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Aggregate Instability under Labor Income Taxation and Balanced-Budget Rules: Preferences Matter*

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Abstract: *We investigate the role of preferences in the existence of expectation-driven instability under a balanced budget rule where government spendings are financed by a tax on labor income. Considering a one-sector neoclassical growth model with a large class of preferences, we find that expectation-driven fluctuations are more likely when consumption and labor are Edgeworth substitutes. Under this property, an intermediate range of tax rates and a sufficiently low elasticity of intertemporal substitution in consumption lead to instability. Numerical simulations of the model support the conclusion that labor income taxation is a plausible source of instability in most OECD countries.*

Keywords: *Indeterminacy, expectation-driven business cycles, labor income taxes, balanced-budget rule, infinite-horizon model.*

Journal of Economic Literature Classification Numbers: C62, E32, E62.

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1. Introduction

During the Summer 2011, the European stock market downswing leads policy-makers to implement rules that aim to balance government budget. This revival of balanced-budget rules can be understood as a mean to reduce risks of an exploding public debt but also as a signal sent to the rating agencies that public finances are kept sane. Indeed, in countries facing a too large debt or uncontrolled public account, long-term growth may be crowded out and borrowing on financial market may be harder since rising interest rates increases the burden of public debt. This leads therefore to unsustainable situation as in Greece, Spain or Portugal in the recent years. In this context, the European Union enforces the "European Fiscal Compact".

Although an extensive literature addresses the question of fiscal rules through its procyclical effect, a key argument has been stressed in the seminal contribution of Schmitt-Grohé and Uribe [21]. They show that in a standard one-sector Ramsey model with a constant stream of wasteful government expenditures financed by a distortionary tax on labor income, a balanced-budget rule may be a source of aggregate instability. Indeed, tax rates larger than the capital share of income and lower than the tax rate associated to the peak of the Laffer curve involve expectation-driven fluctuations. The mechanism behind instability relies on the volatility of agents' expectations and goes as follows. An increase in the expected tax rate implies a reduction in future employment and therefore of capital returns. Consequently, investment decreases and households need to work less. The tax rate being decreasing in hours worked, the governement has to increase the tax rate to maintain the budget balanced and expectations are therefore self-fulfilling.

Several contributions extend this framework, but provide contradictory conclusions. Ghilardi and Rossi [5] generalize the technology with a CES production function assuming the same preferences as Schmitt-Grohé and Uribe [21]. They show that instability is more likely when capital and labor are substitutes. In constrast, Linnemann [14] keeps a Cobb-Douglas technology, but considers a particular class of non-separable preferences and shows that instability is unlikely. In this paper, we propose to reexamine the destabilizing effect of labor income taxation under a more general approach and emphasize particularly the role of preferences. Our investigation is motivated by two points. On the one hand, the role of preferences in the occurence of indeterminacy is a cornerstone for several contributions. For instance, Nishimura *et al.* [17] consider the occurrence of indeterminacy in a model with aggregate externalities. They show that the conditions for the emergence of instability strongly depend on the properties of preferences, since some particular utility functions are not compatible with indeterminacy. On the other hand, there is a growing interest in the impact of fiscal policies in presence of specific preferences, especially when non-separable utility functions are assumed. The empirical investigation of Trabandt and Uhlig [22] examines the shapes of the Laffer curve with linearly homogeneous and King-Plosser-Rebelo [12] (KPR) preferences. Bilbiie [2] investigates how non-separability of preferences explains the observed increase in private consumption in response to fiscal shocks.

We examine the interplay between preferences and technology on the existence of tax rates that generate indeterminacy. This is explored in a neoclassical infinite-horizon growth model embedding most popular preferences used by macroeconomists. We consider three classes of utility function: i) additively separable preferences with non-unitary elasticity of intertemporal substitution in consumption and elastic labor supply, ii) a linearly homogeneous utility function, iii) a Jaimovich-Rebelo [9] (JR) formulation where the degree of income effect can be controlled and admits two polar cases. In absence of income effect, a Greenwood-Hercowitz-Huffman [7] (GHH) utility function is considered. On the contrary, with a maximized degree of income effect, the preferences are characterized by a King-Plosser-Rebelo [12] (KPR) formulation. A generalized production function describes the technology of the firms so as to encompass the results of Ghilardi and Rossi [5] on inputs substitution. Governement sector is finally characterized by the same balanced-budget rule considered by Schmitt-Grohé and Uribe [21] for which the tax rate is counter-cyclical with respect to the tax base.

Our general investigation identifies a robust property to obtain indeterminacy. Indeed, we find that indeterminacy is more likely when the preferences exhibit Edgeworth substitutability between consumption and labor, such that a marginal increase in labor decreases the marginal utility of consumption. This property is always satisfied if the utility function is linearly homogeneous, but requires a large enough degree of income effect for JR preferences. Furthermore, with the three specifications we consider, a low enough elasticity of intertemporal substitution in consumption is necessary to get indeterminacy. Under these properties, an intermediate range of tax rates is destabilizing. The intuition behind our results comes from the fact that intertemporal and intratemporal effects need to be in accordance. Coming back to the intuition of Schmitt-Grohé and Uribe described above, when households decrease their labor supply in current period, at the same time, they must decrease their consumption since they have less income. However, this is only compatible with a marginal utility of consumption that is increasing in leisure i.e. Edgeworth substitutability between consumption and labor.

Finally, we study the empirical robustness of the model. A calibrated version of the model based on plausible estimates of structural parameters emphasizes that labor income taxes under a balanced-budget rule are a potential source of instability for most OECD countries. This concerns particularly European countries since they experience the highest tax rates and stand within the range of destabilizing tax rates for most of the calibrations considered.

In the next section, we present the model and derive the optimal choices of households and firms. Section 3 is devoted to prove the existence of a normalized steady state. In Section 4, we provide the dynamic analysis with our main results and a discussion with the related literature, while empirical illustrations are given in Section 5. Economic interpretations are discussed in Section 6. Finally, Appendix presents all the proofs.

2. The model

In this section, we describe our economy with a standard neoclassical growth model. First, we define the policy rule implemented by the government. Then, we state how the agents choose the amount of good consumed and hours worked, and finally, we describe the technological structure.

2.1. Government

Following Schmitt-Grohé and Uribe [21], we assume that the government chooses a constant level of public spendings G, that neither affect the preferences nor the technology. Since the budget is balanced, it is equal to the total tax revenue $\Omega(t)$ generated by a tax rate, $\tau(t)$, applied on labor income, $w(t)l(t)$, with $w(t)$ the wage rate and $l(t)$ the labor supplied:

$$G = \Omega(t) = \tau(t)w(t)l(t) \tag{1}$$

Equivalently, the balanced-budget rule can be written as:

$$\tau(t) = \frac{G}{w(t)l(t)} \tag{2}$$

Since public spendings are constant, the tax rate is counter-cyclical with its tax base, i.e. a decrease in the labor income ends up in an increase in the tax rate.[1]

[1] Giannitsarou [6] considers the same type of balanced-budget rule but focuses on consumption taxes.

2.2. Households' behavior

We consider an economy populated by a large number of identical infinitely-lived agents. We assume without loss of generality that the total population is constant and normalized to one. At each period an agent supplies elastically an amount of labor $l \in [0, \bar{l}]$, with $\bar{l} > 1$ his time endowment. He then derives utility from consumption c and leisure $\mathcal{L} = \bar{l} - l$ according to the instantaneous utility function $U(c, \mathcal{L}/B)$, where $B > 0$ is a scaling parameter, which satisfies:

Assumption 1. *$U(c, \mathcal{L}/B)$ is $\mathbf{C}^r$ over $\mathbb{R}_{++} \times (0, \bar{l})$ for r large enough, increasing with respect to each argument and concave. Moreover, $\frac{U_{c\mathcal{L}}}{U_c}\frac{\mathcal{L}}{B} - \frac{U_{\mathcal{L}\mathcal{L}}}{U_{\mathcal{L}}}\frac{\mathcal{L}}{B} \neq 1$, $\lim_{X \to 0} XU_X(c,X)/U_c(c,X) = 0$ and $\lim_{X \to +\infty} XU_X(c,X)/U_c(c,X) = +\infty$, or $\lim_{X \to 0} XU_X(c,X)/U_c(c,X) = +\infty$ and $\lim_{X \to +\infty} XU_X(c,X)/U_c(C,X) = 0$.*

This assumption will ensure existence of a normalized steady state. In addition to these general properties, we introduce the definition of Edgeworth substitutability between consumption and labor:

Definition 1. *If the marginal utility of consumption is increasing in leisure such that $U_{c\mathcal{L}}(c, \mathcal{L}/B) > 0$, then consumption and labor are Edgeworth substitutes.*

Following Definition 1, Edgeworth complementarity between consumption and labor is obviously obtained when $U_{c\mathcal{L}}(c, \mathcal{L}/B) < 0$.

