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Abstract

This paper shows how to recover behavioral biases from revealed preference ranking implied by choices. The approach formalizes and unifies known behavioral models, including salience thinking, inattention, and logarithmic perception, thereby accounting for many well-documented choice puzzles. I show that this approach provides a way to filter out choice data from behavioral biases explaining rationality breaches before fitting parametric utility models. The approach is applied to workhorse data sets of the literature on choice under risk and scanner consumer choices.
JEL D91, D11, D81

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1 Introduction

Behavioral biases are systematic patterns of deviation from rationality in judgment that fundamentally influence human decision-making. From the intricacies of financial markets to everyday consumer choices, behavioral biases subtly skew logic, often leading to outcomes that deviate from classical economic predictions. These biases, pivotal in the emergence and evolution of experimental economics, have been rigorously examined in hundreds of papers. Yet, the methodological approach to behavioral biases inherently limits the scope of analysis, focusing on recovering specific biases within specific contexts and

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failing to capture the broader, interconnected landscape of human behavior. As a result, the interplay and cumulative impact of these biases remain largely uncharted. Integrating behavioral biases within a unified framework appears as a crucial step toward a more complete and nuanced view of decision-making processes.

This paper proposes a novel approach toward this objective. The key premise of the paper is that since behavioral biases are deviations from rationality, a unified approach to recover behavioral biases is only limited by unified rationality measures. Since there exist several unified rationality measures, we can theoretically build a unified approach to behavioral biases. Concretely, consider a dataset $\{A^i, x^i\}_{i\in\mathcal{I}}$ where $x^i \in A^i$ is the element chosen from the set of alternatives $A^i \subset \mathbb{R}^K_+$ in observation i, with $\mathcal{I}$ the set of observations and $\mathbb{R}^K_+$ the consumption space. Datasets in the form of $\{A^i, x^i\}_{i\in\mathcal{I}}$ are commonly found across a wide spectrum of economic disciplines, including finance, consumer behavior, and choices under risk. My approach consists in arguing that if an agent was free of behavioral influences, the experimenter would observe the data $D_{\phi^{-1}} = \{B^i, \tilde{x}^i\}_{i\in\mathcal{I}}$ instead of $D_{id} = \{A^i, x^i\}_{i\in\mathcal{I}}$, with ϕ a transformation function that measures the effect of behavioral biases on decisions. Given that behavioral biases systematically distort decision-making without being rooted in preference, we should expect to see consistent patterns of rationality breach in the dataset D_{id}. This consistency provides a framework for inferring the transformation ϕ that maximizes a given rationality measure in that dataset.

I focus the main analysis on transformations ϕ that correspond to several broad categories of behavioral biases: inattention, salience thinking, and logarithmic perception. Inattention biases capture many themes in behavioral economics, and share a common structure: people anchor on a simple perception of the world and partially adjust toward it (Gabaix (2019)). Salience describes phenomena where a decision maker's attention is attracted by a salient stimuli, due to high contrast, surprising nature, or prominence (Bordalo, Gennaioli and Shleifer (2022)). Finally, logarithmic perception is a bias where decision-makers perceive quantities or prices in a logarithmic rather than linear manner. I show that it is possible to recover these biases in fairly general choice environments whose main characteristics prevent infinite consumption and allow for free disposal. Moreover, I demonstrate that the approach can be extended to recover these behavioral biases when they affect the perception of both prices and quantities. Finally, I show that the framework is flexible enough to recover sequential or mixed biases made of salience thinking, inattention, and logarithmic perception.

A well-known result of the revealed preferences literature is that no violation of rationality is a necessary and sufficient condition for the existence of a utility model rationalizing the decision-maker's choices. This result, known as Afriat's theorem, gives a sharp condition for fitting utility models to choice data.[1] However, experimenters often fit utility models to data where decision-makers are almost rational but still violate rationality. I show that by recovering behavioral biases, it is possible to filter out choice data from behavioral biases explaining rationality breaches before estimating a utility model. That way, utility estimation can fulfill the sharp condition of Afriat's theorem, potentially improving the quality of the utility estimation. With this approach, experimenters estimate the *augmented-utility models* $v \circ \phi : \mathbb{R}^K_+ \to \mathbb{R}$ rationalizing the observed data D_{id} rather than a utility model v directly, with ϕ measuring the influence of behavioral biases.

I apply the model to two workhorse datasets. First, I study the data from the portfolio choice experiment of Choi et al. (2014). Second, I study household-level scanner panel data, the Stanford Basket dataset, used, among others, by Echenique, Lee and Shum (2011), Shum (2004), and Hendel and Nevo (2006). The first key result of this analysis is that restricting the analysis to transformations that correspond to either inattention or salience explains more than 99% of rationality violations for all decision-makers across the two datasets. Moreover, for most decision-makers, it seems that salience thinking better explains rationality violations than inattention. These first results raise the critical question as to how easy it is to violate rationality once behavioral biases are filtered out from the data, using the methodology described above. Indeed, it is possible that by maximizing a given rationality measure in the dataset $D_{\phi^{-1}}$, the approach described above decreases the instances where rationality can be violated in that dataset relative to the observed dataset D_{id}. To answer this point, I use Bronars' test, measuring the probability that a decision-maker with a random behavior would violate the Generalized Axiom of Revealed Preferences (GARP). The Bronars' score does not differ much from the original dataset D_{id} to the datasets corrected from behavioral biases $D_{\phi^{-1}}$. This suggests that rationality is not artificially achieved in the datasets corrected by behavioral biases by decreasing instances where rationality can be violated.

The second result relates more specifically to risk preferences. First, I estimate a utility model and respondents' risk aversion using the original dataset of Choi et al. (2014). Correlations between risk aversion and sociodemographic variables reflect well-known reg-

[1] Various generalizations of this theorem exist. See, among others, Forges and Minelli (2009), Nishimura, Ok and Quah (2017), Halevy, Persitz and Zrill (2018), and Seror (2024).

ularities: females, older, and low-educated respondents are more risk-averse. Second, I filter salience and inattention biases from the original data and estimate the corresponding augmented utility model. I find that risk aversion is systematically overestimated when behavioral biases are not filtered out. Moreover, the correlations between risk aversion on the one hand, and gender and age on the other become much weaker. The main reason is that females and older respondents are, according to the analysis, significantly more affected by behavioral influences. Hence, these respondents might diverge less from the riskless choices, not because they are risk averse, but because they perceive a higher contrast between their choices and the riskless default. That behavioral tendency makes these respondents violate rationality in systematic ways, so risk aversion might not be a feature of their risk preferences but the outcome of a biased utility estimation.

The third result relates to the scanner data. I recover salience thinking and inattention relative to a default that corresponds to the consumer's memory of her purchasing history. Similar to the portfolio choice data, it seems that the majority of the respondents are subject to salience thinking rather than inattention. Additionally, decision-makers from larger households seem to be less subject to behavioral biases than decision-makers from smaller households. Older and more educated decision-makers have significantly longer memories when they assess their past purchasing history. Finally, I use the approach to recover consumers' left-digit bias. The estimation suggests that left-digit bias might not be sufficient to explain all consumers' rationality breaches.

This paper contributes to the literature in several ways. First, it contributes to the large literature on behavioral biases and decision-making. On the theoretical side, Bordalo, Gennaioli and Shleifer (2012, 2013*a*,*b*, 2020) proposed formal models of salience thinking, showing that these models offer a way to unify many behavioral biases, including the effect of contrast[2], surprise[3], and prominence[4] on choice inconsistencies. Similarly, Gabaix (2019) argues that much behavioral economics reflects a form of inattention.[5] This paper

[2] On contrast and decision-making, see, for example, Savage (1954), Kahneman and Tversky (1979), Tversky and Simonson (1993), Bodner and Prelec (1994), Sydnor (2010), Barseghyan et al. (2013), Chiappori et al. (2019), Lockwood et al. (2021)

[3] On surprise and decision-making: Thaler (1985), Tversky, Slovic and Kahneman (1990), Simonsohn and Loewenstein (2006), Bushong, Rabin and Schwartzstein (2021), Lian, Ma and Wang (2019).

[4] On prominence and decision-making: Loewenstein, O'Donoghue and Rabin (2003), Conlin, O'Donoghue and Vogelsang (2007), Kahneman and Tversky (1979), Dessaint and Matray (2017), Célérier and Vallée (2017).

[5] According to the review of Gabaix (2019) and DellaVigna (2009), there are five ways to measure inattention: deviating from an optimal action (Chetty, Looney and Kroft (2009*a*), Taubinsky and Rees-Jones (2019), Bronnenberg et al. (2015), deviations from normative cross-partials (Aguiar and Riabov (2018), Abaluck

contributes to these studies by giving a methodology to recover both salience thinking and inattention biases that can be applied to a large array of choice data. That way, any choice inconsistency classified as either salience thinking or inattention can be recovered with the same methodology, allowing to capture the broader, interconnected landscape of behavioral biases. Finally, the method is flexible enough to account for other forms of biases, including logarithmic perception, and sequential biases.

This paper also contributes to the literature on revealed preferences. More specifically, Varian (1990), and then Halevy, Persitz and Zrill (2018) use rationality indices as measures of misspecification of utility models rationalizing choice data. I complement these studies by showing that inconsistency measures can also be exploited to identify systematic patterns of rationality violations in choice data. These patterns can help experimenters recover behavioral biases in a unified manner. In the paper, I measure rationality using Afriat (1972) CCEI Index, Houtman and Maks (1985) Index, and Echenique, Lee and Shum (2011) Money Pump Index. These indices have been developed to study inconsistencies in the consumption choice environment. Using the formalism introduced by Forges and Minelli (2009), I show that all these indices can also be used in more general choice environments, whose main characteristics prevent infinite consumption, and allow for free disposal (Forges and Minelli (2009)).

Through the applications, the paper finally contributes to the large literature seeking to uncover systematic patterns of preference heterogeneity and decision-making quality.[6] I contribute to this literature by showing that preference heterogeneity might be critically confounded by behavioral bias heterogeneity. Concretely, estimating risk preference from the portfolio choice data of Choi et al. (2014) shows well-known regularities: women, older and high-educated decision-makers are more risk averse than men, low-educated, and younger decision-makers. Estimating the same utility model once salience thinking and inattention are recovered and filtered out from the data shows that all the previous

and Adams (2017)), physical measurement (Payne, Bettman and Johnson (1993), Gabaix et al. (2006), Reutskaja et al. (2011), Kahneman (1973)), surveys (De Bartolomé (1995), Liebman and Zeckhauser (2004)), and qualitative measures like impact of reminders, or advice (Huberman and Regev (2001), Karlan et al. (2016)), Hanna, Mullainathan and Schwartzstein (2014)). The semi-parametric approach of this paper leverages deviation from rational behavior to recover behavioral biases.

[6]There has been studies measuring variation in preferences across small-scale societies, Henrich et al. (2010, 2006, 2001), Apicella et al. (2014), or university students Rieger, Wang and Hens (2015), Talhelm et al. (2014), Vieider et al. (2015). Other studies have focused on the individual-level determinants of some aspects of social preferences Dohmen et al. (2010, 2011), Croson and Gneezy (2009), Gneezy, Leonard and List (2009), while others have focused on country-level differences in preferences Falk et al. (2018); Hofstede (1984, 2001); Norris and Inglehart (2019); Ronald and Norris (2003).

patterns are explained by choice inconsistencies. Women, older, and less-educated decision-makers are significantly more affected by salience thinking or inattention. Additionally, this paper augments a line of work initiated by Choi et al. (2014), which is concerned with explaining heterogeneity in the quality of decision-making.[7] I find that behavioral biases are significantly affected by gender, age, education, occupation, and household composition.

2 The Model

Let $\mathcal{I} = \{1, \ldots I\}$ denote the index set of observation, $\mathbf{p^i} = \{p_1^i, \ldots, p_K^i\}$ the ith observation of the prices of K goods, $\mathbf{x^i} = \{x_1^i, \ldots, x_K^i\}$ be the associated observations, with $\mathbf{p^i} \in \mathbb{R}_+^K$ and the consumption space is $X \subseteq \mathbb{R}_+^K$. In observation i, the feasible set is A^i. $A^0 = \{A^i\}_{i \in \mathcal{I}}$ is a collection of nonempty subsets of X and corresponds to the set of all feasible sets from which a decision-maker is observed to make a choice. Let $X^o = \{\mathbf{x^i}\}_{i \in \mathcal{I}} \subset X$ denote the set of observations, and $D = \{\mathbf{x^i}, A^i\}_{i \in \mathcal{I}}$ the data set. Finally, I denote $\mathcal{A}$ the set of subsets of X.

