x_R^A(s) > 0$. For any given $s > 1$, we have $x_R^A(s) > x_R^B(s) > 0$. Thus, given $x_R^A(s)$, shareholders will optimally prefer to recapitalize the bank rather than to be deprived of equity. At the same time, in order to ensure that, faced with $x_R^A(s)$, shareholders will not recapitalize the bank at any $x > x_R^A(s)$, we must have:

$$g_{W^*}(x) \equiv E_{W^*}(sx) - E_{W^*}(x) - K(x) < 0, \qquad \text{(B30)}$$

where $W^*(x)$ is a terminal payoff to the manager.

We have $g'_{W^*}(x) < 0$, $g''_{W^*}(x) > 0$ and $g_{W^*}(x_R^A(s)) = 0$. Thus, (B30) holds for any $x > x_R^A(s)$.

P3.3. Optimality

Since $E_{W^*}(x)$ is decreasing on x_R, $x_R^A(s)$ is a solution of the regulatory maximization problem for any $s > 1$ and $b < \bar{b}$.[20] Then, the choice $s^{**} = \arg\min x_R^A(s)$ completes the solution of the maximization problem.

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[20]For $\bar{b} < b < \delta$ the incentive condition never holds in the neighborhood of the recapitalization threshold, whereas for $b > \delta$ the optimal incentive contract would be too costly for shareholders.

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