In our investigation, we will consider three different specifications of preferences commonly used in the literature:

i) An additively separable utility function

$$U(c, (\bar{l} - \mathcal{L})/B) = \frac{c^{1-\frac{1}{\varepsilon_{cc}}}}{1 - \frac{1}{\varepsilon_{cc}}} - \frac{((\bar{l} - \mathcal{L})/B)^{1+\frac{1}{\varepsilon_{ll}}}}{1 + \frac{1}{\varepsilon_{ll}}} \tag{3}$$

where ε_{cc} is the elasticity of intertemporal substitution in consumption and ε_{ll} is the inverse of the wage elasticity of labor. Schmitt-Grohé and Uribe [21] consider this formulation with $\varepsilon_{cc} = 1$ and $\varepsilon_{ll} = +\infty$. Obviously, these preferences exhibit neither Edgeworth substitutability nor complementarity since $U_{c\mathcal{L}}(c, \mathcal{L}/B) = 0$.

ii) A linear homogeneous utility function $U(c, \mathcal{L}/B)$ characterized by the share of consumption within total utility $\alpha(c, \mathcal{L}/B) \in (0, 1)$ defined by:

$$\alpha(c, \mathcal{L}/B) = \frac{U_c(c, \mathcal{L}/B)c}{U(c, \mathcal{L}/B)} \tag{4}$$

while the share of leisure is given by $1 - \alpha(c, \mathcal{L}/B)$. A particular property of these preferences is that consumption and labor are always Edgeworth substitutes $U_{c\mathcal{L}}(c, \mathcal{L}/B) > 0$.

iii) A Jaimovich-Rebelo [9] formulation such that

$$U(c, \mathcal{L}/B) = \frac{(c + (\mathcal{L}/B)^{1+\chi} c^{\gamma})^{1-\theta}}{1-\theta} \tag{5}$$

with $\theta, \chi > 0$ and $\gamma \in [0, 1]$. These preferences are characterized by the parameter γ that controls the degree of income effect and encompass two standard formulations. On the one hand, in absence of income effect ($\gamma = 0$), the GHH formulation is obtained and yields a labor supply independent of consumption. On the other hand, when the income effect is the largest ($\gamma = 1$), the utility function is a KPR formulation which is compatible with balanced growth and stationnary hours worked. According to Definition 1, Edgeworth substitutability between consumption and labor requires $\gamma > \theta$ while Edgeworth complementarity is obtained when $\gamma < \theta$. Linnemann [14] considers a particular restriction of this specification assuming a KPR formulation with $\theta > 1$ that implies Edgeworth complementarity.

Finally, all these utility functions satisfy normality of consumption and labor. In addition, additively separable and linear homogeneous specifications also satisfy concavity but this is not necessarily the case with JR preferences when $\gamma \neq 0$ (see Section 4.3 for further details).

The intertemporal maximization program of the representative agent is given by:

$$\begin{aligned}
\max_{c(t), l(t), K(t)} \quad & \int_{t=0}^{+\infty} e^{-\rho t} U\left(c(t), (\bar{l} - l(t))/B\right) \\
s.t. \quad & c(t) + \dot{K}(t) + \delta K(t) = r(t)K(t) + (1 - \tau(t))w(t)l(t) \\
& K(0) > 0 \text{ given}
\end{aligned} \tag{6}$$

where $r(t)$ is the rental rate of capital, $\rho > 0$ the discount rate, $K(t)$ the capital stock and $\delta > 0$ the depreciation rate of capital. Moreover, we assume in the following that each household considers as given the tax rate $\tau(t)$ on labor income.

Let us introduce the Hamiltonian in current value:

$$\mathcal{H} = U(c(t), (\bar{l} - l(t))/B) + \lambda(t)\Big[r(t)K(t) + (1-\tau(t))w(t)l(t) - c(t) - \delta K(t)\Big]$$

with $\lambda(t)$ the shadow price of capital $K(t)$. Considering the prices (11)-(12) and the tax rate $\tau(t)$ as given, we derive the following first order conditions:

$$U_c(c(t), (\bar{l} - l(t))/B) = \lambda(t) \quad (7)$$

$$(1/B)U_{\mathcal{L}}(c(t), (\bar{l} - l(t))/B) = \lambda(t)(1-\tau(t))w(t) \quad (8)$$

$$\dot{\lambda}(t) = -\lambda(t)[r(t) - \rho - \delta] \quad (9)$$

Any solution needs also to satisfy the transversality condition:

$$\lim_{t \to +\infty} e^{-\rho t}\lambda(t)K(t) \quad = \quad 0 \quad (10)$$

2.3. The production structure

Considering a competitive economy, a continuum of firms of unit size produces a single good Y using capital K and labor l. The firms' technology is a constant returns to scale production function $Y = AF(K,l)$, with $A > 0$ a scaling parameter. We define the intensive stock of capital $a = K/l$ for any $l > 0$ and the intensive production function writes $Y/l = Af(a)$.

Assumption 2. *$f(a)$ is $\mathbf{C}^r$ over $\mathbb{R}_{++}$ for r large enough, increasing ($f'(a) > 0$) and concave ($f''(a) < 0$).*

From the profit maximisation of a firm, we obtain the wage rate $w(t)$ and the rental rate of capital $r(t)$ as:

$$r(t) \quad = \quad Af'(a(t)) \quad (11)$$

$$w(t) \quad = \quad A[f(a(t)) - a(t)f'(a(t))] \quad (12)$$

We also compute the share of capital in total income:

$$s(a) = \tfrac{af'(a)}{f(a)} \in (0,1) \quad (13)$$

and the elasticity of capital-labor substitution:

$$\sigma(a) = -\tfrac{(1-s(a))f'(a)}{af''(a)} > 0 \quad (14)$$

Assumption 3. *Capital and labor are sufficiently strong substitutes, such that $\sigma(a) > s(a)$.*

This last assumption implies that labor income is increasing with the quantity of hours worked. Extending the analysis of Schmitt-Grohé and Uribe [21] who assume a Cobb-Douglas production function, Ghilardi and Rossi [5] also study the impact of substitutability between capital and labor on the range of destabilizing tax rates. However, we generalize both previous contributions they assume a logarithmic utility for consumption and an infinitely elastic labor within additively-separable preferences.

2.4. Intertemporal equilibrium

In order to derive the intertemporal equilibrium, let

$$\tau \equiv \tilde{\tau}(K,l) = \frac{G}{w(K(t)/l(t))l(t)}$$

and substitute $\tilde{\tau}(K,l)$ and the wage rate (12) in the first order conditions (7) and (8). Given K and λ, the system obtained can be solved to express the consumption demand and labor supply functions $c(K(t),\lambda(t))$ and $l(K(t),\lambda(t))$. Plugging the latter in the expression of the tax rate, one obtains:

$$\tilde{\tau}(K(t),l(K(t),\lambda(t))) \equiv \tau(K(t),\lambda(t)) \tag{15}$$

Using (11)-(12), we get the equilibrium values for the rental rate of capital $r(t)$ and the wage rate $w(t)$ with $a(t) = K(t)/l(K(t),\lambda(t))$:

$$\begin{aligned} r(t) &= Af'(a(t)) \equiv r(K(t),\lambda(t)) \\ w(t) &= A[f(a(t)) - a(t)f'(a(t))] \equiv w(K(t),\lambda(t)) \end{aligned} \tag{16}$$

Substituting the expressions obtained for prices, tax rate, consumption demand and labor supply in the equation of capital accumulation (6) and in the Euler equation (9), we obtain the following system of differential equations in K and λ:

$$\begin{aligned} \dot{K}(t) &= r(K(t),\lambda(t))K(t) + (1-\tau(K(t),\lambda(t)))w(K(t),\lambda(t))l(K(t),\lambda(t)) \\ &- \delta K(t) - c(K(t),\lambda(t)) \\ \dot{\lambda}(t) &= -\lambda(t)\left[r(K(t),\lambda(t)) - \rho - \delta\right] \end{aligned} \tag{17}$$

An intertemporal equilibrium is a path $\{K(t),\lambda(t)\}_{t\geq 0}$, with $K(0) > 0$, that satisfies equations (17) and the transversality condition (10).

3. Normalized steady state

A steady state is a 4-tuple (a^*, l^*, c^*, τ^*), with $a^* = K^*/l^*$, satisfying:

$$\delta + \rho = Af'(a^*) \tag{18}$$

$$c^* = l^*\left[(Af'(a^*) - \delta)a^* + (1-\tau^*)A(f(a^*) - a^*f'(a^*))\right] \tag{19}$$

$$\frac{U_{\mathcal{L}}(c^*,(\bar{l}-l^*)/B)}{BU_c(c^*,(\bar{l}-l^*)/B)} = (1-\tau^*)A[f(a^*) - a^*f'(a^*)] \tag{20}$$

$$\tau^* = \frac{G}{A[f(a^*) - a^*f'(a^*)]l^*} \tag{21}$$

We use the scaling parameters $A > 0$ and $B > 0$ to ensure the existence of a normalized steady state (NSS), $a^* = 1$ and $l^* = 1$, which remains invariant with respect to preferences and technological parameters.

Proposition 1. *Let Assumptions 1-2 hold. Then there exist unique values A^* and B^* such that when $A = A^*$ and $B = B^*$, $(a^*, l^*) = (1, 1)$ is a NSS.*

Proof: See Appendix 8.1.

□

Remark: Using a continuity argument, we conclude from Proposition 1 that there exists an intertemporal equilibrium for any initial capital stock $K(0)$ in the neighborhood of K^*.