Definition 1 *I define a transformation of the data as a function ϕ from $\tilde{X}^0$ to X^0 such that for any observation i, $\mathbf{x}^i = \phi(\tilde{\mathbf{x}}^\mathbf{i})$, with $\tilde{X}^0 = \{\tilde{\mathbf{x}}^i\}_{i \in \mathcal{I}}$. The transformation ϕ is such that for any observation i, there exists a unique $\tilde{\mathbf{x}}^\mathbf{i} \in \mathbb{R}_+^K$ with $\tilde{\mathbf{x}}^\mathbf{i} = \phi^{-1}(\mathbf{x}^i)$.*

Was an agent free of behavioral biases, she would choose $\tilde{\mathbf{x}}^\mathbf{i}$ in observation i rather than $\mathbf{x^i}$. Concretely, if the agent was paying more attention at the supermarket, she might have chosen to purchase healthier and more expensive food items rather than unhealthy cheap snacks placed at eye level. It is possible to define the budget set of an unbiased decision-maker B^i as follows:

$$B^i = \{\mathbf{z} \in \mathbb{R}_+^K / \exists \mathbf{x} \in A^i \text{ and } \mathbf{z} = \phi^{-1}(\mathbf{x})\}.$$

Definition 1 focuses on the biased perception of the object of the decision problems faced by the decision-maker. Although this already covers a large array of behavioral influences, it is possible to generalize Definition 1 to account for behavioral biases affecting the perception of other aspects of the decision problem. I generalize the approach later on to account for behavioral biases affecting the perception of prices.

[7] See, among others, Andersson et al. (2016), Carvalho, Meier and Wang (2016), Enke (2020), and D'Acunto et al. (2022).

2.1 An Optimization Problem

Let $\mathbf{F}$ denote a set of transformations. For each transformation $g \in \mathbf{F}$, there exists a functional form ϕ and a vector of parameter β belonging to some vector space such that $g = \phi(.;\beta)$. Concretely, $\mathbf{F}$ might be a set of transformations corresponding to inattention biases, and β a parameter measuring the degree of inattention. I denote $e(D_{\phi^{-1}(.;\beta)}) \in [0,1]$ the index that measures rationality in data set $D_{\phi^{-1}(.;\beta)}$, with $\phi(.;\beta) \in \mathbf{F}$. In its most basic formulation, the recoverability of β can be expressed as:

$$\beta = \underset{\phi(.;\beta)\in\mathcal{F}}{\arg\max}\, e(D_{\phi^{-1}(.;\beta)}). \tag{1}$$

This approach leverages rationality violations to parametrically estimate the behavioral bias that best explains rationality violations within a family of biases $\mathbf{F}$. The feasibility of this optimization problem hinges on three key dimensions. First, in the dataset $D_{\phi^{-1}(;\beta)}$, the decision-maker must be able to violate rationality. This can be particularly limiting for scanner data that often lack the power to reject the revealed preference conditions (Blundell, Browning and Crawford (2003)). Second, there must exist rationality measures for the transformed dataset $D_{\phi^{-1}(;\beta)}$. Third, the set of transformations $\mathbf{F}$ must cover a large array of potential biases. One typical solution to the first point is to use expansion paths to improve the power of non-parametric tests (Blundell, Browning and Crawford (2003)). Hence, the first issue will not be discussed further in this paper. Below, I show how to address the second and third issues.

2.2 Inconsistency Indices

Consumption Choice Environment

This subsection introduces and discusses several known measures of rationality in the standard consumption choice environment. Uninterested readers can skip to the next subsection. In the classical consumption choice environment, the choice sets are linear:

Definition 2 *A choice set B^i is linear if there exists $\tilde{\mathbf{p}}^i \in \mathbb{R}_+^K$ and $R^i \in \mathbb{R}_+$ such that $B^i = \{\mathbf{z} \in \mathbb{R}_+^K$ such that $\mathbf{z}.\tilde{\mathbf{p}}^i \leq R^i\}$.*

When the transformed budget sets B^i, $i \in \mathcal{I}$ are linear, the revealed preference relations in the transformed data set can be defined as follows:

Definition 3 *Let* $\mathbf{e} \in [0,1]^I$. *Given a transformed set of observations* $\{\tilde{\mathbf{x}}^{\mathbf{i}}, B^i\}_{i\in\mathcal{I}}$, $\tilde{\mathbf{x}}^{\mathbf{i}}$ *is*

i $\mathbf{e}$*-directly revealed preferred to* $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, *denoted* $\tilde{\mathbf{x}}^{\mathbf{i}} R^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$, *if* $e^i \tilde{\mathbf{x}}^{\mathbf{i}}.\tilde{\mathbf{p}}^{\mathbf{i}} \geq \tilde{\mathbf{x}}^{\mathbf{j}}.\tilde{\mathbf{p}}^{\mathbf{i}}$.

ii $\mathbf{e}$*-strictly directly revealed preferred to* $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, *denoted* $\tilde{\mathbf{x}}^{\mathbf{i}} P^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$, *if* $e^i \tilde{\mathbf{x}}^{\mathbf{i}}.\tilde{\mathbf{p}}^{\mathbf{i}} > \tilde{\mathbf{x}}^{\mathbf{j}}.\tilde{\mathbf{p}}^{\mathbf{i}}$.

iii $\mathbf{e}$*-revealed preferred to* $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, *denoted* $\tilde{\mathbf{x}}^{\mathbf{i}} R_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$, *if* $\tilde{\mathbf{x}}^{\mathbf{i}} R^0_{\mathbf{e}} \mathbf{x}^{\mathbf{n}} R^0_{\mathbf{e}} \mathbf{x}^{\mathbf{m}} \ldots R^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$ *for some sequence of observations* $(\tilde{\mathbf{x}}^{\mathbf{i}}, \mathbf{x}^{\mathbf{n}}, \mathbf{x}^{\mathbf{m}}, \ldots, \tilde{\mathbf{x}}^{\mathbf{j}})$.

iv $\mathbf{e}$*-strictly revealed preferred to* $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, *denoted* $\tilde{\mathbf{x}}^{\mathbf{i}} P_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$, *if* $\tilde{\mathbf{x}}^{\mathbf{i}} R^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{n}} R^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{m}} \ldots R^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$ *for some sequence of observations* $(\tilde{\mathbf{x}}^{\mathbf{i}}, \tilde{\mathbf{x}}^{\mathbf{n}}, \tilde{\mathbf{x}}^{\mathbf{m}}, \ldots, \tilde{\mathbf{x}}^{\mathbf{j}})$ *and at least one of them is strict.*

When $\phi = id$ and $\mathbf{e} = \mathbf{1}$, Definition 3 reduces to the standard definition of revealed preference relation. When $\phi \neq id$, Definition 3 applies the standard direct revealed preference relations to a transformed set of observations. Definition 4 below gives the axiom of rational choice theory, applied to a transformed data set.

Definition 4 *Let* $\mathbf{e} \in [0,1]^I$. *For a transformation of the data* ϕ, *the transformed set of observations* $D_{\phi^{-1}(.;\beta)}$ *satisfies the general axiom of revealed preference (GARP*$_{\mathbf{e}}$*) if for every pair of observed bundles,* $\tilde{\mathbf{x}}^{\mathbf{i}} R_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{j}}$ *implies not* $\tilde{\mathbf{x}}^{\mathbf{j}} P^0_{\mathbf{e}} \tilde{\mathbf{x}}^{\mathbf{i}}$.

When $\mathbf{e} < \mathbf{1}$, violations of GARP in the transformed data will not necessarily lead to violations of GARP$_{\mathbf{e}}$. Using the previous formalism, Afriat (1972) and Houtman and Maks (1985) inconsistency indices can be defined as follows:

- Afriat (1972) inconsistency index is

$$\mathbf{e}_A(D_{\phi^{-1}(.;\beta)}) = \inf_{\mathbf{e}\in\{\mathbf{v}\in[0,1]^I:\mathbf{v}=v\mathbf{1}\}, D_{\phi^{-1}(.;\beta)} \text{ satisfies } GARP_{\mathbf{e}}} 1 - e \tag{2}$$

- Houtman and Maks (1985) inconsistency index is

$$\mathbf{e}_{HM}(D_{\phi^{-1}(.;\beta)}) = \inf_{\mathbf{e}\in\{0,1\}^I, D_{\phi^{-1}(.;\beta)} \text{ satisfies } GARP_{\mathbf{e}}} I - \sum_{i\in\mathcal{I}} e^i \tag{3}$$

Afriat (1972) and Houtman and Maks (1985) are the most prevalent inconsistency indices in experimental and empirical studies. Afriat's inconsistency index measures the extent of utility-maximizing behavior in the data. The main idea behind this index is that if expenditures at each observation are sufficiently "deflated", then violations of GARP will disappear. Houtman and Maks's index is based on the maximal subset of observations that satisfies GARP.[8]

Echenique, Lee and Shum (2011) introduced the "Money Pump Index" that measures the magnitude of GARP violations. Concretely, let a sequence $\{\tilde{x}^{k_1}, \dots, \tilde{x}^{k_n}\} \in V(D_{\phi^{-1}(.;\beta)})$ define a violation of GARP, with $V(D_{\phi^{-1}(.;\beta)})$ the set of GARP violations in the transformed data set. The *worst MPI* can be defined as follows:

$$e_{MPI}(D_{\phi^{-1}(.;\beta)}) = 1 - \max_{\{\tilde{x}^{k_1},\dots,\tilde{x}^{k_n}\} \in V(D_{\phi^{-1}(.;\beta)})} \frac{\sum_{l=1}^{n} p^{k_l}.(\tilde{x}^{k_l} - \tilde{x}^{k_{l+1}})}{\sum_{l=1}^{n} p^{k_l}.\tilde{x}^{k_l}}. \tag{4}$$

The worst MPI index measures the maximum amount of money that one can obtain from a consumer who violates GARP. It equals one when there is no GARP violation in the transformed data, similar to the two previous indices.

More general choice environments

I now consider compact and comprehensive choice sets, meaning that if $\mathbf{y} \in B^i$, then $\mathbf{z} \in B^i$ for any $\mathbf{z} \in \mathbb{R}_+^K$. This is a rather large class of budget sets, as its main characteristics prevent infinite consumption, and allow for free disposal (Forges and Minelli (2009)). As demonstrated by Forges and Minelli (2009), if a choice set B^i is compact and comprehensive, it is possible to characterize it in the form $B^i = \{\mathbf{x} \in \mathbb{R}_+^K : g^i(\mathbf{x}) \leq 0\}$, with $g^i : \mathbb{R}_+^K \to \mathbb{R}$ an increasing, continuous function, and $g^i(\tilde{x}^i) = 0$ for all $i \in \mathcal{I}$.

Consider a vector $\mathbf{e} \in (0,1]^I$, and the function $g^i(.;e^i)$ such that $g^i(x;e^i) = g^i(x/e^i)$ for any $x \in X$. Function $g^i(.;e^i)$ represents a transformation of the original function $g^i(.)$, preserving its shape and properties. It is possible to generalize Definition 3 as follows:

Definition 5 *Let $\mathbf{e} \in [0,1]^I$. Given a transformed set of observations $\{\tilde{\mathbf{x}}^{\mathbf{i}}, B^i\}_{i\in\mathcal{I}}$ with B^i compact and comprehensive for all $i \in \mathcal{I}$, $\tilde{\mathbf{x}}^{\mathbf{i}}$ is*

i $\mathbf{e}$*-directly revealed preferred to $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, denoted $\tilde{\mathbf{x}}^{\mathbf{i}} R^0 \tilde{\mathbf{x}}^{\mathbf{j}}$, if $0 \geq g^i(\tilde{x}^j; e^i)$.*

ii $\mathbf{e}$*-strictly directly revealed preferred to $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, denoted $\tilde{\mathbf{x}}^{\mathbf{i}} P^0 \tilde{\mathbf{x}}^{\mathbf{j}}$, if $0 > g^i(\tilde{x}^j; e^i)$.*

iii $\mathbf{e}$*-revealed preferred to $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$, denoted $\tilde{\mathbf{x}}^{\mathbf{i}} R \tilde{\mathbf{x}}^{\mathbf{j}}$, if $\tilde{\mathbf{x}}^{\mathbf{i}} R^0 \mathbf{x}^{\mathbf{n}} R^0 \mathbf{x}^{\mathbf{m}} \dots R^0 \tilde{\mathbf{x}}^{\mathbf{j}}$ for some sequence of observations $(\tilde{\mathbf{x}}^{\mathbf{i}}, \mathbf{x}^{\mathbf{n}}, \mathbf{x}^{\mathbf{m}}, \dots, \tilde{\mathbf{x}}^{\mathbf{j}})$.*

[8] Other indices exist, such as Varian (1990) inconsistency index, and the minimum cost inconsistency index (Dean and Martin (2016)).

iv $\mathbf{e}$*-revealed strictly preferred to* $\tilde{\mathbf{x}}^{\mathbf{j}} \in X$*, denoted* $\tilde{\mathbf{x}}^{\mathbf{i}} P \tilde{\mathbf{x}}^{\mathbf{j}}$*, if* $\tilde{\mathbf{x}}^{\mathbf{i}} R^0 \tilde{\mathbf{x}}^{\mathbf{n}} R^0 \tilde{\mathbf{x}}^{\mathbf{m}} \dots R^0 \tilde{\mathbf{x}}^{\mathbf{j}}$ *for some sequence of observations* $(\tilde{\mathbf{x}}^{\mathbf{i}}, \tilde{\mathbf{x}}^{\mathbf{n}}, \tilde{\mathbf{x}}^{\mathbf{m}}, \dots, \tilde{\mathbf{x}}^{\mathbf{j}})$ *and at least one of them is strict.*

The consumption choice environment corresponds to function $g^i(\mathbf{x}) = \mathbf{p^i}.(\mathbf{x} - \tilde{\mathbf{x}}^i)$. In this environment, function $g^i(.; e^i)$ is $g^i(\mathbf{x}) = \mathbf{p^i}.(\mathbf{x}/e^i - \tilde{\mathbf{x}}^i)$, so $0 \geq g^i(\tilde{\mathbf{x}}^j; e^i)$ is verified if and only if $e^i \tilde{\mathbf{x}}^{\mathbf{i}}.\tilde{\mathbf{p}}^{\mathbf{i}} \geq \tilde{\mathbf{x}}^{\mathbf{j}}.\tilde{\mathbf{p}}^{\mathbf{i}}$. In words, Definition 5 reduces to Definition 3 when $g^i(\mathbf{x}) = \mathbf{p^i}.(\mathbf{x} - \tilde{\mathbf{x}}^i)$.