Let us introduce the following elasticities:

$$\begin{array}{rclrcl} \varepsilon_{cc} & = & -\frac{U_c(c,\mathcal{L})}{U_{cc}(c,\mathcal{L})c}, & \varepsilon_{lc} & = & -\frac{U_{\mathcal{L}}(c,\mathcal{L})}{U_{\mathcal{L}c}(c,\mathcal{L})c}, \\ \varepsilon_{cl} & = & -\frac{U_c(c,\mathcal{L})}{U_{c\mathcal{L}}(c,\mathcal{L})l}, & \varepsilon_{ll} & = & -\frac{U_{\mathcal{L}}(c,\mathcal{L})}{U_{\mathcal{L}\mathcal{L}}(c,\mathcal{L})l} \end{array} \quad (22)$$

Normality of consumption and leisure states that $\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}} \geq 0$ and $\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}} \geq 0$ and holds for any preferences we consider. Concavity of preferences implies $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}} \geq 0$. This property is satisfied for both additive separable and linear homogeneous preferences. However, JR formulation requires further restrictions to satisfy concavity. Since we are interested in the local dynamics, Lemma 2 in Section 4.3 provides additionnal restrictions to ensure concavity in the neighboorhood of the steady state. Moreover, according to Definition 1, note that when $\frac{1}{\varepsilon_{cl}}$ and $\frac{1}{\varepsilon_{lc}}$ are negative (positive), consumption and labor are Edgeworth substitutes (complements).

Finally, in the rest of the paper, we evaluate all the shares and elasticities previously defined at the NSS. From (4), (13) and (14), we denote indeed $\alpha(c^*, (\bar{l} - 1)/B^*) = \alpha$, $s(1) = s$ and $\sigma(1) = \sigma$.

4. Instability with balanced-budget rules and labor income taxes

This section investigates the properties of preferences that enhance the likelihood of indeterminacy when a general technology is considered. In Appendix 8.2, we linearize the dynamic system in the neighborhood of the NSS and compute the trace and the determinant of the associated Jacobian matrix. As the dynamic system (17) has one predetermined and one forward variable, indeterminacy requires a negative trace and a positive determinant. From the expression of the trace given in Appendix 8.2, we can directly derive a necessary condition for the existence of indeterminacy whatever the specification of preferences:

Lemma 1. *Let Assumptions 1-3 hold. A necessary condition for local indeterminacy of the NSS is $\tau > \underline{\tau}$, with:*

$$\underline{\tau} = \frac{\frac{s}{\sigma} + \varepsilon_{cc}\left(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right)}{1 + \varepsilon_{cc}\left(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right)} \tag{23}$$

Proof: See Appendix 8.2

Given that the term $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}$ measures the degree of concavity of the utility function, we conclude that stronger degrees of concavity imply that indeterminacy requires higher tax rates on labor income. Lemma 1 therefore underlines the importance of preferences for the destabilizing impact of labor income taxes. Besides, a weak factor substitutability increases the lower bound on tax rate. This last point holds for any given specification of preferences and is particularly discussed in Ghilardi and Rossi [5] who consider the restricted case of additively-separable preferences with logarithmic utility function of consumption and infinitely elastic labor. Their related condition is $\tau > \frac{s}{\sigma}$ while, in Schmitt-Grohé and Uribe's [21] framework with a Cobb-Douglas technology, it becomes $\tau > s$.

4.1. Additively separable preferences

We first focus on a generalized version of Schmitt-Grohé and Uribe [21] with an additively separable utility function. More precisely, we do not restrict the elasticity of intertemporal substitution in consumption or the elasticity of capital-labor substitution to be unitary (i.e. ε_{cc}, $\sigma \neq 1$). Furthermore, labor supply is assumed to be elastic such that $\frac{1}{\varepsilon_{ll}} \in (0, +\infty)$. Note that this class of preferences is characterized by $\frac{1}{\varepsilon_{cl}} = \frac{1}{\varepsilon_{lc}} = 0$ meaning that consumption and labor are neither Edgeworth substitutes nor complements. From Lemma 1, we derive $\underline{\tau} = \frac{s/\sigma + \frac{1}{\varepsilon_{ll}}}{1+\frac{1}{\varepsilon_{ll}}}$ and a first result follows:

Proposition 2. *Under Assumptions 1-3, let $U(c, \mathcal{L}/B)$ be given by (3) and $\underline{\tau} = \frac{s/\sigma + \frac{1}{\varepsilon_{ll}}}{1+\frac{1}{\varepsilon_{ll}}}$. There exist $\bar{\rho} \in (0, +\infty]$, $\bar{\varepsilon}_{cc} > 0$ and $\bar{\tau} \in (\underline{\tau}, 1)$ such that the NSS is locally indeterminate if and only if $\rho \in (0, \bar{\rho})$, $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$ and $\tau \in (\underline{\tau}, \bar{\tau})$.*

Proof: See Appendix 8.3.

This proposition highlights the existence of an interval of tax rates that leads to indeterminacy. On the one hand, the lower bound of this interval depends on the capital share of income s, the capital-labor elasticity of substitution σ and the inverse of the wage elasticity of labor $\frac{1}{\varepsilon_{ll}}$. It is straightforward to prove that

$\underline{\tau}$ is increasing in $\frac{1}{\varepsilon_{ll}}$. As a consequence, infinitely elastic labor (i.e.: $\frac{1}{\varepsilon_{ll}} = 0$), as considered in Schmitt-Grohé and Uribe [21] and Ghilardi and Rossi [5], is the less restrictive case since $\underline{\tau} = \frac{s}{\sigma}$. To ensure that $\underline{\tau} < \bar{\tau}$, the elasticity of intertemporal substitution in consumption has to be low enough. Namely, ε_{cc} has to be lower than $\bar{\varepsilon}_{cc}$ with:

$$\bar{\varepsilon}_{cc} = \frac{\left(\frac{1-s/\sigma}{1+\frac{1}{\varepsilon_{ll}}}\right)\left[(\rho+\delta)(1-s)+s\rho\right]}{\frac{s}{\sigma}\left[\left(\frac{1-s/\sigma}{1+\frac{1}{\varepsilon_{ll}}}\right)(\rho+\delta)(1-s)+s\rho\right]}$$

It is worth pointing out that this upper bound is decreasing with $\frac{1}{\varepsilon_{ll}}$. More precisely, when $\frac{1}{\varepsilon_{ll}} = 0$, $\bar{\varepsilon}_{cc}$ is the largest. This argument reinforces the conclusion of Schmitt-Grohé and Uribe [21] that within additively-separable preferences, instability is more likely when the labor supply is infinitely-elastic.

4.2. Linear homogeneous preferences

A linear homogeneous specification is characterized by $\frac{1}{\varepsilon_{lc}}, \frac{1}{\varepsilon_{cl}} < 0$ such that consumption and labor are always Edgeworth substitutes. Moreover, notice that $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}} = 0$ and we obtain therefore from (23) $\underline{\tau} = \frac{s}{\sigma}$. Looking for conditions ensuring the existence of a continuum of equilibrium paths around the steady state, we obtain the next proposition:

Proposition 3. *Under Assumptions 1-3, let $U(c, \mathcal{L}/B)$ be linear homogeneous and $\underline{\tau} = \frac{s}{\sigma}$. There exist $\bar{\rho} \in (0, +\infty]$, $\bar{\tau} \in (\underline{\tau}, 1)$ and $\bar{\varepsilon}_{cc} > 0$ such that the NSS is locally indeterminate if and only if $\rho \in (0, \bar{\rho})$, $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$ and $\tau \in (\underline{\tau}, \bar{\tau})$.*

Proof: See Appendix 8.4.

As in Proposition 2, we show that the equilibrium is locally indeterminate for tax rates within a bounded interval. The lower bound on tax rates increases with the capital share of income and decreases with the elasticity of capital-labor substitution. Note that in order to have $\underline{\tau} < \bar{\tau}$, we impose a low enough elasticity of intertemporal substitution in consumption such that $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$. This condition is equivalent to the one obtained in the additively separable case. Nevertheless, it is not restrictive because $\bar{\varepsilon}_{cc}$ tends to $+\infty$ when the share of consumption in total utility α tends to unity. Finally, the restriction on the elasticity of intertemporal substitution in consumption has important implications on the wage elasticity of labor. Indeed, combining equations (35) and (37) in Appendix 8.4, one shows that a sufficiently low ε_{cc} implies a low enough wage elasticity of labor ε_{ll} such that $\varepsilon_{ll} < \bar{\varepsilon}_{ll}$ with:

$$\bar{\varepsilon}_{ll} = \frac{(1-\alpha)[(\rho+\delta)(1-s)+s\rho]+\alpha(1-\frac{s}{\sigma})(\rho+\delta)(1-s)}{\alpha^2\frac{s}{\sigma}(\rho+\delta)(1-s)} \tag{24}$$

This conclusion contrasts therefore with Schmitt-Grohé and Uribe [21] and Ghilardi and Rossi [5] that assume additively-separable preferences but also with Linnemann [14], that considers a particular KPR utility function. All these contributions require a large enough wage elasticity of labor to obtain indeterminacy.

4.3. Jaimovich-Rebelo preferences

With JR preferences, we can control the degree of the income effect through the parameter $\gamma \in [0,1]$. It is worth stressing that these preferences exhibits Edgeworth substitutability between consumption and labor if $\gamma > \theta$ and Edgeworth complementarity when $\gamma < \theta$.

In order to ensure concavity in the neighborhood of the NSS, we add the following restriction:

Lemma 2. *Let $U(c,\mathcal{L}/B)$ be given by* (5) *and Assumption 1 holds. A necessary and sufficient condition to obtain concavity in the neighborhood of the NSS is $\theta \geq \underline{\theta}(\tau,\gamma,\chi)$ with:*

$$\underline{\theta}(\tau,\gamma,\chi) = \frac{\gamma\mathcal{C}(\tau)(\gamma+\chi)(1+\chi-(1-\gamma)\mathcal{C}(\tau))}{(1+\chi)^2[\chi+\gamma\mathcal{C}(\tau)\big(2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\big)]}$$

Proof: See Appendix 8.5.

Contrary to the previous cases, it is not possible to derive from (23) an explicit expression of $\underline{\tau}$ since it is implicitly given by $\underline{\tau} = h(\underline{\tau})$ with:

$$h(\tau) = \frac{\frac{s}{\sigma} + \frac{\theta(1+\chi)^2\left[\chi+\gamma\mathcal{C}(\tau)\left(2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\right)\right]-\gamma\mathcal{C}(\tau)(\gamma+\chi)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{\theta(1+\chi)^2-\gamma(1-\gamma)\mathcal{C}(\tau)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}}{1 + \frac{\theta(1+\chi)^2\left[\chi+\gamma\mathcal{C}(\tau)\left(2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\right)\right]-\gamma\mathcal{C}(\tau)(\gamma+\chi)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{\theta(1+\chi)^2-\gamma(1-\gamma)\mathcal{C}(\tau)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}}$$

As shown in Appendix 8.6, there exists a unique $\underline{\tau} \in (0,1)$ such that Lemma 1 holds. We get then the next proposition:

Proposition 4. *Under Assumptions 1-3, let $U(c,\mathcal{L}/B)$ be given by* (5)*. There is a critical value $\underline{\gamma} \in (0,1)$ for which for any given $\gamma \in (\underline{\gamma},1]$, there exist $\bar{\rho} \in (0,+\infty]$, $\bar{\theta} \in (\underline{\theta},+\infty]$, $\underline{\sigma} \in (s,+\infty)$, $\underline{\tau} \in (0,1)$ and $\bar{\tau} \in (\underline{\tau},1)$ such that the NSS is locally indeterminate if and only if $\rho \in (0,\bar{\rho})$, $\theta \in (\underline{\theta},\bar{\theta})$, $\sigma > \underline{\sigma}$ and $\tau \in (\underline{\tau},\bar{\tau})$.*

Proof: See Appendix 8.6.

This proposition, jointly with Propositions 2 and 3, highlights the robustness of the existence of an intermediate range of destabilizing tax rates. Moreover, a direct outcome of Proposition 4 is that indeterminacy is more likely when consumption and labor are Edgeworth substitutes. Indeed, we can show (see Appendix 8.5) that γ is always larger than $\underline{\theta}$. As a result, Proposition 4 implies Edgeworth substitutability or weak complementarity. More precisely, local indeterminacy is ruled out with a GHH specification characterized by the absence of income effect ($\gamma = 0$) and a strong Edgeworth complementarity. On the contrary, with KPR preferences ($\gamma = 1$), consumption and labor are obviously Edgeworth substitutes if $\theta < 1$. In this case, the existence of a range of destabilizing tax rates is ensured. Otherwise, when consumption and labor become weak Edgeworth complements, indeterminacy may still hold but requires higher tax rates. This conclusion explains therefore the result of Linnemann [14] about the lack of plausibility of indeterminacy since he assumes $\theta > 1$, i.e. a strong Edgeworth complementarity between consumption and labor.

We have highlighted the role of preferences on the emergence of indeterminacy in a Ramsey model with a balanced-budget rule financed by a labor income tax. We find that Edgeworth substitutability between consumption and labor and a low elasticity of intertemporal substitution in consumption are crucial for the existence of a range of destabilizing tax rates. Next section is devoted to compare our conclusions to the related literature.

4.4. Comparison with the related literature

Schmitt-Grohé [20] and Schmitt-Grohé and Uribe [21] claim that there is a close correspondence between indeterminacy in models with productive externalities and models with balanced-budget. We argue that this equivalence is not a general property when one assumes non additively separable preferences. The contribution of Nishimura *et al.* [17] considers the occurrence of indeterminacy in a model with small aggregate externalities. They also find that indeterminacy requires an increasing marginal utility of consumption with respect to leisure, i.e. $U_{c\mathcal{L}}(c, \mathcal{L}) > 0$. According to Definition 1, this implies that consumption and labor are Edgeworth substitutes. Nevertheless, additively separable and linear homogeneous preferences display indeterminacy provided that the elasticity of intertemporal substitution in consumption is sufficiently large. Since in the case of labor income tax we require a low enough elasticity of intertemporal substitution in consumption to obtain indeterminacy, the close correspondence discussed by Schmitt-Grohé [21] does not hold.

The literature on balanced-budget rules has also focused on the destabilizing role of consumption taxes. Assuming an additive separable utility function, Giannitsarou [6] show that consumption tax has stabilizing effect since saddle-path stability is always ensured, contrarily to Schmitt-Grohé and Uribe [21] with a labor income tax. However, Nourry *et al.* [18] discuss this conclusion considering non-separable preferences with varying income effect. They find that in presence of an intermediate or a low degree of income effect, consumption taxes lead to instability. In other words, in contrast to labor income tax, the key element for a destabilizing consumption tax is Edgeworth complementarity between consumption and labor. It follows that even though consumption and labor income taxes introduce similar distortions in the consumption-leisure trade-off, they require opposite properties of preferences in order to be destabilizing.

In the next section, we investigate the numerical properties of our results in order to discuss their empirical plausibility.

5. Empirical illustration

To give better insights of our results, we proceed to a numerical exercise. We first divide countries between four groups according to the range of their tax rates on labor income. The classification is based on the contribution of Trabandt and Uhlig [22] that computes the effective tax rates up to 2008 using the methodology of Mendoza *et al.* [15]. The four groups of countries are given in Table 1.

$\tau \in (0.25, 0.30)$	Japan (0.27), U.S. (0.28), U.K. (0.28), Ireland (0.27)
$\tau \in (0.30, 0.40)$	Portugal (0.31), Spain (0.36)
$\tau \in (0.40, 0.50)$	Belgium (0.49), Denmark (0.47), EU-14 (0.41), France (0.46), Germany (0.41), Italy (0.47), Netherland (0.44),
$\tau \in (0.50, 0.60)$	Austria (0.50), Sweden (0.56)

Table 1: Estimated labor income tax rates

We need now to fix the values of the structural parameters. On the basis of quarterly data, we consider the benchmark values $(\rho, \delta, s) = (0.01, 0.025, 0.3)$. According to the empirical literature, there is no clear agreement on the size of the elasticity of capital-labor substitution. Nevertheless, the higher estimates of this elasticity stand in the interval (1.24, 3.24) as shown in Duffy and Papageorgiou [4] and Karagiannis *et al.* [10]. There is also no consensus on the elasticity of intertemporal substitution in consumption. Several contributions provides the range (0.2,0.8) (see Campbell [3] and Kocherlakota [13]), while Mulligan [16] and more recently Vissing-Jorgensen and Attanasio [23] show evidences for higher estimates with an interval (2,3). Finally, many contributions consider that labor is infinitely elastic. However, Rogerson and Wallenius [19] investigate aggregate participation in the labor market at the macro level and find an interval of the wage elasticity of labor that stands between 2.25 and 3.

From the discussion above, we focus on the following intervals for $\varepsilon_{cc} \in [0.66, 2]$, $\varepsilon_{ll} \in [2.5, +\infty]$. We also assume $\sigma \in [0.8, 1.4]$. This interval allows to consider the estimates given by the empirical literature but also extend to the case of complementarity between inputs since the conclusions of the literature are still uncertain.[2] We first calibrate an additively-separable utility function characterized by $\varepsilon_{cc} = 0.67$ and $\varepsilon_{ll} \in (2.85, +\infty)$.

	σ =0.8	σ =1.4
$\varepsilon_{ll} = +\infty$	(0.38,0.87)	(0.21,0.87)
ε_{ll}=2.86	(0.54,0.74)	(0.42,0.74)

Table 2: Range of destabilizing tax rates ($\underline{\tau}$,$\bar{\tau}$) with additively-separable preferences

Table 2 reports the intervals of destabilizing tax rates in the additively-separable case. We observe that the lower bound $\underline{\tau}$ is between 0.21 and 0.54 while the upper bound $\bar{\tau}$ is higher than 0.74. According to Table 1, this stresses the plausibility of our results. Indeed, except when $\sigma = 0.8$ and $\varepsilon_{ll} = 2.86$, most countries in Europe are destabilized. This concerns particularly Sweden, Austria, Belgium, Denmark, Italy and France. Moreover, the whole sample of countries stands inside the range of destabilizing tax rates when labor supply is infinitely elastic and the substitutability between capital and labor is strong enough.

When the utility function is homogeneous linear, we calibrate the parameters $\varepsilon_{cc} = 0.66$, $\alpha = 0.65$ and $\varepsilon_{cc} = 2$, $\alpha = 0.51$ in order to match an interval of $\varepsilon_{ll} \in (2.5, 2.9)$ given the admissible values of τ in Table 1.

As shown in Table 3, the homogeneous linear formulation displays an even better outline. The interval of $\underline{\tau}$ becomes in this case [0.21,0.38] and $\bar{\tau}$ is inside

[2]Note that we assume a high enough elasticity of capital-labor substitution such that $\sigma > (1 - s)$ which implies that capital income is increasing with respect to capital.