Given Definition 5 and the characterization of $g^i(.; e^i)$, it is possible to define GARP$_{\mathbf{e}}$, Afriat (1972), and Houtman and Maks (1985) exactly like in the standard consumption choice environment. The worst MPI index writes

$$e_{MPI}(D_{\phi^{-1}(.;\beta)}) = 1 - \max_{\{\tilde{x}^{k_1}, \dots, \tilde{x}^{k_n}\} \in V(D_{\phi^{-1}(.;\beta)})} \frac{\sum_{l=1}^{n} g^{k_l}(\tilde{x}^{k_{l+1}})}{\sum_{l=1}^{n} g^{k_l}(0)}. \tag{5}$$

With the function g^i that corresponds to the consumption choice environment, $g^i(\mathbf{x}) = \mathbf{p^i}.(\mathbf{x} - \tilde{\mathbf{x}}^i)$, one finds that (4) and (5) are equivalent.

3 Behavioral Biases

In this section, I introduce a set of transformations to recover behavioral biases. Throughout the section, the sets are built with two main objectives. First, the transformations must depend on a few parameters with clear behavioral interpretations. Second, the sets must cover a large array of possible biases.

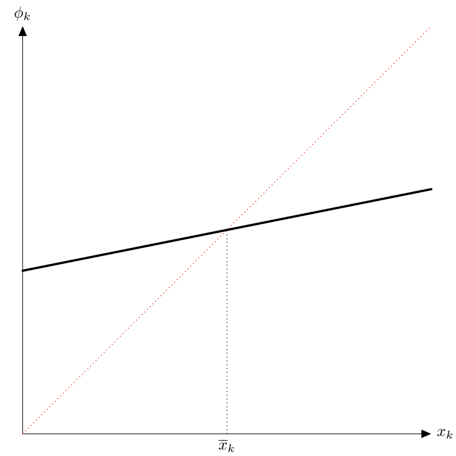
Salience thinking and Inattention

I propose the following transformation $\phi(.; \mathbf{m}, \overline{\mathbf{x}}) : \tilde{X}^0 \to X^0$ where $\mathbf{m} \in \mathbb{R}^K_+$ is a vector of parameters, $\overline{\mathbf{x}} \in X$ a reference consumption vector, and $\phi(\tilde{\mathbf{x}}^{\mathbf{i}}; \mathbf{m}, \overline{\mathbf{x}}) = (\{\phi_k(\tilde{x}^i_k; m_k, \overline{\mathbf{x}})\}_{k \in \mathcal{K}})$ with

$$\phi_k(\tilde{x}^i_k; m_k, \overline{x}_k) = \max(m_k \tilde{x}^i_k + (1 - m_k)\overline{x}_k, 0). \tag{6}$$

Depending on its sign, parameter $m_k \in (0, 1]$ can either be interpreted as inattention or salience-thinking relative to the default $\overline{x}_k$. When $m_k = 1$, there is no distortion as $\phi_k(x^i_k) = x^i_k$. When $0 < m_k < 1$, the agent does not pay full attention and partially perceives attribute k. In the limit case where m_k tends to zero, the agent "does not think

Figure 1: Transformation ϕ_k, with $0 < m_k < 1$.



about attribute k", and replaces x_k^i with the default $\overline{x}_k^i$. When $m_k > 1$, the agent is a salience thinker, as the agent perceives an amplified contrast between x_k^i and the reference level $\overline{x}_k$.[9] Below, I discuss inattention and salience thinking in the context of the model.

Inattention. Inattention biases share a common structure: people anchor on a simple perception of the world and partially adjust toward it (Gabaix (2019)). As argued by Gabaix (2019), this simple form captures many themes in behavioral economics, from limited attention to inattention to the true probability, or left-digit bias. Attention may be allocated to only a subset of attributes or goods.[10] Building on the inattention models of Gabaix and Laibson (2006), Chetty, Looney and Kroft (2009*b*), DellaVigna (2009), Gabaix (2014) or Gabaix (2019).

Figure 1 represents the transformation ϕ_k characterized in equation (6) when $0 < m_k < 1$ and $\overline{x}_k^i > 0$. The decision-maker perceives that attribute x_k is closer to the default option than it is. Figure 2 represents the transformed budget set B^i when $K = 2$, $\overline{\mathbf{x}} = \mathbf{0}$, and $m_2 < m_1 < 1$. The transformed budget set B^i is linear,[11] as by definition of A^i, $\tilde{x} \in B^i$ if

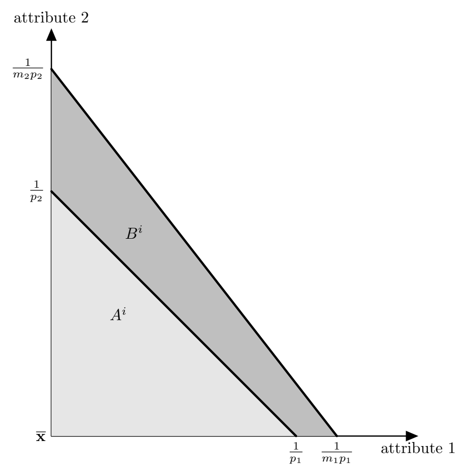
$$\tilde{\mathbf{p}}^{\mathbf{i}}.\tilde{\mathbf{x}} \leq 1 - (\mathbf{p}^{\mathbf{i}} - \tilde{\mathbf{p}}^{\mathbf{i}}).\overline{\mathbf{x}}^i,$$

[9] According to Bordalo, Gennaioli and Shleifer (2012, 2013*a*, 2020), a second property is required for salience thinking: changes in stimuli need to be perceived with diminishing sensitivity. This second property is not verified with a linear transformation characterized in (6), but can follow from the combined effect of a logarithmic perception, as modeled next, and the transformation (6).

[10] Sims (2003), Woodford (2012), Gabaix (2014), Woodford (2020), Khaw, Li and Woodford (2020) provide parametric models of rational inattention and sparsity in decision-making. Gabaix (2019) provides an overview of the behavioral economics literature on inattention.

[11] The reader might have noticed that when $(\overline{x}_1, \overline{x}_2) = (0, 0)$, all elements of A^i have an antecedent in B^i.

Figure 2: Transformed budget sets when $\overline{\mathbf{x}} = (0, 0)$.



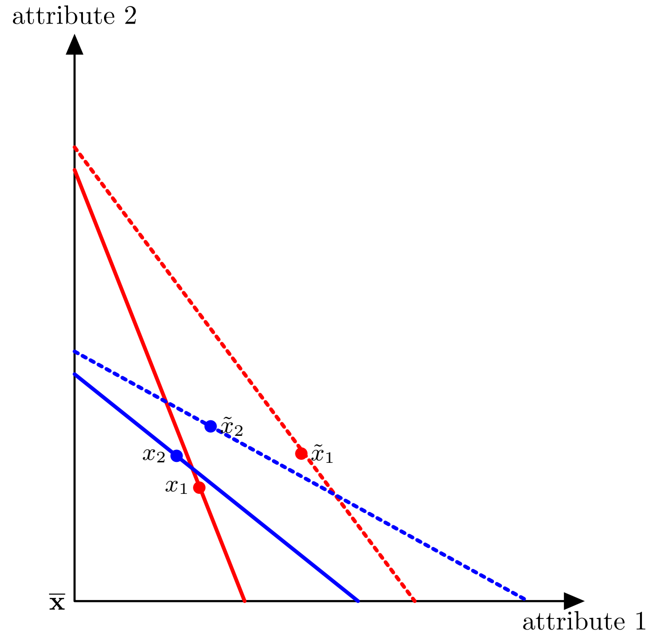
with $\tilde{\mathbf{p}}^{\mathbf{i}} = \mathbf{m} \circ \mathbf{p}^i$ the element-wise product of the vectors $\mathbf{p}^i$ and $\mathbf{m}$. Comparing the budget set of a biased decision-maker A^i with the budget set of an unbiased decision-maker B^i, it is as if a biased decision-maker was facing inflated prices. As a result, she spends less than she would if she was unbiased.

Inattention and GARP violation. To illustrate how inattention can systematically lead to GARP violations, consider the example of a decision-maker that makes two decisions, $D = \{(\mathbf{x^1}, A^1), (\mathbf{x^2}, A^2)\}$. Suppose also that $\mathbf{x^1}$ is better (resp. worse) than $\mathbf{x^2}$ on attribute 1 (resp. 2). In observation 1, the agent chooses $\mathbf{x^1}$ when $\mathbf{x^1}$ and $\mathbf{x^2}$ are available. In observation 2, the agent chooses $\mathbf{x^2}$ although both $\mathbf{x^1}$ and $\mathbf{x^2}$ are available. This is a textbook violation of GARP.

Consider now a canonical transformation ϕ where the decision-maker is inattentive to attribute 1 ($m_1 < m_2 = 1$), and her default option is the average consumption vector in each choice set. In both observations, the decision-maker spends too little on attribute 1. Correcting for this, the GARP violation can be eliminated, as represented in Figure 3 below.

Salience thinking. Salience describes phenomena where a decision maker's attention is involuntarily attracted by a salient stimulus. For example, in the context of consumer demand, a large amount of evidence suggests that choices are context-dependent, as a consumer's attention is drawn to salient features of what they consume. As a simple illustration, experimental subjects thinking of buying a calculator for \$15 and a jacket for \$125 are more likely to agree to travel for 10 minutes to save \$5 on the calculator than to travel 10 minutes to save \$5 on the jacket (Kahneman and Tversky (1984), Kahneman

Figure 3: GARP violation and Inattention.



(2011)). The salience theory has been developed in a series of papers by Bordalo, Gennaioli and Shleifer (2012, 2013a,b, 2022), and offers a way to unify many choice instabilities.

Figure 4 represents the transformation induced by salience thinking, and Figure 5 represents the budget set B^i when $K = 2$ and $1 < m_1 < m_2$. Here, it is as if a salient thinker was perceiving that the two attributes are cheaper than they are. As a result, a salient thinker with $\overline{x} = (0, 0)$ spends more than she would if she was not biased.

Example: Salience thinking and GARP violation. As for inattention, it can be useful to illustrate how salience can lead to systematic violations of GARP. Consider again the example of the previous decision-maker violating GARP in the dataset $D = \{(\mathbf{x^1}, A^1), (\mathbf{x^2}, A^2)\}$. Consider now that the decision-maker is a salient thinker, especially relative to attribute 1 ($m_1 > m_2 > 1$), and her default option is the average consumption vector in each choice set. The decision-maker spent too much on attribute 1, and this is especially true in observation 1 where $\mathbf{x^1}$ is far from the average consumption vector. Correcting for salience-thinking removes the textbook GARP violation, as represented in Figure 6.

Inattention nudges perception closer to a default. Salience amplifies the perceived differences between choices and a default. Attentive readers might have noticed that in this model, correcting a decision process from salience thinking is equivalent at making the

Figure 4: Transformation ϕ_k, with $m_k > 1$ and $\overline{x}_k > 0$ (black line); 45° line (red line).

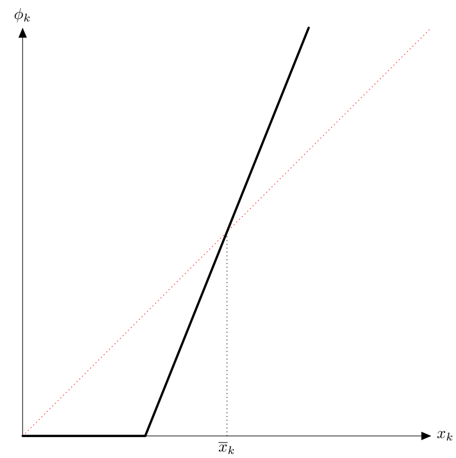


Figure 5: Transformed budget sets when $\overline{\mathbf{x}} = (0, 0)$ and $1 < m_1 < m_2$.