	σ=0.8	σ=1.4
ε_{cc}=0.66, α=0.65	(0.38,0.97)	(0.21,0.97)
ε_{cc}=2, α=0.51	(0.38,0.81)	(0.21,0.81)

Table 3: Range of destabilizing tax rates ($\underline{\tau}$,$\bar{\tau}$) with homogeneous linear preferences

the interval (0.81,0.97). Consequently, all countries with tax rates above 0.40 in Table 1 are now in the range of the destabilizing tax rates.

Finally, with the JR formulation, considering that the tax rates stands between (0.27,0.57) in Table 1, $\theta = 0.5$ and $\chi = 0$ match $\varepsilon_{ll} \in (2.3, 2.5)$. Figure 1 shows the lower and the upper bound on τ as a function of the degree of income effect γ.

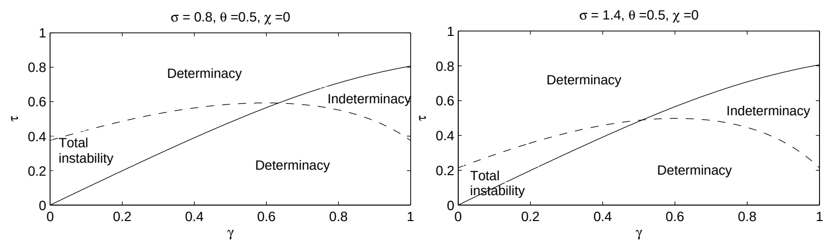

Figure 1: Destabilizing tax rates in the case with Jaimovich-Rebelo preferences. Dash lines: $\underline{\tau}$, solid lines: $\bar{\tau}$

Under our calibration, Figure 1 shows that the minimum level of $\underline{\gamma}$ is in the interval [0.51,0.63]. Moreover, Table 4 reports the interval of $\underline{\gamma}$ for countries considered in Table 1 taking their tax rates as given.

$\underline{\gamma} \in (0.6, 0.75]$	Austria, Belgium, Denmark, France, Italy, Sweden
$\underline{\gamma} \in (0.8, 0.97]$	Germany, EU-14, Netherlands
$\underline{\gamma} \in (0.90, 0.94]$	Portugal, Spain
$\underline{\gamma} \in (0.95, 0.96]$	Japan, U.S., U.K., Ireland

Table 4: Instability with Jaimovich-Rebelo preferences

This shows that a large range of values of γ covers indeterminate tax rates. Furthermore, the intervals given in Table 4 fits the upper estimates of Kahn and Tsoukalas [11]. Using Bayesian estimations, they report a distribution of γ with mean 0.81 and a 10-90 percentiles interval of [0.69,0.95]. Our numerical exercise illustrates therefore that most OECD countries may experience instability for plausible values of structural parameters.

6. Economic intuition

To understand the economic mechanisms, let us assume that agents expect a larger future tax rate. Following (8) and (11), future labor supply decreases and yields a lower interest rate that reduces income in future period. Consequently, investment decreases and since they need to work less, households decrease their labor supply in current period. The decrease in the tax base forces the governement to adjust the budget by increasing tax rates such that volatility in agent's expectations are self-fulfilling. Nevertheless one question remains: why indeterminacy occurs under some class of preferences while it is ruled out with others ? A crucial point to understand our results comes from the fact that the cross-elasticity ε_{lc} needs to be negative or weakly positive, i.e. consumption and labor need to be Edgeworth substitutes or weak Edgeworth complements. In our interpretation, the decrease in interest rate involves $\dot{\lambda} > 0$ since $r < \delta + \rho$. Indeterminacy is obtained if it is associated with a decrease of labor supply in current period that is larger than the decrease of labor supply in the next period.[3] Because they have less income in present period, they must also decrease their current consumption. Self-fulfilling expectations implies therefore $\frac{\dot{c}}{c} > 0$ and $\frac{\dot{l}}{l} > 0$. Since capital is predetermined, we have $\frac{\dot{w}}{w} = -\frac{s}{\sigma}\frac{\dot{l}}{l}$. Taking then the derivative of equation (8) with respect to time, indeterminacy occurs if the following equality is satisfied:

$$(\frac{s}{\sigma} + \frac{1}{\varepsilon_{ll}})\frac{\dot{l}}{l} - \frac{1}{\varepsilon_{lc}}\frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda} \tag{25}$$

Because $(\frac{s}{\sigma} + \frac{1}{\varepsilon_{ll}})$ is positive, this equation is satisfied if $\frac{1}{\varepsilon_{lc}}$ is negative or positive but sufficiently low such that the first term on the left-hand side dominates the second one. It is straightforward to show it is always the case for an additively separable and a linear homogeneous preferences since $\frac{1}{\varepsilon_{lc}} = 0$ in the former and $\frac{1}{\varepsilon_{lc}} < 0$ in the latter.

In the case of Jaimovich-Rebelo preferences, the sign of $\frac{1}{\varepsilon_{lc}}$ is ambiguous and depends on the size of τ, γ:

$$\frac{1}{\varepsilon_{lc}} = \frac{(\theta - \gamma)(1 + \chi) + \gamma(1 - \gamma)\mathcal{C}(\tau)}{1 + \chi - (1 - \gamma)\mathcal{C}(\tau)} \tag{26}$$

[3] This allows to construct a stable dynamic path that explains indeterminacy.

with $\mathcal{C}'(\tau) < 0$.

The denominator in equation (26) being positive, the sign of $\frac{1}{\varepsilon_{lc}}$ is given by the numerator. Consider first that $\gamma > \theta$. Since $\mathcal{C}'(\tau) < 0$, the numerator of (26) is negative for a large enough τ. Consequently, JR preferences display Edgeworth substitutability and equation (25) is always satisfied. In contrast, when $\gamma < \theta$, the numerator is positive and consumption and labor are therefore Edgeworth complement. It follows that the intertemporal mechanisms described in equation (25) is less likely to be satisfied and therefore requires much higher tax rates to obtain self-fulfilling expectations.

7. Concluding comments

This paper contributes to the debate dealing with the (de-)stabilizing properties of balanced-budget rules financed by a labor income tax. More particularly, we emphasize the mechanisms in preferences leading to indeterminacy. Focusing on three commonly used utility functions, we prove that Edgeworth substitutability between consumption and labor increases the likelihood of a destabilizing labor income tax. When the elasticity of intertemporal substitution in consumption is sufficiently low, an intermediate range of tax rates is destabilizing. Finally, a numerical exercise supports our findings according to the empirical evidence underlying the plausibility of balanced-budget rule as a source of instability in most OECD countries.

8. Appendix

8.1. Proof of Proposition 1

To establish the existence of a normalized steady state $(a^*, l^*, c^*, \tau^*) = (1, 1, c^*, \tau^*)$, we have to prove the existence and uniqueness of solutions A^* and B^* satisfying:

$$\delta + \rho = A^* f'(1) \tag{27}$$

$$\tau^* = \frac{G}{A^*[f(1) - f'(1)]} \tag{28}$$

$$c^* = (1 - \tau^*)A^*[f(1) - f'(1)] + A^* f'(1) - \delta \tag{29}$$

$$\frac{U_{\mathcal{L}}(c, (\bar{l} - 1)/B^*)}{B^* U_c(c, (\bar{l} - 1)/B^*)} = (1 - \tau^*)A^*[f(1) - f'(1)] \tag{30}$$

From equation (27), we derive that $A^* = \frac{\rho+\delta}{f'(1)}$ which gives, once substituted in equations (28) and (29), a unique τ^* and c^* rewritten as:

$$\begin{aligned} \tau^* &= \tfrac{s(1)G}{(\rho+\delta)(1-s(1))} \\ c^* &= \tfrac{s(1)\rho+(1-\tau)(\rho+\delta)(1-s(1))}{s(1)} \end{aligned}$$

Considering A^*, τ^* and c^*, we get the following equation from (30):

$$\tilde{g}(B) \equiv \tfrac{U_{\mathcal{L}}(c,(\bar{l}-1)/B)}{BU_c(c,(\bar{l}-1)/B)} = \tfrac{(1-\tau^*)(\rho+\delta)(1-s(1))}{s(1)} \tag{31}$$

Existence of a unique value B^* satisfying equation (31) requires that the marginal rate of substitution $\tilde{g}(B)$ does not have a derivative equal to zero and satisfies appropriate boundaries conditions. Since under Assumption 1, $\lim_{B\to 0}\tilde{g}(B) = 0$ and $\lim_{B\to+\infty}\tilde{g}(B) = +\infty$, or $\lim_{B\to 0}\tilde{g}(B) = +\infty$ and $\lim_{B\to+\infty}\tilde{g}(B) = 0$, the existence of B^* is guaranteed. Moreover, as $B\tilde{g}'(B)/\tilde{g}(B) \neq 0$, uniqueness of B^* also follows.

□

8.2. Proof of Lemma 1

To provide an analysis of local stability, we linearize (17) around the NSS. We then derive the characteristic polynomial by considering the elasticities evaluated at the NSS. We need first to derive a relationship between the cross-elasticities, ε_{cl} and ε_{lc}. Using (22) and the first order conditions (7) and (8), we get $\varepsilon_{cl} = \frac{(1-\tau)wL}{c}\varepsilon_{lc}$. Using the expression of w at the NSS given in (16) together with (13) and (18) we find $wL = K(1-s)(\delta+\rho)/s$. Since at NSS, $c = l[\rho a + (1-\tau)w]$, it follows:

$$\varepsilon_{cl} = \tfrac{(1-\tau)(\delta+\rho)(1-s)+s\rho}{(1-\tau)(\delta+\rho)(1-s)}\varepsilon_{lc} \tag{32}$$

Differentiating $\tau(K(t),\lambda(t))$ as given by (15), one obtains the elasticities of the tax rate with respect to K and λ:

$$\begin{aligned} \varepsilon_{\tau k} = \tfrac{d\tau}{dK}\tfrac{K}{\tau} &= -\tfrac{(1-\tau)s}{\sigma}\tfrac{[\sigma\Delta\varepsilon_{cc}+\sigma-s]}{(1-\tau)\sigma\Delta\varepsilon_{cc}+\tau(s-\sigma)} \\ \varepsilon_{\tau\lambda} = \tfrac{d\tau}{d\lambda}\tfrac{\lambda}{\tau} &= -\tfrac{(1-\tau)(\sigma-s)\varepsilon_{cc}}{(1-\tau)\sigma\Delta\varepsilon_{cc}+\tau(s-\sigma)}\left(\tfrac{1}{\varepsilon_{cc}} - \tfrac{1}{\varepsilon_{lc}}\right) \end{aligned}$$

Using (22), the Implicit Function Theorem gives the partial derivatives of the functions $c(K(t),\lambda(t))$ and $l(K(t),\lambda(t))$ evaluated at the NSS:

$$\begin{aligned} \tfrac{dc}{dK} &= \tfrac{c}{K\Delta\varepsilon_{cl}}\left(\tfrac{s}{\sigma} - \tfrac{\tau\varepsilon_{\tau k}}{1-\tau}\right), \quad \tfrac{dc}{d\lambda} = -\tfrac{c}{\lambda\Delta}\left[\tfrac{1}{\varepsilon_{ll}} - (1-\tfrac{\tau\varepsilon_{\tau\lambda}}{1-\tau})\tfrac{1}{\varepsilon_{cl}} + \tfrac{s}{\sigma}\right] \\ \tfrac{dl}{dK} &= \tfrac{l}{K\Delta\varepsilon_{cc}}\left(\tfrac{s}{\sigma} - \tfrac{\tau\varepsilon_{\tau k}}{1-\tau}\right), \quad \tfrac{dl}{d\lambda} = \tfrac{l}{\lambda\Delta}\left[(1-\tfrac{\tau\varepsilon_{\tau\lambda}}{1-\tau})\tfrac{1}{\varepsilon_{cc}} - \tfrac{1}{\varepsilon_{lc}}\right] \end{aligned}$$

with $\Delta = \frac{1}{\varepsilon_{cc}}\left(\frac{1}{\varepsilon_{ll}} + \frac{s}{\sigma}\right) - \frac{1}{\varepsilon_{cl}\varepsilon_{lc}}$. From these results and (16) we also derive at the NSS:

$$\frac{dr}{dK} = -\frac{r(1-s)}{K\sigma}\Big[1 - \frac{1}{\Delta\varepsilon_{cc}}\big(\frac{s}{\sigma} - \frac{\tau\varepsilon_{\tau k}}{1-\tau}\big)\Big], \quad \frac{dr}{d\lambda} = \frac{r(1-s)}{\lambda\Delta\sigma}\Big[\big(1 - \frac{\tau\varepsilon_{\tau\lambda}}{1-\tau}\big)\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}}\Big]$$

$$\frac{dw}{dK} = \frac{ws}{K\sigma}\Big[1 - \frac{1}{\Delta\varepsilon_{cc}}\big(\frac{s}{\sigma} - \frac{\tau\varepsilon_{\tau k}}{1-\tau}\big)\Big], \quad \frac{dw}{d\lambda} = -\frac{ws}{\lambda\Delta\sigma}\Big[\big(1 - \frac{\tau\varepsilon_{\tau\lambda}}{1-\tau}\big)\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}}\Big]$$

Linearizing the system (17) around the NSS, using (32) and the above results, gives:

$$\begin{aligned}
\frac{d\dot{K}}{dK} &= \rho - \frac{(\delta+\rho)(1-s)}{s}\Big\{\tau\Big[\varepsilon_{\tau k} + \frac{s}{\sigma}\big[1 - \frac{1}{\Delta\varepsilon_{cc}}\big(\frac{s}{\sigma} - \frac{\tau\varepsilon_{\tau k}}{1-\tau}\big)\big]\Big] - \frac{1-\tau}{\Delta\varepsilon_{cc}}\big(\frac{s}{\sigma} - \frac{\tau\varepsilon_{\tau}k}{1-\tau}\big)\Big\} \\
&- \frac{(1-\tau)(1-s)(\delta+\rho)}{s\Delta\varepsilon_{cl}}\big(\frac{s}{\sigma} - \frac{\tau\varepsilon_{\tau k}}{1-\tau}\big) \\
\frac{d\dot{K}}{d\lambda} &= \frac{(1-\tau)(1-s)(\delta+\rho)K}{s\Delta\lambda}\Big[\frac{1}{\varepsilon_{ll}} + \frac{s}{\sigma} - \big(1 - \frac{\tau\varepsilon_{\tau\lambda}}{1-\tau}\big)\frac{1}{\varepsilon_{cl}}\Big] + (1-\tau)\Big[\big(1 - \frac{\tau\varepsilon_{\tau\lambda}}{1-\tau}\big)\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}}\Big] \\
&+ \frac{(\delta+\rho)(1-s)K}{s\lambda}\Big\{\tau\Big[\Delta\varepsilon_{\tau\lambda} - \frac{s}{\sigma}\big[\big(1 - \frac{\tau\varepsilon_{\tau\lambda}}{1-\tau}\big)\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}}\big]\Big]\Big\} \\
\frac{d\dot{\lambda}}{dK} &= -\frac{\lambda(\delta+\rho)(1-s)}{K\sigma}\Big[\Delta + \frac{1}{\Delta\varepsilon_{cc}}\big(\frac{s}{\sigma} - \frac{\tau\varepsilon_{\tau k}}{1-\tau}\big)\Big] \\
\frac{d\dot{\lambda}}{d\lambda} &= -\frac{(\delta+\rho)(1-s)}{\Delta\sigma}\Big[\big(1 - \frac{\tau\varepsilon_{\tau\lambda}}{1-\tau}\big)\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}}\Big]
\end{aligned}$$

After tedious computations and straightforward simplifications, using (32), the expressions of $\varepsilon_{\tau k}$, $\varepsilon_{\tau\lambda}$ as given above, we get the following characteristic polynomial:

$$\mathcal{P}(\lambda) = \lambda^2 - \mathcal{T}\lambda + \mathcal{D} = 0 \tag{33}$$

with

$$\mathcal{T} = \rho - \frac{(\rho+\delta)(1-s)\tau}{\sigma\tau - s - (1-\tau)\sigma\varepsilon_{cc}[\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}]}$$

and

$$\mathcal{D} = \frac{(\rho+\delta)(1-s)\varepsilon_{cc}\Big[[(1-\tau)(\rho+\delta)(1-s)+s\rho]\Big[(1-\tau)\Big(\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}} + \frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\Big) - \tau\Big] + \tau(1-\tau)(\rho+\delta)(1-s)\Big(\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}}\Big)\Big]}{s\sigma\Big[\sigma\tau - s - (1-\tau)\sigma\varepsilon_{cc}[\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}]\Big]}$$

where $\mathcal{T}$ and $\mathcal{D}$ are respectively the trace and the determinant of the associated Jacobian matrix. Local indeterminacy requires $\mathcal{T} < 0$ and $\mathcal{D} > 0$. A necessary condition to have a negative trace is $\tau > \underline{\tau}$ with:

$$\underline{\tau} = \frac{\frac{s}{\sigma} + \varepsilon_{cc}\big(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\big)}{1 + \varepsilon_{cc}\big(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\big)}$$

□

8.3. Proof of Proposition 2

In the case of additively separable preferences, the expression of the trace is:

$$\mathcal{T} = \rho - \frac{\tau(\rho+\delta)(1-s)}{\sigma\tau - s - \frac{(1-\tau)\sigma\chi}{\varepsilon_{ll}}}$$

Following Lemma 1, we derive directly the lower bound on τ:

$$\underline{\tau} = \frac{\frac{s}{\sigma} + \frac{1}{\varepsilon_{ll}}}{1 + \frac{1}{\varepsilon_{ll}}} \tag{34}$$

Moreover, in order to ensure a negative trace, we also need the following expression to be satisfied:

$$\rho(\sigma\tau - s - (\frac{1-\tau)\sigma}{\varepsilon_{ll}}) - (\rho+\delta)(1-s)\tau > 0$$