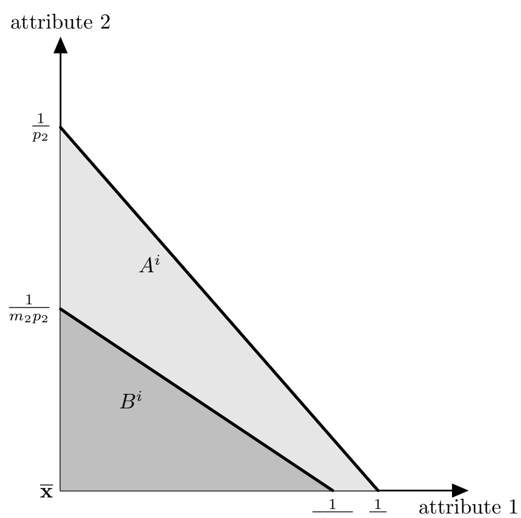
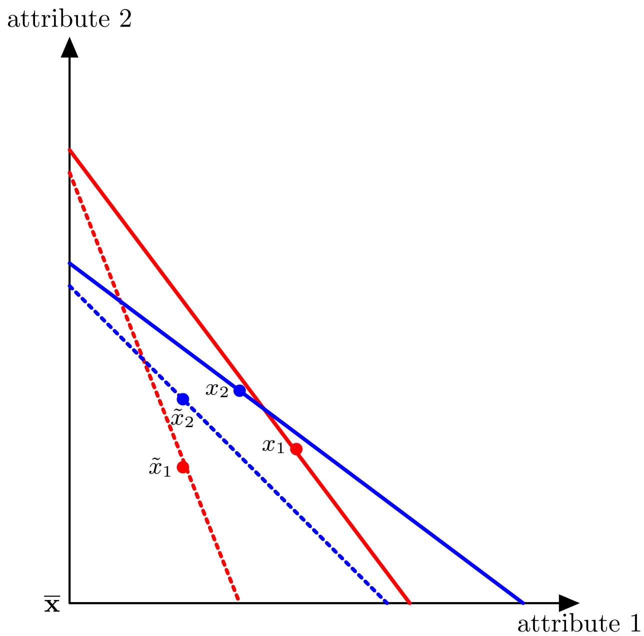


Figure 6: GARP violation and salience thinking.



decision-maker pay less attention to her decision! Here is a metaphor to understand this point. Myopia is a condition that makes the eyes focus light in front of, not on the retina. Glasses correct this by diverging the light before it enters the eyes. This adjustment ensures that light focuses directly on the retina. Salience is like myopia, and inattention is like glasses. To correct from salience thinking, the decision-maker needs to pay less attention to the contrast between the attributes and their reference level.

Finally, it is possible to recover the behavioral bias with the functional form (6) that explains GARP violations by running the optimization problem (1) for the following set of transformations:

$$\mathcal{F}(\mathbf{m}, \overline{\mathbf{x}}) = \{\phi(.; \mathbf{m}, \overline{\mathbf{x}}) : \tilde{X}^0 \to X^0 \text{ with } \mathbf{m}, \overline{\mathbf{x}} \in \mathbb{R}^K_+, \text{ and } \phi \text{ characterized in (6)}\}.$$

Since the transformations in $\mathcal{F}$ keep the budget sets linear, the optimization (1) can be based on the CCEI, the Houtman and Maks, or the MPI indices. Moreover, since both inattention and salience thinking biases belong to $\mathcal{F}(\mathbf{m}, \overline{\mathbf{x}})$, the optimization (1) necessarily horse-race these two types of biases. Additionally, it is possible that given the structure of the data at hand, there is an easy candidate for the reference level $\overline{\mathbf{x}}$, in which case only the vector $\mathbf{m}$ needs to be estimated. Finally, it is straightforward to build sequential biases by composing transformations like (6).

Logarithmic perception

Logarithmic perception is a principle from cognitive psychology that states people often perceive various quantities in a logarithmic rather than a linear manner. Logarithmic perception is found in human perception of numbers, time estimation, prices, or quantities. To capture logarithmic perception, I propose the following transformation, $\phi(.;\mathbf{l}) : \tilde{X}^0 \rightarrow X^0$ where $\mathbf{l} \in \mathbb{R}^K_+$ is a vector that measures the strength of the logarithmic perception for the K attributes, and $\phi(\tilde{\mathbf{x}}^{\mathbf{i}};\mathbf{l}) = (\{\phi_k(\tilde{x}^i_k; l_k)\}_{k\in\mathcal{K}})$ with

$$\phi_k(\tilde{x}^i_k; l_k) = \frac{1}{l_k}\ln(1 + l_k\tilde{x}^i_k) \tag{7}$$

and the choice set B^i can be characterized as:

$$B^i = \{\mathbf{z} \in \mathbb{R}^K_+ / \exists \mathbf{x} \in A^i \text{ such that } \phi_k^{-1}(z_k; l_k) = x_k, \forall k \in \mathcal{K}\}. \tag{8}$$

As ϕ_k is monotonic in $\tilde{x}^i_k$, ϕ_k^{-1} exists. Parameter $l_k \geq 0$ captures the strength of the logarithmic perception for attribute k. When $l_k \rightarrow 0$, there is no logarithmic distortion as $\phi_k(x^i_k) \approx x^i_k$. A higher value of l_k means the function ϕ_k becomes relatively flat more quickly, implying a lower sensitivity to changes in quantities. The budget sets B^i are not linear but they are compact and comprehensive. To recover the behavioral bias with the functional form (7), it is possible to run the optimization problem (1) for the following set of transformations:

$$\mathcal{F}(\mathbf{l}) = \{\phi(.;\mathbf{l}) : \tilde{X}^0 \rightarrow X^0 \text{ with } \phi(.;\mathbf{l}) \text{ characterized in (7)}\}.$$

Left-digit bias

The approach has developed so far how to recover behavioral biases affecting decision-makers' perception of quantities. It can equally be applied to biases affecting the perception of prices. Here, I detail how the approach can be extended to recover left-digit bias, a well-studied behavioral inconsistency affecting the perception of prices.[12]

[12]On left-digit bias, see, among others, Busse et al. (2013), Lacetera, Pope and Sydnor (2012), DellaVigna and Gentzkow (2017), and Strulov-Shlain (2023).

While logarithmic perception captures the general cognitive bias towards perceiving quantities in a non-linear manner, another specific and pervasive cognitive bias in numerical cognition is the left-digit bias. Left-digit bias refers to the disproportionate impact that the left-most digit of a number has on people's perception of numerical values. This bias is most commonly observed in consumer behavior, where prices just below a round number (like \$2.99 instead of \$3.00) are perceived as being significantly cheaper, even though the difference is minimal. Such bias can significantly influence decision-making processes in economic environments.

To capture left-digit bias, I generalize Definition 1 below to account for a biased perception of prices.

Definition 6 *I define a price transformation as a function γ from $\mathbb{R}_+^K$ to $\mathbb{R}_+^K$ such that for any observation i, $\gamma(\mathbf{p^i}) = \tilde{\mathbf{p}}^{\mathbf{i}}$.*

Was an agent free of behavioral influences, she would perceive the price as they are disclosed, $\mathbf{p^i}$. Instead, she perceives $\gamma(\mathbf{p^i})$. Following Strulov-Shlain (2023), one way to capture left-digit bias is to characterize $\gamma(.;\Delta,\theta)$ as follows:

$$\gamma(\mathbf{p^i};\Delta,\theta) = \theta\mathbf{p^i} + (1-\theta)(\lfloor \mathbf{p^i} \rfloor + \Delta), \tag{9}$$

where $\lfloor . \rfloor$ is the floor operator. When $\theta = 1$, the decision-maker correctly perceives prices. When $\theta < 1$, it is as if the agent was not paying full attention. She partially replaces prices with a default $\lfloor \mathbf{p^i} \rfloor + \Delta$. Here, $\Delta \in [0,1)$ corresponds to the decision-maker's focal ending. Concretely, if $p = 4.99$, $\theta = 0.2$ and $\Delta = 0$ for example, instead of perceiving $p = 4.99$, the decision-maker perceives a price closer to 4, $\gamma(p) = 4.20$. It is possible to consider more general functional forms for the left-digit bias transformation, accounting for different levels of bias, or a good-specific transformation similar to transformation (6).

Similar to the previous analysis, it is possible to define the perceived data $D_{\gamma(.;\Delta,\theta)}$. Assuming that the decision-maker is rational given how she perceives prices, it is possible to recover parameters Δ and θ through the following optimization:

$$(\Delta,\theta) = \underset{\gamma(.;\Delta,\theta)\in\mathcal{F}_{LD}(\Delta,\theta)}{\arg\max}\ e(D_{\gamma(.;\Delta,\theta)}), \tag{10}$$

with

$$\mathcal{F}_{LD}(\Delta,\theta) = \{\gamma(.;\Delta,\theta) : \mathbb{R}_+^K \to \mathbb{R}_+^K \text{ with } \Delta \in [0,1), \theta \in [0,1] \text{ and } \gamma \text{ characterized in (9)}\}.$$

Optimizations (24) and (1) work similarly. Here, it is the discrepancy between prices' perceived values and prices' real values that leads to systematic violations of rationality.

Sequential Behavioral Biases

In employing the methodology of this paper, we can discern and reconstruct complex behavioral biases. These biases are often an amalgamation of various factors affecting the perception of both prices and quantities. For instance, a decision-maker could be influenced by salience thinking while simultaneously exhibiting a logarithmic perception of prices, compounded by a left-digit bias. Although these biases may appear intricate, they can be represented as a composite function of different transformations, as outlined in the preceding section. This process is somewhat analogous to the decomposition of waves in physics. No matter how complex a wave might be, it can be broken down into a series of simpler, fundamental waves, each carrying different amplitudes and frequencies. This section aims to illuminate the feasibility of deconstructing complex behavioral inconsistencies by leveraging the behavioral dimensions identified earlier.

Instead of developing a full-fledged model, I illustrate sequential biases by considering the case of two behavioral biases sequentially affecting the decision-maker. The general case easily follows from the analysis. A transformation $\phi(.;\mathbf{m_1},\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{1},\overline{\mathbf{x}}_\mathbf{2})$ characterize a sequential bias, with $\phi(\tilde{\mathbf{x}}^\mathbf{i};\mathbf{m_1},\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{1},\overline{\mathbf{x}}_\mathbf{2}) = (\{\phi_k(\tilde{x}_k^i;\mathbf{m_1},\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{1},\overline{\mathbf{x}}_\mathbf{2})\}_{k\in\mathcal{K}})$ and

$$\phi_k(.;m_{1,k},m_{2,k},\overline{x}_{1,k},\overline{x}_{2,k}) = \phi_k(.;m_2,\overline{x}_{2,k}) \circ \phi_k(.;m_1,\overline{x}_{1,k}), \tag{11}$$

and both $\phi_k(.;m_2,\overline{x}_{2,k})$ and $\phi_k(.;m_1,\overline{x}_{1,k})$ are characterized by (6).

Here, the decision-maker is first subject to the behavioral bias of parameters $(\mathbf{m_1},\overline{\mathbf{x}}_\mathbf{1})$, and then to the behavioral bias of parameters $(\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{2})$. Concretely, provided that a decision-maker pays attention (first bias), she might be subject to salience thinking (second bias). To recover the behavioral bias with the functional form (11), it is possible to run the optimization problem (1) for the following set of transformations:

$$\mathcal{F}(\mathbf{m_1},\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{1},\overline{\mathbf{x}}_\mathbf{2}) = \{\phi(.;\mathbf{m_1},\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{1},\overline{\mathbf{x}}_\mathbf{2}) : \tilde{X}^0 \to X^0 \text{ with } \phi(.;\mathbf{m_1},\mathbf{m_2},\overline{\mathbf{x}}_\mathbf{1},\overline{\mathbf{x}}_\mathbf{2}) \text{ characterized in (11)}\}.$$

One key feature of the sequential transformation (11) is that it nests two simpler behavioral models: one where the decision-maker is only affected by a behavioral bias of parameters $(\mathbf{m_1}, \overline{\mathbf{x}}_\mathbf{1})$, and one where the decision-maker is only affected by a behavioral bias of parameters $(\mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{2})$, as $\mathcal{F}(\mathbf{m_1}, \overline{\mathbf{x}}_\mathbf{1}) \subset \mathcal{F}(\mathbf{m_1}, \mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{1}, \overline{\mathbf{x}}_\mathbf{2})$ and $\mathcal{F}(\mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{2}) \subset \mathcal{F}(\mathbf{m_1}, \mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{1}, \overline{\mathbf{x}}_\mathbf{2})$. Hence, experimenters can run optimization (1) over the set of sequential biases to assess whether simpler biases keep their explanation power when larger sets of transformations are considered. That way, the robustness of simple models can be assessed.

To conclude this section, a large array of behavioral biases can be recovered using simple transformations that depend on a few parameters with clear behavioral interpretations. I suggested several types of transformations, covering inattention and salience thinking, logarithmic perception, and left-digit bias. Building on these canonical transformations, it is possible to build more complex transformations by composing different transformation functions. I discuss more specifically sequential biases where the decision-maker is affected by both inattention and salience thinking but it is straightforward to compose other types of transformations. While these complicated cases are theoretically possible - and potentially useful to assess the robustness of simple biases - whether they actually explain decision-making is an empirical question. The partial answer to this question, from the application section, is that inattention and salience thinking are already sufficient to explain most if not all rationality violations.

4 Augmented-utility model

A well-known result of the revealed preferences literature is that no violation of rationality is a necessary and sufficient condition for the existence of a utility model rationalizing the decision-maker's choices. This result, known as Afriat's theorem, gives a sharp condition for fitting utility models to choice data: the absence of GARP violations. However, experimenters often fit utility models to data where decision-makers are almost rational but still violate rationality. Using the approach of this paper, I show in this section that it is possible to filter out choice data from behavioral biases before estimating a utility model.