This leads to an upper bound on ρ such that:

i) $\rho < \bar{\rho} = \frac{\delta(1-s)\tau}{\sigma\tau - s - \frac{(1-\tau)\sigma}{\varepsilon_{ll}} - s - (\rho+\delta)(1-s)\tau)}$ *if* $\sigma\tau - s - \frac{(1-\tau)\sigma}{\varepsilon_{ll}} - (\rho+\delta)(1-s)\tau) > 0$

ii) $\rho \in (0, +\infty)$ *,otherwise*

Considering the determinant, we get the following expression:

$$\mathcal{D} = \frac{(\delta+\rho)(1-s)}{\sigma s\left(\sigma\tau - s - \frac{(1-\tau)\sigma}{\varepsilon_{ll}}\right)} P(\tau)$$

where

$$\begin{aligned} P(\tau) = &\Big[s\rho + (\delta+\rho)(1-s)\Big](\frac{1}{\varepsilon_{cc}} + \frac{1}{\varepsilon_{ll}}) \\ -\tau\Big[&\big[s\rho + (\delta+\rho)(1-s)\big](\frac{1}{\varepsilon_{cc}} + \frac{1}{\varepsilon_{ll}}) + \frac{(\rho+\delta)(1-s)}{\varepsilon_{ll}} \\ &+ s\rho + (\delta+\rho)(1-s)\Big] + \tau^2(1-s)(\rho+\delta)(1+\frac{1}{\varepsilon_{ll}}) \end{aligned}$$

The denominator of $\mathcal{D}$ is positive when $\tau > \underline{\tau}$. Consequently, the sign of the determinant is given by the sign of $P(\tau)$. The latter function is positive when $\tau = 0$ while negative when $\tau = 1$. Since $P(\tau)$ is strictly decreasing in $\tau \in (0, 1)$, there exists therefore a unique $\bar{\tau} \in (0, 1)$ such that $P(\tau) > 0$ when $\tau < \bar{\tau}$. The condition $\tau \in (\underline{\tau}, \bar{\tau})$ implies therefore a positive determinant.

Finally, we need to ensure that $\underline{\tau} < \bar{\tau}$. This is the case if and only if $P(\tau)$ evaluated at $\underline{\tau}$ is positive. After some simplifications, we derive:

$$\begin{aligned} P(\underline{\tau}) \quad &= \quad \left(\frac{1-s/\sigma}{1+\frac{1}{\varepsilon_{ll}}}\right)\left[(\rho+\delta)(1-s) + s\rho\right]\frac{1}{\varepsilon_{cc}} \\ &- \quad \frac{s}{\sigma}\left[\left(\frac{1-s/\sigma}{1+\frac{1}{\varepsilon_{ll}}}\right)(\rho+\delta)(1-s) + s\rho\right] > 0 \end{aligned}$$

which is satisfied if and only if $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$ with:

$$\bar{\varepsilon}_{cc} = \frac{\left(\frac{1-s/\sigma}{1+\frac{1}{\varepsilon_{ll}}}\right)\left[(\rho+\delta)(1-s) + s\rho\right]}{\frac{s}{\sigma}\left[\left(\frac{1-s/\sigma}{1+\frac{1}{\varepsilon_{ll}}}\right)(\rho+\delta)(1-s) + s\rho\right]}$$

□

8.4. Proof of Proposition 3

Note that with linear homogeneity, all the preferences elasticities are written as function of ε_{cc} such that

$$\begin{array}{c}\varepsilon_{lc} = -\varepsilon_{cc}\frac{(1-\alpha)}{\alpha}, \quad \varepsilon_{cl} = -\varepsilon_{cc}\frac{(1-\alpha)}{\alpha}\frac{(1-\tau)(\delta+\rho)(1-s)+s\rho}{(1-\tau)(1-s)(\rho+\delta)}, \\ \varepsilon_{ll} = \varepsilon_{cc}\frac{(1-\alpha)^2[(1-\tau)(\rho+\delta)(1-s)+s\rho)}{\alpha^2(1-\tau)(\rho+\delta)(1-s)}\end{array} \tag{35}$$

Since linear homogeneity yields $\frac{1}{\varepsilon_{cc}\varepsilon_{ll}} - \frac{1}{\varepsilon_{lc}\varepsilon_{cl}} = 0$, the lower bound on tax rate is given from (23) by:

$$\underline{\tau} = \frac{s}{\sigma} \tag{36}$$

Under this condition, we conclude that $\mathcal{T} < 0$ when:

$$\begin{array}{lll} \textit{i)} & \rho < \bar{\rho} = \frac{\delta(1-s)\tau}{\sigma\tau - s - (1-s)\tau} & \textit{if } \sigma\tau - s - (1-s)\tau > 0 \\ \textit{ii)} & \rho \in (0, +\infty) & \textit{otherwise} \end{array}$$

Considering $\mathcal{D}$ and using the expressions in (35), the determinant is written:

$$\mathcal{D} = \frac{(1-\tau)(\delta+\rho)(1-s)}{(1-\alpha)s\sigma(\sigma\tau-s)}P(\tau)$$

with

$$\begin{array}{rcl} P(\tau) & = & [(\rho+\delta)(1-s)+s\rho] + \frac{\alpha}{1-\alpha}(1-\tau)(\rho+\delta)(1-s) \\ & - & \frac{\tau[(1-\tau)(\rho+\delta)(1-s)+s\rho](1-\alpha)\varepsilon_{cc}}{(1-\tau)} \end{array}$$

Moreover, we derive:

$$\frac{\partial P(\tau)}{\partial \tau} = -\frac{\alpha}{1-\alpha}(\rho+\delta)(1-s) - \frac{(1-\alpha)\varepsilon_{cc}}{(1-\tau)^2}[(1-\tau)^2(\rho+\delta)(1-s)+s\rho] < 0$$

The polynomial $P(\tau)$ is positive when $\tau = 0$ and negative when $\tau = 1$. Since $P(\tau)$ is monotonically decreasing in τ, there exists therefore a unique solution $\bar{\tau} \in (0,1)$ such that $P(\tau) > 0$ if $\tau < \bar{\tau}$. Since the denominator is positive when $\tau > \underline{\tau}$, the determinant is positive if and only if $\tau \in (\underline{\tau}, \bar{\tau})$.

Finally, the condition $\underline{\tau} < \bar{\tau}$ has to be ensured. Substituting $\underline{\tau} = \frac{s}{\sigma}$ into $P(\tau)$, the interval $(\underline{\tau}, \bar{\tau})$ is non-empty if and only if $P(\underline{\tau}) > 0$, i.e. ε_{cc} is low enough such that:

$$\varepsilon_{cc} < \bar{\varepsilon}_{cc} = \frac{(1-s/\sigma)[(\rho+\delta)(1-s)+s\rho+(1-s/\sigma)(\rho+\delta)(1-s)\frac{\alpha}{(1-\alpha)}]}{(1-\alpha)\frac{s}{\sigma}[(1-s/\sigma)(\rho+\delta)(1-s)+s\rho]} \tag{37}$$

$\square$

8.5. Proof of Lemma 2

In the case of Jaimovich-Rebelo preferences, the elasticities in (22) write:

$$\begin{array}{ll}\frac{1}{\varepsilon_{cc}}=\theta\frac{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}-\gamma(1-\gamma)\frac{\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}, & \frac{1}{\varepsilon_{ll}}=\theta\frac{\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}+\chi, \\ \frac{1}{\varepsilon_{cl}}=\frac{\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}\left[\theta\frac{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}-\gamma\right], & \frac{1}{\varepsilon_{lc}}=\theta\frac{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}-\gamma,\end{array} \tag{38}$$

Using these expressions and the relationship between ε_{cl} and ε_{lc} at NSS given by equation (32), one derives:

$$\frac{\frac{(l/B)^{1+\chi}}{1+\chi}c^{\gamma-1}}{1-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^{\gamma-1}}=\frac{(1-\tau)(\rho+\delta)(1-s)}{(1-\tau)(\rho+\delta)(1-s)+s\rho}.$$

Let $\mathcal{C}(\tau)=\frac{(1-\tau)(\rho+\delta)(1-s)}{(1-\tau)(\rho+\delta)(1-s)+s\rho}$ and solve the previous equation such that:

$$\frac{(l/B)^{1+\chi}}{1+\chi}c^{\gamma-1}=\frac{\mathcal{C}(\tau)(1+\chi)}{1+\chi+\gamma\mathcal{C}(\tau)}$$

Then, the following expressions holds:

$$\frac{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}=\frac{1+\chi}{1+\chi-(1-\gamma)\mathcal{C}(\tau)},\quad \frac{\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}{c-\gamma\frac{(l/B)^{1+\chi}}{1+\chi}c^\gamma}=\frac{(1+\chi)\mathcal{C}(\tau)}{1+\chi-(1-\gamma)\mathcal{C}(\tau)}$$

The elasticities rewrite therefore:

$$\begin{array}{ll}\frac{1}{\varepsilon_{cc}}=\theta\frac{1+\chi}{1+\chi-(1-\gamma)\mathcal{C}(\tau)}-\gamma(1-\gamma)\frac{\mathcal{C}(\tau)}{1+\chi}, & \frac{1}{\varepsilon_{ll}}=\theta\frac{(1+\chi)\mathcal{C}(\tau)}{1+\chi-(1-\gamma)\mathcal{C}(\tau)}+\chi, \\ \frac{1}{\varepsilon_{lc}}=\theta\frac{1+\chi}{1+\chi-(1-\gamma)\mathcal{C}(\tau)}-\gamma, & \frac{1}{\varepsilon_{cl}}=\frac{\mathcal{C}(\tau)}{\varepsilon_{lc}},\end{array} \tag{39}$$

According to this, local concavity of the utility function is ensured when $\frac{1}{\varepsilon_{cc}}\geq 0$ and $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}}-\frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\geq 0$. Straighforward computations show that these two inequalities are satisfied if and only if:

$$\theta\geq\underline{\theta}(\tau,\gamma,\chi)\equiv\frac{\gamma\mathcal{C}(\tau)(\gamma+\chi)(1+\chi-(1-\gamma)\mathcal{C}(\tau))}{(1+\chi)^2[\chi+\gamma\mathcal{C}(\tau)\big(2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\big)]} \tag{40}$$

Finally, note that Edgeworth substitutability holds if $\gamma\geq\underline{\theta}(\tau,\gamma,\chi)$. This inequality is satisfied when $\gamma=0$ and $\gamma=1$. It follows that it will be satisfied for any $\gamma\in(0,1)$ if $1\geq\tilde{\underline{\theta}}(\tau,\gamma,\chi)$ with :

$$\tilde{\underline{\theta}}(\tau,\gamma,\chi)=\frac{\mathcal{C}(\tau)(\gamma+\chi)(1+\chi-(1-\gamma)\mathcal{C}(\tau))}{(1+\chi)^2[\chi+\gamma\mathcal{C}(\tau)\big(2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\big)]}$$

Straightforward computations show that $\tilde{\underline{\theta}}(\tau,\gamma,\chi)\in(0,1)$ for any $\gamma\in(0,1)$.