The function $\Phi^{-1} : X \to \tilde{X}$ with $\tilde{X} \subset X$ is said to extend a function $\phi^{-1} : X^0 \to \tilde{X}^0$ if the two functions agree on X^0, i.e., $\Phi^{-1}(x) = \phi^{-1}(x)$ for any $x \in X^0$. Finally, I introduce the set $\phi^{-1}(X) = \{\tilde{x} \in X / \exists x \in X \text{ such that } \Phi^{-1}(x) = \tilde{x}\}$. The following definition of rationalizability relates the revealed preference information implied by observed choices to the ranking induced by a utility function on the transformed set of observations:

Definition 7 *A utility function v monotonically rationalizes the data $D_{\phi^{-1}}$ if for every $\tilde{\mathbf{x}}^{\mathbf{i}} \in D_{\phi^{-1}}$, $\tilde{\mathbf{x}}^{\mathbf{i}} R \tilde{\mathbf{x}}$ implies that $v(\tilde{\mathbf{x}}^{\mathbf{i}}) \geq v(\tilde{\mathbf{x}})$. $D_{\phi^{-1}}$ is rationalizable if such v exists.*

Definition 7 applied to $\phi = id$ is the standard definition. Similarly, Afriat's (1967) seminal theorem can be written as follows:

Theorem 1 *If the transformation ϕ is such that B^i is linear for all $i \in \mathcal{I}$, then the following statements are equivalent*

- *$D_{\phi^{-1}}$ satisfies GARP.*
- *The data set $D_{\phi^{-1}}$ has a continuous, concave, and strictly monotonic rationalization $v : \phi^{-1}(X) \to \mathbb{R}$.*
- *There are strictly positive real numbers U^k and λ^k such that*

$$U^k \leq U^l + \lambda^l \mathbf{p}^{\mathbf{l}}.(\phi^{-1}(\mathbf{x}^{\mathbf{k}}) - \phi^{-1}(\mathbf{x}^{\mathbf{l}})) \tag{12}$$

for each pair of observations $(\mathbf{x}^{\mathbf{k}}, A^k)$ and $(\mathbf{x}^{\mathbf{l}}, A^l)$ in D.

Restricting to transformations that keep the budget sets linear makes it possible to apply Afriat's theorem to the transformed data set $D_{\phi^{-1}}$. Forges and Minelli (2009) generalizes Afriat's theorem for compact and comprehensive sets. In this more general case, the convexity of the rationalizing utility function is not always guaranteed but the Theorem is essentially similar (see the Appendix).[13] The following Corollary is direct from Theorem 1 and Definition 7:

Corollary 1 *If a transformation ϕ is such that B^i is linear for all $i \in \mathcal{I}$ and one of the conditions of Theorem 1 is satisfied, then for any $\mathbf{x}^{\mathbf{i}} \in X^0$, $\mathbf{x}^{\mathbf{i}} = \arg\max_{\mathbf{x} \in Q^i} v \circ \Phi^{-1}(\mathbf{x})$, with $Q^i = \{\mathbf{x} \in X / \exists \mathbf{z} \in B^i$ and $\mathbf{z} = \Phi^{-1}(\mathbf{x})\}$.*

Proof. As v is a rationalization of $D_{\phi^{-1}}$ from Theorem 1,

$$v(\tilde{\mathbf{x}}^{\mathbf{i}}) \geq v(\mathbf{z}) \text{ for any } \mathbf{z} \in B^i, \tag{13}$$

[13] Matzkin (1991) and Nishimura, Ok and Quah (2017) generalize Afriat's theorem to more general choice sets. Seror (2024) generalizes Afriat's theorem to recover single-peaked preferences.

By construction of set B^i, given that $\mathbf{z} \in B^i$, there exists a unique $\mathbf{x} \in A^i$ such that $\mathbf{z} = \Phi^{-1}(\mathbf{x})$, so the previous equation is equivalent to

$$v \circ \Phi^{-1}(\mathbf{x^i}) \geq v \circ \Phi^{-1}(\mathbf{x}) \text{ for any } \mathbf{x} \text{ such that } \exists \mathbf{z} \in B^i \text{ and } \mathbf{z} = \Phi^{-1}(\mathbf{x}) \tag{14}$$

and the result follows. ■

According to Corollary 1, the consumption $\mathbf{x^i}$ can be interpreted as generated by the maximization of an *augmented utility function*. The function v is the concave utility function that would be maximized if the decision-maker was free of behavioral influences. The function Φ^{-1} measures the effect of behavioral biases in decision-making.[14]

Example

To illustrate the previous results, I consider the classical consumption choice environment and characterize the augmented-utility model of decision-makers subject to inattention and salience thinking relative to the origin when $K = 2$: $\overline{\mathbf{x}} = (0, 0)$. According to Corollary 1, $\mathbf{x^i} = (x_1^i, x_2^i)$ results from the maximization of $v \circ \Phi^{-1}$, with $\Phi_k^{-1} : [0, \infty) \to \mathbb{R}^+$,

$$\Phi_k^{-1}(x_k) = x_k/m_k, \tag{15}$$

so the decision problem faced by a biased decision-maker can be written as

$$max_{x_1,x_2} v \circ \Phi^{-1}(x_1, x_2) \text{ given that } x_1 p_1 + x_2 p_2 \leq 1.$$

Consider the case of a a CES utility function v:

$$v(x_1, x_2) = (\alpha x_1^\rho + (1 - \alpha) x_2^\rho)^{1/\rho}, \tag{16}$$

with $\alpha \in [0, 1]$, and $\rho \leq 1$. In the augmented-utility model, the Marshallian demands can be characterized as follows:

$$\begin{cases} x_1^I = \left(\frac{\alpha}{p_1}\right)^\sigma \frac{m_1}{m_1 \alpha^\sigma p_1^{1-\sigma} + m_2 (1-\alpha)^\sigma p_2^{1-\sigma}} \\ x_2^I = \left(\frac{1-\alpha}{p_2}\right)^\sigma \frac{m_2}{m_1 \alpha^\sigma p_1^{1-\sigma} + m_2 (1-\alpha)^\sigma p_2^{1-\sigma}} \end{cases}$$

[14] Note that decision $\mathbf{x^i}$ is not interpreted as the global maximum over the whole set A^i, but only when taking into account a subset Q^i of A^i, that encompasses all the elements of A^i that have an antecedent in the budget set B^i.

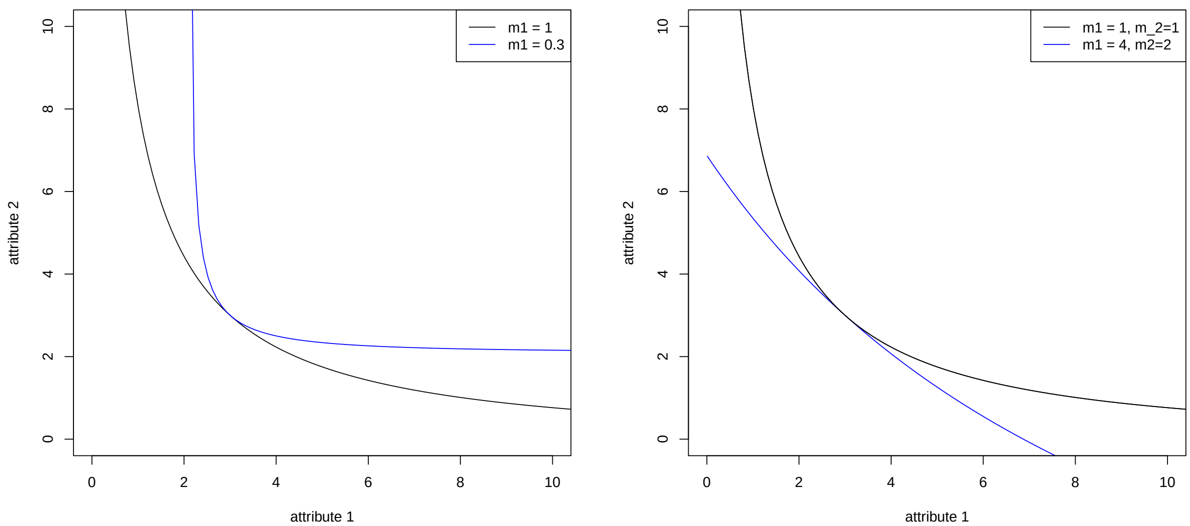
which looks similar to the Marshallian demands of an unbiased decision-maker, except for the effect of vector (m_1, m_2). Here, fitting the utility model v to the data instead of $v \circ \Phi^{-1}$ leads to biased estimates of the utility parameters α and ρ. The discrepancy between the utility and the augmented-utility models can be substantial. In the limit case where m_1 tends to zero for example, the demand for attribute 1 tends to 0 in the augmented-utility model, while it is predicted equal to $\left(\frac{\alpha}{p_1}\right)^{\sigma} \frac{1}{\alpha^{\sigma} p_1^{1-\sigma} + (1-\alpha)^{\sigma} p_2^{1-\sigma}}$ when behavioral biases are not accounted for.

Figure 7 represents several indifference curves. The black curve in both panels gives the indifference curve associated with the standard utility function v. The blue curve gives the indifference curve associated with the augmented utility function $v \circ \Phi^{-1}$. In the left panel, the decision-maker is assumed to pay partial attention to both attributes, $(m_1, m_2) = (0.3, 0.3)$. In the right panel, the decision-maker is assumed to be a salient thinker, with $(m_1, m_2) = (4, 2)$. Comparing the indifference curves, two remarks are in order. First, as for inattention, the black and the blue indifference curves cross when the consumption vector (x_1, x_2) is equal to the default consumption $(\overline{x}_1, \overline{x}_2)$. Second, the indifference curve of the inattention-augmented utility is steeper than the indifference curve associated with the standard utility function. At the opposite, the salience-augmented utility function is less steep than the indifference curve of the standard utility function. This means that salience makes decisions more sensitive to prices, so a salient thinker will more easily make decisions closer to the corners. Inattention has the exact opposite effect. Deviating from the default implies a higher utility loss than in the standard model, so optimal consumption choices are closer to the default for a large set of prices.

5 Applications

I study two types of data in this section. First, I study the data from the portfolio choice experiments in Choi et al. (2007), Choi et al. (2014), and Halevy, Persitz and Zrill (2018). Second, I study household-level scanner panel data, the Stanford Basket Dataset, which contains grocery expenditure data for 494 households from four grocery stores in an urban area of a large U.S. midwestern city. This data was also used, among others, by Echenique, Lee and Shum (2011), Shum (2004), and Hendel and Nevo (2006).

Figure 7: Indifference Curve of a CES utility function of parameters $\alpha = 0.5$ and $\rho = 0.1$ when $(m_1, m_2) = (1, 1)$ (black curve), and $(m_1, m_2) = (0.3, 0.3)$ (blue curve, left panel) and $(m_1, m_2) = (5, 2)$ (blue curve, right panel).

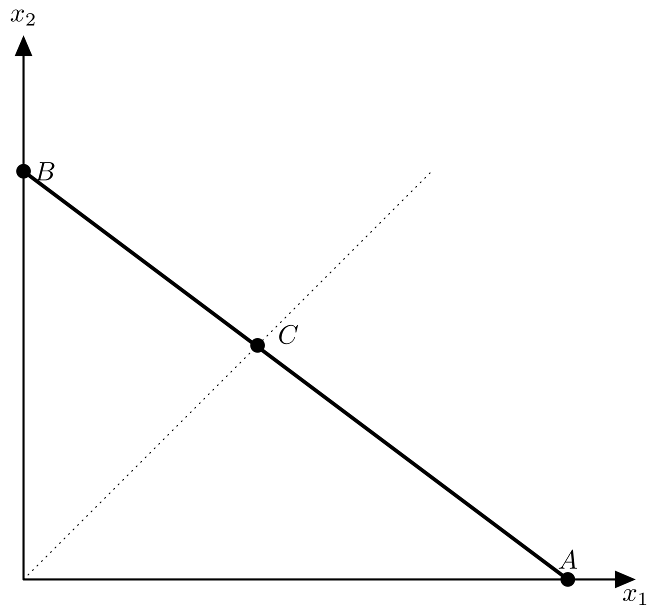


5.1 Portfolio Choice Experiments Data

The data are taken from the experiment conducted by Choi et al. (2014). The data were collected from the CentERpanel. It includes 1,182 adult members. Table 1 gives the summary statistics of individual characteristics.

In the experiment of Choi et al. (2014), participants engaged in a series of decision-making tasks involving risk. Each task required making a selection from a set of options presented on a two-dimensional budget line, where the chosen allocation of points between accounts x and y (representing the horizontal and vertical dimensions, respectively) determined their potential gains. The final rewards were based on the distribution of points across these accounts, with the actual payoff coming from either account x or y, chosen randomly with equal probability. Figure 8 illustrates an example of such a budget line, marked as AB. The intersection point C on the 45-degree line denotes an evenly balanced, certain outcome, while points A and B indicate choices favoring one account. Choices along segment AC involve greater risk due to reduced returns in state x but enhanced returns in state y, leading to a potentially higher expected yield than the balanced point C. Conversely, selections along segment BC are characterized by a lower expected yield compared to C.