□

8.6. Proof of Proposition 4

Using the general expressions for the Trace and the Determinant, we obtain with JR preferences:

$$\mathcal{T} = \rho - \frac{(\rho+\delta)(1-s)\tau}{\sigma\tau - s - (1-\tau)\sigma\varepsilon_{cc}\left[\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right]}$$

and

$$\mathcal{D} = \frac{(\rho+\delta)(1-s)\varepsilon_{cc}\left[\frac{\gamma(1-\tau)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{1+\chi}[(\rho+\delta)(1-s)+s\rho]+[(1-\tau)(\rho+\delta)(1-s)+s\rho][\gamma(1-\tau)\mathcal{C}(\tau)+\chi-\tau(1+\chi)]\right]}{s\sigma\left[\sigma\tau - s - (1-\tau)\sigma\varepsilon_{cc}\left[\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right]\right]}$$

with

$$\varepsilon_{cc}\left[\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right] = \frac{\theta(1+\chi)^2\left[\chi+\gamma\mathcal{C}(\tau)\left(2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\right)\right]-\gamma\mathcal{C}(\tau)(\gamma+\chi)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{\theta(1+\chi)^2-\gamma(1-\gamma)\mathcal{C}(\tau)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}$$

Considering $\gamma = 0$ (GHH case), the trace and the determinant are given by:

$$\mathcal{T} = \rho - \frac{\tau(\rho+\delta)(1-s)}{\sigma\tau - s - (1-\tau)\sigma\chi} \tag{41}$$

$$\mathcal{D} = \frac{(\delta+\rho)(1-s)\varepsilon_{cc}[(1-\tau)(\rho+\delta)(1-s)+s\rho]}{\sigma s[\sigma\tau - s - \sigma(1-\tau)\chi]}\left[\chi - \tau(1+\chi)\right] \tag{42}$$

From (41), a necessary condition to obtain a negative trace is that the tax rate is sufficiently large such that $\tau > \underline{\tau}^0$ with:

$$\underline{\tau}^0 = \frac{\frac{s}{\sigma}+\chi}{(1+\chi)}$$

In equation (42), the condition $\tau > \underline{\tau}^0$ implies a positive denominator. The sign of the determinant is therefore determined by the second factor of (42), i.e. $\chi - \tau(1+\chi)$. This expression is positive if and only if $\tau < \bar{\tau}^0$ with:

$$\bar{\tau}^0 = \frac{\chi}{1+\chi}$$

which is lower than $\underline{\tau}^0$. Since $\bar{\tau}^0 < \underline{\tau}^0$, it is not possible to obtain simultaneously a negative trace and a positive determinant. Indeterminacy is therefore ruled out.

When we consider $\gamma = 1$ (KPR case), the trace and the determinant become:

$$\mathcal{T} = \rho - \frac{(\rho+\delta)(1-s)\tau}{\sigma G(\tau)}$$

and

$$\mathcal{D} = \frac{(\rho+\delta)(1-s)\varepsilon_{cc}}{s\sigma G(\tau)}P(\tau)$$

with

$$G(\tau) = \tau - \frac{s}{\sigma} - (1-\tau)\chi - (1-\tau)\mathcal{C}(\tau)(2 - \frac{1}{\theta})$$

and
$$P(\tau) = [(1-\tau)(\rho+\delta)(1-s)+s\rho]\left[\chi - \tau(1+\chi)\right]$$
$$+(1-\tau)^2(\rho+\delta)(1-s) + (1-\tau)[(\rho+\delta)(1-s)+s\rho]$$

Indeterminacy is obtained if and only if both $G(\tau)$ and $P(\tau)$ are positive.

In order to impose $G(\tau) > 0$, one can use Lemma 1. Yet, a lower bound $\underline{\tau}^1$ as given in (23) is not anymore explicit but is implicitly given by $\underline{\tau}^1 = h(\underline{\tau}^1)$ with:

$$h(\tau) = \frac{\frac{s}{\sigma} + \chi + (2-\frac{1}{\theta})\mathcal{C}(\tau)}{1+\chi+(2-\frac{1}{\theta})\mathcal{C}(\tau)} \tag{43}$$

One derives that $h(0) > 0$ while $h(1) = \frac{\frac{s}{\sigma}+\chi}{1+\chi} < 1$. There exists therefore $\underline{\tau}^1 \in (0,1)$ such that $G(\tau) > 0$ if and only if $\tau > \underline{\tau}^1$. Moreover, because of the polynomial form of $G(\tau)$, $\underline{\tau}^1$ is unique.

Considering the expression of the trace, we directly observe that when ρ tends to zero, the trace is negative if $\tau > \underline{\tau}^1$. There exists therefore $\bar{\rho}^1 > 0$ such that $\mathcal{T} < 0$ if and only if $\tau > \underline{\tau}^1$ and $\rho \in (0, \bar{\rho}^1)$.

For the determinant, $P(\tau)$ is positive when $\tau = 0$ and negative when $\tau = 1$. Since $P(\tau)$ is strictly decreasing in $\tau \in (0,1)$, there exists therefore a unique $\bar{\tau}^1 \in (0,1)$ such that $P(\tau) > 0$ if and only if $\tau < \bar{\tau}^1$. The determinant is therefore positive if and only if $\tau \in (\underline{\tau}^1, \bar{\tau}^1)$.

The condition $\underline{\tau}^1 < \bar{\tau}^1$ has still to be fulfilled and is satisfied if and only if $P(\underline{\tau}^1) > 0$. First, note that substituting equation $\theta = \underline{\theta}^1 \equiv \underline{\theta}(\tau, 1, \chi)$ in (43), one derives:

$$\underline{\tau}^1 = \frac{s}{\sigma}$$

When P(τ) is evaluated at $\tau = \frac{s}{\sigma}$, one obtains:

$$\begin{aligned} P(\tfrac{s}{\sigma}) &= [(1-\tfrac{s}{\sigma})(\rho+\delta)(1-s)+s\rho][\chi - \tfrac{s}{\sigma}(1+\chi)] + (1-\tfrac{s}{\sigma})^2(\rho+\delta)(1-s) \\ &+ (1-\tfrac{s}{\sigma})[(\rho+\delta)(1-s)+s\rho] \end{aligned}$$

On the one hand, if $\sigma = s$, we get $P(\frac{s}{\sigma}) = -s\rho < 0$. On the other hand, when σ tends to $+\infty$, $P(\frac{s}{\sigma})$ is positive. There exist therefore $\underline{\sigma}^1 \in (s, +\infty)$ such that $\underline{\tau}^1 < \bar{\tau}^1$ if and only if $\theta = \underline{\theta}^1$ and $\sigma > \underline{\sigma}^1$. By a continuity argument, there exists therefore $\bar{\theta}^1 \in (\underline{\theta}^1, +\infty]$ and $\underline{\sigma}^1 \in (s, +\infty)$ such that $\underline{\tau}^1 < \bar{\tau}^1$ if and only if $\theta \in [\underline{\theta}^1, \bar{\theta}^1)$ and $\sigma > \underline{\sigma}^1$.

Given that indeterminacy occurs when $\gamma = 1$, $\tau \in (\underline{\tau}^1, \bar{\tau}^1)$, $\theta \in [\underline{\theta}^1, \bar{\theta}^1)$, $\rho \in (0, \bar{\rho}^1)$ and $\sigma > \underline{\sigma}^1$ but is ruled out when $\gamma = 0$, there exists therefore $\underline{\gamma} \in (0,1)$ such that for any $\gamma \in (\underline{\gamma}, 1]$, there exists $\underline{\tau} \in (0,1)$, $\bar{\tau} \in (\underline{\tau}, 1)$, $\bar{\rho} \in (0, +\infty]$, $\bar{\theta} \in (\underline{\theta}, +\infty]$ and $\underline{\sigma} \in (s, +\infty)$ such that the NSS is locally indeterminate if and only if $\tau \in (\underline{\tau}, \bar{\tau})$, $\theta \in [\underline{\theta}, \bar{\theta})$, $\rho \in (0, \bar{\rho})$ and $\sigma > \underline{\sigma}$.

□

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