Figure 8: Choi et al. (2014) Experiment: example of a budget constraint



Respondents were presented with 25 different decisions. Each decision problem started with the computer selecting a budget line. Choices were restricted to allocations on the budget constraint and were made using the computer mouse to move the pointer from a random default option to the desired point. Payoffs were proportional to the amount won throughout the experiment. More information and full experimental instructions can be found in Choi et al. (2014) and the online appendix of that paper.

Using the methodology uncovered in the previous Section, I seek to understand the influence of behavioral biases in this portfolio choice experiment. Specifically, I assume that respondents might be affected by a behavioral bias characterized by equation (6). Respondents are not assumed more biased on one of the two dimensions, as I posit that $m_k = m$. Finally, I consider two potential default options $\overline{\mathbf{x}}^{\mathbf{i}}$. The first is the riskless option where the decision-maker allocates the same amount to both accounts, $\overline{\mathbf{x}}^{\mathbf{i}}_{\mathbf{NR}} = (\frac{1}{p_1^i+p_2^i}, \frac{1}{p_1^i+p_2^i})$. The second is the random default option selected by the computer at the beginning of each decision problem, $\mathbf{x}^{\mathbf{i}}_{\mathbf{0}}$.

Under these assumptions, the optimization problem behind the recoverability of the behavioral bias affecting portfolio choices can be written as:

$$m = \underset{\phi(.;m)\in\mathcal{F}(m)}{\arg\max}\ e(D_{\phi^{-1}(.;m)}), \tag{17}$$

where e is either the CCEI index or the lower bound of the MPI index, $\overline{\mathbf{x}}^{\mathbf{i}} = (\frac{1}{p_1^i + p_2^i}, \frac{1}{p_1^i + p_2^i})$ or $\overline{\mathbf{x}}^{\mathbf{i}} = \mathbf{x}_{\mathbf{0}}^{\mathbf{i}}$, and

$$\mathcal{F}(m) = \{\phi(.; m) : \tilde{X}^0 \to X^0 \text{ with } \phi(\tilde{\mathbf{x}}^{\mathbf{i}}; m) = \max(m\tilde{\mathbf{x}}^{\mathbf{i}} + (1-m)\overline{\mathbf{x}}^{\mathbf{i}}, \mathbf{0})\}.$$

I use a Genetic Algorithm for the optimization, as the function e might be non-monotonic with m, so methods based on gradient descents might be impracticable. When several values of m solve (17), I select the weakest bias, so the value of m solving (17) the closest to 1.

Table 2 presents the summary statistics for the rationality indices. Columns 1 and 4 present the result in the original dataset of Choi et al. (2014). The average CCEI score reaches 88%, and the lower bound of the MPI 83%. Columns 2 and 5 present the rationality indices in the data corrected from a canonical bias m solving (17) when the default is the riskless option. Strikingly, both the CCEI index and the lower bound of the MPI index reach an average above 0.99. This means that for almost *all* respondents in the data, a simple behavioral bias characterized by equation (6) can explain rationality violations. Columns 3 and 6 present the rationality indices in the data corrected from a canonical bias m solving (17) when the default is the random option selected by the computer at the beginning of each decision problem. The patterns are less clear relative to Columns 2 and 5, although the corrected data still show more rational behavior than the original data.

Since subjects are almost rational in the corrected data, it raises the question of how easy it is to violate rationality in the corrected data relative to the original data. I use Bronars' test, measuring the probability that a decision-maker with a random behavior would violate GARP. The results are reported in Table 3. The Bronars's score is equal to 1 in all the datasets, suggesting that the optimization (17) does not artificially achieve higher rationality scores by decreasing instances were rationality can be violated.

Table 4 presents the summary statistics of the behavioral bias parameter m that solves (17). About 75% of the respondents are salient thinkers, while the remaining 25% are inattentive. This pattern holds independently from the rationality index considered, and the default option.

Table 5 presents the correlates of the behavioral bias parameter m that solves (17). From Columns 1 and 2, when the default is assumed equal to the riskless option, older respondents are significantly more subject to behavioral influences than younger participants. Similarly, more educated, single, and wealthier participants are less subject to behavioral influences. From Columns 3 and 4, although less significant, the patterns are relatively similar when the default option is assumed equal to the random answer initially selected by the computer.

The relatively high consistency of subjects' choices tells us that there exists a well-behaved utility function that rationalizes their choices (Afriat (1967)). Following Choi et al. (2007), I consider the following utility specification over portfolios (x_1, x_2):

$$U(x_1, x_2) = \min\{\alpha u(x_1) + u(x_2), u(x_1) + \alpha u(x_2)\}, \text{ with } u(x) = \frac{x^{1-\rho}}{1-\rho}, \quad (18)$$

where $\alpha \geq 1$ captures the loss/disappointment aversion, and ρ is the Arrow-Pratt measure of relative risk aversion. Choi et al. (2007) consider the following measure of the risk aversion:

$$r = \frac{\alpha - 1}{\alpha + 1} + \rho \frac{2\alpha}{(\alpha + 1)^2}. \quad (19)$$

Following Choi et al. (2007), I estimate r at the individual level using a Non Linear Least Square method. I do so for both the dataset of Choi et al. (2014) and the datasets corrected from the behavioral bias estimated using equation (17). For these datasets, the estimated parameter r corresponds to the parameter of an augmented utility model, as discussed in Section 3.

Table 6 presents the summary statistics of the risk aversion parameter r in the original data (Column 1), and the corrected data (Columns 2, 3, 4, 5). From columns 1, 2, and 3, the risk aversion parameter is systematically overestimated in the original data, compared to the data corrected from a behavioral model where the default is the riskless option. From columns 1, 4, and 5, the patterns are less obvious, comparing the risk aversion parameter estimated in the original data on the one hand, and the data corrected from a behavioral model where the default is the initial random option.

Table 7 presents the correlates of the risk aversion parameter r. Column 1 reports the correlates of the results using the data of Choi et al. (2014). The results in column 1 reflect well-known regularities in experiments on risk aversion. First, females are more risk-averse than males. Second, older participants are more risk-averse than younger participants. Finally, more educated respondents are less risk-averse. These regularities are much weaker once the data are corrected from behavioral influences. Columns 2 and 3 report the correlates of risk aversion, once the data are corrected from the canonical behavioral bias that solves (17). Age no longer significantly predicts risk aversion, and the impact of gender and education on risk aversion is much weaker. Columns 4 and 5 show similar patterns. The main reason explaining these results, from Table 5 is that females, older, and less educated participants are significantly more subject to behavioral influences. Concretely, they might diverge less from the riskless option, not because they are risk averse, but because they tend to keep in mind the riskless option when making decisions. This behavioral tendency leads to both rationality violations and risk-averse decisions. Using the rationality violations to recover behavioral biases, the higher risk aversion of these participants disappears.

As a final exercise, I consider the case of two behavioral biases sequentially affecting the decision-maker, and performed the following optimization:

$$(m_1, m_2) = \underset{\phi(.;m_1,m_2)\in\mathcal{F}(m_1,m_2)}{\arg\max}\; e(D_{\phi^{-1}(.;m_1,m_2)}), \tag{20}$$

where e is either the CCEI index or the lower bound of the MPI index, and

$$\mathcal{F}(m_1, m_2) = \{\phi(.; m_1, m_2) : \tilde{X}^0 \to X^0 \text{ with } \phi(.; m_1, m_2) = \phi_2(.; m_2) \circ \phi_1(.; m_1)\},$$

$$\begin{cases} \phi_1(\mathbf{x}; m_1) = m_1\mathbf{x} + (1 - m_1)\overline{\mathbf{x}}_\mathbf{1} \\ \phi_2(\mathbf{x}; m_2) = m_2\mathbf{x} + (1 - m_2)\overline{\mathbf{x}}_\mathbf{2}, \end{cases} \tag{21}$$

where $\overline{x}_1 = \mathbf{x^i_0}$ is the initial option randomly selected by the computer at the beginning of observation i, and $\overline{x}_2 = \mathbf{x^i_{NR}}$ is the riskless option in observation i. This sequential bias is recovered for two reasons. First, it is reasonable to expect that decision-makers might be affected by such a bias. Provided that they pay attention to the experiment, then they might be subject to a bias where the reference is the riskless option. Second, recovering such a sequential bias might help select the simpler behavioral model that works best. Second, the sequential transformation (11) nests two simpler behavioral models: one where the decision-maker is only affected by a behavioral bias of parameters $(\mathbf{m_1}, \overline{\mathbf{x}}_\mathbf{1})$, and one where the decision-maker is only affected by a behavioral bias of parameters $(\mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{2})$. Hence, if the outcome of the optimization over set $\mathcal{F}(m_1, m_2)$ converges to the outcome of the optimization over set $\mathcal{F}(m_2)$ for example, then it would mean that the behavioral model of parameters $(\mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{2})$ better explain the decision-making process than the behavioral model of parameters $(\mathbf{m_2}, \overline{\mathbf{x}}_\mathbf{2})$.

The results of the optimization are reported in Tables A.1 and A.2. While the sequential behavioral model does not definitively align with any of the simpler models, from Table A.2, the behavioral model with a random default option shows a diminishing association with the heterogeneity found in the sample's sociodemographic characteristics. Conversely, the behavioral model that employs a riskless option as the default seems more tightly connected to the sociodemographic data. Therefore, this analysis tends to favor the behavioral model where the respondents keep in mind the riskless option when making decisions.

5.2 Household-level Scanner Data

The Stanford Basket Dataset contains grocery expenditure for 494 households from four grocery stores in an urban area of a large U.S. midwestern city, between June 1991 and June 1993 (104 weeks). This dataset was collected by Information Resources, Inc. I use the same data as Echenique, Lee and Shum (2011). The data focuses on households' expenditures on food categories: bacon, barbecue, butter, cereal, coffee, crackers, eggs, ice cream, nuts, analgesics, pizza, snacks, and sugar. Moreover, the data include 103,345 transactions of 4,082 items. Transactions are aggregated by brand name and category, while coupons are not incorporated into the analysis as they are partially observed.[15] Table 8 presents the population distributions of the demographic variables.

Using the methodology uncovered in the previous Section, I seek to understand the influence of the behavioral bias characterized by equation (6), where $m_k = m$ for any good k, and the default consumption of good k at time i is given by:

$$\overline{x}_k^i(\beta, \delta) = \sum_{t<i} x_k^t \beta^{i-t} e^{-\delta(p_k^i - p_k^t)^2}, \tag{22}$$

where $\beta \geq 0$ captures the time preference of the decision-maker, and $\delta \geq 0$ captures the importance of similarity when the agent recalls period $t < i$ during period i (Bordalo, Gennaioli and Shleifer (2020)). The general optimization problem (1) can be rewritten as

$$(m, \beta, \delta) = \underset{\phi(.;) \in \mathcal{F}}{\arg\max}\, e(D_{\phi^{-1}(.;m,\beta,\delta)}), \tag{23}$$

where e is either the CCEI Index, or the lower bound of the MPI index. I use a Genetic Algorithm for the optimization, as the function e might be non-monotonic with vector

[15] The data construction is discussed more extensively by Echenique, Lee and Shum (2011).

(m, β, δ). When several values of vector (m, β, δ) solve (23), I select the minimum bias with the value of m the closest to 1.

Table 9 presents the summary statistics for the rationality indices in the data, and the data corrected from the canonical bias that solves equation (23). From columns 1 and 3, the rationality scores are already fairly high in the original data, making the approach relatively more limited as only few instances of rationality violations can be leveraged to recover behavioral influences.[16] However, from columns 2 and 4, it seems that a bias solving (23) can explain all rationality violations in the data. Finally, from columns 5, 6, and 6, it does not seem that the optimization algorithm (23) reaches higher rationality scores by decreasing instances where rationality can be violated, since the Bronars' score is not differ much between the original data set and the corrected data sets.

Table 10 presents the correlates of the behavioral bias parameters m, β, and δ that solve (23). Several patterns emerge. Overall, similar to the risk data, subjects seem to be more subject to salient thinking than to inattention, as parameter m averages 2.6. Moreover, decision-makers from larger families seem to be less subject to salience thinking. Additionally, the large estimated values of parameter δ suggest that decision-makers might simply have in mind 0 as a default option rather than their past purchasing history. Nevertheless, it seems that older and more educated decision-makers have a significantly longer time horizon. These patterns hold for both an optimization (23) based on the CCEI index, and an optimization (23) based on the worst MPI index.

Finally, I recover left-digit bias in the previous dataset, using the following optimization algorithm:

$$\theta = \underset{\gamma(.;\theta)\in\mathcal{F}_{LD}}{\arg\max}\, e(D_{\gamma(.;\theta)}), \tag{24}$$

with

$$\mathcal{F}_{LD} = \{\gamma(.;\theta) : \mathbb{R}^K_+ \to \mathbb{R}^K_+ \text{ with } \theta \in [0,1] \text{ and } \gamma(\mathbf{p^i};\theta) = \theta\mathbf{p^i} + (1-\theta)\lfloor\mathbf{p^i}\rfloor\}.$$

From Table A.3, it seems that left-digit bias alone cannot explain all rationality violations, suggesting that a more appropriate behavioral model might cover a larger set of behavioral influences. Nevertheless, using this model as a preliminary benchmark for fu-

[16] See Blundell, Browning and Crawford (2008, 2003) for a methodology to partially address this issue, using Engel curves.

ture research, from Table A.4, the average value of the left-digit bias parameter θ is 0.83 in the sample. As a comparison, Strulov-Shlain (2023) finds an average value of 0.78 using a different methodology, and different data. Finally, the analysis does not reveal systematic patterns explaining the heterogeneity on the left-digit bias estimated parameter θ.

6 Conclusion

In this paper, I propose a unified methodology to recover behavioral biases from choice data. The key premise of the analysis is that since behavioral biases systematically distort decision-making without being rooted in preference, we should expect to see consistent patterns of rationality's breach in observed choice data. This consistency provides a framework for inferring the behavioral model that can best explain decisions.

I show how to recover broad categories of behavioral biases, including inattention, salience thinking, logarithmic perception, and any sequential composition of these biases. Moreover, I demonstrate how this approach can be used to filter behavioral biases from choice data before fitting the data with utility models. That way, measuring behavioral biases in choice data might also improve the quality of utility estimation, which theoretically requires sharp conditions on choice consistency to be satisfied.

I apply the model to two workhorse datasets. First, I study the data from portfolio choice experiments in Choi et al. (2007), Choi et al. (2014), and Halevy, Persitz and Zrill (2018). Second, I study household-level scanner panel data, the Stanford Basket dataset, used, among others, by Echenique, Lee and Shum (2011), Shum (2004), and Hendel and Nevo (2006). The first key result of this analysis is that restricting the analysis to transformations that correspond to either inattention or salience explains more than 99% of rationality violations for all decision-makers across the two datasets. Moreover, systematic heterogeneity patterns seem to emerge from the analysis, as behavioral biases are affected by gender, age, education, wealth, and household composition. Moreover, I show that well-known patterns of preference heterogeneity may be confounded by patterns of behavioral bias heterogeneity. Concretely, estimating risk preference from the portfolio choice data of Choi et al. (2014) shows well-known regularities: women, older and high-educated decision-makers are more risk averse than men, low-educated, and younger decision-makers. Estimating the same utility model once salience thinking and inattention are recovered and filtered out from the data shows that these patterns are explained by heterogeneity in salience thinking and inattention.

A pivotal yet unresolved aspect of this research concerns the selection of the most fitting behavioral model to account for choice inconsistencies. In this paper, I have adopted a pragmatic approach, prioritizing simplicity and generality. Future research could potentially develop a more formalized criterion for model selection, one that hinges on the alignment of estimated behavioral biases across various transformation sets. This challenge is not unique to this study but is a common hurdle in any endeavor to align parametric specifications with empirical data. Finally, while there might be ambiguity regarding whether the estimated models precisely mirror the cognitive processes of individuals, these models nonetheless can guide researchers in their relentless exploration of human decision-making.

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Tables and Figures

Table 1: Sociodemographic Variables: Risk Data from Choi et al. (2014).

Variables	Number of Participants
Female	537
Age	
16-34	219
35-49	309
50-64	421
65+	233
Education	
Low	397
Medium	351
High	430
Household monthly income	
€0-2,500	269
€2,500-3499	302
€3,500-4,999	345
€5,000+	266
Occupation	
Paid work	628
House work	137
Retired	247
Others	170
Partner	956
Observations	1182

From Choi et al. (2014): The low, medium, and high education levels correspond to primary or prevocational secondary education, preuniversity secondary education or senior vocational training, and vocational college or university education, respectively. Choi et al. (2007) use household monthly gross income-level categories such that the proportions of participants in each category are approximately equal. The classification of levels of completed education and occupations are based on the categorization of Statistics Netherlands (Centraal Bureau voor de Statistiek).

Table 2: Summary Statistics: Rationality Scores

	CCEI Index			MPI Index		
	(1)	(2)	(3)	(4)	(5)	(6)
Mean	0.88	1.00	0.95	0.83	1.00	0.92
Std	0.14	0.01	0.07	0.17	0.01	0.10
p5	0.60	0.99	0.79	0.51	0.98	0.70
p25	0.81	1.00	0.93	0.72	1.00	0.86
p50	0.93	1.00	0.99	0.88	1.00	0.96
p95	1.00	1.00	1.00	1.00	1.00	1.00
Behavioral Bias:						
None	Yes	No	No	Yes	No	No
Riskless Default	No	Yes	No	No	Yes	No
Initial Bundle	No	No	Yes	No	No	Yes

Column (1) gives the summary statistics of Afriat's Critical Cost Efficiency Index (CCEI) of the risk data in Choi et al. (2007). Column (2) (resp. (3)) gives the summary statistics of the CCEI of the risk data corrected for a behavioral bias that takes the riskless bundle as a default (resp. the initial bundle randomly selected by the computer at the beginning of each individual-level observation). Column (4) gives the summary statistics of Echenique, Lee and Shum's Money Pump Index. Columns (4) and (5) compute the summary statistics for the same index in the dataset corrected from a canonical behavioral bias where the default is the riskless choice (Column (5)), and the initial bundle randomly selected by the computer at the beginning of each individual-level observation (Column (6)).

Table 3: Summary Statistics: Bronars' Scores

	Bronars' score				
	(1)	(2)	(3)	(4)	(5)
Mean	1.00	1.00	1.00	1.00	1.00
Std	0.00	0.00	0.00	0.00	0.00
p5	1.00	1.00	1.00	1.00	1.00
p25	1.00	1.00	1.00	1.00	1.00
p50	1.00	1.00	1.00	1.00	1.00
p95	1.00	1.00	1.00	1.00	1.00
Behavioral Bias:					
None	Yes	No	No	No	No
Riskless Default	-	Yes	Yes	No	No
Initial Bundle	-	No	No	Yes	Yes
Optimization algorithm:					
CCEI	-	Yes	No	Yes	No
MPI	-	No	Yes	No	Yes

Column (1) gives the Bronars' score of the risk data in Choi et al. (2007). Columns (2), (3) (resp. (4), (5)) give the Bronars' score of the risk data corrected for a behavioral bias that takes the riskless bundle as a default (resp. the initial bundle randomly selected by the computer at the beginning of each individual-level observation). Columns (2), (4) (resp. (3), (5)) are based on an optimization algorithm that maximizes the CCEI index (resp. the MPI index) of the corrected data.

Table 4: Summary Statistics: Behavioral Bias

	m			
	(1)	(2)	(3)	(4)
Mean	3.90	4.04	1.83	1.77
Std	3.54	3.63	1.95	1.81
p5	0.15	0.20	0.67	0.75
p25	1.00	1.00	1.00	1.00
p50	2.28	2.37	1.13	1.14
p75	6.66	7.26	1.72	1.70
p95	9.99	9.99	6.53	5.12
Behavioral Bias Default:				
Riskless Bundle	Yes	Yes	No	No
Initial Bundle	No	No	Yes	Yes
Optimization algorithm:				
CCEI	Yes	No	Yes	No
MPI	No	Yes	No	Yes

Columns (1) and (3) (resp (2) and (4)) give the summary statistics of the behavioral parameter value m, given that the optimization algorithm maximizes the CCEI index (resp. (worst MPI index) of the corrected data. In columns (1) and (2) (resp. (3) and (4)), the default option is assumed equal to the riskless choice (resp. initial bundle randomly selected by the computer).

Table 5: Behavioral Bias Explained by Sociodemographic Variables: Risk Data

	m			
	(1)	(2)	(3)	(4)
Constant	4.044***	4.248***	1.807***	1.947***
	(0.579)	(0.597)	(0.325)	(0.302)
Female	0.430*	0.537**	0.282**	0.126
	(0.225)	(0.231)	(0.126)	(0.117)
Age				
35-49	0.322	0.342	0.098	0.182
	(0.320)	(0.329)	(0.179)	(0.167)
50-64	1.119***	1.056***	0.184	0.218
	(0.314)	(0.324)	(0.176)	(0.164)
65+	0.645	0.447	0.263	0.254
	(0.484)	(0.499)	(0.272)	(0.253)
Education				
Medium	−0.717***	−0.684**	−0.101	−0.202
	(0.261)	(0.269)	(0.147)	(0.136)
High	−0.701***	−0.825***	−0.162	−0.399***
	(0.266)	(0.274)	(0.149)	(0.139)
Income				
€2,500-3,499	−0.070	−0.066	−0.183	−0.192
	(0.295)	(0.304)	(0.166)	(0.154)
€3,500-4,999	−0.277	−0.300	−0.158	0.039
	(0.300)	(0.309)	(0.168)	(0.156)
€5,000+	−0.655*	−0.621*	−0.149	0.026
	(0.335)	(0.345)	(0.188)	(0.175)
Occupation				
Paid Work	−0.856**	−1.011**	−0.253	−0.379*
	(0.406)	(0.419)	(0.228)	(0.212)
House Work	−1.559***	−1.823***	−0.237	−0.357
	(0.465)	(0.479)	(0.261)	(0.242)
Others	−1.581***	−1.762***	−0.154	−0.367
	(0.460)	(0.474)	(0.258)	(0.240)
Household Composition				
Partner	0.669**	0.782***	0.156	0.177
	(0.283)	(0.292)	(0.159)	(0.148)
Number of Kids	0.102	0.087	0.015	−0.025
	(0.112)	(0.116)	(0.063)	(0.059)
Behavioral Bias Default:				
Riskless Bundle	Yes	Yes	No	No
Initial Bundle	No	No	Yes	Yes
Optimization algorithm:				
CCEI	Yes	No	Yes	No
MPI	No	Yes	No	Yes
R^2	0.054	0.055	0.015	0.026
Observations	1,182	1,182	1,182	1,182

The dependent variable is the estimated behavioral bias parameter m characterized in equation (17). $*$ ($p < 0.05$), ** ($p < 0.01$), *** ($p < 0.001$).

Table 6: Summary Statistics: Risk aversion

	r				
	(1)	(2)	(3)	(4)	(5)
Mean	0.94	0.79	0.75	0.97	0.92
Std	1.12	1.19	0.90	1.44	1.37
p5	0.18	0.18	0.19	0.18	0.19
p25	0.46	0.42	0.42	0.44	0.45
p50	0.66	0.55	0.54	0.59	0.60
p75	1.18	0.79	0.78	0.92	0.90
p95	2.15	1.59	1.60	2.67	2.31
Behavioral Bias:					
None	Yes	No	No	No	No
Riskless default	No	Yes	Yes	No	No
Initial Bundle	No	No	No	Yes	Yes
Optimization algorithm:					
CCEI	No	Yes	No	Yes	No
MPI	No	No	Yes	No	Yes

Column (1) gives the summary statistics of the risk aversion parameter r estimated using the data of Choi et al. (2014). Columns (2) and (4) (resp (3) and (5)) give the summary statistics of the risk aversion parameter r in filtered data, given that the optimization algorithm recovering behavioral biases maximizes the CCEI index (resp. (worst MPI index) of the corrected data. In columns (2) and (3) (resp. (4) and (5)), the default option is assumed equal to the riskless choice (resp. initial bundle randomly selected by the computer).

Table 7: Risk Aversion Explained by Sociodemographic Variables

	r				
	(1)	(2)	(3)	(4)	(5)
Constant	0.555***	0.862***	0.704***	1.030***	0.851***
	(0.188)	(0.203)	(0.153)	(0.244)	(0.231)
Female	0.264***	0.029	0.105*	0.150	0.137
	(0.073)	(0.078)	(0.059)	(0.094)	(0.089)
Age					
35-49	0.104	-0.028	0.090	-0.110	0.074
	(0.104)	(0.112)	(0.084)	(0.134)	(0.127)
50-64	0.242**	-0.091	0.006	-0.040	0.005
	(0.102)	(0.110)	(0.083)	(0.132)	(0.125)
65+	0.483***	0.030	0.115	0.041	0.113
	(0.157)	(0.169)	(0.127)	(0.203)	(0.193)
Education					
Medium	-0.193**	-0.035	-0.034	-0.218**	-0.133
	(0.085)	(0.091)	(0.069)	(0.110)	(0.104)
High	-0.173**	-0.189**	-0.111	-0.209*	-0.207*
	(0.086)	(0.093)	(0.070)	(0.112)	(0.106)
Income					
2,500-3,499	-0.017	0.060	0.014	-0.079	-0.029
	(0.095)	(0.103)	(0.077)	(0.123)	(0.117)
3,500-4,999	0.061	0.186*	0.157**	0.182	0.210*
	(0.097)	(0.105)	(0.079)	(0.126)	(0.119)
5,000+	0.037	0.197*	0.020	0.133	0.159
	(0.109)	(0.117)	(0.088)	(0.141)	(0.134)
Occupation					
Paid work	0.271**	-0.009	0.018	0.142	0.156
	(0.132)	(0.142)	(0.107)	(0.171)	(0.162)
House work	-0.040	-0.023	-0.082	0.018	-0.027
	(0.151)	(0.162)	(0.122)	(0.195)	(0.185)
Others	0.189	0.074	0.075	0.169	0.312*
	(0.149)	(0.160)	(0.121)	(0.193)	(0.183)
Household composition					
Partner	-0.070	-0.158	-0.112	-0.139	-0.199*
	(0.092)	(0.099)	(0.075)	(0.119)	(0.113)
Number of children	0.050	0.044	0.031	-0.006	0.027
	(0.037)	(0.039)	(0.030)	(0.047)	(0.045)
Behavioral Bias:					
None	Yes	No	No	No	No
Riskless default	No	Yes	Yes	No	No
Initial Bundle	No	No	No	Yes	Yes
Optimization algorithm:					
CCEI	No	Yes	No	Yes	No
MPI	No	No	Yes	No	Yes
Observations	1,167	1,167	1,167	1,167	1,167
R^2	0.026	0.011	0.015	0.014	0.017

The dependent variable is the estimated risk aversion parameter r characterized in equation (19). * ($p < 0.05$), ** ($p < 0.01$), ***($p < 0.001$).

Table 8: Sociodemographic Variables: Scanner Data from Echenique, Lee and Shum (2011)

Variable	Number of Households
Family size	
Mid size (3,4 members)	187
Large size (>4 members)	65
Income	
$20,000-45,000	200
$45,000+	141
Age	
30-65	201
65+	157
Education	
High school	197
College	255
Observations	494

From Echenique, Lee and Shum (2011): The age categories correspond to the average age of the spouses in the households. Annual income are reported. If both spouses are present in a household, the average education of both spouses is reported.

Table 9: Summary Statistics: Rationality Scores

	CCEI Index		MPI Index		Bronars Score		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Mean	0.97	1.00	0.94	1.00	0.22	0.20	0.21
Std	0.05	0.01	0.07	0.02	0.13	0.17	0.18
p5	0.88	1.00	0.82	1.00	0.05	0.00	0.00
p25	0.95	1.00	0.91	1.00	0.13	0.06	0.07
p50	0.99	1.00	0.96	1.00	0.21	0.18	0.19
p75	1.00	1.00	1.00	1.00	0.29	0.30	0.30
p95	1.00	1.00	1.00	1.00	0.47	0.52	0.54
Behavioral Bias:							
	No	Yes	No	Yes	No	Yes	Yes
Optimization algorithm:							
CCEI	No	Yes	No	No	No	Yes	No
MPI	No	No	No	Yes	No	No	Yes

Column (1) (resp. (3)) gives the summary statistics of Afriat's Critical Cost Efficiency Index (CCEI) (resp. MPI Index) of the scanner data in Echenique, Lee and Shum (2011). Column (2) (resp. (4)) gives the summary statistics of the CCEI of the scanner data corrected for the behavioral bias that solves optimization problem (23) when the rationality index is the CCEI index (resp. MPI index). Columns (5), (6), and (7) give the Bronars scores of the original data, the data corrected using the CCEI rationality index, and the data corrected using the MPI index respectively.

Table 10: Behavioral Bias Explained by Sociodemographic Variables: Scanner Data

	s		β		δ	
	(1)	(2)	(3)	(4)	(5)	(6)
Constant	2.677***	2.657***	0.524	0.156	35.768***	32.080***
	(0.587)	(0.615)	(0.473)	(0.520)	(6.658)	(6.572)
Family size						
Mid Size (3,4 members)	−0.565**	−0.509*	0.075	-0.037	-2.586	-1.131
	(0.262)	(0.275)	(0.211)	(0.232)	(2.972)	(2.934)
Large size (> 4 members)	−0.866**	−0.685*	-0.001	0.163	-2.758	-6.437
	(0.365)	(0.382)	(0.294)	(0.323)	(4.134)	(4.080)
Income						
$20,000-45,000	-0.354	-0.134	-0.232	0.168	-0.693	-2.647
	(0.296)	(0.310)	(0.239)	(0.262)	(3.357)	(3.314)
$45,000+	-0.066	0.084	0.038	0.148	-3.216	-1.037
	(0.343)	(0.359)	(0.276)	(0.304)	(3.885)	(3.834)
Age						
30-65	0.015	0.129	0.310	0.424*	-1.058	1.577
	(0.282)	(0.295)	(0.227)	(0.250)	(3.195)	(3.153)
65+	0.197	0.264	0.522*	0.768**	-4.496	0.050
	(0.365)	(0.383)	(0.294)	(0.324)	(4.144)	(4.091)
Education						
High School	0.526	0.314	0.866**	1.007**	-6.128	-3.580
	(0.491)	(0.515)	(0.396)	(0.435)	(5.570)	(5.499)
College	0.527	0.449	0.703*	0.888**	-3.020	-3.350
	(0.508)	(0.532)	(0.409)	(0.450)	(5.758)	(5.684)
Optimization algorithm:						
CCEI	Yes	No	Yes	No	Yes	No
MPI	No	Yes	No	Yes	No	Yes
Observations	480	480	480	480	480	480
R^2	0.033	0.020	0.023	0.024	0.010	0.011

Columns (1), (2), and (3) report the behavioral bias estimated using optimization (23), and the CCEI index to measure rationality. The remaining columns report the behavioral bias estimated using optimization (23) and the worst MPI index to measure rationality. $*$ ($p < 0.05$), ** ($p < 0.01$), *** ($p < 0.001$).

Appendix

A Generalized Version of Theorem 1 and Corollary 1

This section presents a generalization of Theorem 1 to compact and comprehensive transformed budget sets. The following theorem is taken from Forges and Minelli (2009) (their Proposition 3) and adapted to the issues studied in this paper.[17]

Theorem 2 *Let $B^i = \{\mathbf{z} \in \mathbb{R}^K_+ / g^i(\mathbf{z}) \leq 0\}$ with $g^i : \mathbb{R}^K_+ \to \mathbb{R}$ an increasing, continuous function. The following conditions are equivalent*

- *The data $D_{\phi^{-1}}$ satisfies GARP*
- *The data set $D_{\phi^{-1}}$ has a locally nonsatiated, continuous rationalization $v : \phi(X) \to \mathbb{R}$.*
- *There exist numbers U^k and λ^k such that*

$$U^k \leq U^l + \lambda^l g^l \circ \phi^{-1}(\mathbf{x^k})$$

  for each pair of observations $(\mathbf{x^k}, A^k)$ and $(\mathbf{x^l}, A^l)$.

Corollary 2 *If a transformation ϕ is such that B^0 is a collection of budget sets characterized as $B^i = \{\mathbf{z} \in \mathbb{R}^K_+ / g^i(\mathbf{z}) \leq 0\}$ with $g^i : \mathbb{R}^K_+ \to \mathbb{R}$ an increasing, continuous function, and one of the conditions of Theorem 2 is satisfied, then for any $\mathbf{x^i} \in X^0$, $\mathbf{x^i} = \arg\max_{\mathbf{x} \in Q^i} v \circ \Phi^{-1}(\mathbf{x})$, with $Q^i = \{\mathbf{x} \in A^i / \exists \mathbf{z} \in B^i$ and $\mathbf{z} = \Phi^{-1}(\mathbf{x})\}$.*

Corollary 2 is the generalization of Corollary 1 and the proof is identical. Theorem 2 can be applied to a collection of comprehensive and compact budget sets $\{B^i\}_{i \in \mathcal{I}}$. One key difference between the two theorems is that the concavity of the utility function rationalizing unbiased decisions is not guaranteed by Theorem 2. This makes Theorem 2 less amenable to empirical applications, although a larger spectrum of behavioral biases can be covered. Below, I show that two additional transformations can be studied in the context of consumer choice under Theorem 2.

[17] Other generalizations of Afriat's theorem includes Matzkin (1991), and Nishimura, Ok and Quah (2017). Nishimura, Ok and Quah (2017) give the most comprehensive approach. However, in the context of this paper, it does not seem necessary to develop a version of Afriat's theorem as general as Nishimura, Ok and Quah (2017).

B Additional Tables

Table A.1: Summary Statistics: Sequential Behavioral Bias

	m_1		m_2	
	(1)	(2)	(3)	(4)
Mean	1.79	1.54	4.63	2.83
Std	1.37	1.18	2.67	2.45
p5	0.60	0.89	0.84	0.28
p25	1.03	1.05	2.50	1.09
p50	1.41	1.24	4.17	1.91
p75	2.05	1.60	6.53	3.69
p95	4.07	2.94	9.59	8.97
Optimization algorithm:				
CCEI	Yes	No	Yes	No
MPI	No	Yes	No	Yes

Columns (1) and (2) give the summary statistics of the behavioral parameter value m_1. Columns (3) and (4) give the summary statistics of the behavioral parameter value m_2. In columns (1) and (3) (resp. (2) and (4)), the optimization algorithm relies on the CCEI index (resp. worst MPI index)

Table A.2: Sequential Behavioral Bias Explained by Sociodemographic Variables: Risk Data

	m_1		m_2	
	(1)	(2)	(3)	(4)
Constant	1.757***	1.481***	3.922***	1.481***
	(0.228)	(0.197)	(0.439)	(0.197)
	(0.579)	(0.597)	(0.325)	(0.302)
Female	0.211**	0.067	0.423**	0.067
	(0.088)	(0.077)	(0.170)	(0.077)
Age				
35-49	0.014	0.126	0.490**	0.126
	(0.126)	(0.109)	(0.242)	(0.109)
50-64	0.076	0.080	0.981***	0.080
	(0.124)	(0.107)	(0.238)	(0.107)
65+	0.097	0.085	1.317***	0.085
	(0.191)	(0.165)	(0.367)	(0.165)
Education				
Medium	-0.006	-0.130	-0.323	-0.130
	(0.103)	(0.089)	(0.198)	(0.089)
High	-0.083	-0.100	-0.519**	-0.100
	(0.105)	(0.091)	(0.202)	(0.091)
Income				
€2,500-3,499	0.165	0.003	-0.389*	0.003
	(0.116)	(0.101)	(0.223)	(0.101)
€3,500-4,999	0.240**	0.066	-0.386*	0.066
	(0.118)	(0.102)	(0.227)	(0.102)
€5,000+	0.082	-0.050	-0.636**	-0.050
	(0.132)	(0.114)	(0.254)	(0.114)
Occupation				
Paid Work	-0.262	-0.153	-0.100	-0.153
	(0.160)	(0.139)	(0.308)	(0.139)
House Work	-0.359*	-0.059	-0.754**	-0.059
	(0.183)	(0.158)	(0.352)	(0.158)
Others	-0.240	-0.207	-0.265	-0.207
	(0.181)	(0.157)	(0.348)	(0.157)
Household Composition				
Partner	0.057	0.160*	0.634***	0.160*
	(0.112)	(0.097)	(0.215)	(0.097)
Number of Kids	-0.051	0.008	0.097	0.008
	(0.044)	(0.038)	(0.085)	(0.038)
Optimization algorithm:				
CCEI	Yes	No	Yes	No
MPI	No	Yes	No	Yes
R^2	0.021	0.013	0.054	0.013
Observations	1,182	1,182	1,182	1,182

The dependent variable is the estimated behavioral bias parameters m_1 and m_2 of a sequential bias where the decision-maker is first subject to a bias characterized by (6) with the default is the initial basket randomly selected by the computer, and then a bias characterized by (6) where the default is the riskless option. $*$ ($p < 0.05$), ** ($p < 0.01$), *** ($p < 0.001$).

Table A.3: Summary Statistics: Rationality Scores and Left-Digit Bias

	CCEI Index		MPI Index		Bronars score		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Mean	0.97	0.98	0.94	0.97	0.22	0.21	0.21
Std	0.05	0.04	0.07	0.07	0.13	0.13	0.13
p5	0.88	0.92	0.82	0.85	0.05	0.03	0.04
p25	0.95	0.99	0.91	0.95	0.13	0.11	0.11
p50	0.99	1.00	0.96	1.00	0.21	0.19	0.19
p75	1.00	1.00	1.00	1.00	0.29	0.27	0.28
p95	1.00	1.00	1.00	1.00	0.47	0.44	0.43
Behavioral Bias:							
	No	Yes	No	Yes	No	Yes	Yes
Optimization algorithm:							
CCEI	No	Yes	Yes	No	Yes	No	
MPI	No	No	Yes	Yes	No	No	Yes

Column (1) (resp. (3)) gives the summary statistics of Afriat's Critical Cost Efficiency Index (CCEI) (resp. MPI Index) of the scanner data in Echenique, Lee and Shum (2011). Column (2) (resp. (4)) gives the summary statistics of the CCEI of the scanner data corrected for the behavioral bias that solves optimization problem (24) when the rationality index is the CCEI index (resp. MPI index). Columns (5), (6), and (7) give the Bronars scores of the original data, the data corrected using the CCEI rationality index, and the data corrected using the MPI index respectively.

Table A.4: Summary Statistics: Left-Digit Bias

	θ	
	(1)	(2)
Mean	0.83	0.86
Std	0.24	0.23
p5	0.33	0.35
p25	0.72	0.79
p50	0.98	0.99
p75	1.00	1.00
p95	1.00	1.00
Optimization algorithm:		
CCEI	Yes	No
MPI	No	Yes

Column (1) (resp. (2)) gives the summary statistics of the behavioral bias parameter θ, given that the optimization algorithm maximizes the CCEI index (resp. worst MPI index) of the corrected data.